HEDONIC PRICE FUNCTIONS

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cemmap working paper CWP18/06

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September 25, 2006

Abstract

A hedonic price function describes the equilibrium relationship between characteristics of a product and its price. They are used to predict prices of new goods, to adjust for quality change in price indexes, and to measure consumer and producer valuations of differentiated products. They emerge as market outcomes from both competitive and non-competitive markets. The functional form is determined by the distribution of buyers and their preferences, the distribution of sellers and their costs, and the structure of competition in the market.

Hedonic price function

A hedonic price function describes the equilibrium relationship between the economically relevant characteristics of a product or service (or bundle of products) and its price. For example, in a simple labour economics model, the hedonic wage function might describe how the wages of a worker depend on education, experience, and skill. In a simple housing economics model, the hedonic house price might describe how the price of a house depends on geographic location, size, and quality. In each case, the hedonic price function describes equilibrium (not necessarily competitive) valuations of the economically relevant characteristics of the product.

In empirical applications, statistical estimates of hedonic price functions have primarily been used to calculate quality adjusted price indexes for goods and to measure consumer valuations or producer costs of product characteristics. They have been used to study markets for agricultural products, automobiles, labor, houses, computers, and myriad other differentiated commodities. They have been used to measure quality change in private goods markets and to measure consumer valuations of changes in public goods such as clean air, schools or transport infrastructure. In all these applications, hedonic methods are crucial because the goods in question are not homogenous and their value to buyers and sellers varies systematically with characteristics.

Key questions to be answered when developing a hedonic model to analyze a product market are what are the economically relevant characteristics of the product and what is the market environment that generates the hedonic equilibrium price. Given answers to these questions, a key theoretical goal of hedonic analysis is to determine the theoretical relationship between these market equilibrium prices and underlying structural features of the economy such as producer costs and consumer preferences. Two key empirical goals of hedonic analysis are to understand when statistical estimates of hedonic relationships provide good out-of-sample predictions of prices and to understand what structural information these statistical relationships provide about costs and preferences.

1 General hedonic demand

Hedonic models make various assumptions about whether the space of feasible characteristics is discrete or is a continuum, and whether the characteristics embodied in different products can be bundled or unbundled. This section discusses a general model of hedonic demand that encompasses these special cases. The supply side of the market and various notions of equilibrium are discussed in section 2.

Each consumer who participates in the hedonic market derives utility from a vector of characteristics $z \in Z_m \subseteq \mathbf{R}^{n_z}$. The bundle z is obtained either by buying a single product that embodies z or by buying a set of products that together produce z. In either case the hedonic cost or price is $p(z)$. The set Z_m is the feasible set given current market conditions. The set Z_m could be a finite set or it could be a continuum. Each consumer also has the option not to participate in the hedonic market in which case they obtain reservation utility u_0. Assume that characteristics are defined so that utility is increasing in each element of z. Also, assume that utility is decreasing in $p(z)$.

Every consumer is represented by a type $x \in X \subseteq \mathbf{R}^{n_x}$. The space X is the space of all consumer types. The vector x is a vector of consumer characteristics (such as income, education or preference parameters) that affects utility. Consumer heterogeneity is an important feature of hedonic models.

Given hedonic price $p(z)$, consumer x chooses $z \in Z_m$ to maximize utility $u(x, z, p(z))$. That is, they solve

$$\max_{\{z \in Z_m\}} \{u(x, z, p(z))\}. \tag{1}$$

The solution $z = d(x)$ is the hedonic demand function (or correspondence) for consumer x.

Several features of the model are important. First, z is a complete list of the product characteristics that both affect consumer utility and are known to the consumer at time of purchase. In the housing market example, z could measure geographic location, age of the dwelling, lot size, number of rooms, size of the yard, etc. Second, there may be additional characteristics of the good that affect ex post utility but that are not known to the consumer at time of purchase. In such cases, the utility function should be interpreted as the expected utility from purchasing a good with known characteristics z. Third, buyer utility depends on x and on z. Two consumers, x_1 and x_2, with $x_1 \neq x_2$, will generally choose different bundles $(z_1, p(z_1))$ and $(z_2, p(z_2))$ and will obtain different levels of utility.

1.1 Continuous choice version

To specialize to the case where Z_m is a compact convex subset of $\mathbf{R}^n$, both u and p are differentiable and the consumer maximization problem has an interior solution, the first order condition describing the consumer's hedonic demand is

$$\frac{\partial u(x, z, p(z))}{\partial z} + \frac{\partial u(x, z, p(z))}{\partial p} \frac{\partial p(z)}{\partial z} = 0 \qquad (2)$$

which can be rewritten as

$$\frac{\partial p(z)}{\partial z} = -\left(\frac{\partial u(x,z,p(z))}{\partial z} \Big/ \frac{\partial u(x,z,p(z))}{\partial p}\right). \tag{3}$$

The marginal price at z equals the marginal rate of substitution of the consumer x who chooses z. In the quasi-linear utility case $u(x,z,p(z)) = u(x,z) - p(z)$ and equation (3) becomes

$$\frac{\partial p(z)}{\partial z} = \frac{\partial u(x,z)}{\partial z}. \tag{4}$$

These results are the basis for the intuition that the slope of the hedonic price function measures consumers' marginal willingness to pay. Figure 1 illustrates. Consumers x_1 and x_2 optimally choose bundles z_1 and z_2 respectively. At z_1, the marginal price equals the marginal willingness to pay of consumer x_1. However, it is less than the marginal willingness to pay of consumer x_2. At z_2, the marginal price equals the marginal willingness to pay of x_2 but is greater than the marginal willingness to pay of x_1.

The hedonic price function reveals precise information about consumers x_1 and x_2 at points z_1 and z_2 respectively. At all other person-location pairs, it reveals only bounds on willingness to pay. It also reveals very little about how consumers x_1 and x_2 will react to large changes in the shape of the price function. More precise information requires the estimation of consumer preferences.

1.2 Discrete choice version

If the marginal conditions in (3) and (4) are replaced by inequalities, the qualitative interpretations above apply equally to economies in which Z_m is finite. Suppose there are J elements in Z_m. Let z_j be the j'th element in Z_m and let $p_j = p(z_j)$ for $j = 1, ..., J$. In the quasi-linear case, if consumer x chooses z_j, then

$$u(x, z_j) - p_j \geq u(x, z_k) - p_k$$

for all $k \in \{1, ..., J\}$.

Consider the set of consumers who choose z_j and for whom

$$u(x, z_j) - p_j = u(x, z_k) - p_k \tag{5}$$

for some $k \neq j$. These consumers are indifferent between bundle z_j at price p_j and bundle z_k at price p_k. The difference in prices between z_j and z_k exactly compensates for the difference in utilities. For these indifferent consumers, willingness to pay for z_j over z_k is

$$p_j - p_k = u(x, z_j) - u(x, z_k).$$

This is the discrete analog of the marginal willingness to pay.

Equation (5) only holds for those who are indifferent between j and k. For those who are not indifferent, the willingness to pay for z_j over z_k is strictly larger than the price. That is

$$u(x, z_j) - u(x, z_k) > p_j - p_k.$$

When the set of available alternatives Z_m is finite, the hedonic price function provides a precise measure of willingness to pay for consumers who are indifferent between options and provides bounds on willingness to pay for consumers who strictly prefer one option to others.

1.3 Single product demand version

In single product demand models, the vector z measures the characteristics of the unique product type that is chosen. These models assume that households cannot buy two separate products with characteristics z_1 and z_2 and combine their characteristics to obtain some other bundle z_3 (Rosen, 1974). These models do allow consumers to choose both a product type z and a quantity. To see this, rewrite the utility function in (1) as

$$u(x, z, p(z)) = \max_{\{q\}} \{\widetilde{u}(x - qp(z), z, q)\}$$

where q is the quantity of product type z and x is income. This is the primary model used to study location choices and demand for land in urban economic models. See Fujita (1991).

1.4 Home production version

Home production models assume that consumers purchase a vector of goods in quantities $q \in \mathbf{R}^n_+$ at market prices $\pi \in \mathbf{R}^n_+$ and produce the bundle z from the goods purchased. See Gorman (1980), Lancaster (1966), and Muellbauer (1974). In home production hedonic models, consumers have a technology $f : Z \times \mathbf{R}^n \longrightarrow \mathbf{R}^m$ describing the production possibility frontier. Given purchases of q units of market goods, any bundle z that satisfies the restriction $f(z, q) = 0$ is feasible.

Given market prices π and technology f, the cost of obtaining the bundle z is

$$p(z) = \min_{\{q\}} \{\pi \cdot q \text{ subject to } f(z, q) = 0\}. \tag{6}$$

Thus, the hedonic price $p(z)$ is the minimum cost of obtaining bundle z given market prices π and technology f. Given $p(z)$, consumers maximize the utility given in (1). The single product demand model is a special case of the home production model.

In the Gorman/Lancaster version of the model, the technology is linear and $f(z,q) = z - Aq$ where A is a $n_z \times n_q$ matrix. Each market good contains a fixed quantity of characteristics. The total amount available for consumption is the sum of characteristics across all goods purchased.

1.5 Hedonic cost of living index

In each of these models, one can calculate various hedonic cost of living indexes. See Pollak (1989) for details of many alternatives. This section discusses one alternative.

Consider a consumer who purchases a vector of quantities of homogenous goods q with linear prices π and a single differentiated product with characteristics z and hedonic price $p(z)$. When prices are (π, p), the cost of obtaining utility level u_0 is

$$c(\pi, p, u_0) = \min_{\{q,z\}} \{\pi \cdot q + p(z) \text{ subject to } u(q,z) \geq u_0\}. \tag{7}$$

If prices change from (π_0, p_0) to (π_1, p_1), then the constant utility hedonic cost of living index is

$$\frac{c(\pi_1, p_1, u_0)}{c(\pi_0, p_0, u_0)}.$$

This cost index hold utility constant and allows consumers to alter consumption of q and z in response to changing prices. When consumer preferences are unknown, this theoretical index cannot be calculated. With data on prices and quantities, empirical alternatives include the Laspeyres index and the Paasch index.

Let (q_0, z_0) solve (7) when prices are (π_0, p_0) in period zero. Let prices in period one be (π_1, p_1). Then a hedonic Laspeyres index is

$$L(q_1, p_1, q_0, p_0, x_0, z_0) = \frac{\pi_1 \cdot q_0 + p_1(z_0)}{\pi_0 \cdot q_0 + p_0(z_0)} \geq \frac{c(\pi_1, p_1, u_0)}{c(\pi_0, p_0, u_0)}.$$

This index holds the consumption bundle (q_0, z_0) constant at initial levels. Like the standard Laspeyres index, it is an overestimate of the cost of living index because it ignores a consumer's ability to alter consumption in response to changing prices. If some components of z are exogenous (e.g. public goods like air quality or public safety), alternative indexes can be defined by including the time varying exogenous elements of z as arguments in the cost function.

One major problem with the index is that the set of available products often changes rapidly over time. If product z_0 is not traded in period one, then $p_1(z_0)$ will not be observed. Pakes (2003) shows that an estimate of $p_1(z_0)$ based on observed prices is an upper bound under certain circumstances. A better option is to calculate the virtual price $p_1^V(z_0)$ that makes the household indifferent between purchasing z_0 at price $p_1^V(z_0)$ and purchasing z_1 (the product actually chosen in period 1) at price $p_1(z_1)$. The virtual price satisfies

$$p_1^V(z_0) = p_1(z_1) - (u(x, z_1) - u(x, z_0)).$$

Data on prices and quantities can be used to bound the virtual price. Precise results require estimation of consumer preferences.

Another major problem is that statistical authorities, as discussed in section 3, do not observe the elements of z that enter consumer preferences. A third major problem is that time constraints and cost constraints place severe limitations on data collection and analysis for use in practical price index calculations. Triplett (2004) provides a comprehensive overview of these issues.

2 Market equilibrium

Hedonic prices emerge as equilibrium outcomes from a market environment. They might emerge from a purely competitive environment in which neither buyers nor sellers have power to influence prices or they might emerge from an imperfectly competitive environment in which either buyers or sellers have market power. They may be observed in arms-length transactions or unobserved as in black market wage contracts or implicit marriage contracts.

In general, the hedonic price function in a market is a nonlinear function of the characteristics z. Its functional form is determined by the distribution of buyers and their preferences, the distribution of sellers and their costs, and by the type of equilibrium in the market. Special cases exist where more can be said. If bundles of characteristics can be unbundled, arbitrage leads to a linear hedonic price (Rosen, 1974). In the Gorman/Lancaster model, the hedonic price function is piece-wise linear (See Pollak (1983) or Heckman and Scheinkman (1987)). In the Tinbergen (1956) model, the hedonic price is quadratic. When both buyer utility and seller costs depend on z only through an index $q(z)$, the hedonic price function satisfies $p(z) = \widetilde{p}(q(z))$.

2.1 Competitive hedonic equilibrium

Consider a one dimensional Tinbergen/Rosen model in which consumers of type $x \in \mathbf{R}_+$ choose $z \in \mathbf{R}_+$. Assume that consumer utility is $u(x,z) = x\widetilde{u}(z)$ where $x\frac{\partial \widetilde{u}(z)}{\partial z} > 0$. Note that $\frac{\partial^2 u(x,z)}{\partial z \partial x} = \frac{\partial \widetilde{u}(z)}{\partial z} > 0$. Assume that the distribution of consumer types is described by the distribution function $F_x(x)$ with density function $f_x(x)$ and support $\mathbf{R}_+$.

Treat the supply side symmetrically. Assume that firms of type $y \in \mathbf{R}_+$ have costs of producing one unit of product z of $c(y,z) = \frac{\widetilde{c}(z)}{y}$ where $\left(\frac{1}{y}\right)\frac{\partial \widetilde{c}(z)}{\partial z} > 0$. Note that $\frac{\partial^2 c(y,z)}{\partial z \partial y} = \left(\frac{-1}{y^2}\right)\frac{\partial \widetilde{c}(z)}{\partial z} < 0$. The distribution function describing the distribution of firms is $F_y(y)$ with density $f_y(y)$ and support $\mathbf{R}_+$.

Given a differentiable price, consumers solve

$$\max_{\{z\}} \{x\widetilde{u}(z) - p(z)\}.$$

Assume there is a unique interior optimizer. The consumer first order condition is

$$x\frac{\partial \widetilde{u}(z)}{\partial z} - \frac{\partial p(z)}{\partial z} = 0.$$

This equation implicitly defines the buyer demand function $z = d(x)$ and the inverse demand function $x = \widetilde{d}(z) = \left(\frac{\partial p(z)}{\partial z} \Big/ \frac{\partial \widetilde{u}(z)}{\partial z}\right)$. Note that the consumer second order condition implies that $\frac{\partial \widetilde{d}(z)}{\partial z} > 0$. As a result, the distribution function describing the distribution of demand is $F_x\left(\widetilde{d}(z)\right) = F_x\left(\frac{\partial p(z)}{\partial z} \Big/ \frac{\partial \widetilde{u}(z)}{\partial z}\right)$.

By the same reasoning, the firms' first order conditions define the inverse supply function $y = \widetilde{s}(z) = \left(\frac{\partial \widetilde{c}(z)}{\partial z} \Big/ \frac{\partial p(z)}{\partial z}\right)$ which also is monotonic. As a result the distribution of supply can be written $F_y\left(\frac{\partial \widetilde{c}(z)}{\partial z} \Big/ \frac{\partial p(z)}{\partial z}\right)$.

An equilibrium hedonic price function is one that equates the distributions of supply and demand. Formally, a function $p(z)$ is an equilibrium price function if it satisfies the differential equation

$$F_x\left(\frac{\partial p(z)}{\partial z} \Big/ \frac{\partial \widetilde{u}(z)}{\partial z}\right) = F_y\left(\frac{\partial \widetilde{c}(z)}{\partial z} \Big/ \frac{\partial p(z)}{\partial z}\right) \tag{8}$$

for almost all $z \in Z_m$ and if $p(z_{\min})$ ensures that all buyers and sellers obtain at least their reservation utilities.

Some simple conclusions stem from this analysis. First, since $\frac{\partial^2 u(x,z)}{\partial z \partial x} > 0$ and $\frac{\partial^2 c(y,z)}{\partial y \partial z} < 0$, the equilibrium involves positive assortative matching between buyers and sellers. Second, the equilibrium price depends on u, the preferences of buyers, c, the costs of sellers, and on F_x and F_y, the distributions of both types of agents. Third, the price function is the envelope of seller cost and buyer utility.

In more general cases and in cases of higher dimension, the differential equation (8) often does not have nice numerical properties. However, one can solve the equilibrium problem by solving the associated social welfare maximization problem which is an optimal transportation problem (an infinite dimensional linear programming problem with special structure). Recent results in this area include Gretsky, Ostroy and Zame (1999) and Chiappori, McCann, and Nesheim (2006).

2.2 Oligopoly hedonic equilibrium

When there is imperfect competition in hedonic markets, firms set prices to maximize profits. Assume individual demand is derived from the discrete choice model in section 1.2. Let $p = (p_1, ..., p_J)$ and $z = (z_1, ..., z_J)$. Given p and z, let $D_j(p, z, x) \in [0, 1]$ be the demand of consumer x for product j. Let $f_x(x)$ be the density of consumer types with support X.

Aggregate demand for good j is

$$q_j(p, z) = \int_x D_j(p, z, x) f_x(x) \, dx.$$

Given the strategies of all firms $k \neq j$, firm j solves

$$\max_{\{z_j, p_j\}} \{p_j q_j(p, z) - c(j, q_j, z_j)\}$$

The first order conditions are

$$q_j + p_j \frac{\partial q_j}{\partial p_j} - \frac{\partial c_j}{\partial q} \frac{\partial q_j}{\partial p_j} = 0 \tag{9}$$

$$p_j \frac{\partial q_j}{\partial z_j} - \frac{\partial c_j}{\partial q_j} \frac{\partial q_j}{\partial z_j} - \frac{\partial c_j}{\partial z_j} = 0. \tag{10}$$

A pure strategy Nash equilibrium is a set of strategies (z_j, p_j) for each firm $j = 1, ..., J$ such that each firm maximize profits given the strategies of its competitors. In a Nash equilibrium, the equilibrium hedonic prices

p and characteristics z are determined by the distribution of buyers and their preferences, the costs of the competitors and by the competitive structure of the market. Buyers preferences u and the distribution f_x determine the structure of demand. This demand structure combined with the costs of competitors and the number of competitors determine the fierceness of competition. See for example Berry, Levinsohn, and Pakes (1995).

3 Estimating hedonic prices

3.1 Ideal case: z is perfectly observed

The theory of hedonic prices places no restrictions on the hedonic price functional form. The lack of theoretical predictions has led to controversy about functional form in empirical hedonic price work. Different researchers have used linear models, log linear models, Box-Cox models, and fixed effect models. To estimate hedonic quality adjustments for use in price indexes, many statistical authorities adopt the even more restrictive "time-dummy" model in which the hedonic price function takes the form

$$p_t = \beta_0 + \beta_1 z_{1t} + \beta_2 z_{2t} + \beta_3 \cdot D_t + \varepsilon_t \tag{11}$$

where D_t is a vector of time dummies. See Triplett (2004) for a detailed discussion. This version restricts the hedonic price function to be linear in characteristics and to have coefficients that are constant over time. The time-dummy model is rarely theoretically justified and the constant coefficient restriction is usually rejected in empirical tests. Nevertheless, Triplett (2004) argues that in many cases of interest to statistical authorities, the restriction works as an approximation and does not make much empirical difference for estimates of hedonic price indexes.

There is no theoretical justification for restrictive parametric empirical models of hedonic prices unless prior knowledge of the market and the products traded exists to support the restrictions. When datasets are large and the dimension of z is small, there is little empirical justification for parametric models either. In such cases, hedonic price functions should be estimated nonparametrically unless prior knowledge sufficient to restrict the model exists. Such nonparametric regressions can be easily estimated on desktop computers.

When sample size is small or the dimension of z is large, however, then unrestricted nonparametric methods are often impractical. In these cases, prior information should first be used to impose structure on the hedonic relationship. In some cases, it is then feasible to use semiparametric methods to estimate the hedonic relationship without imposing further structure. In many (if not most) cases, however, there is no choice but to impose further structure that is supported neither by data nor by theory. If the primary use of the method is to predict prices out-of-sample, then goodness of fit and stability with respect to changing market conditions can be useful criteria to choose functional form. If the primary use, is to estimate marginal willingness to pay in some dimension, then semiparametric methods that allow for flexibility in the dimension of interest might be of most use. Tests for robustness should be implemented and interpretations of results should consider potential mis-specification biases.

3.2 Practical case: z is imperfectly observed.

Empirical estimates of hedonic price functions may be biased due to omitted variables or mis-measured variables. Assume the goal is to estimate the hedonic price $p(z)$ and that the methods used will rely on estimation of conditional expectations. Discussion of estimation of $\ln(p(z))$ or methods based on other statistics such as the median would proceed along similar lines.

Let $z = (z_1, z_2)$ be the set of all hedonic characteristics and let $\widetilde{z} = (\widetilde{z}_1, \widetilde{z}_2)$ be the set of variables that the econometrician observes. Assume that z_1 is observed without error so that $\widetilde{z}_1 = z_1$. Assume that $\widetilde{z}_2$ is a vector of proxy variables (or instrumental variables) and that $z_2 = g(\widetilde{z}_2, \varepsilon_2)$ where ε_2 is a vector of unobservables. Let $p(z_1, z_2)$ be the theoretical hedonic price function. Observed prices $\widetilde{p}$ satisfy

$$\widetilde{p} = p(\widetilde{z}_1, g(\widetilde{z}_2, \varepsilon_2)) + \eta \tag{12}$$

where η is measurement error, $E(\eta) = 0$, and η is assumed independent of $(\widetilde{z}_1, \widetilde{z}_2, \varepsilon_2)$. The unobserved characteristic case, is the case where $g(\widetilde{z}_2, \varepsilon_2) = \varepsilon_2$ and $f_{\varepsilon_2}(\varepsilon_2 | \widetilde{z}_1, \widetilde{z}_2) = f_{\varepsilon_2}(\varepsilon_2 | \widetilde{z}_1)$. Then ε_2 is the unobserved characteristic of the product.

Under these assumptions, the expectation of $\widetilde{p}$ conditional on $(\widetilde{z}_1, \widetilde{z}_2)$ is

$$\begin{aligned} E(\widetilde{p} | \widetilde{z}_1, \widetilde{z}_2) &= \int p(\widetilde{z}_1, g(\widetilde{z}_2, \varepsilon_2)) f_{\varepsilon_2}(\varepsilon_2 | \widetilde{z}_1, \widetilde{z}_2) d\varepsilon_2 \\ &= h(\widetilde{z}_1, \widetilde{z}_2) \end{aligned} \tag{13}$$

where $f_{\varepsilon_2}(\varepsilon_2 | \widetilde{z}_1, \widetilde{z}_2)$ is the density of ε_2 conditional on $(\widetilde{z}_1, \widetilde{z}_2)$. This is the best predictor (in the integrated squared error sense) of $\widetilde{p}$ given data on $(\widetilde{z}_1, \widetilde{z}_2)$. However, in general $h(\widetilde{z}_1, \widetilde{z}_2) \neq p(\widetilde{z}_1, \widetilde{z}_2)$ and little can be said about the relationship between the two without more information.

Researchers have employed instrumental variables techniques or prior information that places structure on g, on p, or on f_{ε_2} to cope with this problem. See Chay and Greenstone (2005) and Bajari and Benkhard (2005) for examples.

4 Estimating hedonic preferences

In most cases, the full set of consumer characteristics that affect choices is not observed. The econometrician only observes a subset of consumer characteristics such as education, income, age, and household structure. For example, suppose the consumer has two characteristics (x, ε) and x is observed while ε is not. Recall the consumer first order condition

$$\frac{\partial p(z)}{\partial z} = \frac{\partial u(x, \varepsilon, z)}{\partial z}. \tag{14}$$

This equation defines the hedonic demand function $z = d(x, \varepsilon)$.

When data on (x, z, p) are available, u cannot be estimated directly using (14) because z is an endogenous variable. As in Figure 1 where households with different values of x choose different value of z, households with different values of ε will choose different values of z.

Additional restrictions can help identify u. Ekeland, Heckman, and Nesheim (2004) show that the utility function can be identified nonparametrically if $\frac{\partial u}{\partial z}$ is additively separable. That is if,

$$\frac{\partial u(x, \varepsilon, z)}{\partial z} = u_0(x) + u_1(z) + \varepsilon$$

where u_0 and u_1 are arbitrary nonparametric functions.

More generally, Heckman, Matzkin, and Nesheim (2005) prove that the demand function $d(x, \varepsilon)$ can be estimated using data on (z, x) alone if ε is statistically independent of x. They further show that the function u is not identified with data from a single market unless prior information is used to restrict u. For example, if marginal utility is weakly separable so that $\frac{\partial u(x,\varepsilon,z)}{\partial z} = v(q(z, x), \varepsilon)$ where q is a known function, then the function v can be estimated.

Heckman, Matzkin, and Nesheim (2005) also show how to use multi-market data to estimate the unrestricted equation (14). Because cross market variation in prices is tied to cross market variation in the distributions of buyers and sellers, it is functionally independent of within market variation in z and x. As a result, this cross market variation in prices can then be used to identify and estimate the function u.

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Figure 1: Tangency of price and utility