CEMMAP Masterclass: Empirical Models of Comparative Advantage and the Gains from Trade[1]
— Lecture 3: Gravity Models —

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[1] All material based on earlier courses taught jointly with Arnaud Costinot (MIT).

Plan for this Lecture

1. **The Simplest Gravity Model: Armington**
2. Gravity Models and the Gains from Trade: ACR (2012)
3. Beyond ACR's (2012) Equivalence Result: CR (2013)

The Armington Model: Armington (1969)

The Armington Model: Equilibrium

- Labor endowments

$$L_i \text{ for } i = 1, \ldots n$$

- CES utility $\Rightarrow$ CES price index

$$P_j^{1-\sigma} = \sum_{i=1}^{n} \left(w_i \tau_{ij}\right)^{1-\sigma}$$

- Bilateral trade flows follow **gravity equation**:

$$X_{ij} = \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{l=1}^{n} \left(w_l \tau_{lj}\right)^{1-\sigma}} w_j L_j$$

- In what follows $\varepsilon \equiv -\frac{d \ln X_{ij}/X_{jj}}{d \ln \tau_{ij}} = \sigma - 1$ denotes the **trade elasticity**
- Trade balance

$$\sum_i X_{ji} = w_j L_j$$

The Armington Model: Welfare Analysis

- **Question:**
  Consider a foreign shock: $L_i \to L_i'$ *for* $i \neq j$ *and* $\tau_{ij} \to \tau_{ij}'$ *for* $i \neq j$.
  How do foreign shocks affect real consumption, $C_j \equiv w_j / P_j$*?*

- Shephard's Lemma implies

$$d \ln C_j = d \ln w_j - d \ln P_j = -\sum_{i=1}^{n} \lambda_{ij} \left(d \ln c_{ij} - d \ln c_{jj} \right)$$

  with $c_{ij} \equiv w_i \tau_{ij}$ and $\lambda_{ij} \equiv X_{ij} / w_j L_j$.

- Gravity implies

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon \left(d \ln c_{ij} - d \ln c_{jj} \right).$$

The Armington Model: Welfare Analysis

- Combining these two equations yields

$$d \ln C_j = \frac{\sum_{i=1}^{n} \lambda_{ij} \left(d \ln \lambda_{ij} - d \ln \lambda_{jj} \right)}{\varepsilon}.$$

- Noting that $\sum_i \lambda_{ij} = 1 \Longrightarrow \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0$ then

$$d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}.$$

- Integrating the previous expression yields (with $\hat{x} \equiv x'/x$)

$$\hat{C}_j = \hat{\lambda}_{jj}^{-1/\varepsilon}.$$

The Armington Model: Welfare Analysis

- In general, predicting $\hat{\lambda}_{jj}$ requires (computer) work
  - We can use "exact hat algebra" as in DEK (previous lecture)
  - Requires data on initial levels of data $\{\lambda_{ij}, Y_j\}$, and ε
  - Where to get ε? Gravity equation suggests natural (but not the only) way would be to estimate (via OLS):

$$\ln X_{ij} = \alpha_i + \alpha_j - \varepsilon \ln t_{ij} + \nu_{ij}$$

  - With α_i as fixed-effects and t_{ij} some observable and exogenous component of trade costs (that satisfies $\tau_{ij} = t_{ij}\nu_{ij}$).
- But predicting how bad it would be to shut down all trade is easy...
  - In autarky, $\lambda_{jj} = 1$. So

$$C_j^A / C_j = \lambda_{jj}^{1/\varepsilon}$$

  - Thus **gains from trade** can be computed as

$$GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon}$$

The Armington Model: Gains from Trade

- Suppose that we have estimated the trade elasticity using the gravity equation (as described above).
  - Central estimate in the literature is $\varepsilon = 5$; see Head and Mayer (2013) Handbook chapter.
- Using World Input Output Database (2008) to get λ_{jj}, we can then estimate gains from trade:

	λ_{jj}	$\%\ GT_j$
Canada	0.82	3.8
Denmark	0.74	5.8
France	0.86	3.0
Portugal	0.80	4.4
Slovakia	0.66	7.6
U.S.	0.91	1.8

Cheese, really?

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Motivation

- **New Trade Models**
  - Micro-level data have lead to **new questions** in international trade:
    - How many firms export?
    - How large are exporters?
    - How many products do they export?
  - New models highlight **new margins** of adjustment:
    - From inter-industry to intra-industry to intra-firm reallocations
- **Old question:**
  - How large are the gains from trade (GT)?
- **Arkolakis, Costinot and Rodriguez-Clare (AER, 2012) question:**
  - How do new trade models affect the magnitude of GT?

ACR's Main Equivalence Result

- ACR focus on gravity models
  - Perfect comp.: Armington and Eaton & Kortum '02
  - Monopolistic comp.: Krugman '80 and many variations of Melitz '03
- Within that class, welfare changes are ($\hat{x} = x'/x$)

$$\hat{C} = \hat{\lambda}^{1/\varepsilon}$$

- **Two sufficient statistics** for welfare analysis are:
  - Share of domestic expenditure, λ;
  - Trade elasticity, ε
- **Two views** on ACR's result:
  - Optimistic: welfare predictions of Armington model are more robust than you might have thought
  - Pessimistic: within that class of models, micro-level data do not matter

- **CES utility**
  - Consumer price index,

$$P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega,$$

- **One factor of production:** labor
  - $L_i \equiv$ labor endowment in country i
  - $w_i \equiv$ wage in country i

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = \underbrace{q w_i \tau_{ij} \alpha_{ij}(\omega)\, t^{\frac{1}{1-\sigma}}}_{\text{variable cost}} + \underbrace{w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega)\, m_{ij}(t)}_{\text{fixed cost}},$$

q : quantity,
τ_{ij} : iceberg transportation cost,
$\alpha_{ij}(\omega)$: good-specific heterogeneity in variable costs,
ξ_{ij} : fixed cost parameter,
$\phi_{ij}(\omega)$: good-specific heterogeneity in fixed costs.

Primitive Assumptions

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = q w_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega) m_{ij}(t)$$

$m_{ij}(t)$: cost for endogenous destination specific technology choice, t,

$$t \in [\underline{t}, \overline{t}] \ , \ m'_{ij} > 0, \ m''_{ij} \geq 0$$

- Heterogeneity across goods

$$G_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n) \equiv \{\omega \in \Omega \mid \alpha_{ij}(\omega) \leq \alpha_i, \ \phi_{ij}(\omega) \leq \phi_i, \ \forall i\}$$

Primitive Assumptions

Market Structure

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.
- **Monopolistic competition**
  - Either firms in i can pay $w_i F_i$ for monopoly power over a random good.
  - Or exogenous measure of firms, $\overline{N}_i < \overline{N}$, receive monopoly power.
- Let N_i be the measure of goods that can be produced in i
  - Perfect competition: $N_i = \overline{N}$
  - Monopolistic competition: $N_i < \overline{N}$

- Bilateral trade flows are defined as:

$$X_{ij} \equiv \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega)\, d\omega$$

- **R1:** *For any country j,*

$$\sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji}$$

  - Trivial if perfect competition or $\beta = 0$.
  - Non trivial if $\beta > 0$.

- **R2:** *For any country* j,

$$\Pi_j / \left(\sum_{i=1}^{n} X_{ji} \right) \text{ is constant}$$

where Π_j : aggregate profits gross of entry costs, $w_j F_j$, (if any)

  - Trivial under perfect competition.
  - Direct from CES preferences in Krugman (1980)—each firm has sales equal to $\frac{\sigma}{\sigma - 1}$ times costs.
  - Non-trivial in more general environments.

- *Import demand system* defined as:

$$(\mathbf{w}, \mathbf{N}, \boldsymbol{\tau}) \rightarrow \mathbf{X}$$

- **R3:**

$$\varepsilon_j^{ii'} \equiv \partial \ln \left(X_{ij} / X_{jj} \right) / \partial \ln \tau_{i'j} = \begin{cases} \varepsilon < 0 & i = i' \neq j \\ 0 & \textit{otherwise} \end{cases}$$

- Note: symmetry and separability.

Macro-Level Restrictions

Comments on CES Import Demand System

- The *trade elasticity* ε is an *upper-level* elasticity: it combines
  - $x_{ij}(\omega)$ (*intensive margin*)
  - Ω_{ij} (*extensive margin*).
- R3 $\implies$ complete specialization.
- R1-R3 are not necessarily independent
  - E.g., if $\beta = 0$ then R3 $\implies$ R2.

- **R3':** The IDS satisfies

$$X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot (w_i \tau_{ij})^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot (w_{i'} \tau_{i'j})^{\varepsilon}}$$

  where χ_{ij} is independent of $(\mathbf{w}, \mathbf{M}, \boldsymbol{\tau})$.

- Same restriction on $\varepsilon_j^{ii'}$ as R3, but additional structural relationships (R3' $\Rightarrow$ R3 but converse not true)

- Most prominent models that satisfy R3':
  - PC: Armington with CES prefs and EK (2002)
  - MC: Krugman (1980 AER), and Melitz (2003 ECMA) with Pareto-distributed firm-level productivity distribution

Welfare results

- State of the world economy:

$$\mathbf{Z} \equiv (\mathbf{L}, \tau, \xi)$$

- *Foreign shocks*: a change from $\mathbf{Z}$ to $\mathbf{Z}'$ with no domestic change.
  - Effects of a domestic change in L_i would be different between PC and MC models (due to the home-market effect which is at work in the MC models but not in the PC models...more on this in Lecture 5).

Equivalence (I)

- **Proposition 1:** *Suppose that R1-R3 hold. Then for any foreign shock it must be true that*

$$\widehat{W_j} = \widehat{\lambda}_{jj}^{1/\varepsilon}.$$

- Implication: $\widehat{\lambda}_{jj}$ acts as (along with elasticity ε) a sufficient statistic for the fully global (i.e. arbitrarily large changes) GE welfare analysis for any country j
- Note that it is still true that for any of these models we could estimate ε from OLS gravity regression:

$$\ln X_{ij} = \alpha_i + \alpha_j - \varepsilon \ln t_{ij} + \nu_{ij}$$

- New margins affect structural interpretation of ε
  - ...and composition of gains from trade (GT)...
  - ... but size of GT is the same.

Gains from Trade Revisited

- Proposition 1 is an *ex-post* result (based on seeing $\widehat{\lambda}_{jj}$).
- A simple *ex-ante* result (for the case of going to autarky):
- **Corollary 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}.$$

Equivalence (II)

- A stronger ex-ante result for **variable trade costs** (and things that act like variable trade costs, like productivity shocks; but not for other types of foreign shocks) under R1-**R3'**:
- **Proposition 2:** *Suppose that R1-R3' hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$$

where

$$\widehat{\lambda}_{jj} = \left[\sum_{i=1}^{n} \lambda_{ij} \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}\right]^{-1},$$

and

$$\widehat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{\varepsilon}}.$$

- So: ε and $\{\lambda_{ij}\}$ are sufficient to predict $\widehat{W}_j$ (ex-ante) from $\hat{\tau}_{ij}$, $i \neq j$.

Taking Stock

- ACR consider models featuring:
  - (i) CES preferences;
  - (ii) one factor of production;
  - (iii) linear cost functions; and
  - (iv) perfect or monopolistic competition;

  with three macro-level restrictions:
  - (i) trade is balanced;
  - (ii) aggregate profits are a constant share of aggregate revenues; and
  - (iii) a CES import demand system.
- Equivalence for ex-post welfare changes and GT
  - under R3' equivalence carries over to ex-ante welfare changes
  - So if R3' is satisfied, then models in this ACR class agree on all implications of changes in variable trade costs

A note on methodology

- ACR set out to answer the question of whether different trade models predict different GT
- In some of these cases, tempting to think that it is easy to rank GT across these models.
  - E.g., following Melitz and Redding (AER, 2015), note that since Krugman is a strictly nested special case of Melitz for which the firm productivity distribution is degenerate, the Melitz model is Krugman plus an additional margin of adjustment.
  - And since both models are efficient (i.e. correspond to planner's problem, so equilibrium is maximizing something—see Dhingra and Morrow, JPE 2017), an additional margin of adjustment guarantees that the damage done by a negative shock (e.g. move to autarky) is less bad in Melitz than in Krugman.
- But note that this is not the ACR thought experiment.
  - ACR's thought experiment is to ask about GT (or any other counterfactual), conditional on the data $\{\lambda_{ij}, Y_j\}$ and elasticity ε we have.

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Departing from ACR's (2012) Equivalence Result

- Costinot and Rodriguez-Clare (Handbook of International Econ, 2013) has nice discussion of general cases.
- **Other Gravity Models:**
    - Multiple Sectors
    - Tradable Intermediate Goods
    - Multiple Factors
    - Variable Markups (ACDR 2012—more on this in Lecture 5)

Multiple sectors, GT

- Nested CES: Upper level EoS ρ and lower level EoS ε_s

- Recall gains for Canada of 3.8%. Now gains can be much higher: $\rho = 1$ implies $GT = 17.4\%$

Tradable intermediates, GT

- Set $\rho = 1$, add tradable intermediates with Input-Output structure
- Labor shares are $1 - \alpha_{j,s}$ and input shares are $\alpha_{j,ks}$ ($\sum_k \alpha_{j,ks} = \alpha_{j,s}$)

Tradable intermediates, GT

	% GT_j	% GT_j^{MS}	% GT_j^{IO}
Canada	3.8	17.4	30.2
Denmark	5.8	30.2	41.4
France	3.0	9.4	17.2
Portugal	4.4	23.8	35.9
U.S.	1.8	4.4	8.3

Multiple sectors: a combination of micro and macro features

- In Krugman, free entry $\Rightarrow$ scale effects associated with total employment
- In Melitz, additional scale effects associated with sales in each market
- In both models, trade may affect entry and fixed costs
- All these effects do not play a role in the one sector model (since factor supply to that one sector is assumed to be fixed)
- With multiple sectors and traded intermediates, these effects come back (since inter-sectoral factor supply is now endogenous)

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	29.5	11.2	22.5	29.2	8.0
MS, IO, MC (Krugman)	33.0	28.0	41.4	20.8	8.6
MS, IO, MC (Melitz)	**39.8**	**77.9**	**52.9**	**20.7**	**10.3**