Social Welfare in Program Evaluation and Treatment Choice

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April 2021

- We are grateful to Arthur Lewbel for initial discussions
- DB acknowledges financial support from ERC Consolidator Grant EDWEL, Project number 681565.

Outcome vs Utility

- Effect of tuition subsidy
- $\bar{p}$: market price, s: subsidy, y: income
- ATE: $q(\bar{p} - s, y) - q(\bar{p}, y)$
- Tt Assignment: $\max_{s(\cdot)} \int q(\bar{p} - s(y), y)\, dF(y)$ s.t. $\int s(y) \times q(s(y), y)\, dF(y) = M$
- But for each y people have heterogeneous willingness-to-pay, so heterogeneous effects on **utility**
- No **outcome** change but **utility** change

Social Welfare

- In public finance, cost-benefit analysis based on average indirect utility – a functional of individual **Indirect Utilities (**Bergson-Samuelson)
- $W(\mathbf{p}, y, \eta)$: Maximized utility by η type consumer on budget set $(\mathbf{p}, y)$
- $\mathcal{W}(\mathbf{p}, y) = \int H(W(\mathbf{p}, y, \eta))\, dF(\eta)$ e.g. $\mathcal{W}^{\varepsilon}(\mathbf{p}, y) \equiv E_{\eta}\left\{\frac{W(\mathbf{p}, y, \eta)^{1-\varepsilon}}{1-\varepsilon}\right\}$ or $\int \mathcal{W}^{\varepsilon}(\mathbf{p}, y)\, dF(y)$
- Ramsey 1927, Atkinson 1971, Mirrlees 1971, Deaton 1984, Stern 1987....
- No unobservables, i.e. parametric, representative agent

Agenda

- Reconcile **utility**-based CBA with general **unobserved heterogeneity**
- Binary choice with unrestricted heterogeneity
- CDF of indirect utility function, expression for aggregate social welfare
- Connection with Hicksian measures
- Treatment Assignment Problem
- Application to ITN subsidies
- Multinomial Choice, Continuous Choice

Set-Up

- 2 options $\{0,1\}$, utilities $U_0(y,\eta)$, $U_1(y-p,\eta)$; e.g. $\eta=(\eta_0,\eta_1)$, unknown dimension/distribution
- $U_0(\cdot,\eta)$, $U_1(\cdot,\eta)$ strictly increasing, define

$$\begin{aligned}\bar{q}(p,y) &\equiv \int 1\{U_0(y,\eta)\leq U_1(y-p,\eta)\}\,dG(\eta)\\ W(p,y,\eta) &\equiv \max\{U_0(y,\eta),U_1(y-p,\eta)\}\end{aligned}$$

- Slutsky condition: Bhattacharya Ecta, 2021

$$\frac{\partial}{\partial p}\bar{q}(p,y)\leq 0,\qquad \frac{\partial}{\partial p}\bar{q}(p,y)+\frac{\partial}{\partial y}\bar{q}(p,y)\leq 0.$$

Distribution of Indirect Utility

- Normalize $W(p, y, \eta) \equiv \max\left\{y, U_0^{-1}\left(U_1(y-p, \eta), \eta\right)\right\}$: money-metric
- $W(p, y, \eta) \geq y$ w.p. 1, for $c \geq y$,

$$\begin{aligned} & \Pr\left[\max\left\{y, U_0^{-1}\left(U_1(y-p,\eta), \eta\right)\right\} \leq c\right] \\ = & \Pr\left[U_1(y-p,\eta) \leq U_0(c,\eta)\right], \text{ since } U_0(\cdot,\eta) \text{ cont. and strictly} \uparrow \\ = & \Pr\left[U_1(c-(c-y+p),\eta) \leq U_0(c,\eta)\right] \\ = & 1-\bar{q}(c-y+p, c). \end{aligned}$$

- C.D.F. of indirect utility

$$\Pr\left[W(p, y, \eta) \leq c\right] = \begin{cases} 0 \text{ if } c < y \\ 1-\bar{q}(c-y+p, c) \text{ if } c \geq y \end{cases}$$

- Non-decreasing in c since $\frac{\partial}{\partial p}\bar{q}(p, y) + \frac{\partial}{\partial y}\bar{q}(p, y) \leq 0$ (Bhattacharya Ecta, 2021)

Bergson-Samuelson Social Welfare

$$
\begin{aligned}
& E_\eta \left\{ \frac{W(p, y, \eta)^{1-\varepsilon}}{1-\varepsilon} \right\} \\
= \quad & \frac{y^{1-\varepsilon}}{1-\varepsilon} + \int_0^\infty (z+y)^{-\varepsilon} \times \bar{q}(z+p, z+y)\, dz
\end{aligned}
$$

- Utilitarian case, $\varepsilon = 0$

$$
E_\eta \{ W(p, y, \eta) \} = y + \int_0^\infty \bar{q}(z+\bar{p}, z+y)\, dz
$$

Connection with Hicksian Measures

Alternative 1 unavailable $(p = \infty)$ to available at price p

$$
\begin{aligned}
& \underbrace{y + CV\left(y, p, \infty, \eta\right)}_{\text{compensated income}} \\
= \quad & \max\left\{y, U_0^{-1}\left(U_1\left(y - p, \eta\right), \eta\right)\right\} \\
= \quad & \underbrace{W\left(\mathbf{p}, y, \eta\right)}_{\text{Indirect utility}}
\end{aligned}
$$

Connection with Hicksian Measures

By path-independence

$$\begin{aligned} & CV\left(y, p^0, \infty, \eta\right) \\ = \quad & CV\left(y, p^0, p^1, \eta\right) + CV\left(\underbrace{y + CV\left(y, p^0, p^1, \eta\right)}_{\neq y}, p^1, \infty, \eta\right) \\ \neq \quad & CV\left(y, p^0, p^1, \eta\right) + CV\left(y, p^1, \infty, \eta\right). \end{aligned}$$

Therefore,

$$W\left(p^1, y, \eta\right) - W\left(p^0, y, \eta\right) \neq -CV\left(y, p^0, p^1, \eta\right).$$

Critique of Aggregate Hicksian Compensation

- Blackorby and Donaldson 1988, Banks-Blundell-Lewbel 1996
- Expected CV not concave in income
- Planner's preference over *changes* rather than *levels* of utility
- Constant marginal social utility of income

Social Welfare vis-a-vis Hicksian Compensation

Social welfare $V(p,y,\eta) = H(W(p,y,\eta))$, e.g. $H(x) = \frac{x^{1-\varepsilon}}{1-\varepsilon}$. Then

$$\begin{aligned} & \frac{\partial}{\partial p} V(p,y,\eta) \\ = \quad & H'(W(p,y,\eta)) \frac{\partial W(p,y,\eta)}{\partial p} \\ & \overset{\text{Roy's Identity}}{=} -\underbrace{\left[H'(W(p,y,\eta)) \times \frac{\partial W(p,y,\eta)}{\partial y} \right]}_{\text{marginal social utility of income}} q(p,y,\eta) \\ \Longrightarrow \quad & V(p_0,y,\eta) - V(p_1,y,\eta) = \int_{p_0}^{p_1} \left[MSU(p,y,\eta) \times q(p,y,\eta) \right] dp. \end{aligned}$$

Compare with Hicksian compensation

$$\int_{p_0}^{p_1} q^c(p,y,\eta)\, dp$$

Optimal Targeting Problem

- Per capita subsidy cost capped at M
- Cost: $\int C(y, \sigma(y))\, dF_Y(y)$
- Benefit: $\mathcal{W}^{\varepsilon}(p, y) \equiv E_{\eta}\left[W(p, y, \eta)^{1-\varepsilon} / (1-\varepsilon)\right]$
- Optimal allocation for sample

$$\begin{array}{l} \arg\max_{\sigma(\cdot) \in \mathcal{T}} \int \mathcal{W}^{\varepsilon}(\bar{p} - \sigma(y), y)\, dF_Y(y) \\ \text{s.t. } \int C(y, \sigma(y))\, dF_Y(y) = M \end{array}$$

Treatment Choice with Parameter Uncertainty

- Optimal subsidy targeting rule $\sigma(y)$
- θ 'parameter' describing distribution of income and demand function; can be nonparametric
- Loss function

$$
\begin{aligned}
L(\sigma(\cdot), \theta, c) &= \int \mathcal{W}^{\varepsilon}(\bar{p} - \sigma(y), y, \theta)\, dF_Y(y; \theta) \\
&\quad + \lambda \left[M - \int \sigma(y) \times \bar{q}_1(\bar{p} - \sigma(y), y; \theta)\, dF(y; \theta) \right]^2
\end{aligned}
$$

- $P_{post}(\theta | data)$ posterior of θ given data, e.g. sampling distribution θ
- Choose $\sigma(\cdot)$ via

$$
\max_{\sigma(\cdot)} \int L(\sigma(\cdot), \theta, c)\, dP_{post}(\theta | data)
$$

Empirical Illustration: Evaluating a Subsidy

- **Change in average social welfare**

$$\begin{aligned} & \Delta(\bar{p}, \sigma, y; \varepsilon) \\ \equiv & \int_0^\infty (z+y)^{-\varepsilon} \times \left[\bar{q}_1(\bar{p}-\sigma+z, y+z) - \bar{q}_1(\bar{p}+z, y+z)\right] dz. \end{aligned}$$

- **Average treatment effect**

$$T(\bar{p}, \sigma, y) \equiv \bar{q}_1(\bar{p}-\sigma, y) - \bar{q}_1(\bar{p}, y),$$

- **Average cost** $\sigma \times \bar{q}_1(\bar{p}-\sigma, y)$

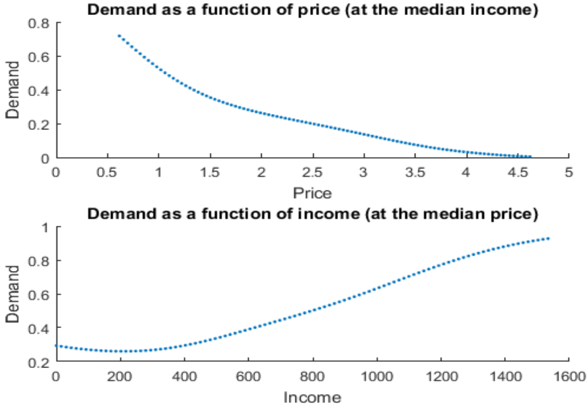
Empirical Illustration

- Experimental price subsidies for insecticide-treated mosquito nets (Dupas 2014)
- 2000 hhds in rural Kenya, price, wealth per month, # children$\leq$10, dummy for bank account
- Price variation $0 to $4.6.

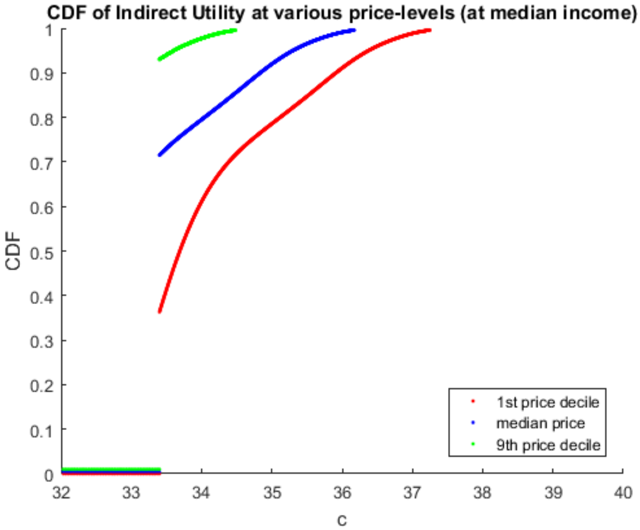
Estimation

- Tensor-product B-spline $\mathcal{T}_{M_1,M_2}(\cdot,\cdot)$ satisfying shape restrictions in price and income

$$\bar{q}(p,y,x) \equiv P(q=1 \mid p,y,x) = \Phi\left(\mathcal{T}_{M_1,M_2}(p,y) + x'\beta\right)$$
$$\frac{\partial}{\partial p}\bar{q}(p,y,x) \leq 0,\ \frac{\partial}{\partial p}\bar{q}(p,y,x) + \frac{\partial}{\partial y}\bar{q}(p,y,x) \leq 0.$$



CDF of Indirect Utility at Different Price

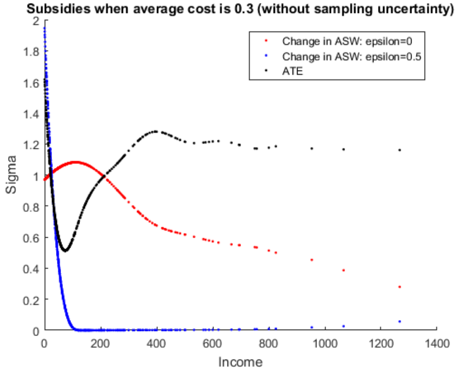


Welfare estimates

Welfare quantity	At median income	Averaged over income
average CV	0.4208	0.4194
	(0.0248)	(0.0242)
ATE	0.2684	0.2790
	(0.0315)	(0.0290)
change in ASW, $\varepsilon = 0$	0.4207	0.4192
	(0.0248)	(0.0242)
change in ASW, $\varepsilon = 0.5$	0.0716	0.0817
	(0.0042)	(0.0049)
change in ASW, $\varepsilon = 1$	0.0122	0.0211
	(0.0007)	(0.0016)

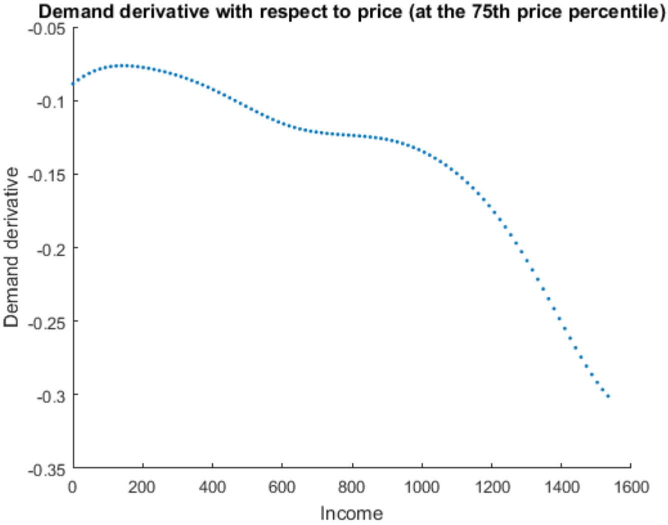
Table: Quantities in (1)-(**??**) calculated at $\bar{p} = 2.7692$ (75th percentile of the price distribution), $\bar{p} - \sigma = 1.2308$ (25th percentile of the price distribution). Covariate values are at their median values. Bootstrap standard errors in parentheses are based on 400 replications.

Optimal Subsidy (Without parameter uncertainty)



- ATE: high price sensitivity at very low income, then less sensitive, then more sensitive plus positive income-effect
- ASW: $U_1(y - p, \eta)$ sharply increasing at low values of $y - p$, so reduce price with income, then flattens – maintain $y - p$ to level marginal utility

Price derivative vs Income



Conclusion

- Reconciles program evaluation tradition in econometrics with aggregate cost-benefit analysis in public finance
- Choice Problems with unrestricted heterogeneity and income effects
- Indirect utility distribution nonparametrically identified from average demand without knowledge of utility functions and heterogeneity dimension/distribution
- Theoretical connection with Hicksian compensation, equal if no income effect
- Empirical cost-benefit analysis with ITN data

Daly-Zachary-Williams Theorem

Additive random utility model

$$W\left(p, y, \boldsymbol{\eta}\right) = \max\left\{y + \eta_0, y - p + \eta_1\right\},$$

then

$$\bar{q}\left(\mathbf{p}, y\right) = \frac{\partial}{\partial\left(y - p\right)} E_\eta\left\{W\left(p, y, \boldsymbol{\eta}\right)\right\}.$$

Average CV and Utilitarian Welfare

- Price changes from $\bar{p}-\sigma$ to $\bar{p}$. The **change in average social welfare** for $\varepsilon=0$ equals

$$\Delta\left(\bar{p},\sigma,y;0\right)\equiv\int_0^\infty\left[\bar{q}_1\left(\bar{p}-\sigma+z,y+z\right)-\bar{q}_1\left(\bar{p}+z,y+z\right)\right]dz. \tag{1}$$

- The **average compensating variation** at income y (c.f. Bhattacharya 2015) equals

$$S\left(\bar{p},\sigma,y\right)\equiv\int_0^\sigma\bar{q}_1\left(\bar{p}-\sigma+z,y+z\right)dz. \tag{2}$$

- Therefore

$$\begin{aligned}&\Delta\left(\bar{p},\sigma,y;0\right)-S\left(\bar{p},\sigma,y\right)\\ =\;&\int_\sigma^\infty\left[\bar{q}_1\left(\bar{p}-\sigma+z,y+z\right)-\bar{q}_1\left(\bar{p}-\sigma+z,y-\sigma+z\right)\right]dz\end{aligned}$$

- Difference large when income effect large
- If no income effect then avg CV=avg EV=Avg SW=Marshallian CS

Multinomial choice

- $\max_j \{U_0(y,\eta), U_1(y-p_1,\eta), ..., U_J(y-p_J,\eta)\}$
- Indirect Utility

$$\begin{aligned} & W(\mathbf{p}, y, \eta) \\ = & \max\left\{y, U_0^{-1}(U_1(y-p_1,\eta),\eta), ..., U_0^{-1}(U_J(y-p_J,\eta),\eta)\right\} \end{aligned}$$

- Distribution of Indirect Utility

$$\Pr[W(\mathbf{p}, y, \eta) \leq c] = \begin{cases} 0 \text{ if } c < y \\ \bar{q}_0(c-y+p_1, ..c-y+p_J, c) \text{ if } c \geq y \end{cases}$$

Continuous choice

- Roy's identity

$$-\frac{\frac{\partial W(p,y,\eta)}{\partial p}}{\frac{\partial W(p,y,\eta)}{\partial p}} = q\left(p, y, \eta\right)$$

- Knowledge of distribution of RHS does not produce distribution of $W\left(p, y, \eta\right)$
- Analogy: Want to know distribution of $h\left(x, \eta\right) \equiv \eta_0 + \eta_1 x$, $\eta_0 \perp \eta_1$, and $\left(\eta_0, \eta_1\right) \perp X$
- Distribution of $\frac{\partial h(x,\eta)}{\partial x} = \eta_1$ tells us nothing about η_0

More Structure

- Random Coefficients: $q(p, y, \eta) = \eta_0 + \eta_1 p + \eta_2 y$
- Beran and Hall 1992: coefficient joint distribution identified
- Solve PDE

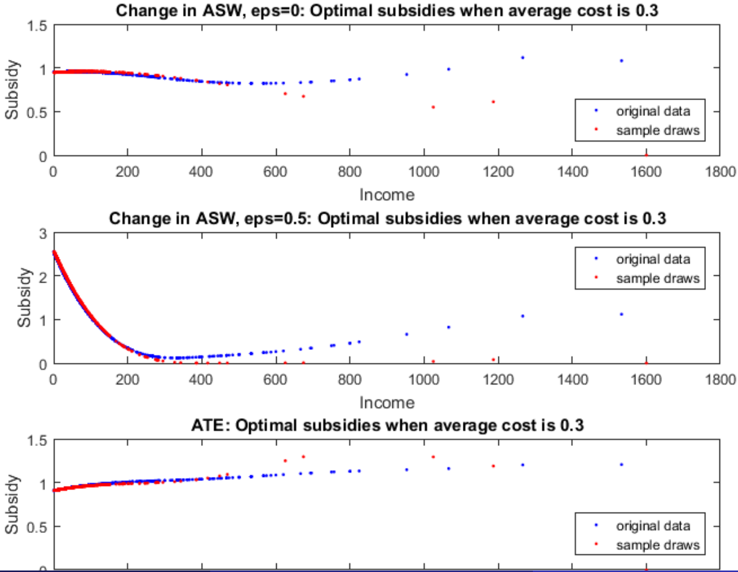
$$-\frac{\partial W(p, y, \eta)}{\partial p} \Big/ \frac{\partial W(p, y, \eta)}{\partial y} = \eta_0 + \eta_1 p + \eta_2 y$$

- Hausman 1981 for (nonrandom coefficients)

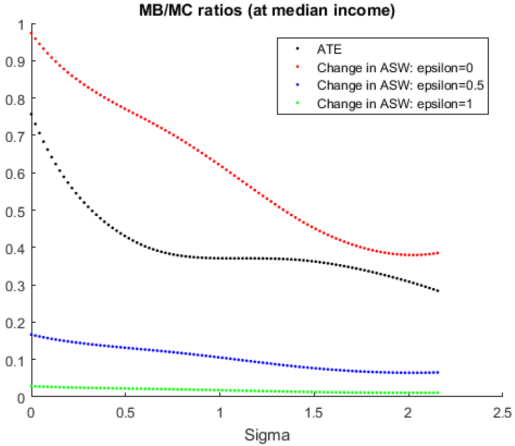
$$W(p, y, \eta) = e^{-\eta_2 p}\left[y + \frac{1}{\eta_2}\left(\eta_1 p + \frac{\eta_1}{\eta_2} + \eta_0\right)\right].$$

- Delta Method

Optimal Subsidy (With parameter uncertainty)



MB/MC



MB/MC ratios (at median income)
ATE
Change in ASW: epsilon=0
Change in ASW: epsilon=0.5
Change in ASW: epsilon=1
Sigma
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5

Marginal Utility of Income

- $W(p, y+S, \eta)$ constant; therefore,

$$
\begin{aligned}
0 &= \frac{\partial}{\partial p} W(p, y+S, \eta) \\
\implies & \frac{\partial S(p, y, \eta)}{\partial p} = \frac{-\frac{\partial}{\partial p} W(p, y+S, \eta)}{\frac{\partial}{\partial y} W(p, y+S, \eta)} = q^c(p, y, \eta) \\
\implies & \int_{p^0}^{p^1} \frac{\partial S(p, y, \eta)}{\partial p} = \int_{p^0}^{p^1} q^c(p, y, \eta)\, dp
\end{aligned}
$$