

# On the dynamics of unemployment and wage distributions

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# ON THE DYNAMICS OF UNEMPLOYMENT AND WAGE DISTRIBUTIONS

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ABSTRACT. Postel-Vinay and Robin's (2002) sequential auction model is extended to allow for aggregate productivity shocks. Workers exhibit permanent differences in ability while firms are identical. Negative aggregate productivity shocks induce job destruction by driving the surplus of matches with low ability workers to negative values. Endogenous job destruction coupled with worker heterogeneity thus provides a mechanism for amplifying productivity shocks that offers an original solution to the unemployment volatility puzzle (Shimer, 2005). Moreover, positive or negative shocks may lead employers and employees to renegotiate low wages up and high wages down when agents' individual surpluses become negative. The model delivers rich business cycle dynamics of wage distributions and explains why both low wages and high wages are more procyclical than wages in the middle of the distribution and why wage inequality may be countercyclical, as the data seem to suggest is true.

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## 1. INTRODUCTION

The initial motivation for this paper is two-fold. First, empirical results in Bonhomme and Robin (2009) and Heathcote, Perri, and Violante (2010), among others, suggest that wage and earnings inequality increase in downturns (while earnings mobility decreases) and that low earnings, and low wages to a lesser extent, are more procyclical than high earnings or wages. The reason why it is so is not totally clear by lack of a theory of the business cycle fluctuations of wage distributions. Second, models of individual earnings dynamics in Lillard and Willis (1978), Abowd and Card (1989), Moffitt and Gottschalk (1995), Baker (1997), and numerous followers, consider extensions of the basic permanent-transitory factor model:

$$y_{it} = p_t \mu_i + \lambda_t v_{it},$$

where  $y_{it}$  is the residual of a regression of log earnings on time dummies, education, etc., and  $v_{it}$  is a stationary (“transitory”) process;  $p_t$  and  $\lambda_t$  are factor loadings, i.e. time-varying parameters to be estimated. Increasingly more complex structures have been proposed in the literature without strong economic rationale.

The aim of this work is to propose a theory of labour markets with heterogeneous workers subject to aggregate and idiosyncratic productivity shocks. A particular effort will be made to propose rational match-formation and wage-setting mechanisms that can explain the strong counter-cyclicalities of unemployment together with the special cyclical patterns of the tails of cross-sectional wage distributions.

I use Postel-Vinay and Robin’s (2002) sequential auctions to model wage formation, in a way that is similar to the model in Lise, Meghir, and Robin (2009) except than I allow for aggregate shocks to productivity instead of firm-specific shocks. Wage contracts are long term contracts that can be renegotiated by mutual agreement only. Employees search on the job and employers counter outside offers. There is no invisible hand to set wages as in a Walrasian equilibrium. Instead, it is assumed that firms have full monopsony power vis-à-vis unemployed workers and hire them at a wage that is only marginally greater than their reservation wage. A worker paid less than the competitive wage then has a strong incentive to look for an alternative employer, and trigger Bertrand competition. In

such an environment, at a steady-state equilibrium, with identical workers and identical firms, there are only two wages in the support of the equilibrium distribution: the lower and the upper bounds of the bargaining set – either the firm gets all the surplus, or the worker.

In a very influential paper, Shimer (2005) argues that the search-matching model of Mortensen and Pissarides (1994) cannot reproduce unemployment dynamics well. A long series of papers have tried to solve the puzzle, essentially by making wages sticky (Hall, 2005, Hall and Milgrom, 2008, Gertler and Trigari, 2009, Pissarides, 2009) or by reducing the match surplus to a very small value Hagedorn and Manovskii (2008). Mortensen and Nagypál (2007) review this literature and consider alternative mechanisms. Interestingly, although endogenous job destruction is at the heart of the Mortensen-Pissarides model, this literature has neglected endogenous job destruction as a possible amplifying mechanism when coupled with worker or match heterogeneity. Yet, we shall see that a small fraction of workers (around 5%) at risk of a negative surplus suffices to amplify the effect of negative productivity shocks on unemployment above and beyond the steady exogenous layoff flows. Realistic unemployment dynamics can be generated with an exogenous layoff rate of 4.3% and an overall job destruction rate of 4.5%. The 0.2% difference is the endogenous part. Exogenous job destruction (idiosyncratic) implies a minimum unemployment rate of about 4% (frictional unemployment). Endogenous job destruction (driven by macroeconomic causes) induces additional unemployment fluctuations between 0 and 5% (classical unemployment).

A few search-matching models with endogenous earnings distribution dynamics have been recently proposed in the literature. Pissarides (2009) suggests a novel approach to solve the unemployment volatility puzzle by assuming that productivity shocks change entry wages in new jobs differently from wages in on-going jobs. Gertler and Trigari (2009) generate wage stickiness using a Calvo-type mechanism such that only a fraction of contracts are renegotiated in every period. Both models generate cross-sectional wage dispersion but they do not address the issue of wage inequality dynamics. Here, wages in new matches and wages in on-going matches may also be different. However, wages result from a state-dependent rent sharing mechanism that is totally independent of

unemployment dynamics – unemployment dynamics depending on the level of the rent (and how it compares to zero), not on how it is split.

Two other recent papers generate wage distribution dynamics. Moscarini and Postel-Vinay (2008, 2009) study the non-equilibrium dynamics of the Burdett-Mortensen wage posting model. Workers are identical but firms are different. This model yields very interesting insights on the business-cycle dynamics of firm size distributions. Menzio and Shi (2009) also consider a wage posting model but they assume undirected search instead of directed search. Neither firms nor workers are intrinsically different but a new match productivity value is drawn after setting up a new partnership.

In this paper, wage dispersion accrues partly because “starting wages” (upon exiting unemployment) differ from “promotion wages” (resulting from Bertrand competition); partly because of workers’ heterogeneous abilities; and partly because the long term nature of wage contracts obviates aggregate state dependence in a very special way. A wage contract is renegotiated after a productivity shock if this shock puts the current contract outside the bargaining set: a low wage suddenly becomes lower than the worker’s reservation wage and the employer is forced to renegotiate the wage upward; a high wage suddenly becomes higher than the employer’s reservation value and the worker is forced to accept a wage cut. This makes both low and high wages more procyclical than wages in the middle of the distribution.

Table 1 shows elasticities of three hourly wage inequality measures (D9/D5, D5/D1, D9/D1 where  $Dx$  stands for the  $x$ th decile). Elasticities are calculated with respect to aggregate unemployment (CPS) and productivity (BLS) using a log-log regression of detrended series.<sup>1</sup> The data seem to comply with the model’s predictions as the elasticity of D9/D5 for wages, with respect to aggregate productivity, is positive and the elasticity of D5/D1 is negative. Table 1 also displays the elasticities of inequality indices of annual earnings. Low earnings are much more procyclical than wages. This indicates that business cycle affects hours worked more than wages (Heathcote, Perri, and Violante, 2010). We shall see that the dynamic sequential auction model can be calibrated in a

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<sup>1</sup>The inequality data are obtained from about 20 years of CPS surveys starting in 1967. I am immensely grateful to Gianluca Violante who passed me these data.

	D9/D5	D5/D1	D9/D1
	Hourly Wage		
Unempl. Rate (linear trend)	-0.048 (0.014)	0.056 (0.018)	0.007 (0.017)
Unempl. Rate (HP-filtered)	-0.095 (0.003)	0.113 (0.011)	0.017 (0.009)
Productivity (linear trend)	0.21 (0.14)	-0.53 (0.16)	-0.32 (0.15)
Productivity (HP-filtered)	0.44 (0.15)	-0.98 (0.15)	-0.53 (0.03)
	Annual Earnings		
Unempl. Rate (linear trend)	0.010 (0.012)	0.25 (0.032)	0.26 (0.033)
Unempl. Rate (HP-filtered)	-0.005 (.009)	0.36 (.024)	0.35 (0.025)
Productivity (linear trend)	-0.10 (0.11)	-1.41 (0.41)	-1.51 (0.43)
Productivity (HP-filtered)	-0.044 (0.16)	-2.46 (0.50)	-2.51 (0.50)

TABLE 1. *Wage and Earnings Inequality* (Source: CPS, 1967-2005. Each case displays the elasticity of the column variable with respect to the row variable. The logged variables are first detrended using a linear trend or HP-filtering.)

way that generates more procyclicality in low wages than in high wages and that also produces a swifter employment response of low ability workers to aggregate productivity.

The paper is organized as follows. A dynamic sequential-auction model with heterogeneous workers and identical firms is first developed. The DSGE model is so simple that it can be exactly simulated. Then, the model's parameters are estimated by simulated GMM, and the results are interpreted.

## 2. THE MODEL

### 2.1. Setup.

**Aggregate shocks.** Time is discrete and indexed by  $t \in \mathbb{N}$ . The global state of the economy is an ergodic Markov chain  $y_t \in \{y_1 < \dots < y_N\}$  with transition matrix  $\mathbf{\Pi} = (\pi_{ij})$  (with a slight abuse of notation,  $y_t$  denotes the stochastic process and  $y_i$  an element of the support). Aggregate shocks accrue at the beginning of each period.

**Workers.** There are  $M$  types of workers and  $\ell_m$  workers of each type (with  $\sum_{m=1}^M \ell_m = 1$ ). Each type is characterized by a time-invariant ability  $x_m$ ,  $m = 1, \dots, M$ , with  $x_m < x_{m+1}$ . Workers are paired with identical firms to form productive units. The per-period output of a worker of ability  $x_m$  when aggregate productivity is  $y_i$  is denoted as  $y_i(m)$ . A natural specification for match productivity is  $y_i(m) = x_m y_i$ . This seems the most

neutral specification as far as the cyclical nature of relative match productivity dispersion is concerned, i. e.  $y_i(m)/y_i(m')$  only depends on worker types  $m$  and  $m'$ , not on the economy's state  $i$ . We denote as  $S_t(m)$  the surplus of a match including a worker of type  $x_m$ , that is, the present value of the match minus the value of unemployment and minus the value of a vacancy (assumed to be nil). Only matches with positive surplus  $S_t(m) > 0$  are viable.

**Turnover.** Matches form and break at the beginning of each period, after the aggregate state has been reset. Let  $u_t(m)$  denote the proportion of unemployed in the population of workers of ability  $x_m$  at the end of period  $t - 1$ , and let  $u_t = \sum_{m=1}^M u_t(m)\ell_m$  define the aggregate unemployment rate. At the beginning of period  $t$ , a fraction  $\mathbf{1}\{S_t(m) \leq 0\}[1 - u_t(m)]\ell_m$  is endogenously laid off and another fraction  $\delta\mathbf{1}\{S_t(m) > 0\}[1 - u_t(m)]\ell_m$  is exogenously destroyed.

I assume that firms cannot direct their search to specific worker types. Also, for simplicity, I assume that workers meet employers at exogenous rates; it is easy to work out an extension of the model with a standard matching function if necessary. Thus, a fraction  $\lambda_0\mathbf{1}\{S_t(m) > 0\}u_t(m)\ell_m$  of employable unemployed workers meet an employer and a fraction  $\lambda_1(1 - \delta)\mathbf{1}\{S_t(m) > 0\}[1 - u_t(m)]\ell_m$  of employed workers meet an alternative employer, where  $\lambda_0$  and  $\lambda_1$  are the respective search intensities of unemployed and employed workers (exogenous).<sup>2</sup>

**Wages.** I assume that employers have full monopsony power with respect to workers. Hence, unemployed workers are offered their reservation wage, the employer taking all the surplus. Rent sharing accrues via on-the-job search, which triggers competition of employers for workers. Because firms are identical and there is no mobility cost, Bertrand competition transfers the whole surplus to the worker who gets paid the firm's reservation value. This wage dynamics is equivalent to the optimal wage-tenure contracts studied by Stevens (2004). She shows that an infinity of wage-tenure contracts are optimal. In particular employers could pay the workers their productivity and charge them an entry fee.

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<sup>2</sup>Unproductive unemployed workers search because they may turn productive in the next period.

All firms being identical, a worker is indifferent between staying with the incumbent employer or moving to the poacher. I assume that the tie is broken in favour of the poacher with probability  $\tau$ .<sup>3</sup>

**Turnover rates.** The following turnover rates can then be computed:

- Exit rate from unemployment:

$$f_{0t} = \lambda_0 \frac{\sum_m \mathbf{1}\{S_t(m) > 0\} u_t(m) \ell_m}{u_t};$$

- Quit rate (job-to-job mobility):

$$f_{1t} = \tau \lambda_1 (1 - \delta) \frac{\sum_m \mathbf{1}\{S_t(m) > 0\} [1 - u_t(m)] \ell_m}{1 - u_t};$$

- Job destruction rate:

$$s_t = \delta + (1 - \delta) \frac{\sum_m \mathbf{1}\{S_t(m) \leq 0\} (1 - u_t(m)) \ell_m}{1 - u_t}.$$

Notice that the quit rate and the job separation rate are thus related by the following deterministic relationship:  $f_{1t} = \tau \lambda_1 (1 - s_t)$ .

## 2.2. Unemployment dynamics.

**The value of unemployment.** Let  $U_i(m)$  denote the present value of remaining unemployed for the rest of period  $t$  for a worker of type  $m$  if the economy is in state  $i$ . We do not index this value by any other state variable but the state of the economy for reasons that will immediately become clear.

An unemployed worker receives a flow-payment  $z_i(m)$  for the period. At the beginning of the next period, the state of the economy changes to  $y_j$  with probability  $\pi_{ij}$  and the worker receives a job offer with some probability. However, because the employer has full monopsony power and takes the whole surplus, the present value of a new job to the worker is only marginally better than the value of unemployment. Consequently, the

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<sup>3</sup>The randomness in the eventual mobility may explain why employers engage in Bertrand competition in the first place if part of the outcome of the production process is non transferable.

value of unemployment solves the following linear Bellman equation:

$$U_i(m) = z_i(m) + \frac{1}{1+r} \sum_j \pi_{ij} U_j(m).$$

**The match surplus.** After a productivity shock from  $i$  to  $j$  all matches yielding negative surplus are destroyed. Otherwise, if the worker is poached, Bertrand competition transfers the whole surplus to the worker whether she or he moves or not. Everything that the worker or the firm will earn in the future is included in the definition of the current surplus. The surplus of a match with a worker of type  $m$  when the economy is in state  $i$  thus solves the following (almost linear) Bellman equation:

$$S_i(m) = y_i(m) - z_i(m) + \frac{1-\delta}{1+r} \sum_j \pi_{ij} \max\{S_j(m), 0\}.$$

This almost-linear system of equations can be solved numerically by value function iteration.

As for the unemployment value, the match surplus only depends on the state of the economy, and in particular not on calendar time. Hence the match surplus process, for workers of type  $m$ ,  $S_t(m)$ , is also a Markov chain with support  $\{S(m), i = 1, \dots, N\}$  and transition matrix  $\Pi$ .

**The unemployment process.** The joint process of  $u_t(m)$  and  $S_t(m)$ , or  $u_t(m)$  and  $\mathbf{1}\{S_t(m) > 0\}$ , is Markovian, the law of motion of individual-specific unemployment rates being:

$$\begin{aligned} u_{t+1}(m) &= 1 - [(1-\delta)(1-u_t(m)) + \lambda_0 u_t(m)] \mathbf{1}\{S_t(m) > 0\} \\ &= \begin{cases} 1 & \text{if } S_t(m) \leq 0, \\ u_t(m) + \delta(1-u_t(m)) - \lambda_0 u_t(m) & \text{if } S_t(m) > 0. \end{cases} \end{aligned}$$

Unemployment dynamics is independent on how the surplus is split between employers and employees.

In state  $i$ 's steady-state equilibrium, the unemployment rate in group  $m$  is

$$u_i(m) = \frac{\delta}{\delta + \lambda_0} \mathbf{1}\{S_i(m) > 0\} + \mathbf{1}\{S_i(m) \leq 0\}.$$

The aggregate unemployment rate is:

$$u_i = \sum_{m=1}^M u_i(m) \ell_m = \frac{\delta}{\delta + \lambda_0} L_i + 1 - L_i = 1 - \frac{\lambda_0}{\delta + \lambda_0} L_i,$$

where  $L_i = \sum_{m=1}^M \ell_m \mathbf{1}\{S_i(m) > 0\}$  is the number of employable workers.

The state-contingent equilibrium unemployment values are bounded from below by  $\frac{\delta}{\delta + \lambda_0}$ . The aggregate unemployment rate is greater than this lower bound when a low aggregate productivity value  $y_i$  induces endogenous job destruction/non participation ( $L_i < 1$  in steady state).

### 2.3. Wages.

**The worker surplus.** Let  $W_i(w, m)$  denote the present value of a wage  $w$  in state  $i$  to a worker of type  $m$ . The surplus flow for the current period is  $w - z_i(m)$ . In the following period, the worker is laid off with probability  $\mathbf{1}\{S_j(m) \leq 0\} + \delta \mathbf{1}\{S_j(m) > 0\}$ , and suffers zero surplus. Otherwise, with probability  $\lambda_1$ , the worker receives an outside offer and enjoys the whole surplus. In absence of poaching (with probability  $1 - \lambda_1$ ) wage contracts may still be renegotiated if a productivity shock moves the current wage outside the bargaining set. We follow MacLeod and Malcomson (1993), and the recent application by Postel-Vinay and Turon (2007) in a similar environment as the one of this paper, and assume that the new wage contract is the closest point in the bargaining set from the old, now infeasible wage. That is, if  $W_j(w, m) - U_j(m) < 0$ , the worker has a credible threat to quit to unemployment and her employer accepts to renegotiate the wage up to the point where the worker obtains zero surplus. If  $W_j(w, m) - U_j(m) > S_j(m)$ , the employer has a credible threat to fire the worker unless she accepts to renegotiate down to the point where she gets the whole surplus and no more.

The worker surplus,  $W_i(w, m) - U_i(m)$ , therefore satisfies the following Bellman equation:

$$W_i(w, m) - U_i(m) = w - z_i(m) + \frac{1 - \delta}{1 + r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) > 0\} \left[ \lambda_1 S_j(m) + (1 - \lambda_1)(W_j^*(w, m) - U_j(m)) \right]$$

where

$$W_j^*(w, m) - U_j(m) = \min\{\max\{W_j(w, m) - U_j(m), 0\}, S_j(m)\}$$

is the renegotiated worker surplus.

Note that here again there is only one aggregate state variable, aggregate productivity. The assumption that the rate of offer arrival is exogenous is important to justify this point. With a matching function the unemployment rate should be included in the state space. However, a very good approximation would be obtained by assuming that the unemployment rate and market tightness jump to their steady-state value after a productivity shock.

**The set of equilibrium wages.** For all aggregate states  $y_i$  and all worker types  $x_m$ , there are only two possible wages. Either the worker was offered a job while unemployed, and he can only claim a wage  $\underline{w}_i(m)$  such that  $W_i(\underline{w}_i(m), m) = U_i(m)$  (his reservation wage); or he was already employed and he benefits from a wage rise to  $\bar{w}_i(m)$  such that  $W_i(\bar{w}_i(m), m) = U_i(m) + S_i(m)$  (the employer's reservation value).

I now explain how these wages can be solved for. For all  $k$ , let us denote the worker surpluses when the economy is in state  $k$  evaluated at wages  $\underline{w}_i(m)$  and  $\bar{w}_i(m)$  as

$$\underline{W}_{k,i}(m) = W_k(\underline{w}_i(m), m) - U_k(m),$$

$$\bar{W}_{k,i}(m) = W_k(\bar{w}_i(m), m) - U_k(m),$$

and let also

$$\underline{W}_{k,i}^*(m) = \min\{\max\{\underline{W}_{k,i}(m), 0\}, S_i(m)\},$$

$$\bar{W}_{k,i}^*(m) = \min\{\max\{\bar{W}_{k,i}(m), 0\}, S_i(m)\}.$$

Making use of the definitions of wages,  $\underline{W}_{i,i}(m) = 0$  and  $\overline{W}_{i,i}(m) = S_i(m)$ . These worker surpluses therefore satisfy the following modified Bellman equations:

$$\begin{aligned} \underline{W}_{k,i}(m) &= \underline{W}_{k,i}(m) - \underline{W}_{i,i}(m) = z_i(m) - z_k(m) \\ &+ \frac{1-\delta}{1+r} \sum_j (\pi_{kj} - \pi_{ij}) \mathbf{1}\{S_j(m) > 0\} \left[ \lambda_1 S_j(m) + (1-\lambda_1) \underline{W}_{j,i}^*(m) \right] \end{aligned}$$

and

$$\begin{aligned} \overline{W}_{k,i}(m) - S_i(m) &= \overline{W}_{k,i}(m) - \overline{W}_{i,i}(m) = z_i(m) - z_k(m) \\ &+ \frac{1-\delta}{1+r} \sum_j (\pi_{kj} - \pi_{ij}) \mathbf{1}\{S_j(m) > 0\} \left[ \lambda_1 S_j(m) + (1-\lambda_1) \overline{W}_{j,i}^*(m) \right]. \end{aligned}$$

Again, value function iteration delivers a simple numerical solution algorithm, using for starting value the solution of the linear system that is obtained by removing the ‘‘stars’’ from the continuation values.

Having determined  $\underline{W}_{k,i}(m)$  and  $\overline{W}_{k,i}(m)$  for all  $k, i$  and  $m$ , wages then follow as

$$\underline{w}_i(m) = z_i(m) - \frac{1-\delta}{1+r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) > 0\} \left[ \lambda_1 S_j(m) + (1-\lambda_1) \underline{W}_{j,i}^*(m) \right]$$

and

$$\overline{w}_i(m) = S_i(m) + z_i(m) - \frac{1-\delta}{1+r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) > 0\} \left[ \lambda_1 S_j(m) + (1-\lambda_1) \overline{W}_{j,i}^*(m) \right].$$

**2.4. The dynamics of wage distributions.** The support of the wage distribution is the union of all sets  $\Omega_m = \{\underline{w}_i(m), \overline{w}_i(m), \forall i\}$ . Let  $g_t(w, m)$  denote the measure of workers of ability  $m$  employed at wage  $w \in \Omega$  at the end of period  $t - 1$ .

Conditional on  $y_t = y_i$  (maybe equal to  $y_{t-1}$ ) at the beginning of period  $t$ , no worker can be employed if  $S_i(m) \leq 0$ . The inflow into the stock of workers paid the minimum wage  $\underline{w}_i(m)$  is otherwise made of all unemployed workers drawing an offer  $(\lambda_0 u_t(m) \ell_m)$  plus all employees paid a wage  $w$  such that  $W_i(w, m) - U_i(m) < 0$  who were not laid off but still were not lucky enough to get poached. The outflow is made of those workers

previously paid  $\underline{w}_i(m)$  who are either laid off or poached. That is,

$$g_{t+1}(\underline{w}_i(m), m) = \mathbf{1}\{S_i(m) > 0\} \left[ \lambda_0 u_t(m) \ell_m + (1 - \delta)(1 - \lambda_1) \left( g_t(\underline{w}_i(m), m) + \sum_{w \in \Omega_m} \mathbf{1}\{W_i(w, m) - U_i(m) < 0\} g_t(w, m) \right) \right].$$

The inflow into the stock of workers paid  $\bar{w}_i(m)$  has two components. First, any employee paid less than  $\bar{w}_i(m)$  (in present value terms) who is contacted by another employer benefits from a pay rise to  $\bar{w}_i(m)$ . Second, any employee paid more than  $\bar{w}_i(m)$  (in present value) has to accept a pay cut to  $\bar{w}_i(m)$  to avoid layoff. The only reason to flow out is layoff. Hence,

$$g_{t+1}(\bar{w}_i(m), m) = \mathbf{1}\{S_i(m) > 0\} (1 - \delta) \left[ \lambda_1 (1 - u_t(m)) \ell_m + (1 - \lambda_1) \left( g_t(\bar{w}_i(m), m) + \sum_{w \in \Omega_m} \mathbf{1}\{W_i(w, m) - U_i(m) > S_i(m)\} g_t(w, m) \right) \right].$$

For all  $w \in \Omega_m \setminus \{\underline{w}_i(m), \bar{w}_i(m)\}$ , only those workers paid  $w$  greater than  $\underline{w}_i(m)$  and less than  $\bar{w}_i(m)$  (in value terms), who are not laid off or poached, keep their wage:

$$g_{t+1}(w, m) = \mathbf{1}\{S_i(m) > 0\} (1 - \delta)(1 - \lambda_1) \times \mathbf{1}\{0 \leq W_i(w, m) - U_i(m) \leq S_i(m)\} g_t(w, m).$$

Note that summing  $g_{t+1}(w, m)$  over all wages and dividing by  $\ell_m$  yields the law of motion for the unemployment rates  $u_t(m)$ :

$$1 - u_{t+1}(m) = \mathbf{1}\{S_i(m) > 0\} [\lambda_0 u_t(m) + (1 - \delta)(1 - u_t(m))].$$

The *joint* process of distributions and surpluses is Markovian with a finite state-space. In principle one can certainly calculate its ergodic distribution, but this is a rather cumbersome calculation. In practice, I shall use simulations to approximate the theoretical moments to match with the data moments used for the estimation of structural parameters.

## 3. PARAMETRIZATION AND ESTIMATION

**3.1. Aggregate shocks.** I use the BLS quarterly series of seasonally adjusted real output per person in the non-farm business sector (BLS series PS85006163) to construct the aggregate productivity process  $y_t$ . The data cover the period 1947q1-2009q1. The raw data are successively log-transformed, HP-filtered, and exponentiated.<sup>4</sup>

I assume that the aggregate productivity process  $y_t$  is an ergodic Markov process. I define the state of the economy as the *rank* of  $y_t$  in its marginal/ergodic distribution,  $F$ . The joint distribution of two consecutive ranks  $F(y_t)$  and  $F(y_{t+1})$  is a copula  $C$  (i.e. the cdf of the distribution of two random variables with uniform margins). For example, the usual Gaussian AR(1) process used in the literature has Gaussian margins and a Gaussian copula. It is commonplace to obtain a discrete approximation of the copula by calculating the transition probability matrix across discretized states (quintiles, deciles, etc.) but fitting a parametric copula (Archimedean, elliptical) is much more economical than fitting all transition probabilities separately.

I use the following two-stage semi-parametric estimation procedure:

- (1) Estimate the marginal distribution  $F$  by kernel smoothing the empirical distribution of  $y_t$ .
- (2) Estimate the copula  $C$  by maximum likelihood on sample  $\{F(y_{t-1}), F(y_t)\}$ . A simple scatterplot gives a good indication regarding to which parametric specification of the copula to choose.<sup>5</sup>

Figure 1, panel (a), shows the marginal distribution of detrended productivity. The kernel density estimate is of course much less dented than the histogram. It resembles a normal density except for the left tail that is fatter than the normal.

Figure 1, panel (b), provides a graphical display of the copula. The actual scatterplot (left) indicates an elliptical distribution with no specific tail-dependence. Hence, I use a

<sup>4</sup>I follow the usual practice since Shimer (2005) and use a smoothing parameter of  $10^5$  instead of the usual 1,600 with quarterly data. The usual smoothing parameter seems to put too much cycle in the trend. This is particularly clear for the nearly non-trended unemployment series (see Figure 4 below).

<sup>5</sup>Chen, Wu, and Yi (2009) argue that a more efficient estimation of the marginal distribution can be obtained using a single-step estimation if the marginal distribution is peaked and the copula displays strong tail dependence. This should be less of a problem here because this two-step procedure is applied to detrended – hence less autocorrelated – data.

$t$ -copula with parameters  $\rho$  (linear correlation coefficient) and  $\nu$  (the number of degrees of freedom; a large  $\nu \geq 30$ , indicates Gaussianity). I estimate  $\rho = 0.89$  and  $\nu = 13.11$ . Parameter  $\nu$  is large, indicating a close-to-Gaussian copula. The other scatterplot (right) shows a simulation of the  $t$ -copula with estimated parameters  $\rho$  and  $\nu$ . No apparent discrepancy with the true one can be easily detected.<sup>6</sup>

Figure 2 displays a simulation of productivity levels. Panel (a) shows the actual series of exponentiated HP-filtered log-productivity. Panel (b) shows two simulations obtained with the same sequence of iid uniform innovations: one uses the semi-parametric estimate and the other one uses a Gaussian AR(1) model. The estimated semiparametric nonlinear process is indeed very similar to a normal AR(1) process.<sup>7</sup>

Finally, a discrete Markov chain approximation can be obtained as follows. Let  $a_0 = \underline{y} < a_1 < \dots < a_N = \bar{y}$  delimit a grid on the support of the productivity distribution. I use equal-sized intervals ( $a_i - a_{i-1} = \frac{\bar{y} - \underline{y}}{N}$ ) and extreme points  $\underline{y}$  and  $\bar{y}$  are chosen according to the estimated marginal distribution  $F$  as  $F(\underline{y}) \simeq 0$  and  $F(\bar{y}) \simeq 1$ . Then,

- (1) Set discrete productivity values as bins' midpoints  $y_i = \frac{a_{i-1} + a_i}{2}$ .
- (2) Estimate marginal state probabilities as  $p_i = F(a_i) - F(a_{i-1})$ .
- (3) Set transition probabilities as:

$$\begin{aligned} \pi_{ij} &= \frac{\Pr \{ [a_{i-1}, a_i] \times [a_{j-1}, a_j] \}}{\Pr \{ [a_{i-1}, a_i] \}} \\ &= \frac{1}{p_i} \left[ C(F(a_i), F(a_j)) - C(F(a_{i-1}), F(a_j)) \right. \\ &\quad \left. - C(F(a_i), F(a_{j-1})) + C(F(a_{i-1}), F(a_{j-1})) \right]. \end{aligned}$$

<sup>6</sup>The simulation algorithm is very simple: given observation  $r_{t-1}$  of the  $(t-1)$ th rank, generate  $r_t$  as

$$t_\nu^{-1}(r_t) = \rho t_\nu^{-1}(r_{t-1}) + e_t \sqrt{\frac{\nu + (t_\nu^{-1}(r_{t-1}))^2}{\nu + 1}} (1 - \rho^2)$$

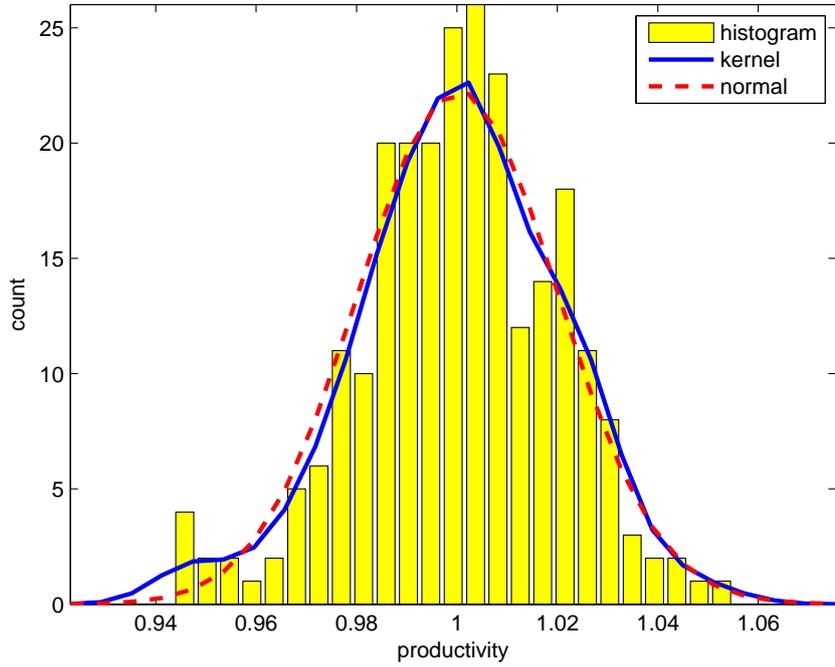
where  $e_t = t_{\nu+1}^{-1}(u)$  with  $u \sim \text{Uniform}[0, 1]$  (or  $e_t \sim t_{\nu+1}$ ). Then generate  $y_t = F^{-1}(r_t)$  for any marginal cdf  $F$ . Note that for  $\nu \rightarrow \infty$ ,  $t_\nu \rightarrow \Phi$  the cdf of the standard normal distribution and the recursive formula for ranks becomes:

$$\Phi^{-1}(r_t) = \rho \Phi^{-1}(r_{t-1}) + \sqrt{1 - \rho^2} e_t$$

where  $e_t \sim N(0, 1)$ . The  $t$ -copula thus operates a different transformation of the raw data and features conditional heteroskedasticity.

<sup>7</sup>For completeness, the autoregression of detrended log-productivity yields an autocorrelation coefficient  $\rho = 0.876$  and a standard deviation of residuals  $\sigma = 0.0097$ .

(a) Marginal productivity distribution



(b) Scatterplot of  $t, t + 1$  productivity ranks

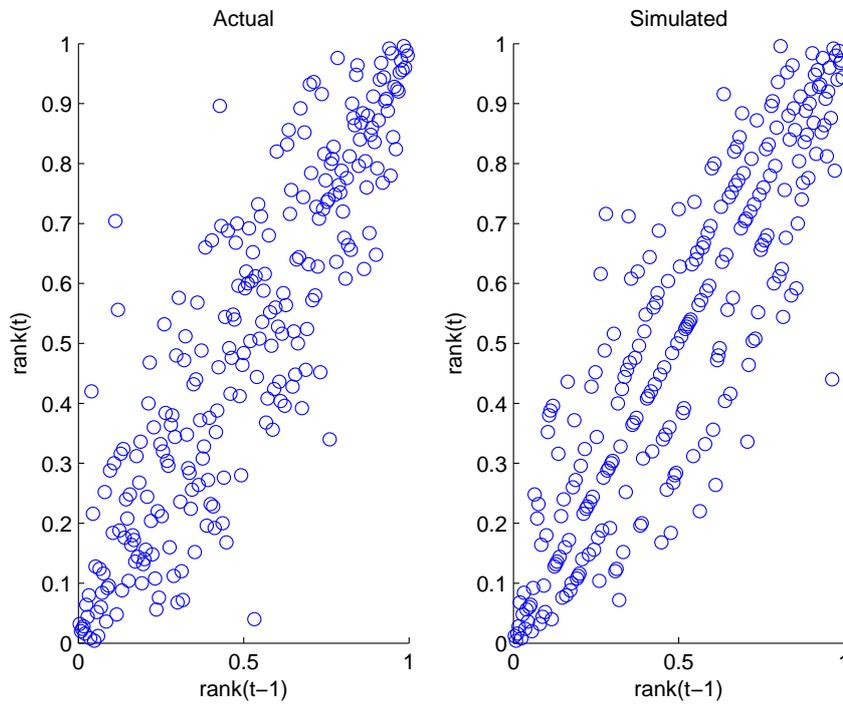
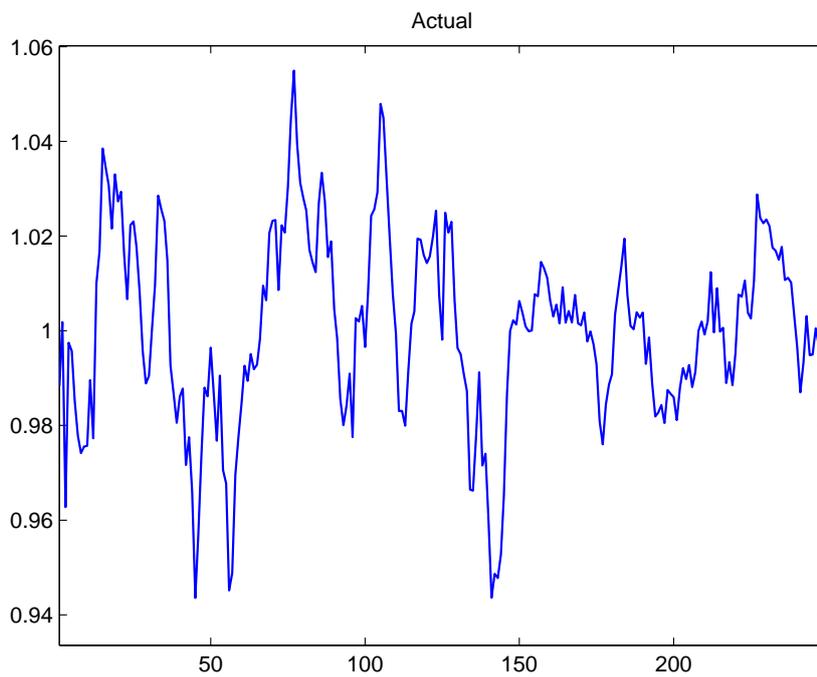


FIGURE 1. *Two-step Estimation of the Aggregate Productivity Process*

(a) Actual HP-filtered productivity series



(b) Simulation

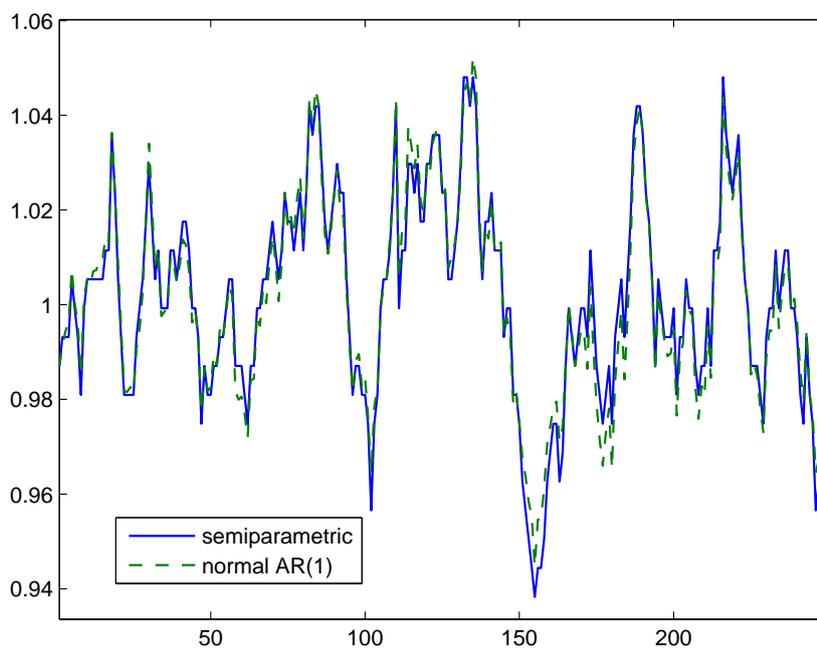


FIGURE 2. *Simulation of Productivity Dynamics*

**3.2. Worker heterogeneity.** I specify match productivity as  $y_i(m) = y_i(Bx_m + C)$ , where  $B$  and  $C$  are two constants parametrizing the support of workers' abilities, and  $x_m \in ]0, 1[$ . Specifically,  $x_m = \frac{m-0.5}{M}$  for  $m = 1, \dots, M$ . The distribution of individual ability is approximately beta-distributed:

$$\begin{aligned} \ell_m &= \text{betacdf}\left(\frac{m}{M}, \mu, 1\right) - \text{betacdf}\left(\frac{m-1}{M}, \mu, 1\right) \\ &\simeq \frac{1}{M} \text{betapdf}(x_m, \mu, 1) \quad (\text{as } M \rightarrow \infty). \end{aligned}$$

The beta distribution allows for a variety of shapes for the density (increasing, decreasing, non monotone, concave or convex).

Note that aggregate productivity is the mean of  $y_t(m)$  across all  $m$  such that  $S_t(m) > 0$ :

$$\begin{aligned} \bar{y}_t &= \frac{\sum_m (1 - u_t(m)) \ell_m y_t(m)}{1 - u_t} \\ &= \frac{\sum_m (1 - u_t(m)) \ell_m (Bx_m + C)}{1 - u_t} y_t \\ &= \bar{x}_t y_t \quad (\text{say}). \end{aligned}$$

The dynamics of  $\bar{y}_t$  differs from the dynamics of  $y_t$  if composition effects make  $\bar{x}_t$ , the mean ability of employees, differ from the mean ability of all workers, employed and unemployed. I will thus calibrate the distribution of  $x_m$  (parameters  $B, C, \mu$ ) so that the mean of  $\bar{x}_t$  is equal to one and the volatility of  $\ln \bar{x}_t + \ln y_t$  is equal to the volatility of  $\ln y_t$  (that is, the variance of  $\ln \bar{x}_t$  and the covariance of  $\ln \bar{x}_t$  and  $\ln y_t$  balance each other out).

Lastly, the opportunity cost of employment (leisure utility, UI benefits, etc.) is specified as  $z_i(m) = z_0 + \alpha [y_i(m) - z_0]$ . I allow for a potential indexation of unemployment flow utility on productivity. Otherwise the reservation wage of high skill workers is lower in booms than in busts as unemployed workers face better future prospects in booms than in busts. Also, if  $\alpha$  is low, high skill workers have lower reservation wages than low skill workers, for exactly the same reason.

**3.3. Estimation/calibration.** I set the unit of time equal to a quarter. The parameters that have to be estimated are the turnover parameters  $\lambda_0, \lambda_1$  and  $\delta$ , the probability

	From Unempl.	Job to Job	Job to Unempl.
Mean rate, %/month (%/qtr)	30%/mth (66%/qtr)	2.1% (6.2%)	1.5% (4.5%)
Elasticity wrt unemployment ( $R^2$ )	-1.01 (75%)	-0.61 (71%)	0.33 (33%)

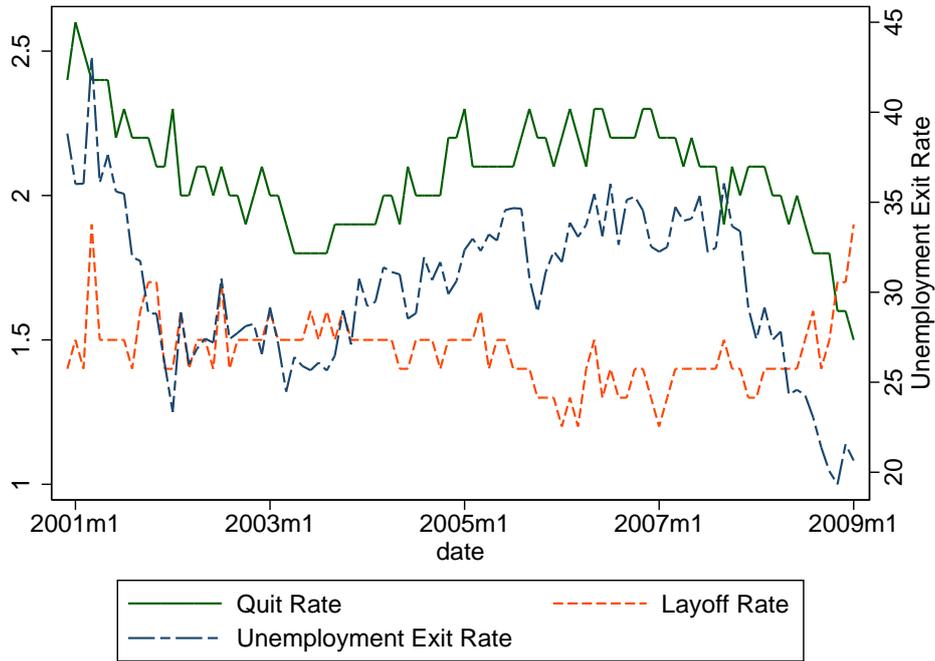
TABLE 2. *Descriptive Statistics on Turnover* (Source: JOLTS)

of moving upon receiving an outside offer  $\tau$ , the leisure cost parameters  $z_0$  and  $\alpha$ , the parameters of the support of worker heterogeneity,  $B$  and  $C$ , and parameter  $\mu$  shaping the distribution of heterogeneity. These parameters will be calibrated so as to fit unemployment, turnover and wage dynamics as I now explain.

I use the Job Openings and Labor Turnover Survey (JOLTS) to measure turnover. The JOLTS provides information on the number of firm hires per month ( $H$ ), the number of quits ( $Q$ ) and involuntary separations (layoffs and discharges), denoted  $L$ . I also use the total employment series ( $E$ ) from the Current Employment Statistics (CES), that is supposedly consistent with the JOLTS series. The number of unemployed ( $U$ ) is extracted from the Current Population Survey (CPS). These are monthly series spanning 2000m12-2009m1.

Assuming that no employee voluntarily quits her job to become unemployed (the exact opposite to assuming that all separations are layoffs) the exit rate from unemployment is  $\frac{H-Q}{U}$  (measuring  $f_{0t}$ ; see section 2.1), the job-to-job mobility rate is the quit rate  $\frac{Q}{E}$  (measuring  $f_{1t}$ ) and the layoff rate is  $\frac{L}{E}$  (measuring  $s_t$ ). The unemployment rate is  $\frac{U}{U+E}$ . Figure 3, panel (a), displays turnover series, and panel (b) graphs the turnover series as a function of the unemployment rate to emphasize the link with the business cycle. As expected, hiring rates are procyclical and the layoff rate is countercyclical, with elasticities reported in Table 2. Shimer (2005, 2007) estimates a separation rate of 3.4% per month from CPS data, which is roughly the same rate that can be calculated using  $\frac{Q+L}{E}$  from JOLTS data, i.e. the sum of the layoff rate and the quit rate. Notice that, as already noticed by Shimer (2007), the elasticity of the layoff rate is not only lower than the other rates (in absolute value), the correlation is also weaker (as indicated by the  $R^2$  of the log-log regression in brackets).

(a) Turnover series



(b) Turnover cycle

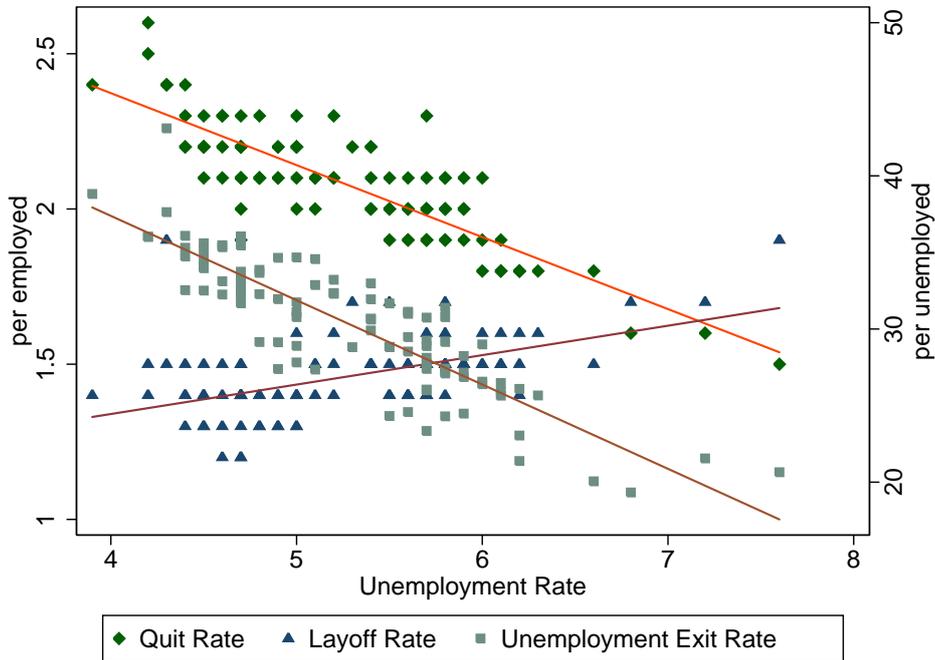


FIGURE 3. *Turnover Dynamics* (Source: JOLTS)

Using results in Jolivet, Postel-Vinay, and Robin (2006), who estimate a wage-posting equilibrium search model on PSID data, I estimate the proportion of employees' contacts

with alternative employers resulting in actual mobility to 53%. I thus set  $\tau = 0.5$ . Also, because the exit rate of unemployment is so high at the quarterly frequency,<sup>8</sup> I arbitrarily set  $\lambda_0 = 1$ . Then, the model predicts a job-to-job mobility rate  $f_{1t}$  such that  $f_{1t} = \tau\lambda_1(1 - s_t)$ . This implies a rate of on-the-job offer arrival of  $\lambda_1 = 0.13$ . This estimate is consistent with estimates from micro studies (see e.g. Jolivet, Postel-Vinay, and Robin 2006). Finally, I set the exogenous job destruction rate  $\delta$  equal to 4.3%. This implies a minimal frictional unemployment rate of  $\delta/(1 + \delta) = 4.12\%$ . This value is slightly greater than the two lows of the pre-1973 period, and it is slightly less than the lows of the post-1973 period (see Figure 4).

I set the number of aggregate states equal to  $N = 50$ , the number of different ability types equal to  $M = 500$  and I simulate very long series of  $T = 5000$  observations so as to match the following moments:<sup>9</sup>

- The mean productivity is 1 and the standard deviation of log productivity is equal to 0.02; the mean unemployment rate is 5.6% and the standard deviation of log unemployment is 0.19.<sup>10</sup>
- The mean layoff rate is 4.5% per quarter (1.5%/month).
- The standard deviation of log wages is 0.017, the elasticity of wages to productivity is 0.53.<sup>11</sup>
- Lastly, the mean values of D9/D5 and D5/D1 for wages are equal to 2.05 and 2.20, and the elasticity of D9/D5 and D5/D1 with respect to productivity are equal to 0.21 and -0.53 (from CPS).

<sup>8</sup>66% using the JOLTS data. Shimer estimates an even higher rate of 83% (45% per month, hence  $1 - (1 - .45)^3 = 83\%$  per quarter) using CPS data.

<sup>9</sup>A high number of worker types is necessary to smooth the dynamics of unemployment (more on this later) and I simulate a large number of observations to reduce the variance of empirical moments.

<sup>10</sup>These moments were calculated using HP-filtered, long (1947q1-2009q1), quarterly series from the BLS as in Shimer (2005).

<sup>11</sup>I use hourly compensation (BLS series PRS85006103) divided by the implicit output deflator (PRS85006113), readjusted per person by multiplying by hours (PRS85006033) and dividing by employment (PRS85006013). One argument in favour of this series is that when I detrend it using the HP-filter with the same smoothing parameter, I obtain exactly the same trend as for productivity, and regressing wages on productivity gives a coefficient of one. Note that the estimated elasticity is close to that calculated by Gertler and Trigari (2009) from CPS data (series posterior to 1967).

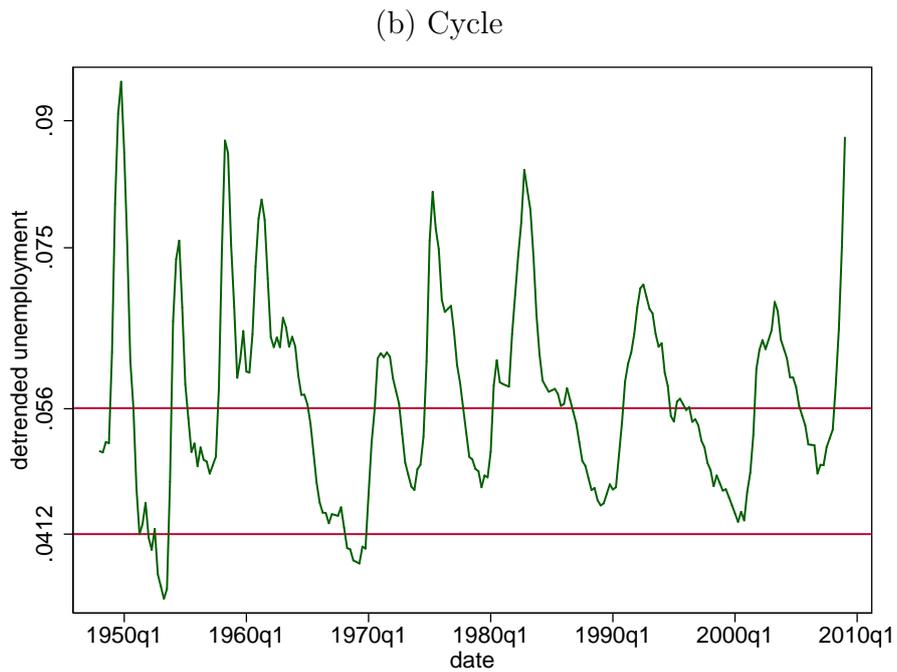
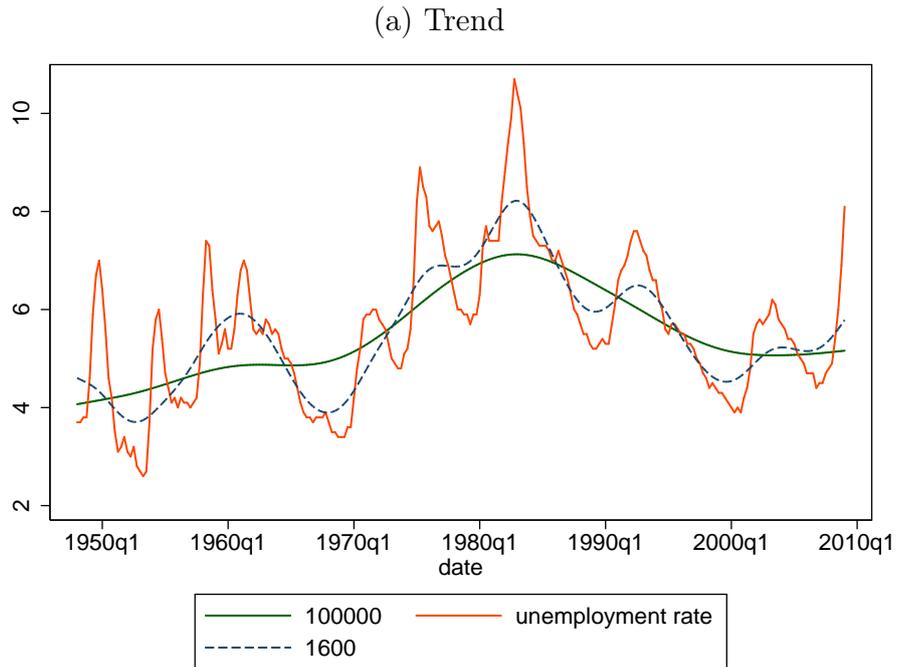
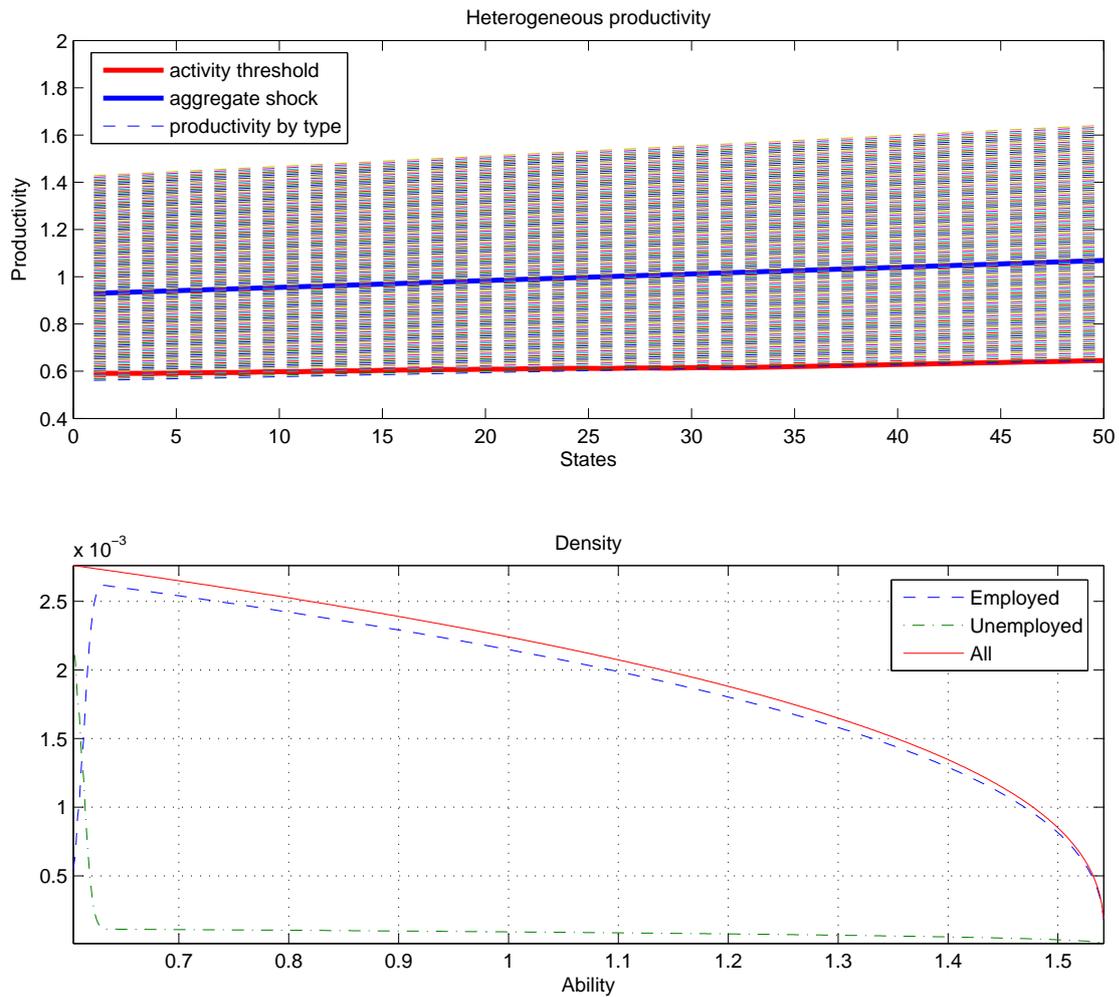


FIGURE 4. *Unemployment series* (The series are first log-transformed before passing the HP-filter. The cycle is then re-exponentiated and multiplied by the mean value of raw levels. This is done in order to avoid negative values in filtered series. The cycle in the bottom panel is obtained for a smoothing parameter of  $10^5$ . The top panel shows that the usual smoothing parameter value of 1,600 generates a very cyclical trend.)

FIGURE 5. *Heterogeneous Productivity*

The wage moments aim at identifying parameter  $\alpha$ , as for any value of  $\alpha$  there is an observationally equivalent value of  $(z_0, B, C)$  yielding the same unemployment values and surpluses, and also at identifying the range of worker heterogeneity  $[C, B + C]$ .

**3.4. Results.** I estimate  $\alpha = 0.5$ ,  $z_0 = 0.6115$ ,  $B = 0.935$ ,  $C = 0.5935$ ,  $\eta = 1.33$ .

Figure 5 shows the distribution of worker heterogeneity and how it affects individual productivity given the state of the economy. Every thin line in the top panel corresponds to a different ability type. The thick line in the middle is the aggregate productivity level  $y_i$ . The other thick line at the bottom indicates the viability threshold: for a given aggregate state  $i$ , all individual types  $m$  such that  $S_i(m) \leq 0$  have their productivity below the threshold. Only very few lines are below the threshold; namely, 16 (out of 500)

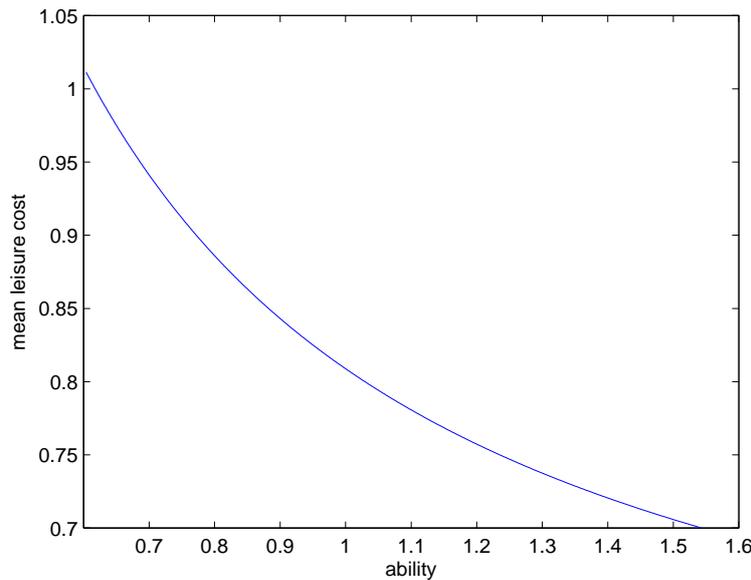


FIGURE 6. *Mean Unemployment Benefit/Output Ratio ( $z_t(m)/y_t(m)$ ) by Ability*

low productivity types, 4.85% of all workers, bear a risk of endogenous layoff. The bottom panel displays the distributions of workers' expected productivity in the whole population, and in the sub-populations of employed and unemployed workers. As expected, low ability workers are over-represented amongst the unemployed.

The mean leisure cost  $z_t(m)$  averaged over worker types and time is 0.80, somewhere between Hagedorn and Manovskii's (2008) calibration, 0.95, and Hall and Milgrom's (2008), 0.70. Workers in the low range of abilities have a mean unemployment benefit/productivity ratio close to one, whereas high productivity workers have one that is close to 0.70 (see Figure 6). We shall see that the ability of the model to match the volatilities of aggregate productivity and unemployment depends on there being a small fraction of workers at risk of endogenous job destruction. So, the argument of this paper does not contradict the small surplus argument of Hagedorn and Manovskii.

**3.5. Employment and turnover.** Table 3 compares various moments calculated on the actual quarterly series as in Shimer (2005) and on the simulated series. The model also predicts an exit rate of unemployment in the right interval albeit with a slightly higher volatility. The moments of the overall separation rate are well reproduced. The model tends to overestimate the correlation with productivity, which induces too much elasticity

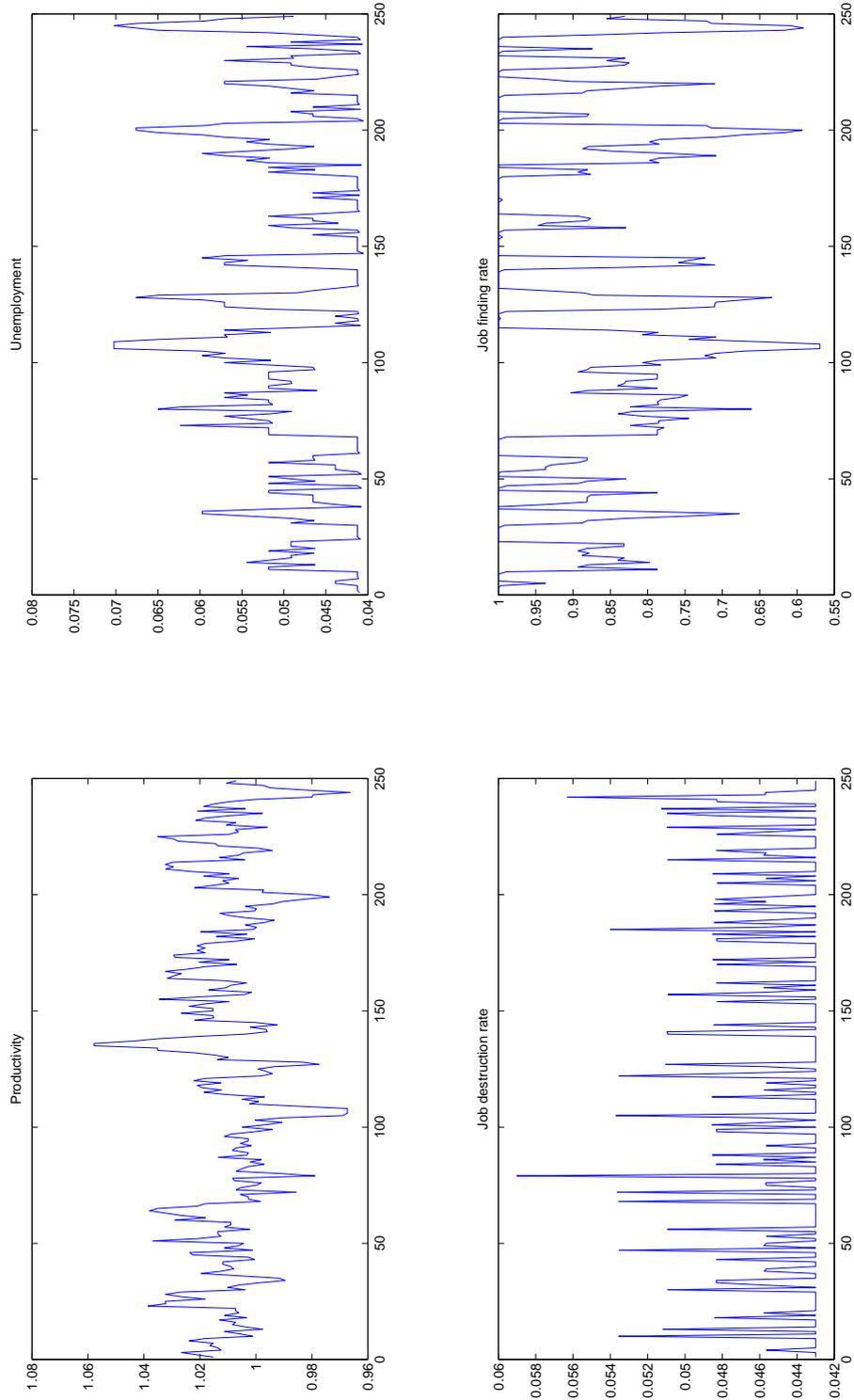


FIGURE 7. *Simulation of Employment and Turnover Dynamics*

given that the volatility is well fitted. Figure 7 shows a simulation of the dynamics of unemployment and turnover resulting from one particular simulated history of aggregate productivity shocks (the number of observations is  $T = 249$  which is the number of quarters in 1947q1-2009q1). The range of unemployment rates is as in the actual series, but the simulated trajectory seems less smooth than the true one (see Figure 4).

The elasticity of the exit rate from unemployment with respect to unemployment is correctly reproduced (close to -1) but the elasticities of the separation rate and, even more so, the job-to-job mobility rate are underestimated with respect to the values that were calculated with the JOLTS series (Table 2). Yet it is remarkable that the model predicts that the elasticity of the exit rate of unemployment is bigger than the elasticity of job separations. This is because the whole volatility of unemployment results from the behaviour of a small fraction of workers (about 5%), that remains small in the stock of employees but makes a large proportion of the stock of unemployed.

The mechanism by which productivity shocks are amplified is simple to understand. In a boom, unemployment is steady, all separations follow from exogenous shocks (there is no endogenous layoff in the last 18 aggregate states ( $32 \leq i \leq 50$ )). When aggregate productivity falls more workers lose their jobs as more match surpluses become negative (see Figure 8). About 4% unemployment accrues because of the 4.3% exogenous layoff rate. One may call this minimum unemployment level frictional unemployment. Classical unemployment, due to business cycle conditions, ranges between 0 and 5% depending on the severity of the recession. Note that the correlation between unemployment rates and productivity shocks is high because the link shown in Figure 8 is smooth and monotone. A smaller correlation requires more nonlinearity which can be obtained by reducing the aggregate productivity threshold that triggers endogenous layoff.

**3.6. Wages.** Table 4 shows that the model can replicate the dynamics of first and second-order statistics of cross-sectional wage distributions (means and inequality indices) well. In particular, the dynamics of wage inequality in the upper part of the distribution is procyclical, and it is counter-cyclical in the bottom part. Overall, countercyclicality dominates. I also compare annual earnings with present values. Given that labour supply

	$\bar{y}_t$	$u_t$	$f_{0t}$	$s_t$	$f_{1t}$
	Actual moments ((Shimer, 2005))				
mean	1	0.056	0.83	0.045	
std	0.020	0.19	0.118	0.075	
corr with $\ln \bar{y}_t$	1	-0.41	0.40	-0.52	
reg on $\ln \bar{y}_t$	1	-4.08	4.56	-1.95	
	Simulated moments				
mean	1	0.054	0.82	0.045	0.062
std	0.018	0.18	0.18	0.067	0.0034
corr wrt $\ln \bar{y}_t$	1	-0.95	0.88	-0.38	0.37
reg on $\ln \bar{y}_t$	1	-9.50	8.86	-1.43	0.072
reg on $\ln u_t$		1	-0.95	0.128	-0.006

TABLE 3. *Fit of Employment and Turnover Moments* (Rows labelled “mean” refer to the mean of levels while the other rows refer to the log of the variable in each column.)

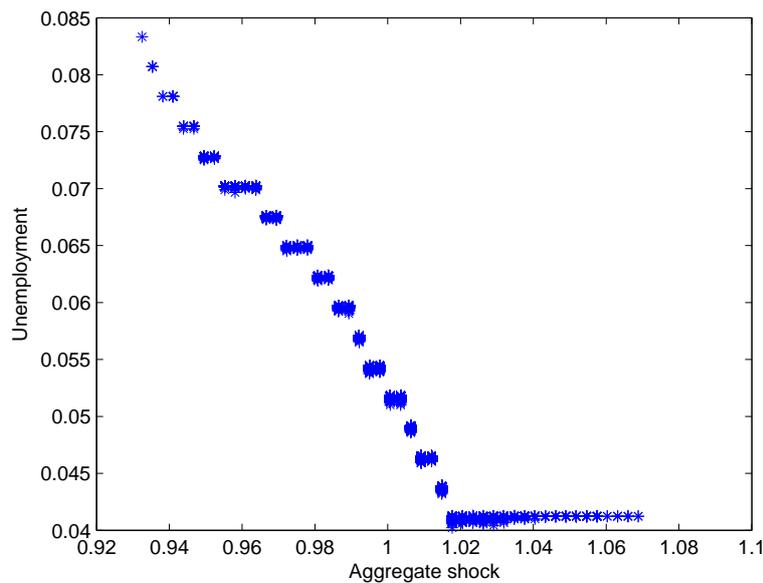


FIGURE 8. *Unemployment Rate as a Function of the Aggregate Shock* (For each step down a new group of low ability workers of about the same size (0.43%) becomes employable as aggregate productivity rises. There are 17 steps because only 16 out of  $M = 500$  workers types, 4.4% of all workers, face endogenous unemployment risk. When the aggregate productivity index reaches about 1.02 all workers have positive surplus. Note that the unemployment rate does not quite jump to its state-contingent equilibrium value, but nearly does.)

is reduced to the extensive margin in the model, it makes sense to consider present values as a way of reconstructing an intensive margin. Countercyclicalities now shows up in both the upper and the bottom parts of the present value distributions. One difference between

	Hourly wages				Annual earnings		
	Mean	D9/D5	D5/D1	D9/D1	D9/D5	D5/D1	D9/D1
Actual series							
mean		2.05	2.20	4.55	2.08	2.88	6.00
std (*)	0.017	0.023	0.030	0.026	0.018	0.078	0.081
corr w/ $\bar{y}_t$ (*)	0.64	0.15	-0.31	-0.21	-0.10	-0.31	-0.32
reg on $\bar{y}_t$ (*)	0.53	0.21	-0.53	-0.32	-0.10	-1.41	-1.51
Simulated series							
	Wages				Present values		
	Mean	D9/D5	D5/D1	D9/D1	D9/D5	D5/D1	D9/D1
mean	0.84	1.57	2.52	3.96	1.22	1.28	1.55
std	0.012	0.0051	0.009	0.0054	0.0028	0.0040	0.0064
corr w/ $\bar{y}_t$	0.86	0.94	-0.84	-0.54	-0.83	-0.85	-0.93
reg on $\bar{y}_t$	0.53	0.27	-0.40	-0.13	-0.13	-0.20	-0.34

TABLE 4. *Fit of Wage Moments* (Column “Mean” either refers to the BLS series (deflated per person compensation) or to the cross-section mean in the simulated data. Columns “D9/D5”, “D5/D1” and “D9/D1” are the decile ratios of either hourly wages and annual earnings calculated from the CPS (panel “Actual series”), or wages and present values,  $U_i(m)$  or  $W_i(m)$ , for simulated data.)

actual and simulated data is that the volatilities of simulated inequality indices are much lower, equal to about one fourth of their actual values. This suggests that there are sources of wage dispersion that the model does not account for (firm heterogeneity for example).

So the model can generate a median wage that is less procyclical than the first and last deciles. Table 5 shows an interesting phenomenon that goes one step forward towards an explanation. The bottom of the wage distribution only includes starting wages ( $\underline{w}_i(m)$ ) and the top only promotion wages ( $\bar{w}_i(m)$ ), and for this calibration at least, the median wage is also a promotion wage. Moreover, starting wages are considerably more procyclical than promotion wages. Lastly, the procyclicality of starting wages diminishes with their rank, while the opposite is true for promotion wages. Consequently, wages in the middle of the distribution are the least procyclical.

Pissarides (2009) builds an argumentation based on wages in new jobs being different from wages in on-going spells. He also documents a long list of empirical papers contrasting the cyclicity of wages at the beginning of job spells and that of wages in on-going spells. Initial wages are usually found more procyclical than on-going wages. Note that

	Starting wages	Promotion wages	All wages
	Mean		
D9	0.59	1.37	1.33
D5	0.40	0.98	0.85
D1	0.15	0.69	0.30
	Cyclicalities (elasticity wrt productivity)		
D9	0.85	0.69	0.65
D5	1.00	0.56	0.39
D1	1.74	0.38	0.78

TABLE 5. *Cyclicalities of Starting vs Promotion Wages* (simulated data)

the opposition between initial and on-going wages that is used in the literature is less marked than the opposition between starting and promotion wages that is introduced here; first, because on-going wages is not a very precise concept, and second, because first wages in new jobs are qualitatively different depending on whether a new job follows unemployment or another job. Therefore, I conjecture the empirical elasticity wedge between starting and promotion wages to be bigger than between initial and on-going wages.

In order to better understand why starting wages and promotion wages have these distinct cyclical patterns, I next consider the following variance decomposition exercise. Let  $w_{it}$  denote the wage of an individual  $i$  at time  $t$ , and  $z_{it}$  some characteristics, then

$$\text{Var } w_{it} = \underbrace{\text{Var } \mathbb{E}(w_{it}|z_{it})}_{\text{between}} + \underbrace{\mathbb{E}\text{Var}(w_{it}|z_{it})}_{\text{within}}.$$

Three variables  $z_{it}$  contribute to wage dispersion (the set  $\{\underline{w}_j(m), \bar{w}_j(m), \forall j, m\}$ ): worker heterogeneity ( $m$ ), aggregate state dependence ( $j$ ) and the worker's threat point in bargaining ( $\underline{w}$  or  $\bar{w}$ ).

Table 6 shows the between and within contributions of each of these three sources of wage dispersion. The threat point explains 60% of wage dispersion; aggregate state dependence, 40%; and ability only 15%. Bertrand competition, via the difference between starting wages and promotion wages is the main determinant of the level of inequality. However, only aggregate state dependence contributes negatively, and strongly, to cyclicalities.

Variance	Total	Between	Within
	Group = ability		
Share	0.127	0.022 (15%)	0.116
Std of log	0.027	0.055	0.022
Elasticity	1.29	2.92	1.00
	Group = aggregate state		
Share	0.127	0.052 (40%)	0.076
Std of log	0.027	0.19	0.11
Elasticity	1.29	-2.42	3.57
	Group = starting/promotion		
Share	0.127	0.083 (60%)	0.055
Std of log	0.027	0.021	0.039
Elasticity	1.29	0.89	1.90

TABLE 6. *Wage Variance Decomposition*

Thus, although the mechanism for amplifying productivity shocks in unemployment volatility that I discuss here bears no relation with wage stickiness, in contradistinction with the previous literature on the subject, wage stickiness seems to be decisive to explain the relative cyclical patterns of different wage quantiles. When productivity increases workers with a low wage (a starting wage) credibly threaten to quit to unemployment as their reservation wage increases with aggregate productivity, and firms are thus forced to renegotiate wages up. This is the main determinant of the stronger procyclicality of low wages. At the other end of the distribution, when aggregate productivity falls (in a downturn), workers with high wages are forced by their employer to accept a cut as the firm surplus becomes negative. This is the main determinant of the stronger procyclicality of high wages. Wage renegotiation without alternative offers therefore has a remarkable effect on wage inequality dynamics.

#### 4. CONCLUSION

We have proposed a simple dynamic search-matching model with cross-sectional wage dispersion and worker heterogeneous abilities. Worker heterogeneity interacts with aggregate shocks to match productivity in a way that allows for endogenous job destruction. It suffices that a small fraction of the total workforce be at risk of a shock to productivity that renders the match surplus negative to amplify productivity shocks enough to

generate the observed unemployment volatility. Moreover, we show that the model can generate inequality dynamics similar to the observed pattern: wages in the middle of the distribution are less procyclical than wages in the bottom and the top. We argue that it reflects the lumpy renegotiation process implied by long-term contracts following productivity shocks. Extreme wages are subject to renegotiation as low wages may become lower than workers' reservation wages after a positive productivity shock and high wages may become greater than firms' reservation wages following a negative shock. Wages in the middle of the distribution are more likely to remain in the bargaining set.

Our prototypical model is extremely simple to simulate outside the steady-state equilibrium and still generates very rich dynamics. This is due to two very strong assumptions: firms have full monopsony power and they are identical. Giving workers some bargaining power as in Cahuc, Postel-Vinay, and Robin (2006) and Dey and Flinn (2005) and allowing for firm heterogeneity as in Lise, Meghir, and Robin (2009), in a macrodynamic model, are very exciting avenues for further research.

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