

Climate and Economics: Tropical Forests

Part II

Jose A. Scheinkman (Columbia University)

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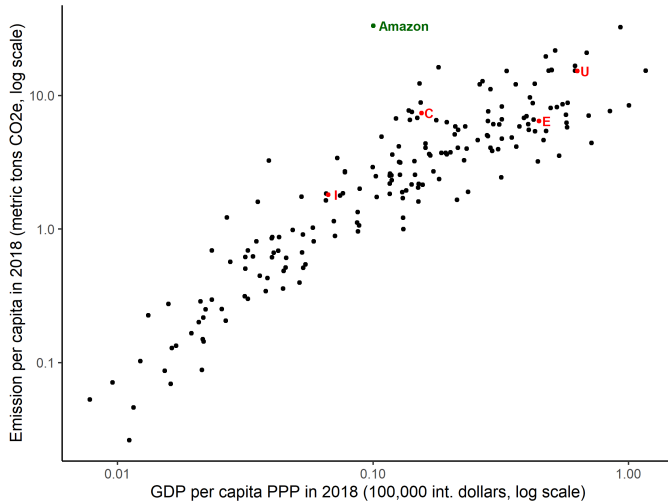
Carbon capture potential of reforestation in tropical forests

- Assunção et al. [2023]
- Studies the problem of a **fictitious social planner** to provide a benchmark for *ad hoc* policy alternatives.
- Uses data on the Brazilian Amazon
- Analyzes a dynamic model across heterogeneous regions in the Amazon.
- Exploits rich panel data set that covers a cross-section of regions in the Amazon.
- Uses numerical methods to achieve a necessary degree of economic and environmental richness to achieve credible results.
- Implements a novel refinement to uncertainty quantification.

Motivation

- The Amazon forest contains 123 ± 31 billion tons of captured carbon that can be released into the atmosphere, equivalent to the historical cumulative emissions of the United States (Malhi et al. [2006], Friedlingstein et al. [2022])
- Brazilian Amazon occupies 60% of the 2.7 million square miles that comprise the Amazon.
- An area the size of Texas has been deforested in the Brazilian Amazon.
- Portions of Amazon have become a source instead of sink for carbon.
- Destruction of forest has not help alleviate to poverty in Brazil
 - Income of agricultural workers in legal Amazon was 829 reais/month in 2019, only 83% of Brazilian minimum wage
 - 85% informal

Emissions curve



Road map

- ① Present model
- ② Short literature review
- ③ Discuss calibration
- ④ Present results
- ⑤ Some added remarks
- ⑥ Conclusions

State and control variables

- Sites are denoted $i = 1, \dots, I$ and the state-vector by (Z, X, P^a) .
- $Z = (Z^1, \dots, Z^I)$, is the vector of site-specific hectares of land used for agriculture.
- $X = (X^1, \dots, X^I)$ is the vector of site-specific stocks of captured carbon (above ground).
- P^a is an index of cattle prices in Brazil in 2017 USD.
 - 85% of deforested land is used for cattle raising.
- P^e is the social price of emissions
- Non-negative controls U^i and V^i with $\dot{Z} = U - V$
 - At optima $U_t^i V_t^i = 0$
- State constraints:

$$0 \leq Z_t^i \leq \bar{z}^i,$$

where \bar{z}^i is the maximum area for agriculture in site i .

State dynamics

- Carbon capture dynamics

$$\dot{X}^i = -\gamma^i U^i - \alpha X^i + \alpha \gamma^i (\bar{z}^i - Z^i)$$

where $\gamma^i > 0$ and $\alpha > 0$.

- Does not allow for interactions across sites.
- q state Markov chain with possible values for the agricultural price, p_1^a, \dots, p_q^a
- An infinitesimal generator given by $q \times q$ matrix $\mathbb{M} = [m_{\ell, \ell'}]$ with non-negative off-diagonal entries and

$$\sum_{\ell' \neq \ell} m_{\ell \ell'} = -m_{\ell \ell} > 0.$$

- $\exp(t\mathbb{M})$ gives transition probabilities over interval t .

Outputs

- Agricultural output

$$A^i = \theta^i P^a Z^i$$

where $\theta^i \geq 0$ is a site specific productivity parameter. and P^a is the national price.

- Net emissions

$$\kappa \sum_{i=1}^I Z_t^i - \sum_{i=1}^I \dot{X}_t^i$$

where $\kappa > 0$ measures the emissions per hectare of land induced by agriculture.

Quadratic adjustment cost

- Aggregate investment/disinvestment in agriculture over sites

$$\sum_{i=1}^I |\dot{Z}^i|$$

- Quadratic adjustment costs

$$\frac{\zeta}{2} \left[\sum_{i=1}^I (U_t^i + V_t^i) \right]^2.$$

- Motivation: Finite amount of resources to change use.
- Today: Only source of interactions across site.

Social planner's objective I

- Planner maximizes

$$\mathbb{E} \int_0^{\infty} \exp(-\delta t) \left[-P^e \left(\kappa \sum_{i=1}^I Z_t^i - \sum_{i=1}^I \dot{X}_t^i \right) + P_t^a \sum_{i=1}^I \theta^i Z_t^i - \frac{\zeta}{2} \left(\sum_{i=1}^I (U_t^i + V_t^i) \right)^2 \right] dt$$

- Assume passive restoration
 - Because of low and diminishing productivity, 7.2 million hectares of deforested land has been abandoned and passively restored for at least 6 years.
- Planner chooses site-specific controls U^i, V^i subject to the state evolution equations and the initial states

Social planner's objective II

- P^e , price of emissions, reflects a market for offsets and/or a planner's own valuation.
- Cross sectional heterogeneity often implies boundary solutions for sites.

Adding parameter uncertainty I

- Assume P_t^a is constant, $\varphi^i := (\gamma^i, \theta^i)$, φ full $2I$ dim parameter vector.
- $\varphi = \varphi(\beta)$, $\dim(\beta) < 2I$.
 - Time taken by numerical solution calculation sensitive to dimension of parameter vector.
- π baseline distribution of β .
- d be the vector of decisions and $f(d, \varphi(\beta))$ for the resulting value given the unknown β .
-

$$\max_d \min_{g, \int g d\pi = 1} \int f(d, \beta) g(\beta) d\pi(\beta) + \xi \int \log g(\beta) g(\beta) d\pi(\beta)$$

- $\xi > 0$ is penalty parameter.
- For given g , max problem can ignore penalty.

Adding parameter uncertainty II

- No learning.
- minimizing g is given by:

$$g_d(\beta) = \frac{\exp \left[-\frac{1}{\xi} f(d, \beta) \right]}{\int_{\mathcal{B}} \exp \left[-\frac{1}{\xi} f(d, \beta) \right] d\pi(\beta)} \quad (1)$$

Deterministic limit case

- Limit case with $\xi = \infty$ corresponds to *ambiguity neutrality*. Decision problem uses objective

$$\max_d \int_{\mathcal{B}} f(d, \beta) g(\beta) d\pi(\beta).$$

- Problem is deterministic with parameters set at the average value of β under the baseline distribution π .

Some references I

- Role of natural solutions
- Griscom et al. [2017], Heinrich et al. [2021] (focus on Brazilian Amazon)
- Franklin Jr and Pindyck [2024] ignores passive restoration, focus on active restoration projects at small scale in Brazil and derive high marginal costs.
- Static discrete-choice models to study the link between agriculture and deforestation, e.g, (Souza-Rodrigues [2019] and Dominguez-lino [2021]).
- Araujo et al. [2022]) develop dynamic model along lines of Souza-Rodrigues [2019] but with dynamics restricted to forward-looking behavior of farmers.

Some references II

- Survey of economics of tropical deforestation, Balboni et al. [2022], contains many references.
 - Most studies on agricultural expansion and deforestation are static.
- This paper provides framework that integrates the impact of carbon prices on deforestation, forest restoration, and agriculture. Also take into account the uncertainty on agricultural productivity and forest carbon measures.

Sites and initial states

- Sites:
 - Fine grid of 1887 sites of $\approx 67 \text{ km} \times 67 \text{ km}$. Of these 1043 have at least 3% ot area in the Brazilian Amazon biome. Featured in results today without price uncertainty and with ambiguity neutrality; solve as a deterministic model.
 - Coarser grid of 78 sites (featured in results with agricultural price uncertainty, solve using MPC methods with a Markov process for prices of agricultural output and in results about parameter ambiguity, and for comparison, deterministic model): Aggregate 16 sites of fine grid to produce sites of $\approx 268\text{km} \times 268\text{km}$.
 - Drop three sites with $< 3\%$ in Brazilian Amazon biome.
- Agricultural areas in 2017 (Z_0^i)
 - Source: MapBiomas
- Total land available in 2017 (\bar{z}^i)
 - Source: MapBiomas
- $X_0^i = \gamma^i(\bar{z}^i - Z_0^i)$.

Cattle productivity I

- Use Cattle data since cattle occupies 85% of areas deforested for agriculture.
- Data on value cattle sales and area of agriculture for 540 municipalities that overlap the biome from 2017 Agriculture Census.
- Missing data (74 out of 540) and unlikely outliers for municipalities with very small agricultural area.
- Regression of log value of cattle per hectare on set of 7 geographical variables R_{θ}^m and the local farmgate-price (proxy for transportation costs) yields smoother and lower dimensional representation $\beta_{\theta} \cdot R_{\theta}^m$ and fills in missing data.
 - Because cattle-grazing area differs a lot across municipalities, weight observations by 2017 pasture area in municipality.
- Calculate site θ^i by averaging over overlapping municipalities (weighted by overlap) of $\exp(\beta_{\theta}^m R_{\theta}^m)$, and dividing by p_{2017}^A .

Cattle productivity II

- Heterogeneity reflects transportation cost and current technology.
 - Historically transportation network increased deforestation but with limited effect on productivity (Gollin and Wolfersberger [2023]).
- Note we are not accounting for labor costs, thus exaggerating productivity of sector.

Cattle price uncertainty and adjustment costs

- Cattle price uncertainty: Fit two-state Markov process as hidden state Markov chain with Gaussian noise, using **hmmlearn** package in python.
 - Fits two states and transition matrix.
 - P^s mean of stationary distribution.
- Adjustment cost, ζ , so marginal cost of changing land use matches forest to pasture cost estimated by Araujo et al. [2022].
 - Need to explore asymmetry

Carbon dynamics I

- γ^i : Extract random sample of 1.2M 30m-pixels and select 893,753 pixels that could be considered **primary forest** in 2018 (pixels with no deforestation since 1985). Add above ground biomass density data for 2017, from ESA Biomass (Santoro and Cartus [2021]). Biomass data comes in a grid format $\sim 100\text{m}$, so spatially match it to sample and calculate average CO_2 density (Mg/ha).
- Calculate mean γ^m for municipalities.
- Regression of $\log \gamma^m$ on set of 5 geographical variables R_γ^m yield smoother representation.
- Calculate site γ^i by average over overlapping municipalities (weighted by overlap) of $\exp(\beta_\gamma^m R_\gamma^m)$.
- α , carbon depreciation parameter, set so convergence of carbon accumulation process is 100 years. (Henrich et al.(2021)).
- κ calibrated from agricultural net annual emission data at the state level available from SEEG.

Baseline distribution

- Uncertainty on $\beta := (\beta_\gamma, \beta_\theta)$ induces uncertainty on $\varphi^i = (\gamma^i, \theta^i)$
 - Lower dimensionality of uncertainty (13 vs. 78)
- Use conjugate prior updating (Raiffa et al. [1961], Hansen and Sargent [2013], Section 5.3) to produce a baseline distribution π of the vector β .
- Baseline for θ . γ analogous except that no weight matrix.
- Let Y be the vector of log value of cattle per hectare for the 466 municipalities with data. Coeffs of least squares with weight W_θ and Gaussian error variance-covariance $\sigma_\theta^2 I$ is same as coeffs of least square regression,

$$Y_\theta = R_\theta \beta_\theta + \varepsilon_\theta, \quad \varepsilon_\theta \sim \mathcal{N}(0, \sigma_\theta^2 W_\theta^{-1}), \quad (2)$$

Baseline distribution I

- If priors

$$\beta_{\theta} \mid \sigma_{\theta}^2 \sim \mathcal{N}(m_0, \sigma_{\theta}^2 Q_0^{-1}) \quad (3)$$

and,

$$\sigma_{\theta}^2 \sim \text{Inv-Gamma}(a_0, b_0) \quad (4)$$

- Posterior is given by Normal-inverse gamma with

$$Q = R'_{\theta} W_{\theta} R_{\theta} + Q_0,$$

$$m = Q^{-1}(R'_{\theta} W_{\theta} y_{\theta} + Q_0 m_0),$$

$$a = a_0 + \frac{n}{2},$$

$$b = b_0 + \frac{1}{2}(y'_{\theta} W_{\theta} y_{\theta} + m'_0 Q_0 m_0 - m' Q m),$$

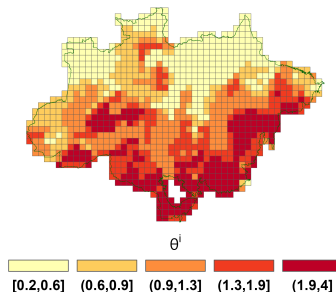
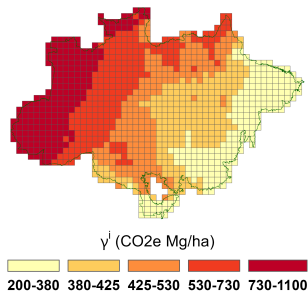
Baseline distribution II

- Impose improper priors:

$$Q_0 = 0 \quad m_0 = 0 \quad a_0 = 0 \quad b_0 = 0,$$

- Implies posteriors inputs are familiar regression statistics.
- Uncertainty on θ larger than on γ .

Site-specific Parameters γ^i and θ^i (1043 sites)



Computational Approach

- Discrete-time (year) approximation
- Deterministic prices (78 or 1043 sites): Interior Point Method: inequalities are approximated with logarithmic penalty functions.
- Uncertain prices (78 sites): Add Model Predictive Control
 - Finite-horizon approximation with two horizons:
 - Relatively short uncertainty horizon (u.h.) where controls are computed as a function of potential shock realizations (five periods);
 - Longer horizon where the control solutions are approximated by eliminating shocks beyond the uncertainty horizon (200 periods).
 - Solve the model again in subsequent periods with the same u.h..
 - Choose u.h.= 5) because value function changes little from u.h. = 4.
 - Interested in first 50 years

Parameter uncertainty

- i) Given a g , solve the maximization problem for a candidate d . May ignore relative entropy penalty.
- ii) For given d , solve the minimization problem to obtain new g .
- iii) Repeat until achieve convergence.
- For step ii) use quasi-analytical solution (1) and a Markov chain Monte Carlo method that is based on Hamiltonian dynamics. Often more efficient for high dimensional problems than Metropolis-Hastings (Neal et al. [2011], Carpenter et al. [2017]).

Planner's own valuation - Inferred shadow price of carbon from aggregate behavior 1995-2008

- 1995 - Begin reliable price data. 2008 - Announcement of Amazon Fund that incentivizes preservation of the forest with resources from Norway. (NOK 8.3 billion in 2009-2018)
- Observe prices P_t^a in 95-08 and find P^{ee} that produces $Z_{2008} = Z_{2008}^o$.
- Price P^{ee} that matches observed deforestation varies with model chosen.
- A model where, implicitly, a planner would act more aggressively against preservation would imply a larger P^{ee} .
- Larger P^{ee} applied to future decisions lowers deforestation (increase reforestation).
- P^{ee} that vary with model brings future trajectories across models closer.
- Similar observation when comparing discount rates.

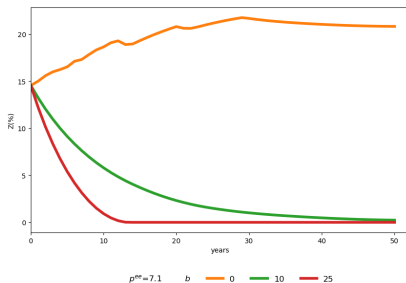
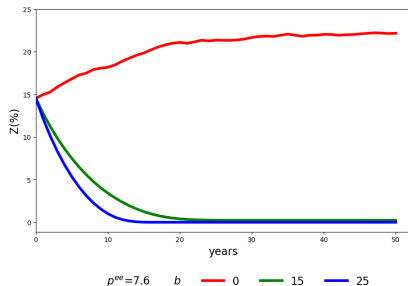
Shadow prices for different specifications

Table: Business-as-usual prices

number of sites	agricultural price	ξ	carbon price (P^{ee})
1043	$P^a = 41.1$	∞	7.6
78	$P^a = 41.1$	∞	7.1
78	$P^a = 41.1$	1	5.3
78	stochastic	∞	6.9

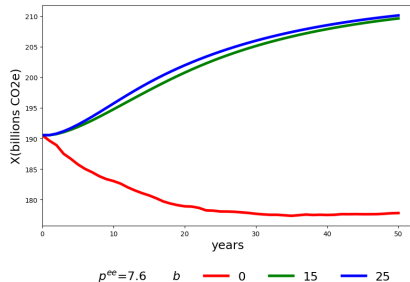
Notes: The agricultural price $P^a = 41.1$ is the mean under the stationary distribution.

Evolution of agricultural area (deterministic case, $P^a = P^s$)

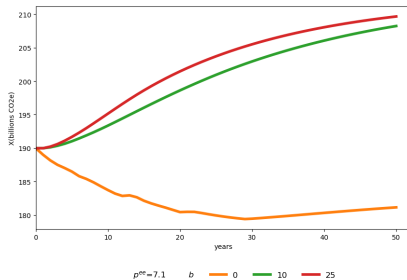


- $P^{ee} = \$7.6$ (\$7.1) for 1043 (78) sites
- Business as usual agr. area $\sim 25\%$ - may result on tipping of east, south and central Amazon (Lovejoy and Nobre [2018])

Evolution of the stock of carbon (deterministic case, $P^a = P^s$)



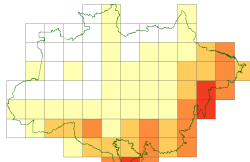
1043 Sites



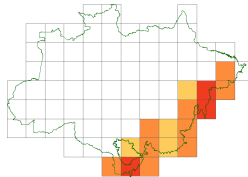
78 Sites

- $P^{ee} = \$7.6$ (\$7.1) for 1043 (78) sites

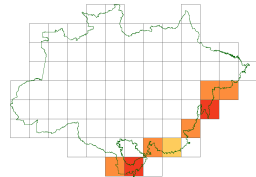
Evolution of occupation by agriculture, 78 sites, $b = 15, P^a = P^s$


 Z_{2017}^i (%)

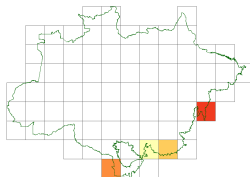
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 Z_{2022}^i (%), $b=15$

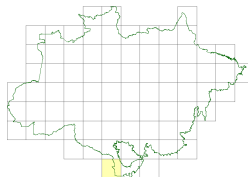
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 Z_{2027}^i (%), $b=15$

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 Z_{2032}^i (%), $b=15$

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 Z_{2037}^i (%), $b=15$

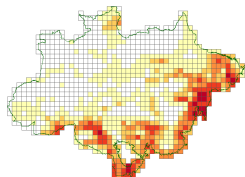
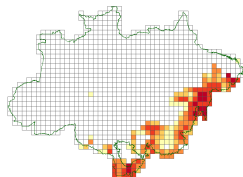
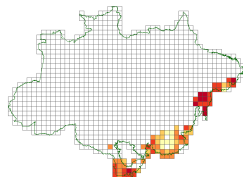
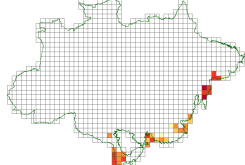
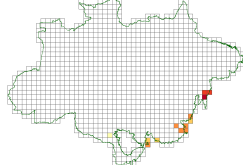
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 Z_{2047}^i (%), $b=15$

[0] [0-20] [20-40] [40-60] [60-80] [80-100]

- Much of the reallocation in 15 years

Evolution of occupation by agriculture, 1043 sites, $b = 15$, $P^a = P^s$


 Z_{2017}^i (%)

 Z_{2022}^i (%), $b=15$

 Z_{2027}^i (%), $b=15$

 Z_{2032}^i (%), $b=15$

 Z_{2037}^i (%), $b=15$

 Z_{2047}^i (%), $b=15$


Evolution of occupation by agriculture: with and without transfers

Planner Value Decomposition (200 years)

Table: 78 Sites - Deterministic case

P^a (\$)	P^e (\$)	b (\$)	Agricultural Output Value (\$ 10^{11})	Net Transfers (\$ 10^{11})	Forest Services (\$ 10^{11})	Adjustment Costs (\$ 10^{11})	Planner Value (\$ 10^{11})
41.1	7.1	0	3.31	0.00	-1.10	0.06	2.14
41.1	17.1	10	0.43	1.22	0.87	0.11	2.41
41.1	22.1	15	0.26	2.022	0.95	0.17	3.06
41.1	27.1	20	0.20	2.78	0.98	0.22	3.75
41.1	32.1	25	0.17	3.54	1.00	0.26	4.45

Notes: \$41.1 is the mean of the agricultural price in the stationary distribution of the two-state Markov chain. Forest services are calculated using baseline shadow price ($b = 0$)

Planner Value Decomposition (200 years)

Table: 1043 Sites - Deterministic case

P^a (\$)	P^e (\$)	b (\$)	Agricultural Output Value (\$ 10^{11})	Net Transfers (\$ 10^{11})	Forest Services (\$ 10^{11})	Adjustment Costs (\$ 10^{11})	Planner Value (\$ 10^{11})
41.1	7.6	0	3.72	0.00	-1.39	0.07	2.26
41.1	17.6	10	0.57	1.16	0.88	0.11	2.51
41.1	22.6	15	0.33	1.98	1.00	0.17	3.14
41.1	27.6	20	0.23	2.76	1.04	0.22	3.82
41.1	32.6	25	0.18	3.52	1.07	0.26	4.52
44.3	7.8	0	4.25	0.00	-1.64	0.08	2.51
44.3	17.8	10	0.73	1.09	0.85	0.10	2.58
44.3	22.8	15	0.40	1.94	1.01	0.16	3.19
44.3	27.8	20	0.26	2.74	1.07	0.21	3.86
44.3	32.8	25	0.20	3.51	1.09	0.26	4.56

Notes: For P^a , 41.1 is the mean of the agricultural price in the stationary distribution and 44.3 is the upper value in the two-state Markov chain.

Transfer cost (30 years, 78 sites)

P^e (\$)	b (\$)	Net Captured Emissions (billion tons of CO ₂ e)	Discounted Net Transfers (\$ 10 ¹¹)	Discounted Effective Cost (\$ per ton of CO ₂ e)
7.1	0	-15.28	0.00	NaN
17.1	10	11.79	0.84	3.13
22.1	15	14.08	1.54	5.25
27.1	20	14.74	2.18	7.27
32.1	25	15.06	2.81	9.26

Notes: \$41.1 is the mean of the agricultural price in the stationary distribution.

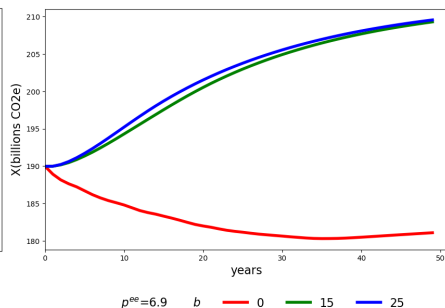
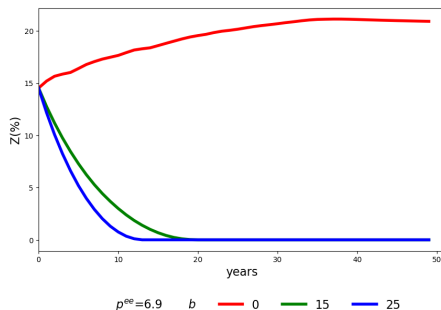
- Gains from trade

Transfer cost (30 years, 1043 sites)

P^e (\$)	b (\$)	Net Captured Emissions (billion tons of CO ₂ e)	Discounted Net Transfers (\$ 10 ¹¹)	Discounted Effective Cost (\$ per ton of CO ₂ e)
7.6	0	-17.66	0.00	NaN
17.6	10	11.67	0.86	2.93
22.6	15	13.85	1.55	4.92
27.6	20	14.62	2.21	6.85
32.6	25	15.00	2.86	8.75

Notes: Agricultural price $P^a = \$41.1$, which is the mean of the agricultural price in the stationary distribution.

Evolution of agricultural area and stock of carbon (Uncertainty on P^a)



● $P^{ee} = \$6.9$

Planner value decomposition (200 years)

Table: 78 Sites - MPC case

(\$ 10 ¹¹)	Agricultural Output Value	Net Transfers	Forest Services	Adjustment Costs	Planner Value
P^a = stochastic					
b = \$0					
10%	3.22	0.00	-1.01	0.05	2.19
50%	3.34	0.00	-1.00	0.05	2.29
90%	3.43	0.00	-0.97	0.05	2.37
b = \$25					
10%	0.16	3.48	0.96	0.27	4.33
50%	0.18	3.48	0.96	0.28	4.34
90%	0.19	3.48	0.96	0.28	4.35
P^a = \$41.1					
b = \$0	3.31	0.00	-1.10	0.06	2.14
b = \$25	0.17	3.54	1.00	0.26	4.45

Notes: The quantiles were computed based on two hundred simulations. Shadow prices are $P^{ee} = 7.1$ for $P^a = 41.1$ and $P^{ee} = 6.9$ for $P^a = \text{stochastic}$.

Transfer cost (30 years)

Table: 78 Sites - MPC case

	Net Captured Emissions (billion tons of CO ₂ e)	Discounted Net Transfers (\$ 10 ¹¹)	Discounted Effective cost (\$ per ton of CO ₂ e)
P^a = stochastic			
b = \$0			
10%	-14.41	0.00	NaN
50%	-13.79	0.00	NaN
90%	-13.33	0.00	NaN
b = \$25			
10%	14.78	2.78	9.49
50%	14.79	2.78	9.70
90%	14.81	2.78	9.86
P^a = \$41.1			
b = \$0	-15.25	0.00	NaN
b = \$25	15.08	2.87	9.47

Notes: The quantiles were computed based on two hundred simulations. Shadow prices are $P^{ee} = 7.1$ for $P^a = 41.1$ and $P^{ee} = 6.9$ for P^a = stochastic.

Parameter uncertainty I

- Hamiltonian Monte Carlo
- Following Stan (software for HMC) recommendation we sample from

$$\exp \left[-\frac{1}{\xi} f(d, \beta) \right] d\pi(\beta_\theta, \sigma_\theta^2 | R_\theta, y_\theta) d\pi(\beta_\gamma, \sigma_\gamma^2 | R_\gamma, y_\gamma) \quad (5)$$

- Baseline marginal for β is a multivariate t -distribution.
- $\rho := (\beta, \sigma_\theta, \sigma_\gamma)$ take logs and change sign get *potential energy*:

$$\begin{aligned} \mathcal{U}(\rho) = & \frac{1}{\xi} f(d, \beta) - \log d\pi(\beta_\theta | \sigma_\theta^2, R_\theta, y_\theta) - \log d\pi(\beta_\gamma | \sigma_\gamma^2, R_\gamma, y_\gamma) \\ & - \log d\pi(\sigma_\theta^2 | R_\theta, y_\theta) - \log d\pi(\sigma_\gamma^2 | R_\theta, y_\gamma) \end{aligned} \quad (6)$$

- *Momentum* ω (same dim as ρ), with $\omega \sim \mathcal{N}(0, M)$ and M symmetric, positive-definite *mass matrix*.

Parameter uncertainty II

- $$\mathcal{H}(\rho, \omega) := \mathcal{U}(\rho) + \frac{1}{2} \omega' M^{-1} \omega \quad (7)$$
- \mathcal{H} is "energy function"
- Induces a unique "canonical probability" with density

$$P(\rho, \omega) = \frac{1}{Z} \exp(-\mathcal{U}(\rho)) \exp\left(-\frac{1}{2} \omega' M^{-1} \omega\right)$$

Iteration I

- Sample from baseline π and initialize φ_0 as the mean of the transformed samples.
- For each iteration $\tau = 0, 1, \dots, \bar{\tau}$:
 - ① Solve the planner's problem for decision vector d_τ using φ_τ .
 - ② Sample $\{\rho_\tau^s\}_{s=1}^{4000}$ using (5) by running HMC simultaneously across 4 independent Markov chains, taking 1000 samples and 500 burn-in samples per chain.
 - ③ Transform marginal samples $\{\beta_\tau^s\}_{s=1}^{4000}$ back into the φ space.
 - ④ Compute $\bar{\varphi}_\tau$ as the mean across samples, and update φ_τ using $\varphi_{\tau+1} := w\bar{\varphi}_\tau + (1 - w)\varphi_\tau$, with $w = 0.25$.
 - ⑤ $\bar{\tau}$ is first occurrence of when $\|\varphi_{\tau+1} - \varphi_\tau\|_\infty < 0.001$. If $\tau < \bar{\tau}$, return to step 1 and repeat.
 - ⑥ Once $\bar{\tau}$ is reached redo steps 2 and 3 with $d_{\bar{\tau}}$, taking 5000 samples, and save the transformed samples $\{\varphi_{\bar{\tau}}^s\}_{s=1}^{20000}$.

Iteration II

- The density figures are plotted using $\{\varphi_{\bar{\tau}}^s\}_{s=1}^{20000}$.
- Trajectory plots and decompositions are computed using $d_{\bar{\tau}}$.
- For $\xi = 1$, we obtain $P^{ee} = 5.3$

HMC algorithm I

- For a given d_τ :
- Initialize ρ_τ^0 .
- For each chain, for $s = 0, \dots, 1000$:
 - ① Sample momentum $\omega_\tau^s \sim N(0, M)$.
 - ② Generate proposal $(\tilde{\rho}_\tau^s, \tilde{\omega}_\tau^s)$ by moving according to (8) , using the leapfrog integrator with step size ϵ and a number of steps L :

$$\frac{d\rho}{dt} = \frac{\partial \mathcal{H}}{\partial \omega}, \quad \frac{d\omega}{dt} = -\frac{\partial \mathcal{H}}{\partial \rho} \quad (8)$$

- ③ Orbits preserve canonical density probability distribution.
- ④ Orbits preserve value of \mathcal{H} .

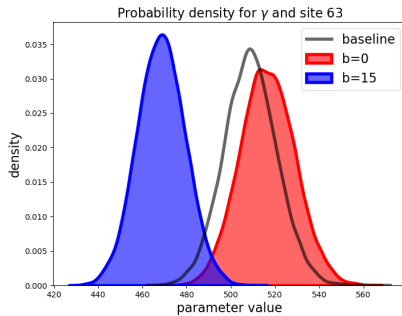
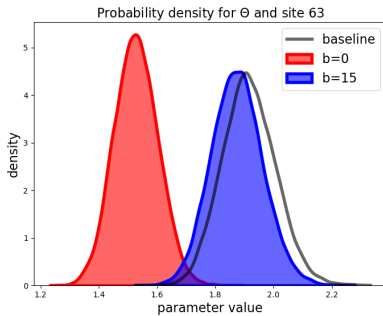
HMC algorithm II

- ⑤ Perform Metropolis test to accept or reject the state update $(\rho_\tau^{s+1}, \omega_\tau^{s+1}) \leftarrow (\tilde{\rho}_\tau^s, \tilde{\omega}_\tau^s)$, with acceptance probability:

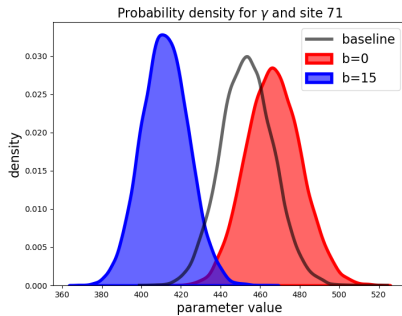
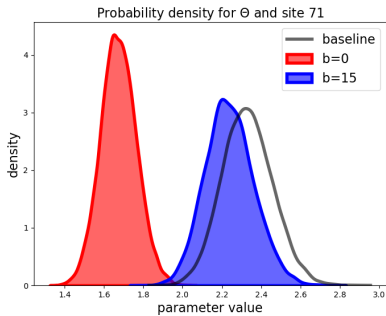
$$\min \{1, \exp(\mathcal{H}(\rho_\tau^s, \omega_\tau^s) - \mathcal{H}(\tilde{\rho}_\tau^s, \tilde{\omega}_\tau^s))\}$$

- In continuous should always accept but discrete approximation.
- In our case accept over 90% of the time

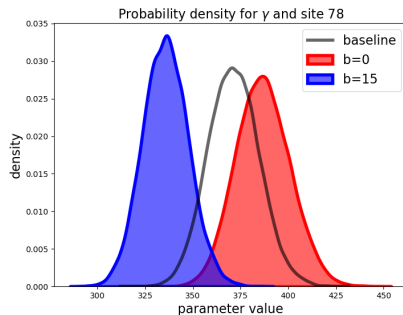
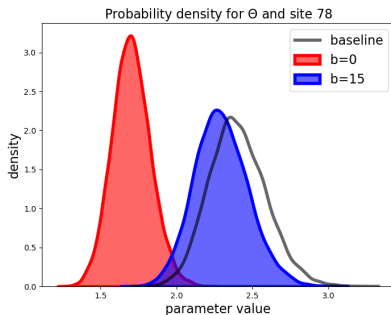
Ambiguity adjustment, $b = 15$, $P^a = P^s$



Ambiguity adjustment, $b = 15$, $P^a = P^s$

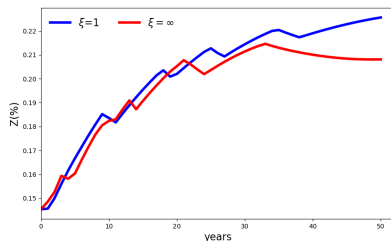
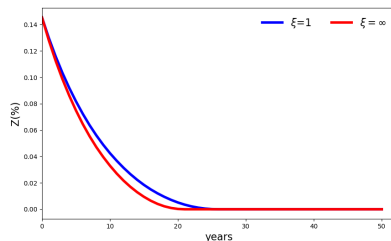


Ambiguity adjustment, $b = 15$, $P^a = P^s$

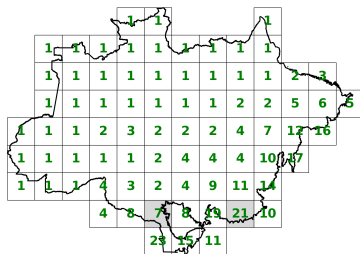


- Larger adjustments if we did not adjust P^{ee} .

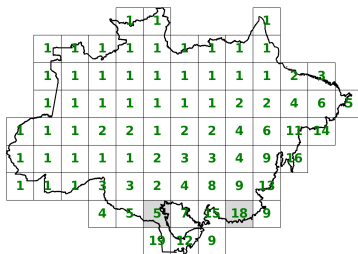
Evolution of agricultural area: productivity ambiguity, $b = 0, 15$, $P^a = P^s$

(a) $b=0$ (b) $b=15$

Year of maximal reforestation in site i ($b = 15$)



ambiguity



no ambiguity

- Larger changes if did not adjust P^{ee} .

Planner Value Decomposition (200 years)

Table: 78 Sites - HMC case

b (\$)	Agricultural Output Value (\$ 10^{11})			Planner Value (\$ 10^{11})		
	ambiguity neutral	ambiguity aversion	percent change	ambiguity neutral	ambiguity aversion	percent change
0	3.31	2.57	-22.4	2.14	1.64	-23.4
10	0.41	0.55	33.6	2.41	2.08	-13.9
15	0.26	0.30	14.2	3.06	2.62	-14.4
20	0.20	0.23	12.9	3.75	3.19	-15.0
25	0.17	0.19	11.9	4.45	3.74	-15.8

Notes: $P^a = \$41.11$, the average price under the stationary distribution. Forest services are calculated using baseline shadow price ($b = \$0$).

- Bigger difference when not adjusting P^{ee} .

Transfer cost (30 years)

Table: 78 Sites - HMC case

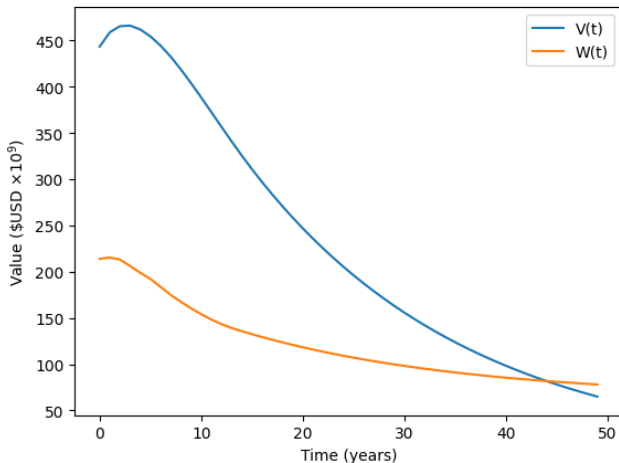
<i>b</i> (\$)	Ambiguity Neutral			Ambiguity Aversion		
	net captured emissions (billion tons of CO ₂ e)	discounted net transfers (\$ 10 ¹¹)	discounted effective cost (\$ per ton of CO ₂ e)	net captured emissions (billion tons of CO ₂ e)	discounted net transfers (\$ 10 ¹¹)	discounted effective cost (\$ per ton of CO ₂ e)
0	-15.25	0.00	NaN	-15.08	0.00	NaN
10	11.91	0.87	3.21	10.18	0.74	2.93
15	14.10	1.57	5.37	12.57	1.39	5.03
20	14.75	2.23	7.43	13.15	1.97	6.97
25	15.08	2.87	9.47	13.29	2.51	8.86

Notes: Agricultural price $P^a = \$41.1$, which is the mean of the agricultural price in the stationary distribution.

Incentive to defect

- Value tables show that shows that the planner would agree to sign an agreement to receive (pay) $b = 25$ dollars for each ton of CO_2 captured (emitted) in the Brazilian Amazon.
- However since mature forests reach an equilibrium, the value of transfers eventually converge to zero.
- Possibility: At some t , Brazil defects and opts to follow the optimal trajectory when $b = 0$, starting from (z_t, x_t) . Call the present value as of t of defecting W_t .
- Defection will occur if $W_t > V_t$.

Value of continuing versus defecting (78 sites deterministic)



Avoiding defection I

- Max present value for $t \leq 50$ is less than \$5 billion.
 - If $b = 30$ max present value $t \leq 50$ is less than \$1.5 billion.
- Defection can be avoided with carrot or stick
- Carrot: Set fund of \$5 billion payable at $t = 50$ if no (substantial) deviation in z_t , for $t \leq 50$. Cost per ton of CO_2 less than \$1/ton.
- Stick: Brazil issues a bond with initial value of \$5 billion that only becomes due if (substantial) deviation in z_t , for $t \leq 50$ is observed
- Many Carbon capture and sequestration (CCS) projects are private, with no clear liability horizon. Limited liability implies that indemnification for loss is only possible up to the value of the firm's assets (Gollier [2005]). Often long term liability for leaks transferred to governments.
 - Australian Commonwealth and Western Australia state agreed to take over liability of Gorgon CCS project from Chevron and partners that include Shell and ExxonMobil after closing of project.

Comparison with other carbon sequestration schemes

- US IRC Section 45Q, states that secure geological storage includes “storage at oil and gas reservoirs” .
- According to CBO, almost all CCS facilities in the US use captured CO₂ for enhanced oil recovery (EOR) to force more oil out of aging oil fields.
 - Occidental Petroleum, which is developing large carbon removal projects in Texas, uses EOR to sell what it calls “net-zero oil”
 - Capture carbon to release captured carbon.
- As of 9/23 total US capacity for CCS amounted to 22 million tons, .4% of US current emissions.
- Under IRA, U.S. 45Q tax credit for EOR carbon capture projects pays \$60/ton (\$130 for direct air capture DAC)) for facilities that start construction before 2033 and pay prevailing wages during the construction phase and during the first 12 years of operation. Amounts adjusted for inflation after 2026.

Carbon prices above \$25 (World Bank)



- Economic efficiency implies all carbon prices should be the same.

Conclusions I

- Posed explicit dynamic model across heterogeneous regions in Amazon to assess potential adverse impact of deforestation.
- Rich panel data set
- Computational challenge because the heterogeneity of subregions requires large number of state variables and state-constraints that bind at optimum.
 - Parameter uncertainty
- With modest prices for CO₂e (compared e.g. with US EOR schemes or with ETS prices in Europe...), Brazilian Amazon would produce noticeable CO₂ capture.
 - Compared to IPCC “budget”
 - Compared to Griscom et al. [2017] that identify and quantify “natural climate solutions” (NCS).

Conclusions II

- In Part III will show the existence of important interactions across sites that are ignored here.
- Predicted path under “business as usual” even more perilous.
- Further results can be found in the Online Appendix.

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