Welfare analysis using nonseparable models

Stefan Hoderlein,

Boston College

and

Anne Vanhems,

Toulouse Business School and Toulouse School of Economics

Shape Restrictions in Non and Semiparametric Estimation of Econometric Models

November 2010

1. Motivation and general context.
   - Application in Microeconomics.
   - Objectives.
2. Nonseparable demand model.
   - Identification.
   - Estimation and convergence.
3. Welfare model.
   - Identification.
   - Estimation and convergence.
4. Conclusion.

Motivation.

- Estimation of the variation of exact consumer surplus associated with a price variation.
- Consider one consumer; y the income; p^0 the price of a unique good; q the demand in good.

  $\lambda(p) = e(p,u) - e\left(p^0,u\right) = e(p,u) - y$, with e the cost

  function and u the utility level.
- Link between q and λ:

$$\begin{cases} v(p,e(p,u)) = & v(p^0,y) \\ \frac{\partial_1 v(p,y)}{\partial_2 v(p,y)} = & -q(p,y) \end{cases}$$ (Roy's Identity)

Motivation.

- Estimation of the variation of exact consumer surplus associated with a price variation.
- Consider one consumer; y the income; p^0 the price of a unique good; q the demand in good.

  $\lambda(p) = e(p,u) - e(p^0,u) = e(p,u) - y$, with e the cost

  function and u the utility level.

- Link between q and λ:

$$\begin{cases} \lambda'(p) & = q(p, y + \lambda(p)) \\ \lambda(p^0) & = 0 \end{cases}$$

Motivation.

Two step methodology

- Nonparametric estimation of the demand function q
  ex: kernel estimation see *Hausman Newey 1995*
- Nonparametric estimation of the exact consumer surplus λ
  ex: numerical resolution of the differential equation like Euler Cauchy algorithm, Runge Kutta method, ...

Application to Gasoline demand

- *Hausman Newey 1995*, data from the US Department of Energy at the household level.
  The gasoline price is the weighted average of purchase price over a month, with variations due to state and city taxes.
- *Yatchew No 2001*, Household gasoline demand in Canada.
- *Blundell Horowitz Parey 2009*, US National Household Travel Survey

Motivation.

Two step methodology

- Nonparametric estimation of the demand function q
  ex: kernel estimation see *Hausman Newey 1995*
- Nonparametric estimation of the exact consumer surplus λ
  ex: numerical resolution of the differential equation like Euler Cauchy algorithm, Runge Kutta method, ...

Application to Gasoline demand

- *Hausman Newey 1995*, data from the US Department of Energy at the household level.
  The gasoline price is the weighted average of purchase price over a month, with variations due to state and city taxes.
- *Yatchew No 2001*, Household gasoline demand in Canada.
- *Blundell Horowitz Parey 2009*, US National Household Travel Survey

Motivation.

First demand model

Prices are exogenous-separable model

$$\begin{cases} Q_i = \varphi\left(P_i, Y_i, \tilde{Z}_i\right) + A_i, i = 1, ..., n \\ E\left[A \middle| P, Y, \tilde{Z}\right] = 0 \end{cases}$$

$\tilde{Z}$: exogenous observable characteristics, $Z = (Y, \tilde{Z})$

$\varphi(p, z) = E[Q | P = p, Z = z]$

Nonparametric estimation

$$\begin{cases} \hat{\varphi}(p, z) = \frac{\sum_{i=1}^{n} Q_i K\left(\frac{p-P_i}{h_n}\right) K\left(\frac{z-Z_i}{h_n}\right)}{\sum_{i=1}^{n} K\left(\frac{p-P_i}{h_n}\right) K\left(\frac{z-Z_i}{h_n}\right)} \end{cases} \quad \begin{cases} \hat{\lambda}'(p) & = \hat{\varphi}\left(p, y + \hat{\lambda}(p), \tilde{z}\right) \\ \hat{\lambda}(p^0) & = 0 \end{cases}$$

Consistency and asymptotic normality for $\hat{\varphi}$ and $\hat{\lambda}$

Hausman and Newey 1995; Vanhems 2006

Motivation.

First demand model

Prices are exogenous-separable model

$$\begin{cases} Q_i = \varphi\left(P_i, Y_i, \tilde{Z}_i\right) + A_i, i = 1, ..., n \\ E\left[A \middle| P, Y, \tilde{Z}\right] = 0 \end{cases}$$

$\tilde{Z}$: exogenous observable characteristics, $Z = (Y, \tilde{Z})$
$\varphi(p, z) = E[Q | P = p, Z = z]$

Nonparametric estimation

$$\begin{cases} \hat{\varphi}(p, z) = \dfrac{\sum_{i=1}^{n} Q_i K\left(\frac{p-P_i}{h_n}\right) K\left(\frac{z-Z_i}{h_n}\right)}{\sum_{i=1}^{n} K\left(\frac{p-P_i}{h_n}\right) K\left(\frac{z-Z_i}{h_n}\right)} \end{cases} \quad \begin{cases} \widehat{\lambda}'(p) & = \widehat{\varphi}\left(p, y + \widehat{\lambda}(p), \tilde{z}\right) \\ \widehat{\lambda}(p^0) & = 0 \end{cases}$$

Consistency and asymptotic normality for $\widehat{\varphi}$ and $\widehat{\lambda}$

Hausman and Newey 1995; Vanhems 2006

Motivation.

First demand model

Prices are exogenous-separable model

$$\begin{cases} Q_i = \varphi\left(P_i, Y_i, \tilde{Z}_i\right) + A_i, i = 1, ..., n \\ E\left[A \middle| P, Y, \tilde{Z}\right] = 0 \end{cases}$$

$\tilde{Z}$: exogenous observable characteristics, $Z = (Y, \tilde{Z})$

$\varphi(p, z) = E[Q | P = p, Z = z]$

Nonparametric estimation

$$\left\{ \hat{\varphi}(p, z) = \frac{\sum_{i=1}^{n} Q_i K\left(\frac{p-P_i}{h_n}\right) K\left(\frac{z-Z_i}{h_n}\right)}{\sum_{i=1}^{n} K\left(\frac{p-P_i}{h_n}\right) K\left(\frac{z-Z_i}{h_n}\right)} \right. \qquad \begin{cases} \widehat{\lambda}'(p) & = \widehat{\varphi}\left(p, y + \widehat{\lambda}(p), \tilde{z}\right) \\ \widehat{\lambda}\left(p^0\right) & = 0 \end{cases}$$

Consistency and asymptotic normality for $\widehat{\varphi}$ and $\widehat{\lambda}$

Hausman and Newey 1995; Vanhems 2006

Motivation.

Second demand Model

- Prices are endogenous-separable model

$$\begin{cases} Q = \varphi(P, Z) + A \\ E[A|P, Z] \neq 0 \end{cases}$$

- Example of price endogeneity in demand function (Yatchew and No 2001)
- Nonparametric estimation using *Instrumental variables* or *control functions*

  Darolles, Florens and Renault (2002), Newey and Powell (2003), Hall and Horowitz (2005), Gagliardini and Scaillet (2007), Blundell and Horowitz (2007), Blundell, Chen and Kristensen (2007), Carrasco, Florens, Renault (2008), Chen and Reiss (2008), Johannes, Van Bellegem, Vanhems (2008)
- Consistency and asymptotic properties *Vanhems 2010*

Motivation.

Third demand Model

- (exogenous / endogenous) nonseparable model

$$Q = \varphi(P, Z, A)$$

A: unobservable heterogenous preference parameters

Literature

Blundell Powell 2003, Chesher 2003, 2005, 2007, Matzkin 2003, 2005, 2007, 2008, Altonji and Matzkin 2005, Horowitz Lee 2007, Hoderlein and Mammen 2007, Blundell Horowitz Parey 2009, Imbens and Newey 2009, Blundell Kristensen Matzkin 2010

Objectives.

Study the interest parameter λ solution of:

$$\begin{cases} \lambda'(p) & = \varphi(p, y + \lambda(p), \tilde{z}, a) \\ \lambda(p^0) & = 0 \end{cases}$$

where the function φ is defined by:

$$Q = \varphi(P, Z, A)$$

and P exogenous / endogenous variable

Two inverse problems to solve

Objectives.

Study the interest parameter λ solution of:

$$\begin{cases} \lambda'(p) & = \varphi(p, y + \lambda(p), \tilde{z}, a) \\ \lambda(p^0) & = 0 \end{cases}$$

where the function φ is defined by:

$$Q = \varphi(P, Z, A)$$

and P exogenous / endogenous variable

Two inverse problems to solve

Objectives.

Study the interest parameter λ solution of:

$$\begin{cases} \lambda'(p) & = \varphi(p, y + \lambda(p), \tilde{z}, a) \\ \lambda(p^0) & = 0 \end{cases}$$

where the function φ is defined by:

$$Q = \varphi(P, Z, A)$$

and P exogenous / endogenous variable

Two inverse problems to solve

Nonseparable demand model.

Demand model.

$$Q = \varphi(P, Z, A)$$

with $Q \in R$ observed demand, $Z = (Y, \tilde{Z}) \in R^{1+L}$ observed characteristics, and A unobserved individual heterogeneity, uniformly distributed on $[0, 1]$.

First case: P exogenous and A scalar

Classical setting of nonseparable model, Matzkin 2003

Second case: P endogenous and correlated with A

Chesher 2003, Imbens and Newey 2009

Nonseparable demand model.

Demand model.

$$Q = \varphi(P, Z, A)$$

with $Q \in R$ observed demand, $Z = (Y, \tilde{Z}) \in R^{1+L}$ observed characteristics, and A unobserved individual heterogeneity, uniformly distributed on $[0, 1]$.

First case: P exogenous and A scalar

Classical setting of nonseparable model, Matzkin 2003

Second case: P endogenous and correlated with A

Chesher 2003, Imbens and Newey 2009

Nonseparable demand model.

Demand model.

$$Q = \varphi(P, Z, A)$$

with $Q \in R$ observed demand, $Z = (Y, \tilde{Z}) \in R^{1+L}$ observed characteristics, and A unobserved individual heterogeneity, uniformly distributed on $[0, 1]$.

First case: P exogenous and A scalar

Classical setting of nonseparable model, Matzkin 2003

Second case: P endogenous and correlated with A

Chesher 2003, Imbens and Newey 2009

First case: P exogenous and A scalar

Identification

Following Matzkin 2003, under the assumptions that:

(i) A independent from (P, Z)

(ii) for all (p, z), $\varphi(p, z, .)$ is strictly increasing

= > φ is identified and $\varphi(p, z, a) = F^{-1}_{Q|P=p,Z=z}(a)$

Estimation

- i.i.d. observations $\{(Q_i, P_i, Z_i) : i = 1, ..., n\}$ where $Z_i = (Y_i, \tilde{Z}_i)$
- $\hat{\varphi}(p, z, a) = \hat{F}^{-1}_{Q|P=p,Z=z}(a)$

Asymptotic properties

Following Matzkin 2003, the estimator is consistent and converge asymptotically to a normal distribution with a rate of convergence in $\sqrt{nh^{L+2}}$

First case: P exogenous and A scalar

Identification

Following Matzkin 2003, under the assumptions that:

(i) A independent from (P, Z)

(ii) for all (p, z), $\varphi(p, z, .)$ is strictly increasing

$=>$ φ is identified and $\varphi(p, z, a) = F^{-1}_{Q|P=p,Z=z}(a)$

Estimation

- i.i.d. observations $\{(Q_i, P_i, Z_i) : i = 1, ..., n\}$ where $Z_i = (Y_i, \tilde{Z}_i)$
- $\hat{\varphi}(p, z, a) = \hat{F}^{-1}_{Q|P=p,Z=z}(a)$

Asymptotic properties

Following Matzkin 2003, the estimator is consistent and converge asymptotically to a normal distribution with a rate of convergence in $\sqrt{nh^{L+2}}$

First case: P exogenous and A scalar

Identification

Following Matzkin 2003, under the assumptions that:

(i) A independent from (P, Z)

(ii) for all (p, z), $\varphi(p, z, .)$ is strictly increasing

= > φ is identified and $\varphi(p, z, a) = F^{-1}_{Q|P=p,Z=z}(a)$

Estimation

- i.i.d. observations $\{(Q_i, P_i, Z_i) : i = 1, ..., n\}$ where $Z_i = (Y_i, \tilde{Z}_i)$
- $\hat{\varphi}(p, z, a) = \hat{F}^{-1}_{Q|P=p,Z=z}(a)$

Asymptotic properties

Following Matzkin 2003, the estimator is consistent and converge asymptotically to a normal distribution with a rate of convergence in $\sqrt{nh^{L+2}}$

Second case: P endogenous and correlated with A

The model

- $A = (A_1, A_2) \in R^2$, P is endogenous and correlated with A_1
- $P = h(Z, W, A_1)$ with $W \in R$ exogenous instrumental variable. (Chesher 2003, Imbens and Newey 2009)

Identification of h

(iii) A_1 independent from (Z, W)

(iv) For all (z, w), $h(z, w, .)$ is strictly increasing

$=>$ h is identified and $A_1 = F_{P|Z,W} = r(P, Z, W)$

Identification of φ

(v) A_2 independent from (P, Z, W)

(vi) For all (p, z, a_1), $\varphi(p, z, a_1, .)$ is strictly increasing

(vii) For all (p, z, w), r is differentiable w.r.t w and $\frac{\partial r}{\partial w}(p, z, w) \neq 0$

$=>$ φ is identified, $\varphi(p, z, a_1, a_2) = F^{-1}_{Q|P=p,Z=z,A_1=a_1}(a_2)$.

Second case: P endogenous and correlated with A

The model

- $A = (A_1, A_2) \in R^2$, P is endogenous and correlated with A_1
- $P = h(Z, W, A_1)$ with $W \in R$ exogenous instrumental variable. (Chesher 2003, Imbens and Newey 2009)

Identification of h

(iii) A_1 independent from (Z, W)

(iv) For all (z, w), $h(z, w, .)$ is strictly increasing

= > h is identified and $A_1 = F_{P|Z,W} = r(P, Z, W)$

Identification of φ

(v) A_2 independent from (P, Z, W)

(vi) For all (p, z, a_1), $\varphi(p, z, a_1, .)$ is strictly increasing

(vii) For all (p, z, w), r is differentiable w.r.t w and $\frac{\partial r}{\partial w}(p, z, w) \neq 0$

= > φ is identified, $\varphi(p, z, a_1, a_2) = F^{-1}_{Q|P=p,Z=z,A_1=a_1}(a_2)$.

Second case: P endogenous and correlated with A

The model

- $A = (A_1, A_2) \in R^2$, P is endogenous and correlated with A_1
- $P = h(Z, W, A_1)$ with $W \in R$ exogenous instrumental variable. (Chesher 2003, Imbens and Newey 2009)

Identification of h

(iii) A_1 independent from (Z, W)

(iv) For all (z, w), $h(z, w, .)$ is strictly increasing

= > h is identified and $A_1 = F_{P|Z,W} = r(P, Z, W)$

Identification of φ

(v) A_2 independent from (P, Z, W)

(vi) For all (p, z, a_1), $\varphi(p, z, a_1, .)$ is strictly increasing

(vii) For all (p, z, w), r is differentiable w.r.t w and $\frac{\partial r}{\partial w}(p, z, w) \neq 0$

= > φ is identified, $\varphi(p, z, a_1, a_2) = F^{-1}_{Q|P=p,Z=z,A_1=a_1}(a_2)$.

Second case: P endogenous and correlated with A

Estimation

- $A_{1i} = F_{P|Z=Z_i,W=W_i}(P_i) : i = 1, ..., n$ with the conditional cdf $F_{P|Z,W}$ and denote by $\hat{A}_{1i} = \hat{F}_{P|Z=Z_i,W=W_i}(P_i)$ the associated kernel estimator
- $\hat{A}_{1i} = \frac{\sum_{j=1,j\neq i}^{n} \tilde{K}(\frac{P_i - P_j}{h}) K(\frac{Z_i - Z_j}{h}, \frac{W_i - W_j}{h})}{\sum_{j=1,j\neq i}^{n} K(\frac{Z_i - Z_j}{h}, \frac{W_i - W_j}{h})}$ where $\tilde{K}(u) = \int_{-\infty}^{u} K(s)ds$.
- $\hat{\varphi}(p, z, a_1, a_2) = \hat{F}^{-1}_{Q|P=p,Z=z,\hat{A}_1=a_1}(a_2)$

Asymptotic Properties

Following Matzkin 2003, the estimator is consistent and converge asymptotically to a normal distribution with a rate of convergence in $\sqrt{nh^{L+3}}$

Second case: P endogenous and correlated with A

Estimation

- $A_{1i} = F_{P|Z=Z_i,W=W_i}(P_i) : i = 1, ..., n$ with the conditional cdf $F_{P|Z,W}$ and denote by $\hat{A}_{1i} = \hat{F}_{P|Z=Z_i,W=W_i}(P_i)$ the associated kernel estimator
- $\hat{A}_{1i} = \frac{\sum_{j=1,j\neq i}^{n} \tilde{K}(\frac{P_i-P_j}{h})K(\frac{Z_i-Z_j}{h},\frac{W_i-W_j}{h})}{\sum_{j=1,j\neq i}^{n} K(\frac{Z_i-Z_j}{h},\frac{W_i-W_j}{h})}$ where $\tilde{K}(u) = \int_{-\infty}^{u} K(s)ds$.
- $\hat{\varphi}(p, z, a_1, a_2) = \hat{F}^{-1}_{Q|P=p,Z=z,\hat{A}_1=a_1}(a_2)$

Asymptotic Properties

Following Matzkin 2003, the estimator is consistent and converge asymptotically to a normal distribution with a rate of convergence in $\sqrt{nh^{L+3}}$

1. Motivation and general context.
   - Application in Microeconomics.
   - Objectives.

2. Nonseparable demand model.
   - Identification.
   - Estimation and convergence.

3. Welfare model.
   - Identification.
   - Estimation and convergence.

4. Conclusion.

General relation.

Consider both systems:

$$(1)\begin{cases} \lambda'(p) & = \varphi\left(p, y + \lambda(p), \tilde{z}, a\right) \\ \lambda\left(p^0\right) & = 0 \end{cases}$$

$\widehat{\varphi}$ is an estimator of φ.

Goal: Study the convergence of $\widehat{\lambda}$ to λ given $\widehat{\varphi}$ and φ.

General relation.

Consider both systems:

$$(2)\begin{cases} \widehat{\lambda}'(p) & = \widehat{\varphi}\left(p, y + \widehat{\lambda}(p), \tilde{z}, a\right) \\ \widehat{\lambda}(p^0) & = 0 \end{cases}$$

$\widehat{\varphi}$ is an estimator of φ.

Goal: Study the convergence of $\widehat{\lambda}$ to λ given $\widehat{\varphi}$ and φ.

General relation.

Consider both systems:

$$(2)\begin{cases} \widehat{\lambda}'(p) &= \widehat{\varphi}\left(p, y+\widehat{\lambda}(p), \tilde{z}, a\right) \\ \widehat{\lambda}\left(p^0\right) &= 0 \end{cases}$$

$\widehat{\varphi}$ is an estimator of φ.

Goal: Study the convergence of $\widehat{\lambda}$ to λ given $\widehat{\varphi}$ and φ.

General relation.

Consider both systems:

$$(2)\begin{cases} \widehat{\lambda}'(p) & = \widehat{\varphi}\left(p, y + \widehat{\lambda}(p), \tilde{z}, a\right) \\ \widehat{\lambda}\left(p^0\right) & = 0 \end{cases}$$

$\widehat{\varphi}$ is an estimator of φ.

Goal: Study the convergence of $\widehat{\lambda}$ to λ given $\widehat{\varphi}$ and φ.

Identification and overidentification.

Existence and uniqueness of (1) and (2)

Under regularity conditions on φ and $\widehat{\varphi}$, there exists unique solutions λ and $\widehat{\lambda}$. (Cauchy Lipschitz Theorem)

Regularity conditions (1)

Lipschitz conditions on φ and $\widehat{\varphi}$

$$
\begin{aligned}
|\varphi(p, y_2, \tilde{z}, a) - \varphi(p, y_1, \tilde{z}, a) &\leq k|y_2 - y_1|, \text{for all}(p, y_i), \\
|\widehat{\varphi}(p, y_2, \tilde{z}, a) - \widehat{\varphi}(p, y_1, \tilde{z}, a)| &\leq \widehat{k}|y_2 - y_1|, \text{for all}(p, y_i).
\end{aligned}
$$

Identification and overidentification.

Existence and uniqueness of (1) and (2)

Under regularity conditions on φ and $\widehat{\varphi}$, there exists unique solutions λ and $\widehat{\lambda}$. (Cauchy Lipschitz Theorem)

Regularity conditions (1)

Lipschitz conditions on φ and $\widehat{\varphi}$

$$\begin{aligned}
|\varphi(p, y_2, \tilde{z}, a) - \varphi(p, y_1, \tilde{z}, a) &\leq k|y_2 - y_1|, \text{for all}(p, y_i), \\
|\widehat{\varphi}(p, y_2, \tilde{z}, a) - \widehat{\varphi}(p, y_1, \tilde{z}, a)| &\leq \widehat{k}|y_2 - y_1|, \text{for all}(p, y_i).
\end{aligned}$$

Identification and overidentification.

Regularity conditions (2)

$sup_{p,y}|\frac{\partial}{\partial y}\widehat{\varphi}(p,y) - \frac{\partial}{\partial y}\varphi(p,y)|$ converges in probability to 0.
(<=> $\widehat{k} \rightarrow k$)

Stability

The solutions are stable: $d\left(\widehat{\lambda}, \lambda\right) \leq C.d\left(\widehat{\varphi}, \varphi\right)$

Well-posed Inverse Problem

$$\begin{aligned} \lambda(p) &= \Phi\left[F\right](p) \\ \widehat{\lambda}(p) &= \Phi\left[\widehat{F}\right](p) \end{aligned}$$

with F cdf of (Q, P, Z)

(or (Q, P, Z, W) for the endogenous case)

Identification and overidentification.

Regularity conditions (2)

$sup_{p,y}|\frac{\partial}{\partial y}\widehat{\varphi}(p,y) - \frac{\partial}{\partial y}\varphi(p,y)|$ converges in probability to 0.
(<=> $\widehat{k} \rightarrow k$)

Stability

The solutions are stable: $d\left(\widehat{\lambda},\lambda\right) \leq C.d\left(\widehat{\varphi},\varphi\right)$

Well-posed Inverse Problem

$$\begin{aligned} \lambda(p) &= \Phi\left[F\right](p) \\ \widehat{\lambda}(p) &= \Phi\left[\widehat{F}\right](p) \end{aligned}$$

with F cdf of (Q,P,Z)

(or (Q,P,Z,W) for the endogenous case)

Identification and overidentification.

Regularity conditions (2)

$sup_{p,y} \left| \frac{\partial}{\partial y}\widehat{\varphi}(p,y) - \frac{\partial}{\partial y}\varphi(p,y) \right|$ converges in probability to 0.

(<=> $\widehat{k} \rightarrow k$)

Stability

The solutions are stable: $d\left(\widehat{\lambda},\lambda\right) \leq C.d\left(\widehat{\varphi},\varphi\right)$

Well-posed Inverse Problem

$$\lambda(p) = \Phi\left[F\right](p)$$
$$\widehat{\lambda}(p) = \Phi\left[\widehat{F}\right](p)$$

with F cdf of (Q,P,Z)

(or (Q,P,Z,W) for the endogenous case)

Nonparametric estimator.

- No closed form solution
- Several algorithms to approximate the solution: Euler-Cauchy, Runge-Kutta, etc...

Example of Euler-Cauchy algorithm

- Consider one consumer with fixed characteristics: $y, \tilde{z}, a$, and a price value p_0.
- Consider a path of prices from p_0 to p as $p_0, p_1, ..., p_{T-1}, p_T = p$ and compute $\hat{\lambda}(p)$ recursively through

$$\widehat{\lambda}(p_i) = \widehat{\lambda}(p_{i-1}) + (p_i - p_{i-1})\,\hat{\varphi}(p_{i-1}, y + \widehat{\lambda}(p_{i-1}), \tilde{z}, a)$$

$$\text{for } \; i = 1, .., T.$$

Nonparametric estimator.

- No closed form solution
- Several algorithms to approximate the solution: Euler-Cauchy, Runge-Kutta, etc...

Example of Euler-Cauchy algorithm

- Consider one consumer with fixed characteristics: $y, \tilde{z}, a$, and a price value p_0.
- Consider a path of prices from p_0 to p as $p_0, p_1, ..., p_{T-1}, p_T = p$ and compute $\hat{\lambda}(p)$ recursively through

$$\widehat{\lambda}(p_i) = \widehat{\lambda}(p_{i-1}) + (p_i - p_{i-1})\,\hat{\varphi}(p_{i-1}, y + \widehat{\lambda}(p_{i-1}), \tilde{z}, a)$$
$$\text{for} \;\; i = 1, .., T.$$

Linearization of the problem.

Theorem

For all $p \in V(p^0)$, we have:

$$\begin{aligned}\widehat{\lambda}(p) - \lambda(p) &= \Phi\left[\widehat{F}\right](p) - \Phi[F](p) \\ &= \underbrace{d\Phi[F]\left(\widehat{F} - F\right)(p)}_{\text{linear part}} + \underbrace{R}_{\text{residual term}}\end{aligned}$$

where:

$$d\Phi[F]\left(\widehat{F} - F\right)(p) = \int_{p^0}^{p} \frac{\int \left[(\hat{f} - f)(s,t,y+\lambda(t),\tilde{z}).(a - 1(s \leq \varphi(t,y+\lambda(t),\tilde{z},a)))\right] ds}{f(t,y+\lambda(t),\tilde{z})} .\gamma(p,t,\tilde{z},a)dt$$

and

$$\gamma(p,t,\tilde{z},a) = \frac{e^{\left[\int_t^p \frac{\partial}{\partial e_2}\varphi(u,y+\lambda(u),\tilde{z},a)du\right]}}{f_{Q|P,Y,\tilde{Z}}(\varphi(t,y+\lambda(t),\tilde{z},a),t,\lambda(t),\tilde{z})}$$

Convergence results.

Theorem

Under the previous assumptions, $\hat{\lambda}(p)$ converges in probability to $\lambda(p)$ and

$$\sqrt{nh^{L+1}}(\hat{\lambda}(p) - \lambda(p)) \to N(0, V) \text{ in distribution}$$

where $V = \|K\|_2^2 \int_{p^0}^{p} var\left[1(Q \leq \varphi(t, y + \lambda(t), \tilde{z}, a)) | P = t, Y = \lambda(t), \tilde{Z} = \tilde{z}\right] \gamma^2(p, t, \tilde{z}, a) dt$

Remark

- Compared to the convergence of $\widehat{\varphi}$, there is a gain in the obtained rate of convergence for $\widehat{\lambda}$.
- The result is similar for the endogenous case with a rate of convergence in $\sqrt{nh^{L+2}}$

Convergence results.

Theorem

Under the previous assumptions, $\hat{\lambda}(p)$ converges in probability to $\lambda(p)$ and

$$\sqrt{nh^{L+1}}(\hat{\lambda}(p) - \lambda(p)) \to N(0, V) \text{ in distribution}$$

where $V = \|K\|_2^2 \int_{p^0}^{p} var\left[1(Q \leq \varphi(t, y + \lambda(t), \tilde{z}, a))|P = t, Y = \lambda(t), \tilde{Z} = \tilde{z}\right] \gamma^2(p, t, \tilde{z}, a)dt$

Remark

- Compared to the convergence of $\widehat{\varphi}$, there is a gain in the obtained rate of convergence for $\widehat{\lambda}$.
- The result is similar for the endogenous case with a rate of convergence in $\sqrt{nh^{L+2}}$

Empirical study.

- dataset on gasoline demand from the 2001 US National Household Travel Survey (NHTS) (see *Blundell, Horowitz, Parey 2009*)
- 22,204 observations
- households randomly sampled over different geographical areas of the U.S.
- key variables: annual gasoline consumption, price per gallon of gasoline (weighted average prices, including taxes at the state level)
- household characteristics: household income, education, size
- Instrumental variable used in *Blundell Horowitz Parey 2009*: distance measure from the source of supply in the Gulf of Mexico to capital of the state.

Empirical study.

Table 1: Summary Table

	Mean	10%	Median	90%	Stdv
Gasoline Demand in 100 Gallons	12.03	2.63	9.75	23.66	10.12
Gasoline Price in $ per Gallon	1.33	1.24	1.34	1.44	0.08
Annual HH Income in 1000 $	53.77	17.50	47.50	120.00	33.85
Distance From Gulf in 1000 kma	1.73	0.88	1.59	2.86	0.72
# of Drivers per HH	1.92	1.00	2.00	3.00	0.74
HH Size	2.64	1.00	2.00	4.00	1.36
Mean Age of Drivers	48.15	29.50	45.33	72.00	15.91
Some College Educationb	0.67	0.00	1.00	1.00	0.47
Rail in MSAb	0.23	0.00	0.00	1.00	0.42
Pop. Dens. [100 Pers./Block]	38.61	0.50	15.00	70.00	53.85
Rural Area	0.23	0.00	0.00	1.00	0.42
Small Town	0.24	0.00	0.00	1.00	0.43
Suburban Area	0.24	0.00	0.00	1.00	0.43
Second City	0.18	0.00	0.00	1.00	0.38
Urban Area	0.10	0.00	0.00	1.00	0.31

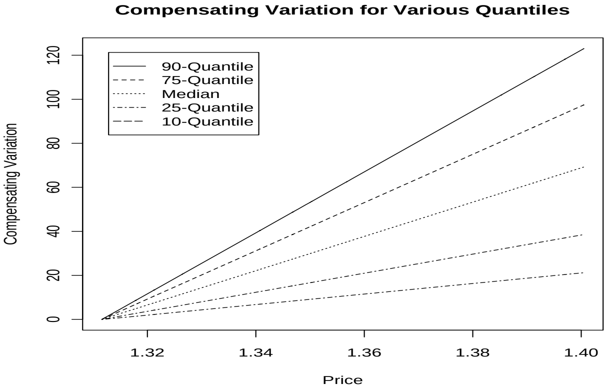
Empirical study.

- choice of exogenous characteristics $\tilde{Z}$: two first principal component factors
- Estimation of $\widehat{A}_1 = \widehat{F}_{P|Z,W}(P; Z, W)$ using second order Epanechnikov kernel
- Estimation of the demand function φ using kernel quantile estimator
- Consider a path of prices from p_0 to p as $p_{0,}p_1, ..., p_{T-1}, p_T$ $p_T = p$, and computing $\hat{\lambda}(p)$ recursively through

$$\widehat{\lambda}(p_i) = \widehat{\lambda}(p_{i-1}) + (p_i - p_{i-1})\,\hat{\varphi}(p_{i-1}, y + \widehat{\lambda}(p_{i-1}), \tilde{z}, a_1, a_2)$$
$$i = 1, ..., T.$$

  for a consumer with $Y = y, \tilde{Z} = \tilde{z}, \hat{A}_1 = a_1$ and a quantile level a_2.
- We evaluate the variation of welfare of a particular consumer $\widehat{\lambda}(p) = \widehat{\lambda}(p; \overline{Y}, \overline{\tilde{Z}}, \overline{\hat{A}_1}, a_2)$

Empirical study.



Conclusion.

- Analysis of welfare allowing for non separability in demand equation
- Study of both exogenous and endogenous cases for price variable.
- Solving the differential equation allows to improve the rate of convergence compared to the nonparametric demand estimator

- Study the distribution of welfares among different characteristics of consumers
- Further investigation on the database...

Conclusion.

- Analysis of welfare allowing for non separability in demand equation
- Study of both exogenous and endogenous cases for price variable.
- Solving the differential equation allows to improve the rate of convergence compared to the nonparametric demand estimator

- Study the distribution of welfares among different characteristics of consumers
- Further investigation on the database...

Idea of the proof.

- Define the two operators Ψ and A such that

$$\begin{aligned}\varphi(p,z,a) &= F^{-1}_{Q|P=p,Z=z}(a)\\ &= \Psi(F)(p,z,a)\end{aligned}$$

$$A : (G,u) \mapsto u'(.) - \Psi(G)(.,y+u(.),\tilde{z},a)$$

- Use the Implicit Function Theorem on the operator A and $A(F,\lambda) = 0$

- $\exists U$ open subset around F; $\forall u \in U, A(u, \Phi[u]) = 0$.

- Differentiate this relation over U in (F,λ) -> linear differential equation to solve