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Nonlinear Difference-in-Differences in Repeated Cross Sections with Continuous Treatments

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Abstract

This paper studies the identification of nonseparable models with continuous, endogenous regressors, also called treatments, using repeated cross sections. We show that several treatment effect parameters are identified under two assumptions on the effect of time, namely a weak stationarity condition on the distribution of unobservables, and time variation in the distribution of endogenous regressors. Other treatment effect parameters are set identified under curvature conditions, but without any functional form restrictions. This result is related to the difference-in-differences idea, but does neither impose additive time effects nor exogenously defined control groups. Furthermore, we investigate two extrapolation strategies that allow us to point identify the entire model: using monotonicity of the error term, or imposing a linear correlated random coefficient structure. Finally, we illustrate our results by studying the effect of mother's age on infants' birth weight.

Keywords: identification, repeated cross sections, nonlinear models, continuous treatment, random coefficients, endogeneity, difference-in-differences.

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1 Introduction

Using the time dimension to correct for the influence of correlated but time invariant unobservables has a long tradition in econometrics. Panel data in combination with a fixed effects or first differencing transformation are common methods to purge the data from the influence of unobserved heterogeneity across many applications. However, panel data sets are rare. For many questions arising in applications they simply do not exist. Moreover, they have several drawbacks. Most prominently, they suffer from nonrandom attrition, a phenomenon that is hard to control.[1] Also, they frequently cover only short time spans.

An alternative is to rely on repeated cross sections, i.e., a data set that covers the same population, but not necessarily the same individual, repeatedly. More formally, as econometricians we have access to the distributions $(F_{Y_1,X_1}, ..., F_{Y_T,X_T})$ of outcomes and explanatory variables, but contrary to panel data the joint distribution $F_{Y_1,X_1,...,Y_T,X_T}$ is not identified. Many prominent data sets are repeated cross sections, e.g., the FES in the UK, or the CEX in the US. We argue in this paper that many of the strong identification results obtained for panel data, e.g., concerning the correlated random coefficients panel data model (Chamberlain, 1982, Graham & Powell, 2012) have close correspondences in repeated cross section (RCS). In particular, in a RCS it is possible to obtain causal effects of a continuous variable of interest X_t on an outcome Y_t, while allowing for an unobservable A_t that is contemporaneously arbitrarily correlated with X_t, and does not even need to be time invariant. Note that the data structure precludes the use of past X_t for the same individual to construct control variables, as in Altonji & Matzkin (2005).

Formally, we consider as a general framework the single-equation structure of the form

$$Y_t = g_t(X_t, A_t) \qquad t = 1, ..., T \tag{1.1}$$

where $Y_t \in \mathbb{R}$ is the outcome, $X_t = (X_{1t}, ..., X_{kt}) \in \mathbb{R}^k$ is a set of endogenous explanatory variables, and A_t are unobserved heterogeneous factors which may be correlated with X_t. Observe

[1] See nonetheless, among others, Hausman & Wise (1979), Hirano et al. (2001), Bhattacharya (2008) or Sasaki (2013) for proposals on how to deal with endogenous attrition.

that the structural function g_t is allowed to depend on the time period t, e.g., whether we are in a boom or an in a crisis in the business cycle. However, we do place restrictions on the time evolution of g_t by requiring that it is comprised of a monotonic transformation m_t and a time invariant base function g, i.e. $Y_t = m_t(g(X_t, A_t))$. This transformation extends typical additive time dummy specifications that are meant to capture macro shocks, and allows for the macro shocks to have different effects on different parts of the distribution of the "detrended" variable $\widetilde{Y}_t \equiv g(X_t, A_t)$, say affects individuals with high $\widetilde{Y}_t$ only.

In this setup, we focus on the identification of several parameters that take the form of average and quantile treatment effects. Generally, we establish that several treatment effect parameters are point identified. This requires in a first step to establish identification of the time dependent transformation function m_t. Based on this result, we establish point identification of a number of treatment effect parameters. Some parameters, like average partial effects at arbitrary positions $X_t = x$, however, are not covered by our point identification results. In those cases we derive bounds under a local curvature condition.

Finally, we clarify the role of a linear correlated random coefficients specification, as in Chamberlain (1982), Wooldridge (2003), Murtazashvili & Wooldridge (2008) or Graham & Powell (2012), in that it allows to extrapolate and thus obtain point identification of average partial effects across the entire population.[2] A similar remark applies to assuming monotonicity in a scalar A_t - we are again able to point identify a structural model across the entire population.

Our key identifying assumption is that for almost all v in the support of V_t defined below and all $(s,t) \in \{1,...,T\}^2$,

$$A_t|V_t = v \sim A_s|V_s = v,$$

where $V_t = \mathbf{F}_t(X_t) = (F_{X_{1t}}(X_{1t}), ..., F_{X_{kt}}(X_{kt}))$. To illustrate the content of this assumption, consider the following textbook example with a scalar X_t: Suppose that Y_t is unemployment duration, A_t is ability and X_t is unemployment benefits, and suppose that, as in many countries, the benefit is tied to income through a fixed ratio. For simplicity, let us suppose that they are

[2]We also show identification of treatment effects in a polynomial correlated random coefficient model provided that the order of the polynomial is less or equal to T. This result is related to Florens et al. (2008) except that time is discrete and does not act as a standard instrument here.

actually equal (i.e., the fixed ratio is 1). Then V_t denotes the rank in the income distribution and has always an uniform distribution on the unit interval $[0, 1]$. The assumption now requires the following: suppose that the rank is fixed in periods s and t, so that $V_t = V_s = v$. At this rank, the distribution of ability ought to be the same in both periods s and t. To give an example, the distributions of abilities have to be the same at median income in 1985 an 2000. Median income ("middle class") people are similar in terms of their raw abilities over time, at least for the relatively short time horizons in a repeated cross section. At least in this example, we think of this assumption as more realistic as requiring time invariance with respect to income X_t itself, or the income process $X_1, ..., X_T$, as is commonly assumed in the panel data literature, because individuals who earned 30 K a year in 1985 (or had a certain trajectory leading to 30 K) could be quite distinct from those who earned 30 K in the year 2000, especially in a country with high growth or inflation.

While this assumption is in place of a traditional exogeneity condition, we also need the source of exogenous variation in our model, time itself, to have some effect on the continuous explanatory variable. Since we are not following individuals over time, these variations should be at the distributional level. Namely, $\mathbf{F}_t \neq \mathbf{F}_s$ is necessary to obtain nontrivial identification results on the effect of X_T. We also require that there be at least one crossing point x^* between $\mathbf{F}_t$ and $\mathbf{F}_s$. Note that this is a testable property, analogous to a rank condition in IV.

The main idea behind our identification result is to first isolate the effect of time, in our notation the monotonic function m_t. This effect is obtained by realizing that we can construct a control group at the crossing point x^*. Since X_t is time invariant at this point, and the associated rank and hence the distribution of unobservables A_t does not change either, we conclude that any effect on the outcome distribution must have been generated by time itself. The combination of this insight with the monotonicity assumption allows one to recover m_t. The next step is purging Y_t from the influence of time, thus removing from the treated groups the effect that time alone had on them. The new variable, $\tilde{Y}_t$ can now be used to point, respectively, set identify any of the various causal effects parameters described below. At this stage, the key insight is that under our condition, in particular the fact that A_t has a time independent conditional distribution, time plays the role of an instrument. We can therefore

use exogenous variation in the distribution of X_t over to time to identify causal effects.

Related Literature: Our setup is most related to the difference-in-difference (DiD) framework introduced by Ashenfelter & Card (1985). In its standard version, the difference-in-difference method also works with repeated cross sections, though it applies to binary treatments, and assumes a linear fixed coefficients structure. The idea is that there are two well-defined groups, namely the control and treatment group, and while none of them are treated at period 1, the treatment group becomes treated at period 2. Then if the effect of time is the same for both groups ("the common trend assumption"), it can be identified using the control group. The effect of the treatment is then obtained using the treatment group and the detrended variable.[3] The broad identification strategy we develop here is similar, though there are important differences, most notably that we consider a continuous endogenous regressor (treatment). But this is by no means the only important difference. Other crucial differences include the following: First, our model is fully nonlinear in both the continuous regressor and the potentially high dimensional unobservables. Second, the effect of time in particular is allowed to be nonadditive in our model. The only other reference that allows for a nonlinear (yet binary) treatment model with nonadditive time effects is Athey & Imbens (2006). Different, however, from the entire literature, including Athey & Imbens (2006), is that the control group is data-dependent in our context, whereas it is defined ex ante in the DiD framework.

As already discussed, we use exogenous variations of X_t due to time. This idea has already been put forward in the literature on repeated cross sections. Previous contributions include Deaton (1985), Moffitt (1993), Verbeek & Nijman (1992, 1993), Verbeek (1996), Collado (1997), McKenzie (2004) and Devereux (2007). Compared to this literature, our contribution is twofold. First, we dispense with the common linear or parametric framework that they consider. Our model is nonlinear and nonparametric, and allows for high dimensional heterogeneity. Second, our identification strategy does not exclude time from affecting the outcome directly. A last important difference between our work and the classical literature on repeated cross sections is the focus. While we are concerned with contemporaneous causal effects, the literature usually

[3]Extensions of this strategy to account for time effects that depend on covariates are considered by Heckman et al. (1997) and Abadie (2005).

focuses on the identification of the joint distribution of $(Y_1, X_1, ..., Y_T, X_T)$, or features of it, from the marginal distributions of (Y_1, X_1),..., (Y_T, X_T), usually to derive dynamic effect, see Moffitt & Ridder (2007) for a survey.

Our work is also related to general work on high dimensional heterogeneity in panel and cross section data, starting with the seminal work by Chamberlain (1982, 1984). Important references in the class of panel data models include Altonji & Matzkin (2005), Graham & Powell (2012), Hoderlein & White (2012) and Chernozhukov et al. (2013). All of these papers consider special cases or similar structures as defined in Equation (1.1), but they do not allow the structural function to depend on time. Instead of our time invariance assumption, all of these references assume, for $(s,t) \in \{1, ..., T\}^2$ and almost all $(x_1, ..., x_T)$

$$A_t | X_1 = x_1, ..., X_T = x_T \sim A_s | X_1 = x_1, ..., X_T = x_T.$$

This condition neither nests nor is nested in our assumption, as we argue below. In addition, Altonji & Matzkin (2005) assume an exchangeability condition that allows to construct a control function that makes A_t conditionally independent of X_t, while Graham & Powell (2012) assume a linear random coefficients structure, arguably a crucial special case that we will also analyze in detail. Evdokimov (2011) imposes the error term to be scalar and to have a monotonic effect. Under monotonicity, we also obtain full identification with only repeated cross sections over two time periods, as opposed to panel data with three periods in his case. On the other hand, we obtain our result under time invariance conditions that are not imposed in his setting. Finally, many of the treatment parameters we are considering appear in these references, but have also figured prominently in the cross section literature, see Imbens & Newey (2009), Schennach et al. (2012), or Hoderlein & Mammen (2007).

Structure of the Paper In section 2, we introduce the model formally, including all major assumptions and the parameters of interest, and discuss them thoroughly. In the third section, we present the main identification result. In the fourth section, we discuss two extrapolation strategies. We consider a linear correlated random coefficient structure and a model where g depends monotonically on a scalar A_t. We show that in both cases, these restrictions yield point identification of the structural effect across the entire population. Finally, in the fifth

section we apply our methodology to the effect of maternal age on birth weight of the first child. This is typically an example where maternal age is endogenous, an instrument might be difficult to find and panel data are useless, because the maternal age at the first birth does not vary within individuals.

2 The Model and Formal Assumptions

In this section, we formally introduce the model and the main assumptions. Since the model is nonparametric and heterogeneous, the parameters of interest are not obvious. We start out by formally introducing these parameters. We then proceed to present and discuss the main assumptions we employ.

2.1 Parameters of interest

We are especially interested in the following average and quantile treatment on the treated effects:

$$\begin{aligned}
\Delta^{ATT}(x,x') &\equiv E\left[g_T(x',A_T) - g_T(x,A_T)|X_T = x\right], \\
\Delta_j^{AME}(x) &\equiv E\left[\frac{\partial g_T}{\partial x_j}(x,A_T)|X_T = x\right], \\
\Delta^{QTT}(\tau,x,x') &\equiv F^{-1}_{g_T(x',A_T)|X_T}(\tau|x) - F^{-1}_{g_T(x,A_T)|X_T}(\tau|x), \\
\Delta_j^{QME}(\tau,x) &\equiv \frac{\partial F^{-1}_{g_T(x',A_T)|X_T}(\tau|x)}{\partial x'_j}|x' = x,
\end{aligned}$$

for any $x = (x_1, ..., x_k)$ and $x' = (x'_1, ..., x'_k)$ in the support of X_T and $j \in \{1, ..., k\}$. These parameters correspond to the effect of exogenous shifts of X_T on Y_T. The first two effects are average effects, while the latter two effects are their quantile analogs. The former two effects are related to treatment effects on the treated in that they provide averages over causal effects for a subpopulation with treatment intensity $X_T = x$. To understand this better, consider the first parameter of interest, $\Delta^{ATT}(x,x')$. To fix ideas, think of A_t as ability in period t, and X_t as schooling. Obviously, we would believe ability to be heterogeneously distributed across the population, as well as contemporaneously correlated with schooling. For an individual with

ability level $A_t = a$ in period t, the effect of changing exogenously the amount of schooling she receives from x to x' would be

$$g_T(x', a) - g_T(x, a).$$

A very natural parameter for a decision maker to be interested in is some form of average across a heterogeneous population. Since X_t and A_t are correlated, the natural question is which type of average one would like to consider. In this paper, we advocate the use of $F_{A_t|X_t}$ as a weighting scheme. The reason is simple, and easily understood in our example. Suppose $X_t = x$ corresponds to 4 years of university, and the question is to determine effect of the introduction of ninth semester (i.e., $x' = x + 0.5$) as a policy measure. In this case it does not make sense to weigh with the unconditional distribution of A_t as there are many individuals, presumably frequently with lower levels of ability, who never complete four years of college. Hence, it is natural to average the causal effect with the weighting scheme $F_{A_t|X_t}(.|x)$, since this is really the subpopulation primarily affected by the policy measure of changing X_t exogenously from x to x'. This corresponds, in period T, to the effect

$$\int (g_T(x', a) - g_T(x, a)) F_{A_T|X_T}(da; x) = E\left[g_T(x', A_T) - g_T(x, A_T)|X_T = x\right].$$

Very analogous arguments apply to the marginal effect $\Delta_j^{AME}(x)$. The analysis of this effect has a long history in econometrics, starting with the seminal work by Chamberlain (1982, 1984), who called this marginal effect the local average response. Important references are Altonji & Matzkin (2005), Wooldridge (2005), Graham & Powell (2012), Hoderlein & White (2012) and Chernozhukov et al. (2013) in the panel data literature, and Hoderlein & Mammen (2007), Imbens & Newey (2009), Schennach et al. (2012) in the IV literature.

An interesting consequence of obtaining $\Delta_j^{AME}(x)(x)$ is that

$$\int \Delta_j^{AME}(x)(x) f_X(x) dx = E\left[\frac{\partial g_T}{\partial x}(X_T, A_T)\right]$$

provides the overall average partial effect (see Chamberlain, 1984). This parameter corresponds to the thought experiment of increasing schooling marginally across the entire population, and averaging the effect across the various levels of eduction and ability.

The quantile effects $\Delta^{QTT}(\tau, x, x')$ and $\Delta_j^{QME}(\tau, x)$ provide causal effects on the counterfactual marginal distributions. This is different from obtaining the distribution of causal effects, but both effects are widely analyzed, see Abadie et al. (2002) and Chernozhukov et al. (2013), amongst many others.

Finally, we consider all effects for period T as we believe there are the most natural to compute in general. However, the result of Theorem 1 below implies that we can actually identify similar effects at any date.

2.2 Assumptions

The broad idea for identifying these parameters is to restrict the way time affects both observed and unobserved variables. More specifically, we impose hereafter three restrictions. The first is a stationarity condition on the observed and unobserved determinants of the outcome. The second restricts the way time is affecting the outcome itself. The third restricts the way the distribution of X_t moves over time. We discuss them in turns, using the notations $\mathbf{F}_t(x) = (F_{X_{1t}}(x_1), ..., F_{X_{kt}}(x_k))$ for any $x = (x_1, ..., x_k)$, $V_t = \mathbf{F}_t(X_t)$ and $\mathcal{V}_t = \text{supp}(V_t)$. The first assumption is:

Assumption 1. *The distribution of X_t is absolutely continuous with a convex support, and for all $(s,t) \in \{1, ..., T\}^2$ and almost all $v \in \mathcal{V}_T$,*

$$A_t | V_t = v \sim A_s | V_s = v.$$

To fix ideas, consider the returns to education example, and suppose that A_t comprises an ability term correlated with education, and an idiosyncratic term independent of ability and education. Assumption 1 means in this context that the distribution of ability conditional on a given rank in the distribution of education remains stable over time.

This stationarity condition is different from the condition

$$A_s | X_1, ..., X_T \sim A_t | X_1, ..., X_T, \tag{2.1}$$

commonly assumed in panel data (see, e.g., Manski, 1987, Honore, 1992, Graham & Powell, 2012 and Chernozhukov et al., 2013). To understand the differences between the two, consider two polar cases. In the first, endogeneity stems from a simultaneity issue while $(A_t, V_t)_{t=1...T}$ are i.i.d. If so, Assumption 1 is satisfied. On the other hand, (2.1) does not hold, unless A_t is independent of V_t, because the distribution of A_s conditional on $(X_1, ..., X_T)$ is a function of X_s only, i.e., $f_{A_s|X_1,...,X_T}(a|x_1, ..., x_T) = f_{A_s|X_s}(a|x_s)$, while the conditional distribution A_t is a function of X_t only, and they do generally not coincide if $x_s \neq x_t$. Assuming (A_s, V_s) independent of (A_t, V_t) is of course often unrealistic, but the same conclusion would hold with, say, a vector autoregressive structure. In the second case, $A_t = (A, U_t)$ where A is a fixed effect potentially correlated with $X_1, ..., X_T$ and $(U_t)_t$ are i.i.d. idiosyncratic shocks that are independent of $(A, X_1, ..., X_T)$. In this case, the condition (2.1) is always satisfied. On the other hand, Assumption 1 holds only under a special correlation structure between A and $(X_1, ..., X_T)$: $A|V_t = v \sim A|V_s = v$, which for instance imposes $Cov(A, V_t) = Cov(A, V_s),\ s \neq t$. While this still allows for arbitrary contemporaneous correlation between A and V_t, respectively V_s, it limits the time evolution of this covariance. It is this type of time invariance of the correlated unobservables that an applied researcher has to check, and, if adopted, defend.

This time invariance is somewhat mitigated by the fact that we allow for the function g_t to vary with time. To see this, let us first state the extent to which we allow for time dependence formally:

Assumption 2. *For all $t \in \{1, ..., T\}$, $g_t = m_t \circ g$, where m_t is strictly increasing. Without loss of generality, we let $m_T(y) = y$ for all $y \in supp(Y_T)$.*

Assumption 2 generalizes the standard translation model $m_t(u) = \delta_t + u$ to allow for heterogeneous effects of time. Allowing for the effect of time on the structural relationship seems quite important. For instance, in the returns to education example, the effect of education on wage may vary according to the state of the business cycle. Our specification allows for these macroeconomic shocks to have heterogeneous effects on individuals. To understand the extent to which is the case, think of $\widetilde{Y}_t = g(X_t, A_t)$ as the latent, long run wage which is free of seasonal or business cycle effects. Then, our specification allows in particular for the effect of

an economic downturn on lower $\widetilde{Y}$ individuals to be stronger (or less strong). But it still places restriction on the way time affects the outcome. In particular, while allowing for contractions and expansions of the wage distribution, we cannot assume that the effect of time is such that the ordering of any two individuals is reversed if neither their observables nor unobservables change over time.

On the positive side, this assumption allows to overcome some of the restrictiveness of the fact that $\text{Cov}(A, V_t) = \text{Cov}(A, V_s)$, $s \neq t$. To understand this, suppose that the structural model is given by $Y_t = \delta_t(\alpha A + \beta h_1(X_t) + \gamma A h_2(X_t)) = \alpha_t A + \beta_t k_1(V_t) + \gamma_t A k_2(V_t)$, where $h_j, k_j, j = 1, 2$ are increasing transformations, and $\gamma_t = \delta_t \gamma$. This specification allows for some interaction effect between between A and V_t, with a time heterogeneous impact on Y_t. In the example of returns to eduction, even if the correlation between ranks in the education distribution and unobserved ability is time invariant, the effect of having high education combined with high ability could be higher in, say, an economic upswing.

Finally, our last assumption concerns the independent variation that identifies the model. Given the highly nonlinear setup we are considering, it comes in form of a distributional assumption. It allows for the construction of a "control group" that identifies the effect of time on the outcome (the function m_t), analogously to the DiD literature.

Assumption 3. *For all $t \in \{1, ..., T\}$, there exists $x_t^* \in \mathbb{R}^k$ such that $\mathbf{F}_t(x_t^*) = \mathbf{F}_T(x_t^*) \in (0, 1)^k$.*

Several remarks are in order: first, Assumption 3 is directly testable in the data. It allows for any change in the distribution of X_t, provided that there is a crossing between the cumulative distribution function of X_{jT} and X_{jt}, for all $j \in \{1, ..., k\}$ and $t \geq 2$.[4] Roughly speaking, this means that time has an heterogenous effect on the distribution of X_t. It fails to hold in the pure location model $X_t = \gamma_t + B_t$, where the distribution of B_t is stationary with support $\mathbb{R}^k$. On the other hand, it holds in the location-scale model $X_t = \gamma_t + \Sigma_t B_t$ if Σ_t is diagonal with

[4]We assume for simplicity crossings between X_T and the other cdf, but actually, $T - 1$ crossings are fine provided that we can "relate" them to each other, for instance if the cdf of X_t crosses the one of X_{t+1} for $1 \leq t < T$. With only one crossing between $\mathbf{F}_s$ and $\mathbf{F}_t$, we can still identify the effect of time between these two periods ($m_t \circ m_s^{-1}$) and then identify some treatment effects.

diagonal terms σ_{jt} that are distinct at each time period. In such a case x_t^* is unique and satisfies

$$x_t^* = \left(\frac{\gamma_t - \gamma_T}{\sigma_{1T} - \sigma_{1t}}, ..., \frac{\gamma_t - \gamma_T}{\sigma_{kT} - \sigma_{kt}} \right).$$

Note that if $\mathbf{F}_t$ remains constant with t, Assumption 3 is satisfied but we identify only trivial parameters such as $\Delta^{ATT}(x, x)$. Nontrivial parameters are identified only when $\mathbf{F}_t$ changes with t.

Identification with repeated cross sections thus requires variation in the distribution of the (continuous) treatment over time. This contrasts with the variation in the individual value of the treatment over time that is typically required with panel data, the fixed effects absorbing any variable that is constant across time. The distribution of X_t can move over time even if X_t is constant for each individual, provided new generations are involved at date t compared to date s. Our application below is an example of such a situation. On the other hand, compared to panel data, we do not identify anything, apart from the time effect m_t, when the treatment changes at an individual level but the distribution of X_t remains constant over time. This is one different aspect of our identification strategy from panel data based strategies.

3 Identification results

3.1 Point Identified Effects

The first idea that drives our results is that the effect of time can be obtained using individuals for which $X_T = X_t = x_t^*$. These individuals, though possibly different across time periods, have under Assumption 1 the same distribution of unobservables and the same value of the treatment. For them, differences between Y_T and Y_t can only stem from the effect of time itself.

This is the reason why we call them the "control group". Formally,

$$\begin{aligned} P\left(Y_T \leq y | X_T = x_t^*\right) &\overset{A.2}{=} P\left(g(x_t^*, A_T) \leq y | V_T = \mathbf{F}_T(x_t^*)\right) \\ &\overset{A.1}{=} P\left(g(x_t^*, A_t) \leq y | V_t = \mathbf{F}_T(x_t^*)\right) \\ &\overset{A.3}{=} P\left(g(x_t^*, A_t) \leq y | V_t = \mathbf{F}_t(x_t^*)\right) \\ &\overset{A.2}{=} P\left(m_t \circ g(x_t^*, A_t) \leq m_t(y) | X_t = x_t^*\right) \\ &\overset{A.2}{=} P\left(Y_t \leq m_t(y) | X_t = x_t^*\right), \end{aligned}$$

the first equality following because m_T is the identity function. As a result, m_t is identified by

$$m_t(y) = F^{-1}_{Y_t|X_t}\left[F_{Y_T|X_T}(y|x_t^*)|x_t^*\right].$$

This transformation is similar in spirit to a transformation in Athey & Imbens (2006). However, it differs in the crucial aspect that we are not exogenously given a treatment and, in particular, a control group, but endogenously obtain the control group through our assumptions. We conjecture that there are more general ways of constructing a control group, in particular if there are more than two time periods available, but we leave this issue for future research.

Next, consider the transformed outcome $\widetilde{Y}_t = m_t^{-1}(Y_t)$, which is purged of the influence of time in the sense that by Assumption 1, time has no direct effect on $\widetilde{Y}_t$. In other words, variations in X_t provided by time are now exogenous in the sense that they do not affect the distribution of unobservables. Time can thus be considered to act like an instrument. As already mentioned, implicitly similar ideas have been used in the panel data literature, though using different and non-nested assumptions (see, e.g., Manski, 1987, Honore, 1992, Graham & Powell, 2012, Hoderlein & White, 2012, Chernozhukov et al., 2013), which all consider the effect of time variations on X_t and Y_t.

To proceed with the identification of our model, let $q_t(x)$ denote the value of X_t (say, income in period t) for an individual at the same rank as another individual whose period T income is $X_T = x$, $x \neq x^*$. Formally, let $q_{jt} = F^{-1}_{X_{jt}} \circ F_{X_{jT}}$ for $j \in \{1, ..., k\}$ and

$$q_t(x) = (q_{1t}(x_1), ..., q_{kt}(x_k)).$$

We have then that

$$\begin{aligned} E\left[\widetilde{Y}_t | X_t = q_t(x)\right] &= E\left[g(q_t(x), A_t) | V_t = \mathbf{F}_T(x)\right] \\ &\overset{A.1}{=} E\left[g(q_t(x), A_T) | V_T = \mathbf{F}_T(x)\right] \\ &= E\left[g(q_t(x), A_T) | X_T = x\right]. \end{aligned}$$

The latter is the mean counterfactual outcome at period T for individuals with $X_T = x$ if X_T was moved exogenously to $q_t(x)$. We can therefore identify $\Delta^{ATT}(x, q_t(x))$, the average effect of moving X_T from their initial value x to $q_t(x)$, by

$$\begin{aligned} \Delta^{ATT}(x, q_t(x)) &\overset{A.2}{=} E\left[g(q_t(x), A_T) - g(x, A_T) | X_T = x\right] \\ &= E\left[\widetilde{Y}_t | X_t = q_t(x)\right] - E\left[\widetilde{Y}_T | X_T = x\right], \end{aligned}$$

where the first equality comes from the normalization in assumption 2 implies that $g_T = m_T \circ g = g$, and hence $ATT(x, x') \equiv E\left[g_T(x', A_T) - g_T(x, A_T) | X_T = x\right] = E\left[g(x', A_T) - g(x, A_T) | X_T = x\right]$. This means that we can obtain $\Delta^{ATT}(x, x')$ for any pair $x, x' = q_t(x)$, and $x \neq x^*$. Note that we cannot point identify $\Delta^{ATT}(x, x')$ for $x' \neq q_t(x)$, but we will show in the following subsection that we can at least set identify these parameters under plausible curvature restrictions. Also, we cannot identify any effect $\Delta^{ATT}(\xi, \xi')$ with $\xi' \neq \xi$ if $\mathbf{F}_t(\xi) - \mathbf{F}_T(\xi) = 0$. As mentioned above, we need the distribution of X_t to change with time.

When X_T is multivariate, it may be difficult to interpret $\Delta^{ATT}(x, q_t(x))$ because it corresponds to the effect of a change of potentially all components of X_T. However, still using the crossing points, we can identify some partial effects. To see this, consider ${}_j\widetilde{x}_t = (x^*_{1t}, ..., x^*_{j-1t}, x_j, x^*_{j+1t}, ..., x^*_{kt})$ for some $x_j \neq x^*_{jt}$. Then, by definition of x^*_t,

$$q_t({}_j\widetilde{x}_t) = \left(x^*_{1t}, ..., x^*_{j-1t}, q_{jt}(x_j), x^*_{j+1t}, ..., x^*_{kt}\right).$$

This means that $\Delta^{ATT}({}_j\widetilde{x}_t, q_t({}_j\widetilde{x}_t))$ corresponds to the average partial effect of exogenously shifting X_{jT} from x_j to $q_{jt}(x_j)$.

For people at the crossing points x^*_t, we do not learn anything from the above reasoning, because $\Delta^{ATT}(x^*_t, q_t(x^*_t)) = \Delta^{ATT}(x^*_t, x^*_t) = 0$ by construction. On the other hand, under mild

regularity condition (see Assumption 4 below), we can identify the average marginal effects for this population provided that q_t differs from the identity function in the neighborhood of x_t^*. The intuition behind the latter result is that we can find values x close to x_t^* and such that $q_t(x) - x$ is close to zero, but not exactly zero. Then, if X_t is univariate (the multivariate case can be handled similarly),

$$\frac{g(q_t(x), A_T) - g(x, A_T)}{q_t(x) - x} \simeq \frac{\partial g}{\partial x}(x_t^*, A_{1t}). \tag{3.1}$$

Moreover, if the conditional distribution of A_T is regular, conditioning on $X_T = x$ becomes the same as conditioning on $X_T = x_t^*$, so that

$$\frac{\Delta^{ATT}(x)}{q_t(x) - x} \simeq \Delta_1^{AME}(x)(x_t^*).$$

Formally, identification of the marginal effect is achieved on the set $\mathcal{X}_0$ defined by

$$\begin{aligned}\mathcal{X}_0 \;=\; \Big\{ & x \in \mathbb{R}^k : \exists (t, (x_n)_{n\in\mathbb{N}}) \in \{1, ..., T-1\} \times \left(\mathbb{R}^k\right)^{\mathbb{N}} : \; q_t(x) = x, \lim_{n\to\infty} x_n = x \\ & \text{and } q_{jt}(x_{jn}) \neq x_{jn} \text{ for all } j = 1, ..., k \Big\}.\end{aligned}$$

$\mathcal{X}_0$ is the union of fixed points of $q_2, ..., q_T$, once we exclude points x^* such that in their neighborhood, $q_{jt}(x_j) = x_j$ for some $j \in \{1, ..., k\}$. See Figure 1 for an illustration in the univariate case. To make the preceding argument rigorous, the following technical conditions are also required.

Assumption 4. *(Regularity conditions) For all points $x_t^* \in \mathcal{X}_0$, there exists a neighborhood $\mathcal{N}$ such that:*

(i) almost surely, $x \mapsto g(x, A_T)$ is continuously differentiable on $\mathcal{N}$.

(ii) the distribution of A_T conditional on X_T is continuous with respect to the Lebesgue measure and $x \mapsto f_{A_T|X_T}(a|x)$ is continuous at x_t^*.

(iii) For all $j \in \{1, ..., k\}$, $\int |\sup_{x'\in\mathcal{N}} \partial g/\partial x_j(x', a)| \left|\sup_{x'\in\mathcal{N}} f_{A_T|X_T}(a|x')\right| da < \infty$.

(iv) For all $x \in \mathcal{N}$ and $j \in \{1, ..., k\}$, $x'^{-1}_{g(x', A_T)|X_T}(\tau|x)$ is differentiable at x_t^*. $(x, x') \mapsto \frac{\partial F^{-1}_{g(x'', A_T)|X_T}(\tau|x)}{\partial x_j''}|x'' = x'$ is continuous on $\mathcal{N}^2$.

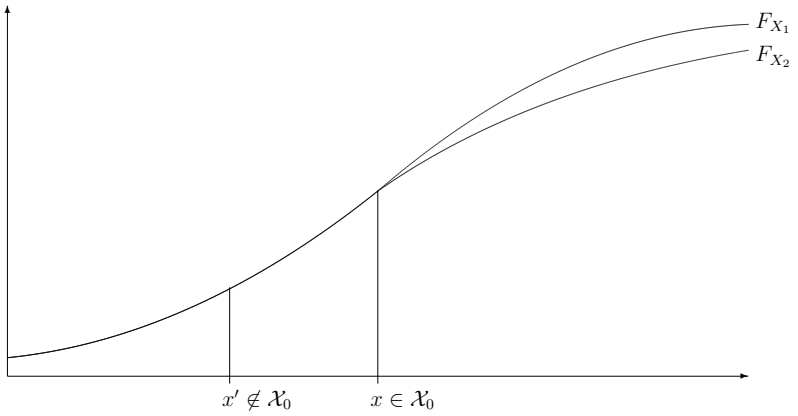


Figure 1: Example of points belonging or not to $\mathcal{X}_0$

Finally, we can apply the same reasoning to the quantile function. We can recover $F^{-1}_{g_T(q_t(x),A_T)|X_T}(\tau|x)$ by $F^{-1}_{\widetilde{Y}_t|X_t}(\tau|q_t(x))$, which implies that $\Delta^{QTT}(\tau, x, q_t(x))$ is identified. We also identify $\Delta^{QME}_j(\tau, x^*_t)$ by a similar argument as above.

Theorem 1 summarizes all findings of this section:

Theorem 1. *Under Assumptions 1-3, we identify, for all $x \in supp(X_T)$, $\tau \in (0,1)$ and $t \in \{1, ..., T-1\}$, the functions m_t and the average and quantile treatment effects $\Delta^{ATT}(x, q_t(x))$ and $\Delta^{QTT}(\tau, x, q_t(x))$. If Assumption 4 holds as well, we also identify $\Delta^{AME}_j(x)(x^*_t)$ and $\Delta^{QME}_j(\tau, x^*_t)$ for all $x^*_t \in \mathcal{X}_0$ and all $j \in \{1, ..., k\}$.*

3.2 Partial Identification of Other Treatment Effects

Theorem 1 implies that we can point identify some but not all average treatment effects $\Delta^{ATT}(x, x')$. Similarly, we point identify the average marginal effects only at some particular points. We show in this subsection that with three or more periods of observation and an univariate X_t, we can get bounds for many other points under a weak local curvature condition.[5] Let us consider the average marginal effect for instance. The idea is that if $g(., A_t)$ is locally

[5]The reasoning developed here also works when X_t is multivariate, but only applies to $\Delta^{ATT}({}_j\widetilde{x}^*_t, {}_j\widetilde{x}^{*\prime}_t)$, where ${}_j\widetilde{x}^*_t$ is defined as before and ${}_j\widetilde{x}^{*\prime}_t$ is similar to ${}_j\widetilde{x}^*_t$, except that its j-th component is x'_j instead of x_j.

concave (say) and $q_t(x) < x$, then $\frac{g(q_t(x),A_T)-g(x,A_T)}{q_t(x)-x}$ is an upper bound of $\frac{\partial g}{\partial x}(x, A_T)$. Similarly, if $q_s(x) > x$, then $\frac{g(q_s(x),A_T)-g(x,A_T)}{q_s(x)-x}$ is a lower bound for $\frac{\partial g}{\partial x}(x, A_T)$ (see Figure 2). By integrating over A_T, we can therefore bound $\Delta_j^{AME}(x)$ by some appropriate $\Delta^{ATT}(x, q_t(x))/(q_t(x) - x)$. The same idea can be used to obtain bounds $\Delta^{ATT}(x, x')$ for $x' \notin \{q_t(x), t = 2...T\}$.

The above argument works even if we do not know a priori whether g is concave or convex. Using the minimum and the maximum of the local discrete treatment effect will be sufficient to obtain bounds, provided that g is locally concave or locally convex around x. We therefore adopt henceforth the following definition.

Definition 1. *g is locally concave or convex on $[\widetilde{x}, \widetilde{x}']$ if $x \mapsto g(x, A_t)$ is twice differentiable*

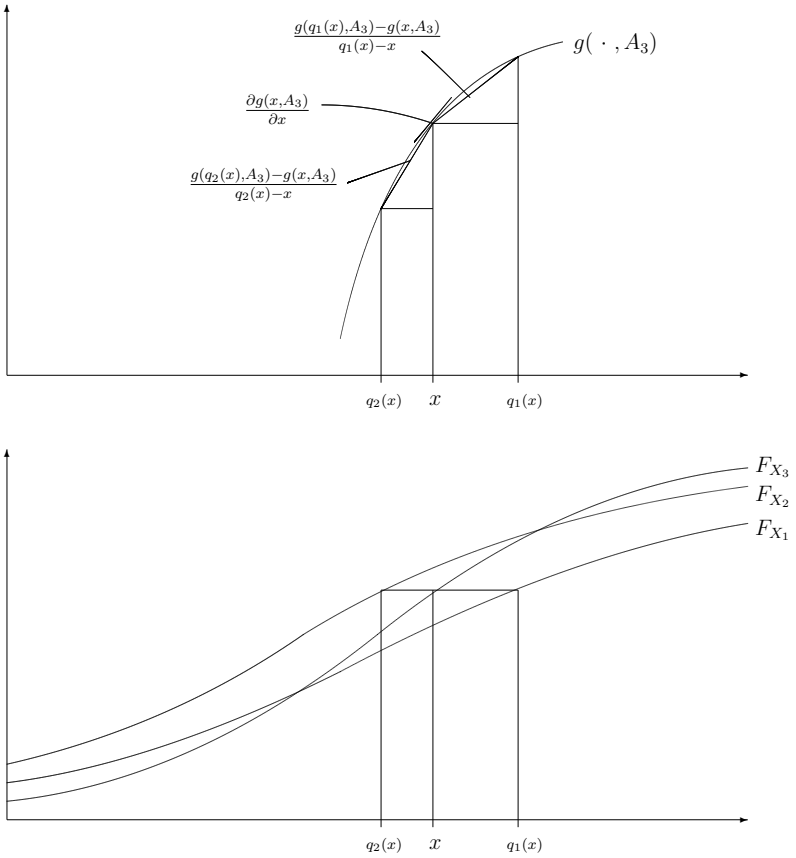


Figure 2: Bounds under the local curvature condition

and

$$\frac{\partial^2 g}{\partial x^2}(x, A_t) \leq 0 \ \forall x \in [\widetilde{x}, \widetilde{x}'] \text{ a.s. or } \frac{\partial^2 g}{\partial x^2}(x, A_t) \geq 0 \ \forall x \in [\widetilde{x}, \widetilde{x}'] \text{ a.s.}$$

Let us introduce, for all $(x, x') \in \text{supp}(X_T)$, $(\underline{x}_T(x'), \overline{x}_T(x'))$ defined by

$$\begin{aligned} \underline{x}_T(x') &= \max\{q_t(x), t \in \{1, ..., T-1\} : \ q_t(x) \neq x \text{ and } q_t(x) < x'\}, \\ \overline{x}_T(x') &= \min\{q_t(x), t \in \{1, ..., T-1\} : \ q_t(x) \neq x \text{ and } q_t(x) > x'\}. \end{aligned}$$

If the sets are empty we let $\underline{x}_T(x') = -\infty$ and $\overline{x}_T(x') = +\infty$.

Theorem 2. *If $k = 1$ and under Assumptions 1-3,*

- for any $x < x'$, if g is locally concave or convex on $[\min(x, \underline{x}_T(x')), \overline{x}_T(x')]$, then

$$\begin{aligned} & (x' - x) \min\left\{ \frac{\Delta^{ATT}(x, \underline{x}_T(x'))}{\underline{x}_T(x') - x}, \frac{\Delta^{ATT}(x, \overline{x}_T(x'))}{\overline{x}_T(x') - x} \right\} \leq \Delta^{ATT}(x, x') \\ \leq \ & (x' - x) \max\left\{ \frac{\Delta^{ATT}(x, \underline{x}_T(x'))}{\underline{x}_T(x') - x}, \frac{\Delta^{ATT}(x, \overline{x}_T(x'))}{\overline{x}_T(x') - x} \right\}. \end{aligned}$$

- If g is locally concave or convex on $[\underline{x}_T(x), \overline{x}_T(x)]$, then

$$\min\left\{ \frac{\Delta^{ATT}(x, \underline{x}_T(x))}{\underline{x}_T(x) - x}, \frac{\Delta^{ATT}(x, \overline{x}_T(x))}{\overline{x}_T(x) - x} \right\} \leq \Delta_1^{AME}(x) \leq \max\left\{ \frac{\Delta^{ATT}(x, \underline{x}_T(x))}{\underline{x}_T(x) - x}, \frac{\Delta^{ATT}(x, \overline{x}_T(x))}{\overline{x}_T(x) - x} \right\}.$$

where the bounds are understood to be infinite when either $\underline{x}_T(x') = -\infty$ or $\overline{x}_T(x') = +\infty$ (whether $x' > x$ or $x' = x$).

Both bounds are finite provided that there exists t, t' such that $q_t(x) < x < q_{1t'}(x)$, which implies that $T \geq 3$. More generally, the bounds improve with T, because $(\underline{x}_T(x'))_{T \in \mathbb{N}}$ and $(\overline{x}_T(x'))_{T \in \mathbb{N}}$ are by construction increasing and decreasing, respectively. The local curvature condition becomes less and less restrictive as T increases, because the interval on which g has to satisfy this condition decreases. It seems particularly credible, if $q_t(x) \mapsto \Delta(x, q_t(x))/(q_t(x) - x)$ is monotonic, because such a pattern is implied by global concavity or global convexity.

To illustrate Theorem 2, we consider the following example:

$$\begin{aligned} Y_t &= 1 - \exp(-0.5(\delta_t + X_t + A_t)) \\ X_t &= \mu_t + \sigma_t \Phi^{-1}(V_t), \end{aligned}$$

where $V_t \sim U[0,1]$ and $A_t | V_t \sim \mathcal{N}(V_t, 1)$. We also suppose that

$$\begin{aligned} \mu_T = 2.5, &\qquad \mu_t \sim \mathcal{N}(\mu_T, 1) \text{ for } t > 1, \\ \sigma_T = 1, &\qquad \sigma_t \sim \chi^2(1) \text{ for } t < T, \\ \delta_T = 0, &\qquad \delta_t \sim \mathcal{N}(0,1) \text{ for } t < T. \end{aligned}$$

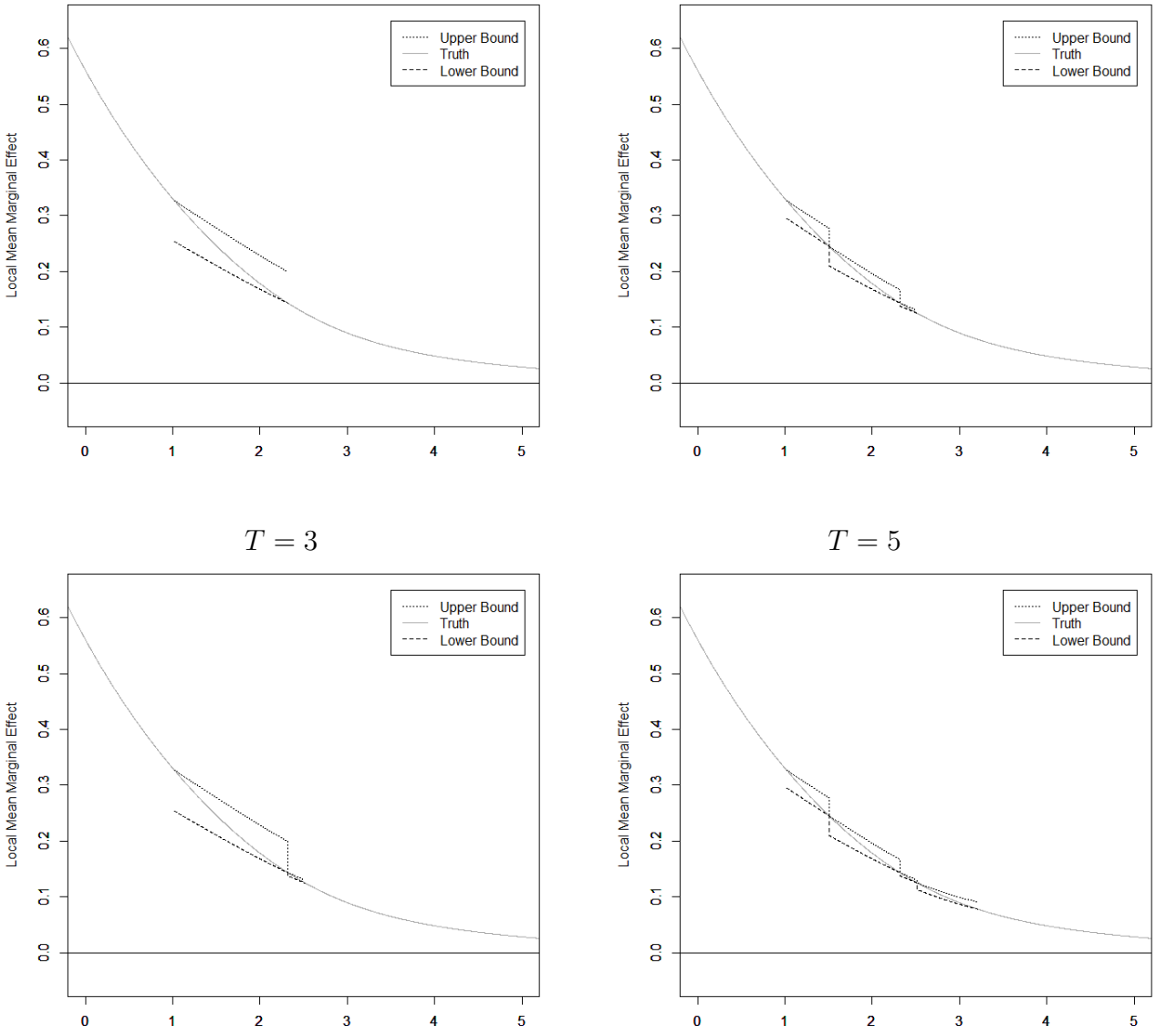
In this example, Assumptions 1, 2 (with $m_t(y) = 1 - \exp(-0.5\delta_t)(1-y)$) and 3 are satisfied, the latter because $\sigma_t \neq \sigma_T$ almost surely. The local curvature condition also holds, since $u \mapsto 1 - \exp(-0.5u)$ is concave. Figure 3 displays the bounds on $\Delta_1^{AME}(x)(x)$ for $T = 3, 4, 5$ and 6. Note that the bounds coincide for $T-1$ points. This simply reflects our previous point identification result. Each $\mathbf{F}_t$ crosses once $\mathbf{F}_T$ and each at a different point. By Theorem 1, point identification is achieved at these $T-1$ crossing points. We also see that in the interval where we get finite bounds, that is to say the interval for which $-\infty < \underline{x}_T(x) < \overline{x}_T(x) < \infty$, the bounds are quite informative even with $T = 3$. Figure 3 also shows that as T increases, both the bounds shrink and the interval on which we get finite bounds increase. For $T = 6$, we get informative bounds for $x \in [1,\ 3.85]$, which corresponds roughly to 85% of the population. This means that we could also obtain finite bounds for the average partial effect for this large fraction of the total population.

3.3 Point Identification with Exogenous Covariates

We consider here the case where exogenous covariates Z_t also affect Y_t, so that the model now writes

$$Y_t = g_t(X_t, Z_t, A_t) \qquad t = 1, ..., T. \tag{3.2}$$

We still focus on the effect of X_t hereafter. In this case, the preceding analysis can be conducted conditionally on Z_t. We briefly discuss this extension here, by considering only the discrete



$T = 3$ $T = 5$

$T = 4$ $T = 6$

Figure 3: Example of bounds on $\Delta_1^{AME}(x)$ for different values of x and $T = 3, 4, 5$ and 6.

average and quantile effects

$$\begin{aligned}\Delta^{ATT}(x,x',z) &\equiv E\left[g_T(x',z,A_T) - g_T(x,z,A_T)|X_T = x, Z_T = z\right], \\ \Delta^{QTT}(\tau,x,x',z) &\equiv F^{-1}_{g_T(x',z,A_T)|X_T,Z_T}(\tau|x,z) - F^{-1}_{g_T(x,z,A_T)|X_T,Z_T}(\tau|x,z).\end{aligned}$$

The marginal effects can be handled similarly.

We first restate our previous conditions in this context. The rank variable is now defined conditionally on Z_t, $V_t = \mathbf{F}_{t|Z_t}(X_t)$ with

$$\mathbf{F}_{t|Z_t}(x) = \left(F_{X_{1t}|Z_t}(X_{1t}|Z_t), ..., F_{X_{kt}|Z_t}(X_{kt}|Z_t)\right).$$

Assumption 1'. *The conditional distributions of $X_t|Z_t = z$ is absolutely continuous with a convex support, $supp((V_t, Z_t))$ does not depend on t and for all $(s,t) \in \{1,...,T\}^2$ and almost all $(v,z) \in supp(V_t, Z_t)$,*

$$A_s|V_s = v, Z_s = z \ \sim \ A_t|V_t = v, Z_t = z.$$

Next, we consider two versions of Assumptions 2 and 3. The trade-off between these two versions is basically between the generality of the model and data requirement. In the first version, we allow for more general time effects but the corresponding crossing condition is more demanding, because we should observe a crossing point for each value of z.

Assumption 2'. *We have either*
(i) for all t, $g_t(X_t, Z_t, A_t) = m_t(Z_t, g(X_t, Z_t, A_t))$, where $m_t(Z_t, .)$ is strictly increasing. Without loss of generality, we let $m_T(z,y) = y$ for all $(y,z) \in supp((Y_T, Z_T))$;
or (ii) for all t, $g_t(X_t, Z_t, A_t) = m_t(g(X_t, Z_t, A_t))$, where m_t is strictly increasing. Without loss of generality, we let $m_T(y) = y$ for all $y \in supp(Y_T)$.

Assumption 3'. *We have either:*
(i) for all $(z,t) \in supp(Z_T) \times \{1,...,T-1\}$, there exists $x_t^*(z)$ such that $\mathbf{F}_{T|Z_T}(x_t^*(z)|z) = \mathbf{F}_{t|Z_t}(x_t^*(z)|z) \in (0,1)$.
or (ii) for all t, there exists (x_t^*, z_t^*) such that $\mathbf{F}_{T|Z_T}(x_t^*|z_t^*) = \mathbf{F}_{t|Z_t}(x_t^*|z_t^*) \in (0,1)$.

These two sets of assumptions lead to the same results, which are qualitatively very similar to those of Theorem 1. The proof, which is very similar to the one of Theorem 1, is omitted.

Theorem 3. *Suppose that Assumption 1' and either Assumptions 2' (i) -3' (i) or Assumptions 2' (ii) -3' (ii) hold. Then, for almost all* $(x, z) \in supp((X_T, Z_T))$*, all* $\tau \in (0, 1)$ *and all* $t \in \{1, ..., T-1\}$*, the functions* m_t *and the average and quantile treatment effects* $\Delta^{ATT}(x, q_t(x), z)$ *and* $\Delta^{QTT}(\tau, x, q_t(x), z)$ *are identified.*

4 Extrapolation

As we have established in Theorem 1, we can point identify several treatment effect parameters under the relatively mild restrictions A1 to A3, but, as pointed out, these are by no means all possible causal effects one may be interested in. As we have seen in the previous section, many more treatment parameters can be set identified under often plausible curvature restrictions, in particular average marginal effects and effects of the form $\Delta^{ATT}(x, x')$. However, in any given application, these bounds may be wide, and to conduct inference may be cumbersome, or even impractical. Hence it makes sense to search for additional assumptions that yield point identification of average structural effects across the entire population, or even of all structural functions.

In the following, we propose two sets of non-nested restrictions that allow us to achieve point identification. The main restriction in the first approach constrains the heterogeneity term A_t to be scalar and have a monotonic effect on g. The main restriction in the second approach constrains X_t to have a linear or polynomial effect on Y_t. On the other hand, the coefficients on the explanatory variables are allowed to be random and correlated with X_t. These two approaches can be seen as providing a trade off. We either limit the extent of unobserved heterogeneity while allowing for flexibility in the way X_t enters the function or impose a functional form restriction on g but allow for a rich heterogeneity structure.

4.1 Scalar Monotonic Heterogeneity

In this subsection, we assume that heterogeneity is scalar and has a monotonic effect on the outcome. More formally:

Assumption 5. *$A_t \in \mathbb{R}$ and $g(X_t, .)$ is strictly increasing in its second argument.*

An example of model satisfying Assumption 5 is the linear quantile regression: $g(X_t, A_t) = X_t'\beta_{A_t}$, where $a \mapsto X_t'\beta_a$ is strictly increasing almost surely (i.e, there is comonotonicity). However, linearity is really not the essence here.

We also rely on the following technical restrictions:

Assumption 6. *(i) $X_t \in \mathbb{R}$ and its support $\mathcal{X} = [\underline{x}, \overline{x}]$ (with $-\infty \leq \underline{x} < \overline{x} \leq +\infty$) does not depend on t.*
(ii) A_t is uniformly distributed.
(iii) $(a, v) \mapsto F_{A_T|V_T}(a|v)$ is continuous on $(0, 1)^2$ and $a \mapsto F_{A_T|V_T}(a|v)$ is strictly increasing on $(0, 1)$ for all $v \in (0, 1)$.
(iv) $g(., .)$ is continuous on $\mathcal{X} \times (0, 1)$.
(v) q_t has a finite number of fixed points.

Under these additional conditions, we obtain

Theorem 4. *Under Assumptions 1-3, 5-6, m_t and g are identified.*

The proof relies on the observation that we have a triangular system

$$\begin{cases} \widetilde{Y}_t &= g(X_t, A_t) \\ X_t &= h(t, V_t) \end{cases}$$

where $h(t, v) = F^{-1}_{X_t}(v)$. This is a nonseparable triangular model where X_t is endogenous and t may be seen as an instrument. In this context, the usual exogeneity condition translates into time invariance of the distribution of (A_t, V_t). Because both $g(X_t, .)$ and $h(t, .)$ are strictly increasing, we can then use the identification results of D'Haultfoeuille & Février (2012) or Torgovitsky (2012). Note that under additional conditions, we could also obtain full identification when X_t is multivariate, using Theorem 5.2 of D'Haultfoeuille & Février (2012).

The reason why monotonicity makes a difference in our context is that we can then directly relate $g(q_t(x), a)$ with $g(x, a)$:

$$g(q_t(x), a) = Q_{q_t(x),x} \circ g(x, a),$$

where $Q_{q_t(x),x}$ is identified. This shows, as before, that $\Delta^{ATT}(x, q_t(x))$ is identified, but also that we can iterate, and relate $g(q_t \circ q_t(x), a)$ to $g(x, a)$, so that $\Delta^{ATT}(x, q_t \circ q_t(x))$ is identified as well. By repeating this argument, and using fixed points of q_t, we can show that the model is fully identified. Because the model is actually identified with $T = 2$, it may well be the case that identification is possible even without any fixed points when $T > 2$. This issue is left for future research.

It is instructive to relate Theorem 4 to results for nonlinear panel data models. The closest paper is the one of Evdokimov (2011), who considers the nonseparable model $Y_t = g_t(X_t, A_t)$ where A_t also satisfies Assumption 5 in his model. Compared to us, he imposes $A_t = U + \varepsilon_t$ and identification is achieved using the entire joint distribution of $(Y_1, X_1, ..., Y_T, X_T)$ and with $T \geq 3$. On the other hand, he does not impose any time invariance restriction on ε_t, nor does he put restriction on the effect of time on Y_t. Other related work is quantile regressions with "fixed effects". Rosen (2012) considers the model $Y_t = X_t'\beta_\tau + \alpha_\tau + \varepsilon_{t\tau}$, with $F^{-1}_{\varepsilon_{t\tau}|X_t,\alpha}(\tau|X_t, \alpha) = 0$ and where α_τ may be correlated with X_t. He shows that β_τ is not point identified for a fixed T. So it might seem surprising that with only $T = 2$, without panel data, and even without assuming linearity, identification can be achieved in such quantile regression models. Once more, the key difference between our setting and the one of Rosen (2012) is the time invariance condition that we impose on the error term.

4.2 Linear Correlated Random Coefficient Model

The second possible route for extrapolation is a random coefficient linear model of the form:

$$Y_t = \delta_t + A_{0t} + X_t'A_t, \tag{4.1}$$

where $A_t = (A_{1t}, ..., A_{kt})'$. Under this structure, the vector $E[A_T|X_T = x]$ is the vector of average marginal effects for individuals at x:

$$E[A_T|X_T = x] = (\Delta_1^{AME}(x), ..., \Delta_k^{AME}(x))'.$$

Moreover,

$$\Delta^{ATT}(x, q_t(x)) = (q_t(x) - x)' E[A_T|X_T = x].$$

Let us define the matrix $\mathbf{Q}(x)$ and the vector $\mathbf{\Delta}(x)$ as

$$\mathbf{Q}(x) = \begin{bmatrix} (q_1(x) - x)' \\ \vdots \\ (q_{T-1}(x) - x)' \end{bmatrix}, \quad \mathbf{\Delta}(x) = \begin{pmatrix} \Delta^{ATT}(x, q_1(x)) \\ \vdots \\ \Delta^{ATT}(x, q_{T-1}(x)) \end{pmatrix}.$$

If $\mathbf{Q}(x)$ is full column rank, we can identify $E[A_T|X_T = x]$ by

$$E[A_T|X_T = x] = (\mathbf{Q}(x)'\mathbf{Q}(x))^{-1} \mathbf{Q}(x)'\mathbf{\Delta}(x). \tag{4.2}$$

Apart from the vector of average marginal effects, we can then identify $\Delta^{ATT}(x, x')$, for any x', by

$$\Delta^{ATT}(x, x') = (x' - x)' E[A_T|X_T = x].$$

Note that the rank condition implies that $T - 1 \geq k$. It also implies that the distribution of X_t differs at each date, so that $q_s(x) \neq q_t(x)$. It makes sense that with several endogenous variables, more time variation on X_t is needed to identify causal effects.

Finally, if $\mathbf{Q}(X_T)$ is full rank almost surely, we point identify the vector of average marginal effect over the whole population, $\Delta^{AME} = (\Delta_1^{AME}, ..., \Delta_k^{AME})'$, by

$$\Delta^{AME} = E[A_{1T}] = E\left[(\mathbf{Q}(X_T)'\mathbf{Q}(X_T))^{-1} \mathbf{Q}(X_T)'\mathbf{\Delta}(X_T)\right].$$

We summarize these finding in the following theorem.

Theorem 5. *Under Assumptions 1-3 and Equation* (4.1)*, δ_t, $\Delta^{ATT}(x, x')$ and $\Delta_j^{AME}(x)$ are identified for all x such that $\mathbf{Q}(x)$ is full column rank, and for any x' and $j \in \{1, .., k\}$. If $\mathbf{Q}(X_T)$ is full column rank almost surely, Δ_j^{AME} is point identified as well for $j \in \{1, .., k\}$.*

Thus, we recover the same parameter as Graham & Powell (2012), who also consider a random coefficient linear model similar to (4.1). They obtain identification with panel data, relying on first-differencing. Compared to them, we rely on variations in the cdf of X_t rather than on individual variations. We rely on a different, non-nested, restriction on the distribution of the error term. In particular, for the same individual, $A_{1t} - A_{1s}$ could be correlated with X_t in our framework.

Apart from identification, Equation (4.2) implies that the linearity assumption can be testable when $T - 1 > k$, because the system of equation is overidentified. In the univariate case, for instance, Equation (4.2) implies

$$\frac{\Delta^{ATT}(x, q_s(x))}{q_s(x) - x} = \frac{\Delta^{ATT}(x, q_t(x))}{q_t(x) - x} \quad \forall s \neq t.$$

We can use additional periods to identify higher moments of the distribution of the coefficients. For instance, with $k = 1$, $V(A_{01}|X_T = x)$, $V(A_{1T}|X_T = x)$ and $\text{Cov}(A_{01}, A_{1T}|X_T = x)$ can be shown to be identified with $T = 3$ as soon as $x, q_{12}(x)$ and $q_{13}(x)$ are distinct. Alternatively (here still with $k = 1$ to simplify), we can identify the random coefficient polynomial model of order T

$$Y_t = \delta_t + A_{0t} + A_{1t}X_t + ... + A_{Tt}X_t^T. \tag{4.3}$$

Identification works the same way as before. At the end, we recover not only average marginal effect, but actually $E(A_{kt}|X_t = x)$ for all $k = 1...T$ and all x such that $(x, q_{12}(x), ..., q_{1T}(x))$ are all distinct. Identification of Model (4.3) was studied before by Florens et al. (2008), but with cross-sectional data and under assumptions that typically rule out discrete instruments (see also Heckman & Vytlacil (1998) for a study of the identification of Model (4.1) with instruments). In contrast, we allow here for a time effect and rely only on a finite number of time periods, which would be equivalent to a discrete instrument.

5 Application to the Effect of Maternal Age on Birth Weight

In most industrialized economies, there is a pronounced trend towards a later age at which a family is established. In particular, mother's childbearing age is steadily increasing. This phenomenon is well documented, and the individual and social costs have been extensively studied (see, e.g., Heffner, 2004, for a medical perspective and Hofferth, 1998), for an economic overview). In this section, we want to focus on one aspect that has received less attention, but which we feel is important: the ceteris paribus effects of mother's age at first birth, denoted X_t, on infant birth weight Y_t. The reason is that infant birth weight plays a very important role in the literature on health economics. In particular, infant birth weights are often thought of as playing a dual role, both as an output and as an input. On the one hand, birth weights are used as a measure of an outcome, namely infant health, that involve maternal behaviors and environments as primitive inputs (see, e.g., Rosenzweig & Schultz, 1983, Corman et al., 1987, Grossman & Joyce, 1990), Geronimus & Korenman, 1992, Rosenzweig & Wolpin, 1991, Rosenzweig & Wolpin, 1995, Evans & Ringel, 1999,Currie & Moretti, 2003, Camacho, 2008). On the other hand, birth weight is itself used as a measure for the initial input, the condition of an individual at birth, that eventually "produces" educational attainment, employment, and earnings as outcomes (see, e.g., Behrman et al., 1994, Currie & Hyson, 1999, Behrman & Rosenzweig, 2004, Black et al., 2007). Both aspects make understanding the causal determinants of a child's birth weight an issue of first order importance.

In most economic approaches, maternal age and the decision to give birth are made endogenously through life cycle plans made by forward-looking decision makers. The key econometric issue is to separate the physiological effects of mother's age from the effects of the economic environment that is associated with a mother's age. The standard panel data approaches will suffer from selection bias, because we cannot observe the same mother giving twice birth to the first child. Having a second child is usually thought of a dynamic decision that depends in parts on the outcome of the first birth, and is hence not a comparable decision. The first pregnancy may also have an effect on subsequent pregnancies. The standard instrumental variable

approach (using an exogenous variable that affects mother's age) extensively used in this literature also seems difficult to justify in our context, because age cannot be exogenously varied by common policy instruments, unlike other treatment variables such as smoking intensity or use of prenatal care. Instead, we will now argue that identification using time stationarity of the conditional distribution of unobservables is well suited for this problem.

In our empirical study, we use extracts from the repeated cross sections of the Natality Vital Statistics System of the National Center for Health Statistics from for years 1990-1999 and 2008. Following our notation, we let X_t and Y_t denote mother's age and infant birth weight, respectively, where t denotes the index for years 90-99 and 08. To exclude any dynamic optimization issues, we focus on the subsample consisting of first births. Table 1 shows summary statistics of (Y_t, X_t), for the repeated cross sections of first births. The displayed values are the sample means of age and birth weight, with the sample standard errors shown in parentheses. These aggregate statistics suggest two time trends – the mean infant birth weight is decreasing over time, and mean age of mother at first birth is increasing over time toward the turn of the century. This simple observation alone, however, does not allow us to conclude the causal effects of mother's age on infant birth weight due to omitted explanatory variables which may also follow certain time trends. Our approach controls for those omitted variables.

To examine whether our approach is applicable, we first consider the time shift of the

t	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2008
Y_t	3306	3298	3300	3288	3283	3285	3277	3275	3272	3271	3177
	(595)	(590)	(588)	(593)	(590)	(597)	(593)	(599)	(599)	(604)	(601)
X_t	24.2	24.2	24.3	24.4	24.4	24.5	24.5	24.7	24.8	24.7	23.8
	(5.5)	(5.6)	(5.7)	(5.8)	(5.9)	(6.0)	(6.0)	(6.0)	(6.1)	(6.1)	(5.6)
N_t	84708	83408	81629	81204	80708	80266	79341	78870	78745	79429	39809

Sources: Natality Vital Statistics System of the National Center for Health Statistics.

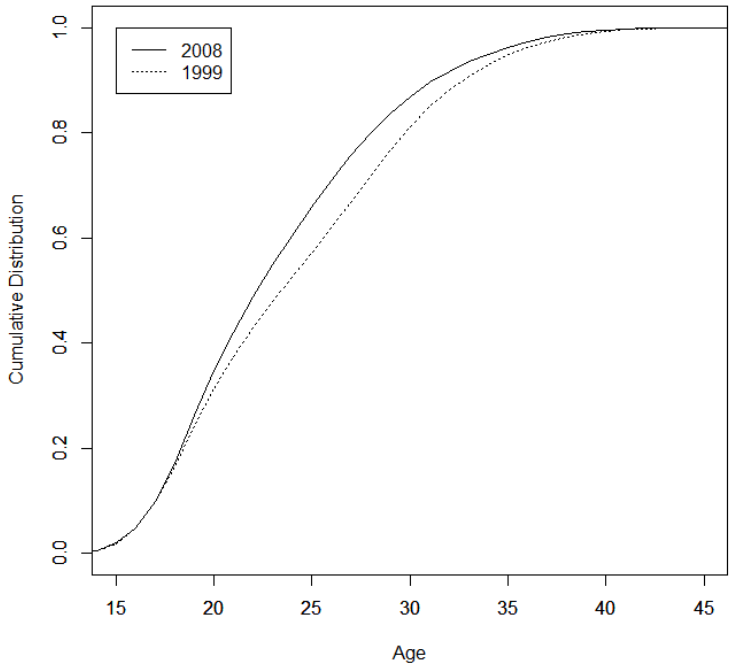
Notes: the displayed values are the sample means, with their sample standard errors shown in the parentheses. The bottom row shows the effective sample sizes of the repeated cross sections for each year.

Table 1: Summary statistics of the first births from 1990 to 1999 and 2008.

cumulative distribution of mother's age at first birth. We focus on the pair of most recent years in our data set, namely 1999 and 2008, but we will later use cross sections of the other years for robustness checks. Figure 4 shows the cdfs of maternal age at first birth in the years 1999 and 2008, i.e., X_{99} and X_{08}, smoothed by interpolation of the discretely supported X_t. Observe that they cross around Age $= 18$, while X_{99} first-order stochastically dominates X_{08} above age 20. Assumption 3 is therefore satisfied, and we can use $x^* = 18$ to form the temporal control group necessary to disentangle the time effects from the effect of age for those mothers older than 18.

Our approach also relies on the conditional stationarity assumption, which states that the distribution of unobservables A_t of all mothers who have rank $V_t = v$ in the first birth age distribution is the same as the distribution of A_s given $V_s = v$. To understand this, think of A_t as variable that captures the healthiness of the lifestyle. Endogeneity arises here because the distribution of healthy lifestyles of mothers who have their first child at 18 (think of teenage pregnancies) is likely to be different from the one of mothers who have their first child at 28,

Figure 4: Cumulative distribution of maternal age at first birth.

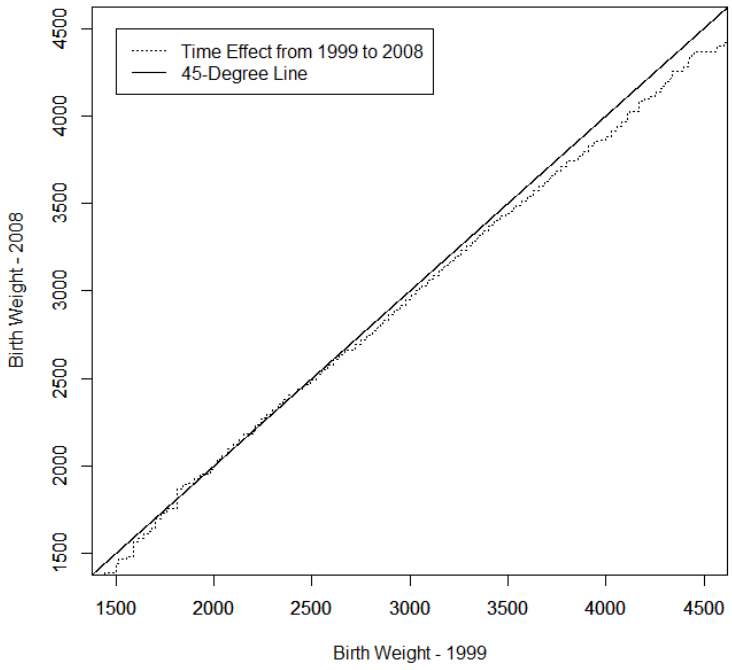


for instance. Then, our identifying assumption says that mothers at the third quartile (say) of first birth age in 1999 (which is 29), have the same distribution in terms of healthy lifestyles, as the mothers at the third quartile of first birth age in 2008 (which is 27). This is plausible if, loosely speaking, these two subpopulations are at the same position in the distribution of smoking and alcohol consumption, physical activity etc, as is likely the case given the close proximity of the two cdfs, and the not too distant time periods.

Given the crossing point $x^* = 18$ that we use to construct the "nontreated" control group, we use the two conditional cumulative distributions, $F_{Y_{99}|X_{99}=18}$ and $F_{Y_{08}|X_{08}=18}$ to identify the effect of time in isolation. Recall that the aggregate summary statistics in Table 1 shows the tendency that the mean birth weights decrease over time from 1999 to 2008 by nearly 100 grams. The same is true in terms of the conditional distribution of birth weight at first birth when mothers were 18 years old. As such, the time effects from 1999 to 2008, identified by $m_{08} \circ m_{99}^{-1} = F_{Y_{08}|X_{08}=18} \circ F^{-1}_{Y_{99}|X_{99}=18}$, is overall smaller than unity, and is illustrated by the Q-Q plot that appear on or below the 45° line shown in Figure 5. In words, for the control group the birth weight decreased. If we think of A_t as the healthiness of lifestyle, which as we argue below is plausibly time invariant for a given rank of the income distribution over such a short time span, this time effect reflects the fact that the structural relationship changes slightly over time. This probably is largely due to the increase of preterm birth rate, which rose by more than 20 percent between 1990 and 2008, and can be attributed to the increase in the medical ability to save lives of even very underweight preterm newborns. Besides, the effect of time appears to be heterogeneous. It is pronounced at both tails of the distribution but insignificant for intermediate quantiles. This shows the potential importance of not restricting oneself to a constant time effect.

Using the estimated time effects, we in turn estimate the marginal effects of interest. As we have $F_{X_{99}}(x) \approx F_{X_{08}}(x)$ for all $x < 21$ and all $x > 37$, it is only for $21 \leqslant x \leqslant 37$ that heterogeneous marginal effects can be obtained, which is of course a very large part of the population. Figure 6 (a) shows the average estimated effects $\Delta^{ATT}(x, q_{12}(x))/(q_{12}(x) - x)$ together with 95% bootstrap confidence intervals. Note that because $q_{12}(x)$ is close to x, these effects are likely to approximate well the average marginal effects $\Delta_j^{AME}(x)(x)$, and with slight

Figure 5: The time effects on birth weight in grams from 1999 to 2008.



abuse of language we refer to them as marginal effects hereafter. The mean estimates are negative throughout the effective domain of mother's age. Furthermore, these marginal effects are significantly negative at the five percent level for 28- through 37-year old mothers, implying that adverse physiological effects of aging on birth weight are likely to exist, at least starting with a maternal age at first birth of 30.

Note that this result accounts for the endogeneity of mother's age at first birth, which may for instance be the result of family planning by forward-looking individuals (crucial is only the conditional stationarity assumption, as discussed above). To see the degree to which this endogeneity would affect estimates of the marginal effects, if not properly taken care off, we also compute a naive cross section estimate of the marginal effects, assuming that mother's age at first birth were exogenous, i.e., the effect of an exogenous shift from x to x' is analyzed using $\mathrm{E}[Y_{08} \mid X_{08} = x'] - \mathrm{E}[Y_{08} \mid X_{08} = x]$, instead of $\mathrm{E}[F^{-1}_{Y_{08}|X_{08}=17} \circ F_{Y_{99}|X_{99}=17}(Y_{99}) \mid X_{08} = x'] - \mathrm{E}[Y_{08} \mid X_{08} = x]$. Figure 6 (b) shows these "naive" estimates. Compared with Figure 6 (a), which accounts for endogeneity, the mean estimates in Figure 6 (b) are much smaller in

absolute value, and are almost never significant. Furthermore, these naively estimated marginal effects are even significantly positive for ages between 20 and 26. One possible explanation of this outcome is that wealthier and more educated women, i.e., women with a healthier lifestyle on average, who may tend to have newborns with higher birth weights are likely to defer childbearing in these early ages. An estimator that does not account for endogeneity might wrongly take as positive marginal effects of mother's age on birth weight.

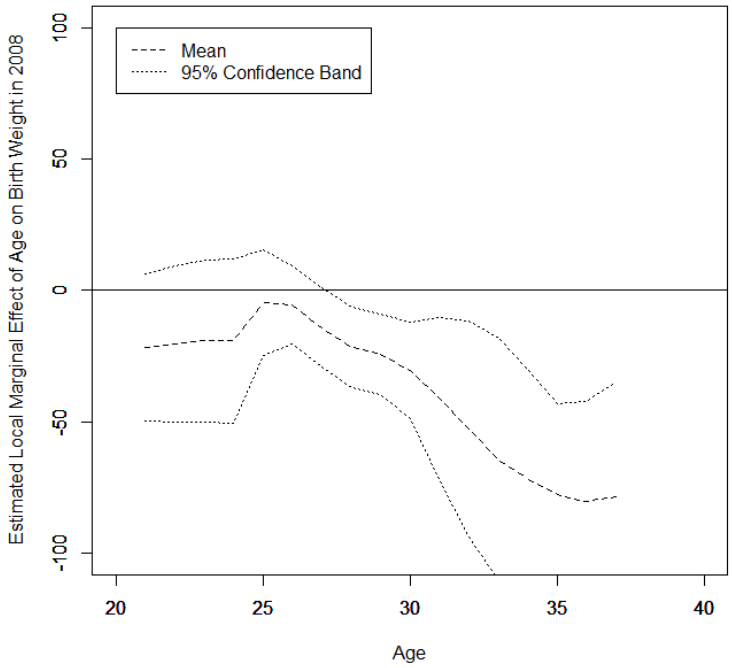
Figure 6 (a) shows our estimates of the marginal effects in 2008 using 1999 as a reference year. We next demonstrate that the qualitative patterns are similar even if we used other years as reference years. The left column of Figure 7 shows analogous estimates of the marginal effects in 2008 using (a) 1996, (b) 1997, and (c) 1998 as reference years, which are computed based on the crossing point $x^* = 18$. The mean estimates are negative almost everywhere throughout the effective domains, and are overall convex-shaped around $X_t = 30$. The negative estimates are robustly significant near or above $X_t = 30$ for all the reference years. There is no single year of age at which these estimates are significantly positive, in contrast to the implausible results we obtain from the naive estimates of Figure 6 (b). The right column of Figure 7 shows the counterparts of these naive estimates using (a) 1996, (b) 1997, and (c) 1998 as reference years. These robust results across multiple reference years support our claim.

Many economies have seen the ongoing trend of delaying marriage and first birth. Social costs of this tendency have been discussed extensively, but we found that there may be costs to the health of children, at least in as far as they are reflected in reduced birth weights. Based on our mean estimates, delaying first birth by a year results in about 50 gram loss of birth weights for 30 year old mothers. This number is statistically significant, and is roughly the same as the weight reduction that could result from 3 additional cigarettes smoked by a smoking pregnant mother per day (see Hoderlein & Sasaki, 2013). Couples may want to consider these potential health costs when making decisions to delay marriage and child birth. Furthermore, since these immediate health costs can have significant losses in the long run (see, e.g.Behrman et al., 1994, Currie & Hyson, 1999, Behrman & Rosenzweig, 2004, Black et al., 2007), our empirical results may inform policy makers on how to improve welfare outcomes by affecting the age of first birth.

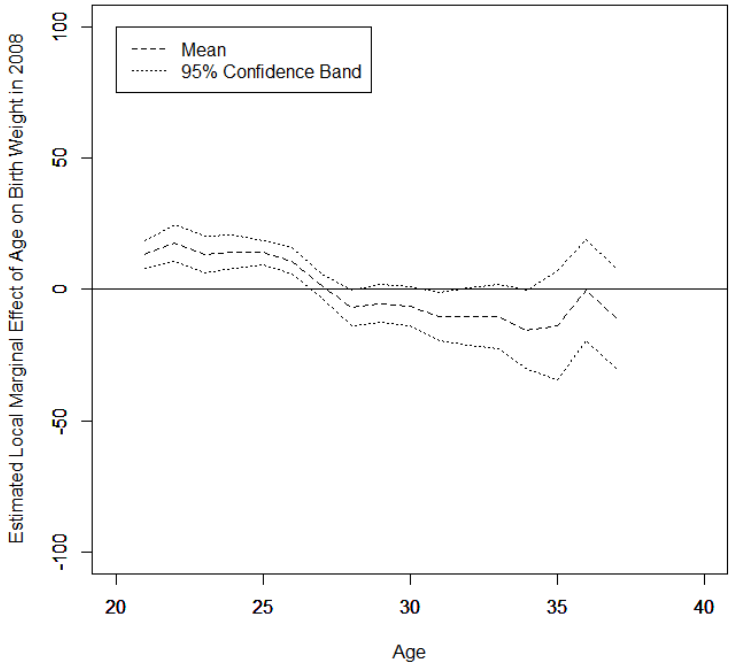
6 Conclusion

Contrary to panel data, repeated cross sections are seldom considered as an alternative to instruments when endogeneity is suspected. Yet, we show that repeated cross sections can resolve this issue even in the case where the explanatory variable of interest, the treatment, is continuous in a way that is reminiscent of difference-in-difference methods. Importantly, this is possible even if time has a nonlinear and heterogeneous effect, meaning that the additive decomposition typically assumed with difference-in-differences is not a necessary condition to conduct such an analysis. However, other conditions are important: The first key assumption is a time invariance condition, which - as we argue - differs from the one usually assumed in panel data models. The second is a crossing condition, which basically holds when time affects the treatment not in a homogeneous way. Under these conditions, several treatment effect parameters are point, while others are set identified. Moreover, we propose two distinct additional set of restrictions that yield point identification of most commonly analyzed treatment effects. The first is a linear correlated random coefficient model recently considered in the panel data literature (see, e.g., Arellano & Bonhomme, 2012), Graham & Powell, 2012). The second does

(a) Endogeneity of mother's age is taken into account.



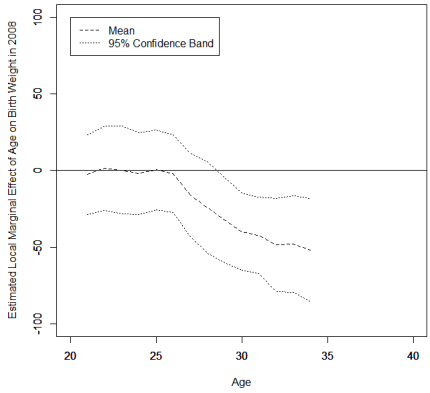
(b) Mother's age is assumed to be exogenous.



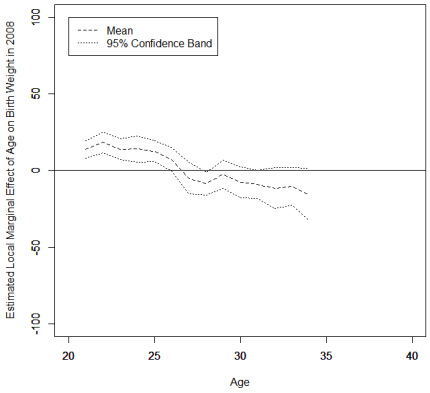
Notes: Effects for first birth in 2008, with bootstrap confidence intervals.

Figure 6: Estimated marginal effects of mother's age on infant birth weights

(a) Endogenous age & 1996 as a reference year



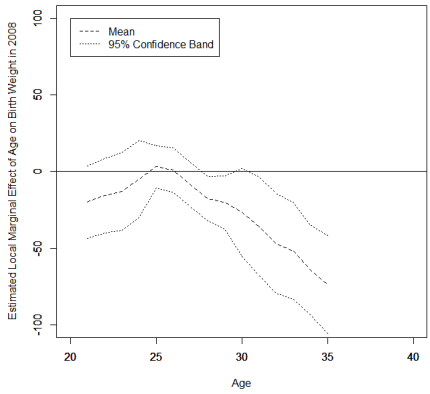
(a) Exogenous age & 1997 as a reference year



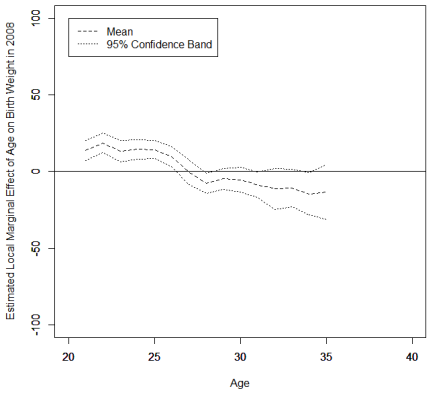
(b) Endogenous age & 1997 as a reference year

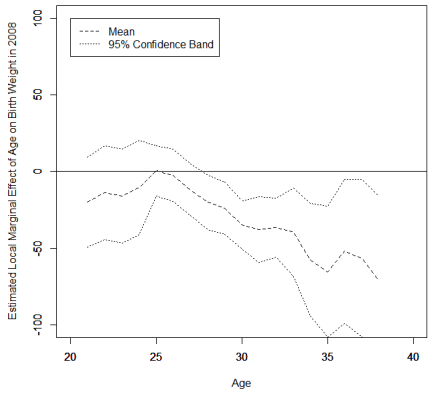
(b) Exogenous age & 1997 as a reference year

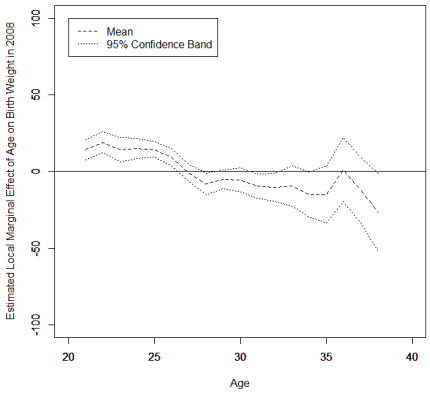
(c) Endogenous age & 1998 as a reference year



(c) Exogenous age & 1998 as a reference year







Notes: Effects for first birth in 2008, with bootstrap confidence intervals. The left column takes endogeneity of mother's age into account, while the right column assumes that mother's age is exogenous. While Figure 6 uses 1999 as a reference year, this figure uses (a) 1996, (b) 1997, and (c) 1998 as reference years.

Figure 7: Estimated marginal effects of mother's age on infant birth weights

not impose linearity, but restricts the error term to be scalar, in line with the literature on non-separable models. We show that such an approach works well in an application that discusses the effect of maternal age at first birth on the birth weight of a newborn, and uncovers, as we feel, interesting details.

A Appendix

A.1 Proof of Theorem 1

The result for m_t and $\Delta^{ATT}(x, q_t(x))$ has already been proved in the text. As for $\Delta^{QTT}(x, q_t(x))$, we have

$$\begin{aligned} F^{-1}_{\widetilde{Y}_t|X_t}(\tau|q_t(x)) &= F^{-1}_{g(q_t(x),A_t)|V_t}(\tau|\mathbf{F}_T(x)) \\ &\overset{A.1}{=} F^{-1}_{g(q_t(x),A_T)|V_T}(\tau|\mathbf{F}_T(x)) \\ &= F^{-1}_{g(q_t(x),A_t)|X_T}(\tau|x). \end{aligned}$$

The result follows.

Now consider marginal effects. Consider a sequence $(x_n)_{n\in\mathbb{N}}$ such that for all $i \in \{1, ..., K\}$, $i \neq j$, $x_{in} = x^*_{it}$ and $q_{jt}(x_{jn}) \neq x_{jn}$. We have

$$\frac{\Delta^{ATT}_j(x_n, q_t(x_n))}{q_{jt}(x_n) - x_{jn}} = \int \frac{g(q_t(x_n), a) - g(x_n, a)}{q_{jt}(x_n) - x_{jn}} f_{A_T|X_T}(a|x_n)da.$$

By Assumption 4-(i) and (ii), we have, for almost all a,

$$\begin{aligned} \frac{g(q_t(x_n), a) - g(x_n, a)}{q_{jt}(x_n) - x_{jn}} f_{A_T|X_T}(a|x_n) &= \frac{\partial g}{\partial x_j}(\widetilde{x}_n, a) f_{A_T|X_T}(a|x_n) \\ &\longrightarrow \frac{\partial g}{\partial x_j}(x^*_t, a) f_{A_T|X_T}(a|x^*_t), \end{aligned}$$

where $\widetilde{x}_n$ is such that $\widetilde{x}_{in} = x^*_{it}$ for all $i \neq j$ and $\widetilde{x}_{jn} \in [x_{jn}, q_{jt}(x_{jn})]$. Moreover, for n large enough, x_n and $\widetilde{x}_n$ belong to the neighborhood $\mathcal{N}$ considered in Assumption 4. Thus, for n large enough,

$$\left| \frac{\partial g}{\partial x_j}(\widetilde{x}_n, a) f_{A_T|X_T}(a|x_n) \right| \leq \left| \sup_{x' \in \mathcal{N}} \frac{\partial g}{\partial x_j}(x', a) \right| \left| \sup_{x' \in \mathcal{N}} f_{A_T|X_T}(a|x') \right|.$$

The right-hand side is integrable by Assumption 4-(iii). Thus, by the dominated convergence theorem,

$$\int \frac{g(q_t(x_n), a) - g(x_n, a)}{q_t(x_n) - x_n} f_{A_T|X_T}(a|x_n)da \longrightarrow \int \frac{\partial g}{\partial x_j}(x^*_t, a) f_{A_T|X_T}(a|x^*_t)da = \Delta^{AME}_j(x^*_t).$$

Finally, let us turn to $\Delta_j^{QME}(\tau, x)$. We have

$$\begin{aligned}\frac{\Delta_j^{QTT}(\tau, x_n, q_t(x_n))}{q_{jt}(x_{jn}) - x_{jn}} &= \frac{F^{-1}_{g(q_t(x_n),A_T)|X_T}(\tau|x_n) - F^{-1}_{g(x_n,A_T)|X_T}(\tau|x_n)}{q_{jt}(x_{jn}) - x_{jn}} \\ &= \frac{\partial F^{-1}_{g(x',A_T)|X_T}(\tau|x_n)}{\partial x'_j}_{|x'=\widetilde{x}'_n},\end{aligned}$$

where $\widetilde{x}'_n$ is such that $\widetilde{x}'_{in} = x^*_{it}$ for all $i \neq j$ and $\widetilde{x}'_{jn} \in [x_{jn}, q_{jt}(x_{jn})]$. By Assumption 4-(iv), the last derivative converges to

$$\frac{\partial F^{-1}_{g(x',A_T)|X_T}(\tau|x^*_t)}{\partial x'_j}_{|x'^*_t} = \Delta_j^{QME}(\tau, x^*_t).$$

A.2 Proof of Theorem 2

Suppose first that g is locally concave on $[\min(x, \underline{x}_T(x')), \overline{x}_T(x')]$. Then, for all $x_1 \leq x' \leq x_2$, almost surely,

$$\frac{g(x_2, A_T) - g(x, A_T)}{x_2 - x} \leq \frac{g(x', A_T) - g(x, A_T)}{x' - x} \leq \frac{g(x_1, A_T) - g(x, A_T)}{x_1 - x}. \tag{A.1}$$

Taking $x_1 = \underline{x}_T(x')$ and $x_2 = \overline{x}_T(x')$ and integrating conditional on $X_T = x$, we obtain

$$(x' - x)\frac{\Delta^{ATT}(x, \overline{x}_T(x'))}{\overline{x}_T(x') - x} \leq \Delta^{ATT}(x, x') \leq (x' - x)\frac{\Delta^{ATT}(x, \underline{x}_T(x'))}{\underline{x}_T(x') - x}.$$

The inequality is simply reverted if g is locally convex. Hence, in either case,

$$\begin{aligned}&(x' - x)\min\left\{\frac{\Delta^{ATT}(x, \underline{x}_T(x'))}{\underline{x}_T(x') - x}, \frac{\Delta^{ATT}(x, \overline{x}_T(x'))}{\overline{x}_T(x') - x}\right\} \leq \Delta^{ATT}(x, x') \\ \leq\ &(x' - x)\max\left\{\frac{\Delta^{ATT}(x, \underline{x}_T(x'))}{\underline{x}_T(x') - x}, \frac{\Delta^{ATT}(x, \overline{x}_T(x'))}{\overline{x}_T(x') - x}\right\}.\end{aligned}$$

The reasoning is the same for marginal effects using, instead of Equation (A.1),

$$\frac{g(x_2, A_T) - g(x, A_T)}{x_2 - x} \leq \frac{\partial g}{\partial x}(x, A_T) \leq \frac{g(x_1, A_T) - g(x, A_T)}{x_1 - x}.$$

A.3 Proof of Theorem 4

m_t is identified by Theorem 1. We now show that we can apply Theorem 4.4 of D'Haultfoeuille & Février (2012). The idea for that is to observe that we have a triangular system

$$\begin{cases} \widetilde{Y}_t &= g(X_t, A_t) \\ X_t &= h(t, V_t) \end{cases}$$

where $h(t,v) = F^{-1}_{X_t}(v)$. This is a nonseparable triangular model where X_t can be seen as the potential endogenous variable corresponding to the value t of an instrument. The only difference between this model and the one considered by D'Haultfoeuille & Février (2012) is that we assume here rank similarity instead of rank invariance. Namely, A_t and V_t are allowed to vary with t here, while the potential error terms corresponding to each value of the instrument are identical in D'Haultfoeuille & Février (2012). But this does not affect the reasoning. In particular,

$$\begin{aligned}
F_{\widetilde{Y}_t|X_t}(g(x,a)|x) &= P(g(x,A_t) \leq g(x,a)|V_t = \mathbf{F}_t(x)) \\
&\overset{A.5}{=} P(A_t \leq a|V_t = \mathbf{F}_t(x)) \\
&\overset{A.1}{=} P(A_{t'} \leq a|V_{t'} = \mathbf{F}_t(x)) \\
&\overset{A.5}{=} P(g(q_{tt'}(x), A_{t'} \leq g(q_{tt'}(x),a)|X_{t'} = q_{tt'}(x)) \\
&= F_{\widetilde{Y}_{t'}|X_{t'}}(g(q_{tt'}(x),a)|q_{tt'}(x)).
\end{aligned}$$

These equalities were also derived by D'Haultfoeuille & Février (2012) (see p.6), the only difference being that they used an independence assumption (Assumption 1 in their paper) in place of our time invariance Assumption 1 here. But both lead to the same conclusion. Note also that the function $q_{tt'}$ plays the role of their function s_{ij}. The rest of the proof is identical, noting that their Assumption 2 holds by Assumption 5, and their Assumptions 3 and 4 are satisfied by Assumption 4.

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