Dynamic Games People Play with their Kids
Borrowing for College, Working for College

Pedro Carneiro[a], Le-Yu Chen[b], Lars Nesheim[c]

[a]University College London and Cemmap
[b]Academia Sinica
[c]University College London and Cemmap

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1 Introduction

An abundant literature documents a strong positive association between family background and schooling attainment (See Haveman and Wolfe (1995) for a survey and the references therein). The impact of family background may arise in various ways. For example, genetic inheritance of ability or other personal traits may lead more capable parents to have more educated children. Alternatively, a family with more financial resources may invest more in their children. Credit constraints may amplify this effect by preventing the financing of college altogether (Haveman and Wolfe (1995), Carneiro and Heckman (2002), Keane and Wolpin (2001), Keane (2002), Stinebrickner and Stinebrickner (2009)). Lack of access to loans may cause youths to fund their college enrollment via part-time or full-time employment. However, in-college employment may have detrimental impacts on students' performance at college and enrollment in a subsequent years (Ehrenberg and Sherman (1987), Stinebricknerand Stinebrickner (2003))[1].

In this paper, we set up and calibrate a dynamic structural model to assess the importance of family resources on the college attendance and labor supply decisions of teenage children. In particular, after controlling for observed family background characteristics, we clarify the relationship between credit constraints, parental financial transfers, child's in-school employment and enrollment and performance in college. Similar to Keane and Wolpin (2001), abbreviated as KW hereafter, we adopt a structural dynamic life cycle model to explicitly account for endogeneity in consumption, family transfers, schooling and labor supply decisions[2].

Our modeling strategy is different from theirs in the following two important aspects. First, we explicitly model the interaction between parents and their child as a dynamic non-cooperative game and then derive the endogenous intra-family transfer function. By contrast, the parental transfer function of KW's model is specified in a reduced form and it is well known that predictions of counterfactual policy experiments may not be sustained when the policies may induce structural change of reduced form objects which are not explicitly accounted for within the model. Second, KW's model does not allow for any effect of in-school working on schooling performance. When making financial transfer decisions, altruistic parents may assess the impact of college employment on the youth's outcomes. Such a channel is allowed in our model. We explicitly account for the relationship between in-school labor supply, human capital accumulation and progression to

[1] It may be argued that acquisition of work experience facilitates the transition from school to work. However, Hotz et al. (2002) demonstrate that the return to work while in school is insignificant after controlling for dynamic selection bias.

[2] See Eckstein and Wolpin (1989), Rust (1994), Aguirregabiria and Mira (2009) and Keane and Wolpin (2009) for excellent surveys of techniques and uses of structural dynamic models in various economic applications.

the subsequent year of college.

We calibrate our structural model using data from the 1998 cohort of the National Longitudinal Survey of Youth (NLSY). This dataset contains detailed longitudinal information on youth including income, assets, schooling, work, proxies of child's ability and parental background. Additionally, the data contains detailed information on parental transfers, co-residency with parents, student loans, grants and costs of college attendance.

The rest of this paper is organized as follows. Section 3 presents some descriptive analysis of the data to motivate the modelling assumptions. Section 2 lays out our model of parental transfers, college attendance and labor supply. Section 4 discusses computational issues. Section 5 discusses results and Section 6 concludes.

2 A Markov game of family transfers, college attendance and labour supply

We set up a dynamic model of the annual choices of a parent (Agent 1) and a child (Agent 2) with the child's age ranging from 18 to 22 and the parent's age ranging from $(18 + gap, 22 + gap)$. Interaction between these two agents (Parent and child) is modeled as a dynamic Markov simultaneous move game[3]. In each period, parents choose their consumption $(c_{1,t})$, savings $(i_{1,t})$, and transfers $(t_{1,t})$ to the child. At the same time, their child chooses his/her consumption $(c_{2,t})$, savings $(i_{2,t})$ and labor supply $(l_{2,t})$, as well as whether to go to college $(e_{2,t})$. Parents are altruistic. Children are not.

We allow parental altruism to operate through two potential channels. First, parents may be altruistic in the sense that their utility is affected by the level of the child's utility[4]. Second, parents may be paternalistic or maternalistic in that they care about the educational attainment of their children.

College costs and qualities may vary. In our model, we categorize colleges into two-year and four-year colleges. This roughly reflects the heterogeneous returns to college education across college types, which are not distinguished in KW's model. The college enrollment decision $e_{2,t} = (e_{2,t}(1), e_{2,t}(2))$ is a vector of two binary valued elements with $e_{2,t}(1)$ and $e_{2,t}(2)$ denoting two-year and four-year

[3]Alternatively, one could consider a sequential move game, a unitary model or a cooperative game model. The period in which teenage children attend college, enter the labor force and move away from home is the period when they leave the control of their parents. In the NLSY sample, more than 50% of children do not live at home. For these reasons, we view the non-cooperative game as a more plausible model in this context. We leave a comparison with alternative models and work testing between them for future work.

[4]See Cox (1987) for an alternative model (the exchange model) that could also motivate intra-household transfer behavior.

college enrollment, respectively. A child's decision of labor supply can be full-time ($l_{2,t}=2$), part-time ($l_{2,t}=1$) or no work ($l_{2,t}=0$). A child's net income depends on human capital (educational attainment) and ability, parental transfers, school fees (net of grants), current educational status and labor supply. The model is a finite horizon dynamic game with T periods.

2.1 Parental problem

Parents have assets $a_{1,t}$, income $y_{1,t}$ and human capital (h_1). Human capital $h_1 = (h_1(1), h_1(2))$ is a time invariant state vector denoting parental accumulated years of two-year ($h_1(1)$) and four-year ($h_1(2)$) college education. The child has assets $a_{2,t}$ and human capital $h_{2,t} = (h_{2,t}(1), h_{2,t}(2))$. Let $q_t = (q_{1,t}, q_{2,t}, q^e_{2,t}, q_{3,t})$ be a vector of *exogenous* stochastic variables that capture parental income shocks ($q_{1,t}$), shocks to the child's wage when unenrolled ($q_{2,t}$) and when enrolled ($q^e_{2,t}$), and a shock to the child's educational performance ($q_{3,t}$) respectively. The shocks are assumed to evolve according to a vector AR(1) process. Let m_1 and m_2 be parental and child permanent heterogeneity (for instance, ability), respectively. Let $s_{1,t}$ be the vector of parental state variables with $s_{1,t} = (a_{1,t}, q_{1,t}, t, h_1, m_1)$ and let $s_{2,t}$ be the vector of child state variables with $s_{2,t} = \left(a_{2,t}, h_{2,t}, q_{2,t}, q^e_{2,t}, q_{3,t}, m_2\right)$. Let $s_t = (s_{1t}, s_{2t})$ be the vector of all state variables in which only $(a_{1,t}, a_{2,t}, h_{2,t})$ evolve endogenously.

Parent's have beliefs about the child's strategy (policy function) as a function of the state variables. Parents believe the child's policy function is

$$(c_{2,t}, l_{2,t}, e_{2,t}) = \pi^b_2 (s_t) .$$

In every period $t \leq T$, given current values of the state variables s_t and given π^b_2, parents choose $(c_{1,t}, i_{1,t}, t_{1,t})$ to maximize utility. The maximal utility obtained is given by the parent's value function $v_1(s_t)$ which equals

$$\max_{\{c_{1,t}, i_{1,t}, t_{1,t}\}} \left\{ u_1(c_{1,t}) + \alpha u_2(c_{2,t}, l_{2,t}, e_{2,t}) + \beta_1 \int v_1(s_{t+1})\, g(q_{t+1}|q_t)\, dq_{t+1} \right\} \tag{1}$$

subject to the eight constraints

$$\begin{aligned}
c_{1,t} + t_{1,t} + i_{1,t} &= a_{1,t} + y_{1,t} && (2)\\
a_{1,t+1} &= r_{t+1} i_{1,t} && (3)\\
a_{2,t+1} &= r_{t+1} i^b_{2,t} && (4)\\
y_{1,t} &= F_1(t, q_{1,t}, m_1, h_1) && (5)\\
h_{2,t+1} &= H_2\left(t, h_{2,t}, e^b_{2,t}, l^b_{2,t}, q_{3,t+1}, m_1, h_1, m_2\right) && (6)\\
c_{1,t} &\geq 0 && (7)\\
t_{1,t} &\geq t_{1L} && (8)\\
i_{1,t} &\geq b_1. && (9)
\end{aligned}$$

The parameters α and β_1 are the parental altruism and discount parameters respectively. The variable r_t is the time varying gross interest rate, t_{1L} is the minimal transfer level and b_1 is the parameter capturing parental credit constraints.

The first constraint is the budget constraint. Constraints two and three describe the law of motion of assets. Constraints 4 and 5 describe income and human capital. Six and seven provide lower bounds on consumption and on the transfer from parent to child. The final constraint is the borrowing constraint.

We use parametric specifications for per-period functions u_1 and u_2, the joint density g of q_{t+1} conditional on q_t, the parental wage function F_1 and the child's human capital production function H_2. We assume that both u_1 and u_2 are twice continuously differentiable and strictly increasing and strictly concave in c. Detailed specifications are presented in Appendix A.

The optimizer of (**??**) is the parental policy function (strategy function) which can be summarized as

$$(i_{1,t}, t_{1,t}) = \pi_1 \left(s_t, \pi_2^b \right).$$

This is the reaction function of the parent.

2.2 Child's problem

The child's problem is analagous. The child has beliefs about parental policies (strategies)

$$(i_{1t}, t_{1t}) = \pi_1^b (s_t).$$

Give π_1^b and s_t, at every period $t \leq T$, a child chooses $(c_{2,t}, l_{2,t}, e_{2,t})$ to maximise utility. Their value function $v_2(s_t)$ equals

$$\max_{\{c_{2,t}, l_{2,t}, e_{2,t}\}} \left\{ u_2 (c_{2,t}, l_{2,t}, e_{2,t}) + \beta_2 \int v_2 (s_{t+1}) \, g (q_{t+1} | q_t) \, dq_{t+1} \right\} \tag{10}$$

subject to the constraints

$$\begin{aligned}
c_{2,t} + i_{2,t} &= a_{2,t} + t_{1,t} + y_{2,t} && (11)\\
a_{1,t+1} &= r_{t+1} i_{1,t} && (12)\\
a_{2,t+1} &= r_{t+1} i_{2,t} && (13)\\
y_{2,t} &= F_2 \left(t, h_{2,t}, e_{2,t}, l_{2,t}, q_{2,t}, q_{2,t}^e, p_{2,t}, p_{4,t}, m_2 \right) && (14)\\
h_{2,t+1} &= H_2 (t, h_{2,t}, e_{2,t}, l_{2,t}, q_{3,t+1}, m_1, h_1, m_2) && (15)\\
c_{2,t} &\geq 0 && (16)\\
i_{2,t} &\geq b_2 && (17)\\
e_{2,t}(1) \, e_{2,t}(2) &= 0. && (18)
\end{aligned}$$

The parameter β_2 is the child's discount factor. The first constraint is the child's budget constraint. The second and third constraints describe the transition of assets. Constraint (14) describes the net income of the child inclusive of earned income and grants and net of all tuition and fees. Constraint (15) describes the evolution of human capital and constraints 6 and 7 provide lower bounds on consumption and borrowing. The final constraint requires the child to enroll in at most 1 educational institution. The parameter b_2 captures the child's credit constraint and $p_{2,t}$ and $p_{4,t}$ denote the net college costs (the difference between schooling subsidies and tuition fees) of two-year and four-year colleges, respectively. The function F_2 is parametric and specifies the child's net income. Further details of the specification are described in Appendix A.

The optimizer of (10) is the child's policy function

$$(c_{2,t}, l_{2,t}, e_{2,t}) = \pi_2\left(s_t, \pi_1^b\right).$$

2.3 Markov perfect equilibrium

A Markov perfect Nash equilibrium is a pair (π_1, π_2) such that almost everywhere

$$\left\{\begin{array}{c} \pi_1 = \pi_1^b \\ \pi_2 = \pi_2^b \end{array}\right\}. \tag{19}$$

It is possible that no pure strategy equilibrium exists. It is also possible (likely) that there are multiple equilibria. We assume that at least one pure strategy equilibrium exists and use computational methods to search for one. We discuss computation in Section 4. First we present some data from the NLSY 1998 that motivates our analysis.

3 Data

The 1997 US National Longitudinal Survey of Youth (NLSY 1997) is a panel survey that contains information on a representative sample of US youth born between 1980 and 1984. The cohort contains 8,984 respondents and provides information on age, ethnic background, gender, residency status, educational attainment and enrollment, ability, labor force status and income, assets, income and education of parents, etc. It also contains information on college costs and loans and grants as well as transfers from family members.

The tables at the end of the paper provide some basic descriptive statistics from the panel.

4 Computation

We compute equilibrium utilities and policy functions in the model by backward induction. In the final period of the model we compute equilibrium utilities, the gradient of utilities with respect to assets and policies conditional on the states for a grid of possible values of the state variables. In fact, we calculate pairs of value functions and gradients for each possible subgame defined by a discrete action of the child (a child can choose 3 distinct enrollment states and 3 distinct labor supply states). This produces pointwise estimates of **conditional** value functions, **conditional on the state and on the discrete subgame**. For the child, the **unconditional** pointwise value function is then the maximum of the **conditional** value functions. For the parent, the unconditional value function is the conditional value function corresponding to the discrete action chosen by the child.[5]

Thus, at a grid of points in the state space we compute pointwise estimates of the equilibrium value functions and gradients. Next we use these pointwise estimates to compute a spline based approximation to the equilibrium value functions and their gradients. Finally, we compute the expected value of these equilibrium value functions using Gaussian quadrature techniques.

In summary, at each time period t, the computational methods involve 4 steps:

1. At each point on a grid in the state space and in each subgame, solve a system of nonlinear equations (the first order conditions to the 2 agents maximization problems) for a fixed point conditional on a fixed state space point. Check that children do not have incentive to deviate.

2. Compute a spline based approximation to the equiibrium value functions.

3. Numerically integrate to obtain the expected value of the equilibrium value functions

4. Compute a spline based approximation to the expected value.

5 Results

Preliminary results from the calibration model are displayed at the end of the paper.

[5] If the game were not a simultaneous move game but instead we assumed that the child chose his/her discrete action first. This would be the subgame perfect equilibrium. Since we assume that parent's and children choose actions simultaneously, this is an equilibrium if the child has no incentive to deviate to a different discrete action. We check this condition in our computation.

6 Conclusion

A Specification

A.1 Parental utility

We assume that parental utility has a CRRA form $c_1 \geq c_{1L} + \kappa_1$. For $c_1 < \kappa_1 + c_{1L}$, we assume that it equals a quadratic Taylor expansion of the CRRA utility function. This specification has the benefits of the CRRA specification when c_1 is not near zero while eliminating the singularity of the CRRA utility function at zero. Utility is strictly increasing and strictly concave. In addition, it is C^2 on the entire real line. When κ_1 is small, agents will only very very rarely choose $c_1 < c_{1L} + \kappa_1$ because the marginal utility of consumption will be a large positive quantity. We interpret negative consumption as the agent spending extra effort to pay off debts.

Algebraically, parental utility is

$$u_1(c_{1t}) = \left\{ \begin{array}{ll} u_{1F}(c_{1t}) & \text{if} \quad c_{1t} - c_{1Lt} \geq \kappa_1 \\ u_{1N}(c_{1t}) & \text{if} \quad c_{1t} - c_{1Lt} < \kappa_1 \end{array} \right\}$$

where

$$u_{1F}(c_{1t}) = \frac{(c_{1t} - c_{1Lt})^{1-\sigma_1} - 1}{1 - \sigma_1}$$

$$\begin{aligned} u_{1N}(c_{1t}) &= u_{1F}(\kappa_1) \\ &\quad + \frac{\partial u_{1F}(\kappa_1)}{\partial c}(c_{1t} - c_{1Lt} - \kappa_1) \\ &\quad + 0.5\frac{\partial^2 u_{1F}(\kappa_1)}{\partial c^2}(c_{1t} - c_{1Lt} - \kappa_1)^2 . \end{aligned}$$

A.2 Child's utility

The child's utility function has the same functional form as the parents but with leisure added as an argument. The child's utility is

$$\begin{aligned} u_2(c_{2t}, l_{2t}, e_{2t}) &= u_{2F}(c_{2t}, l_{2t}, e_{2t}) \\ &= \frac{\left[(c_{2t} - c_{2Lt})^{\rho_2} + \lambda_1 (1 - \lambda_2 l_{2t} - \lambda_3 e_{2t}(1) - \lambda_4 e_{2t}(2))^{\rho_2}\right]^{\frac{1-\sigma_2}{\rho_2}} - 1}{1 - \sigma_2} \end{aligned}$$

if $c_{2t} - c_{2Lt} \geq \kappa_2$.

If $c_{2t} - c_{2Lt} < \kappa_2$, then

$$\begin{aligned} u_{2N}(c_{2t}, l_{2t}, e_{2t}) &= u_{2F}(\kappa_2) \\ &\quad + \frac{\partial u_{2F}(\kappa_2)}{\partial c}(c_{2t} - c_{2Lt} - \kappa_2) \\ &\quad + 0.5\frac{\partial^2 u_{2F}(\kappa_2)}{\partial c^2}(c_{2t} - c_{2Lt} - \kappa_2)^2 . \end{aligned}$$

A.3 Parental Terminal Utility

In the terminal period of the model, the children are 22 and the parents are $22 + gap$. For the baseline results, $gap = 28$, so the parents are 50. The terminal value function is the present value of utility starting from age 50. We assume that the terminal utility depends on parental and children's assets (a_{1T}, a_{2T}) and on the child's human capital h_2. We define a vector of parameters α_{T1} that describe how parental terminal utility depends on children's state variables.

Let $\overline{a}_1$ and $\overline{a}_2$ measure assets plus the present value of future income (including pensions) for parents and children respectively. We assume that if $\frac{\overline{a}_1+\alpha_{T1}(2)\overline{a}_2}{R_{1T}} \geq \kappa_1$, then parental terminal utility is

$$\begin{aligned}\overline{v}_1\left(a_1, a_2, h_2\right) &= \beta_{1T}\frac{\left(\frac{\overline{a}_1+\alpha_{T1}(2)\overline{a}_2}{R_{1T}}\right)^{1-\sigma_{1T1}}-1}{1-\sigma_{1T1}} \\ &\quad+\sum_{i=0}^{2}\alpha_{T1}\left(i+3\right)\mathbf{I}\left[h_2\left(1\right)=i\right] \\ &\quad+\sum_{i=3}^{7}\alpha_{T1}\left(i+3\right)\mathbf{I}\left[h_2\left(2\right)=\left(i-3\right)\right]\end{aligned}$$

where

$$\overline{a}_1 = a_1 + y_{T+1}^1 + \overline{y}_1\left(z_{T+1}^1, m_1\right)$$

and $\overline{y}_1$ is the discounted expected present value of parental future income

$$\begin{aligned}\overline{y}^1\left(z_{T+1}^1, m_1\right) &= \sum_{s=T+2}^{death}\int\frac{y_s^1}{\overline{r}_s}g^1\left(z_s\left|z_{T+1}^1\right.\right)dz_s \\ \overline{r}_s &= \prod_{u=T+1}^{s} r_u.\end{aligned}$$

Also,

$$\overline{a}_2 = a_2 + y_{T+1}^2 + \overline{y}^2\left(z_{T+1}^2, m_2, h_2\right)$$

where

$$\begin{aligned}\overline{y}^2\left(z_{T+1}^2, m_2, h_2\right) &= \sum_{s=T+2}^{death}\int\frac{y_s^2}{\overline{r}_s}g^2\left(z_s\left|z_{T+1}^2\right.\right)dz_s \\ \overline{r}_s &= \prod_{u=T+1}^{s} r_u\end{aligned}$$

is the discounted expected present value of future income.

If $\frac{\overline{a}_1+\alpha_{T1}(2)\overline{a}_2}{R_{1T}} < \kappa_1$ then parental terminal utility is

$$\begin{aligned}\overline{v}_1(a_1,a_2,h_2) &= \beta_{T1}u_{1TN}\left(\frac{\overline{a}_1+\alpha_{T1}(2)\overline{a}_2}{R_{1T}}\right) \\ &\quad +\sum_{i=0}^{2}\alpha_{T1}(i+3)\,\mathbf{I}[h_2(1)=i] \\ &\quad +\sum_{i=3}^{7}\alpha_{T1}(i+3)\,\mathbf{I}[h_2(2)=(i-3)]\end{aligned}$$

where

$$\begin{aligned}u_{1TN}(x) &= \frac{\kappa_1^{1-\sigma_{1T1}}-1}{1-\sigma_{1T1}} \\ &\quad +\kappa_1{}^{-\sigma_{1T1}}(x-\kappa_1) \\ &\quad -0.5\sigma_{1T1}x^{\sigma_{1T1}-1}(x-\kappa_1)^2.\end{aligned}$$

A.4 Child's terminal utility

We assume the terminal utility of children has a similar form.

If $\frac{\alpha_{T2}\overline{a}_1+\overline{a}_2}{R_{T2}} \geq \kappa_2$, then child's terminal utility is

$$\overline{v}_2(a_1,a_2,h_2) = \beta_{2T}\frac{\left(\frac{\alpha_{T2}\overline{a}_1+\overline{a}_2}{R_{T2}}\right)^{1-\sigma_{2T}}-1}{1-\sigma_{2T}}.$$

If $\frac{\alpha_{T2}\overline{a}_1+\overline{a}_2}{R_{2T}} < \kappa_2$, then it equals

$$\overline{v}_2(a_1,a_2,h_2) = \beta_{2T}u_{2TN}\left(\frac{\alpha_{T2}\overline{a}_1+\overline{a}_2}{R_{2T}}\right)$$

where

$$\begin{aligned}u_{2TN}(x) &= \frac{\kappa_2{}^{1-\sigma_{2T}}-1}{1-\sigma_{2T}} \\ &\quad +\kappa_2^{-\sigma_{2T}}(x-\kappa_2) \\ &\quad -0.5\sigma_{2T}\kappa_2^{-\sigma_{2T}-1}(x-\kappa_2)^2.\end{aligned}$$

A.5 Parent's income

Let

$$\begin{aligned}y_{1t} &= f_1(z_{1t}) \\ &= y_{1Ht}\left(\frac{y_{1Lt}+\exp(z_{1t})}{y_{1Ht}+\exp(z_{1t})}\right)\end{aligned} \tag{20}$$

and let

$$z_{1t} = \delta_0 \left(h_1, m_1\right) + \sum_{j=1}^{2} \delta_1 \left(j\right) \left(age_{1t}\right)^j + q_{1t}. \tag{21}$$

where

$$q_{1t} = \delta_2 q_{1t-1} + \varepsilon_{1t} \tag{22}$$

with ε_{1t} i.i.d. over time.

$$\begin{aligned}\delta_0 \left(m_1, h_1\right) &= \sum_{i=0}^{2} \delta_{00} \left(i\right) \mathbf{I} \left[h_1 \left(1\right) = i\right] \\ &\quad + \sum_{i=3}^{7} \delta_{00} \left(i\right) \mathbf{I} \left[h_1 \left(2\right) = i\right] \\ &\quad + \delta_{01} m_1\end{aligned}$$

A.6 Child's income

A child earns labour income, pays taxes, and receives benefits. In addition, a child enrolled in school pays tuition and receives grant aid. Let $w_F \left(h_{2t}, z_{2t}\right)$ be the after-tax full time wage. Let $w_P \left(h_{2t}, z_{3t}\right)$ be the after-tax part time wage. Let p_{2t} be the net cost (fees minus grant aid) of attending 2 year college. Let p_{4t} be the net cost (fees minus grant aid) of attending 4 year college.

A child's net income is

$$\begin{aligned}y_{2t} &= w_F \left(h_{2t}, z_{2t}\right) \mathbf{I} \left(l_{2t} = 1\right) + w_P \left(h_{2t}, z_{3t}\right) \mathbf{I} \left(l_{2t} = 0.5\right) \\ &\quad - p_{2t} \mathbf{I} \left(e_{2t} \left(1\right) = 1\right) - p_{4t} \mathbf{I} \left(e_{2t} \left(2\right) = 1\right).\end{aligned}$$

Let

$$\begin{aligned}w_{Ft} &= w_{FHt} \left(\frac{w_{FLt} + \exp\left(z_{2t}\right)}{y_{FHt} + \exp\left(z_{2t}\right)}\right) \\ w_{Pt} &= w_{PHt} \left(\frac{w_{PLt} + \exp\left(z_{3t}\right)}{y_{PHt} + \exp\left(z_{3t}\right)}\right)\end{aligned}$$

where

$$\begin{aligned}z_{2t} &= \pi_0^F \left(m_2\right) \\ &\quad + \sum_{i=0}^{2} \pi_1^F \left(i\right) \mathbf{I} \left[h_{2t} \left(1\right) = i\right] \\ &\quad + \sum_{i=3}^{7} \pi_1^F \left(i\right) \mathbf{I} \left[h_{2t} \left(2\right) = i\right] \\ &\quad + \pi_{21}^F t + q_{2t}\end{aligned}$$

$$
\begin{aligned}
z_{3t} &= \pi_0^P(m_2) \\
&\quad + \sum_{i=0}^{2} \pi_1^P(i)\,\mathbf{I}(h_{2t}(1) = i) \\
&\quad + \sum_{i=3}^{7} \pi_1^P(i)\,\mathbf{I}(h_{2t}(2) = i) \\
&\quad + \pi_2^P t + \pi_3^P q_{2t}
\end{aligned}
$$

where

$$q_{2t} = \pi_4 q_{2t} + \varepsilon_{2t}$$

with ε_{2t} i.i.d. over time.

$$
\begin{aligned}
\pi_0^F(m_2) &= \pi_{00}^F m_2 \\
\pi_0^P(m_2) &= \pi_{00}^P m_2
\end{aligned}
$$

A.7 Child's net present value of income after T

A.8 Child's income for $t = T + 1$

For $t = T + 1$, the child's income will follow the parental process given in (20), (21), and (22) with h_{2T+1} and with q_{1T+1} and $\delta_0(m_1)$ replaced by

$$
\begin{aligned}
\delta_0(h_2, m_2) &= \sum_{i=0}^{2} \delta_{00}(i)\,\mathbf{I}[h_{2T+1}(1) = i] \\
&\quad + \sum_{i=3}^{7} \delta_{00}(i)\,\mathbf{I}[h_{2T+1}(2) = i] \\
&\quad + \sum_{i=3}^{7} \delta_{01} m_2
\end{aligned}
$$

$$q_{1T+1} = \delta_3 q_{1T}(\widehat{y}_T) + \varepsilon_{1T+1}.$$

where

$$\widehat{y}_T = w_F(h_{2T}, z_{2T})\,\tau(s_{2T}) + (1 - \tau(s_{2T}))\,w_P(h_{2T}, z_{2T})$$

$$\tau\left(s_{2T}\right)=\frac{\exp\left(\begin{array}{c}\sum_{i=0}^{2}\tau_{0}\left(i\right)\mathbf{I}\left[h_{2T}\left(1\right)=i\right]\\+\sum_{i=3}^{7}\tau_{0}\left(i\right)\mathbf{I}\left[\left(h_{2T}\left(2\right)=i\right)\right]\\+\tau_{1}m_{2}\end{array}\right)}{1+\exp\left(\begin{array}{c}\sum_{i=0}^{2}\tau_{0}\left(i\right)\mathbf{I}\left[h_{2T}\left(1\right)=i\right]\\+\sum_{i=3}^{7}\tau_{0}\left(i\right)\mathbf{I}\left[\left(h_{2T}\left(2\right)=i\right)\right]\\+\tau_{1}m_{2}\end{array}\right)}$$

A.9 Child's income for $t \geq T+2$

For $t \geq T+1$, the child's income will follow the parental process given in (20), (21), and (22) but with q_{1T+1} and $\delta_0\left(m_1\right)$ replaced by

$$\begin{aligned}\delta_{0}\left(h_{2},m_{2}\right) &= \sum_{i=0}^{2}\delta_{00}\left(i\right)\mathbf{I}\left[h_{2T}\left(1\right)=i\right]\\&+\sum_{i=3}^{7}\delta_{00}\left(i\right)\mathbf{I}\left[h_{2T}\left(2\right)=i\right]\\&+\sum_{i=3}^{7}\delta_{01}m_{2}\end{aligned}$$

$$q_{1T+1}=\delta_{3}q_{1T}\left(\widehat{y}_{T}\right)+\varepsilon_{1T+1}.$$

where

$$\widehat{y}_{T}=w_{F}\left(h_{2T},z_{2T}\right)\tau\left(s_{2T}\right)+\left(1-\tau\left(s_{2T}\right)\right)w_{P}\left(h_{2T},z_{2T}\right)$$

$$\tau(s_{2T}) = \frac{\exp\begin{pmatrix} \sum_{i=0}^{2} \tau_0(i)\,\mathbf{I}\left[h_{2T}(1) = i\right] \\ +\sum_{i=3}^{7} \tau_0(i)\,\mathbf{I}\left[\left(h_{2T}(2) = i\right)\right] \\ +\tau_1 m_2 \end{pmatrix}}{1 + \exp\begin{pmatrix} \sum_{i=0}^{2} \tau_0(i)\,\mathbf{I}\left[h_{2T}(1) = i\right] \\ +\sum_{i=3}^{7} \tau_0(i)\,\mathbf{I}\left[\left(h_{2T}(2) = i\right)\right] \\ +\tau_1 m_2 \end{pmatrix}}$$

A.10 Child's human capital

Child's human capital is a vector recording the number of years of two year college completed and the number of years of four year college. The first component, $h_{2t}(1)$ may take on three possible values, zero, one or two years of completed two year college. The second component, $h_{2t}(2)$ may take on four possible values: zero, one, two, three or four or more years of completed four year college.

The first component of child's human capital evolves according to

$$\begin{aligned} h_{2,t+1}(1) &= H_{21}(h_{2t}, e_{2t}, l_t, q_{3t+1}, h_1, m_1, m_2) \\ &= h_{2t}(1) + e_{2t}(1) \cdot \mathbf{I}\left(q_{3,t+1} \geq \Gamma_1(l_t, h_1, m_1, m_2)\right) \end{aligned}$$

where

$$\begin{aligned} \Gamma_1(l_t, m_1, m_2) &= \gamma_0^1 + \gamma_1^1 \mathbf{I}(l_t = 0.5) + \gamma_2^1 \mathbf{I}(l_t = 1) \\ &\quad + \gamma_3^1(m_1, h_1) + \gamma_4^1 m_2 \\ &\quad + \sum_{i=0}^{2} \gamma_5^1(i)\,\mathbf{I}(h_{2t}(1) = i) \\ &\quad + \sum_{i=3}^{7} \gamma_5^1(i)\,\mathbf{I}(h_{2t}(2) = i) \end{aligned}$$

as long as $h_{2t}(1) \leq 2$. In other words, child's two year college human capital advances by one if both the child is in two year college ($e_{2t}(1) = 1$) and they have a positive shock.

The second component of child's human capital evolves analogously:

$$\begin{aligned} h_{2,t+1}(2) &= H_{22}(h_{2t}, e_{2t}, l_t, z_{t+1}, m_1, m_2) \\ &= h_{2t}(2) + e_{2t}(1) \cdot \mathbf{I}\left(q_{3,t+1}(4) \geq \Gamma_2(l_t, m_1, m_2)\right) \end{aligned}$$

where

$$\Gamma_2\left(l_t, m_1, m_2\right) = \gamma_0^2 + \gamma_1^2 \mathbf{I}\left(l_t = 0.5\right) + \gamma_2^2 \mathbf{I}\left(l_t = 1\right) + \gamma_3^2\left(m_1, m_2\right)$$

as long as $h_{2t}(2) \leq 4$.

We assume

$$q_{3t} = \gamma_4 q_{3t-1} + \varepsilon_3$$

References

[1] Aguirregabiria, V. and Mira, P. (2007), "Sequential Estimation of Dynamic Discrete Games", *Econometrica*, 75, 1-53.

[2] Aguirregabiria, V. and Mira, P. (2009), "Dynamic discrete choice structural models: A survey", *Journal of Econometrics*, forthcoming.

[3] Bajari, P. Benkard, L. and Levin, J. (2007), "Estimating Dynamic Models of Imperfect Competition", *Econometrica*, 75, 1331-1370.

[4] Carneiro, P. and Heckman, J. J. (2002), "The Evidence on Credit Constraints in Post–secondary Schooling", *Economic Journal*, 112, 705-734.

[5] Cox, D. (1987), "Motives for Private Income Transfers", *Journal of Political Economy*, 95, 508-546.

[6] Eckstein, Z. and Wolpin, K. (1989), "The specification and estimation of dynamic stochastic discrete choice models", *Journal of Human Resources,* 24, 562-598.

[7] Ehrenberg, R.G. and Sherman, D.R. (1987), "Employment While in College, Academic Achievement, and Postcollege Outcomes: A Summary of Results", *Journal of Human Resources*, 22, 152-164.

[8] Tamer, E. (2003), "Incomplete Simultaneous Discrete Response Model with Multiple Equilibria," *Review of Economic Studies*, 70, 147-165.

[9] Haveman, R., and Wolfe, B. (1995), "The Determinants of Childrens Attainments: A Review of Methods and Findings", *Jourmal of Ecomomic Literature*, 33, 1829-78.

[10] Hotz, V.J., Xu, L.C., Tienda, M., and Ahituv, A. (2002), "Are There Returns to the Wages of Young Men from Working While in School?", *Review of Economics and Statistics*, 84, 221-236.

[11] Keane, M. and Wolpin, K. (2001), "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment", *International Economic Review*, 42, 1051-1103.

[12] Keane, M. (2002), "Financial Aid, Borrowing Constraints, and College Attendance: Evidence from Structural Estimates ", *American Economic Review*, 92, 293-297.

[13] Keane, M. and Wolpin, K. (2009), "Empirical Applications of Discrete Choice Dynamic Programming Models", *Review of Economic Dynamics*, 12, 1-22.

[14] Pesendorfer, M. and Philipp, S-D. (2008), "Asymptotic Least Squares Estimators for Dynamic Games", *Review of Economic Studies*, 75, 901-928.

[15] Rust, J. (1994), "Structural estimation of Markov decision models", Chapter 51. In: R. Engle and D. McFadden, Editors, Handbook of Econometrics, vol. 4, North-Holland, Amsterdam.

[16] Stinebrickner, R. and Stinebrickner, T. (2003), "Working during School and Academic Performance", *Journal of Labor Economics*, 2, 473-492.

[17] Stinebrickner, R. and Stinebrickner, T. (2008), "The Effect of Credit Constraints on the College Drop-Out Decision: A Direct Approach Using a New Panel Study", *American Economic Review*, 98, 2163-2184.

Table 1: PROPORTION ENROLLED By Age

	Age											
Status	16	17	18	19	20	21	22	23	24	25	26	Total
Not enrolled, no high shool	414	762	1,195	1,266	1,186	1,047	781	561	312	146	12	7,978
	6.19	10.59	14.92	16.24	15.24	13.86	12.97	12.54	10.97	10.85	15.79	11.12
Not enrolled, GED	4	52	184	291	372	411	346	270	203	111	9	2,254
	0.06	0.72	2.3	3.73	4.78	5.44	5.75	6.03	7.14	8.25	11.84	3.14
Not enrolled, high school	5	59	1,397	2,080	1,994	1,844	1,409	1,041	667	297	18	10,812
	0.07	0.82	17.44	26.68	25.62	24.42	23.4	23.27	23.45	22.07	23.68	15.08
Not enrolled, some college	0	3	120	506	1,043	1,345	1,209	1,034	701	315	18	6,295
	0	0.04	1.5	6.49	13.4	17.81	20.08	23.11	24.65	23.4	23.68	8.78
Not enrolled, 2-year college	0	0	0	5	46	102	145	133	109	50	3	593
	0	0	0	0.06	0.59	1.35	2.41	2.97	3.83	3.71	3.95	0.83
Not enrolled, 4-year college	0	0	0	0	1	38	331	502	393	185	7	1,457
	0	0	0	0	0.01	0.5	5.5	11.22	13.82	13.74	9.21	2.03
Not enrolled, graduate degree	0	0	0	0	0	0	1	8	16	19	1	45
	0	0	0	0	0	0	0.02	0.18	0.56	1.41	1.32	0.06
Enrolled in grades 1- 12	6,256	6,230	2,981	528	140	61	25	19	7	4	0	27,881
	93.53	86.59	37.22	6.77	1.8	0.81	0.42	0.42	0.25	0.3	0	38.88
Enrolled in a 2-year college	8	50	715	1,071	925	689	461	265	128	70	3	4,386
	0.12	0.69	8.93	13.74	11.88	9.12	7.66	5.92	4.5	5.2	3.95	6.12
Enrolled in a 4-year college	2	39	1,417	2,048	2,074	2,005	1,214	514	201	89	4	9,607
	0.03	0.54	17.69	26.27	26.65	26.55	20.16	11.49	7.07	6.61	5.26	13.4
Enrolled in a graduat school	0	0	0	0	2	10	99	127	107	60	1	406
	0	0	0	0	0.03	0.13	1.64	2.84	3.76	4.46	1.32	0.57
Total	6,689	7,195	8,009	7,795	7,783	7,552	6,021	4,474	2,844	1,346	76	71,714
	100	100	100	100	100	100	100	100	100	100	100	100

Table 2: Proportion Working Full/Part time
Hours worked
Wages
Enrolled in 2 year college

	15	16	17	18	19	20	21	22	23	24	25	26
Work Full Time	0	0.125	0.02	0.0842	0.145	0.237	0.261	0.302	0.332	0.344	0.457	0
	(.)	-0.354	-0.141	-0.278	-0.352	-0.425	-0.44	-0.459	-0.472	-0.477	-0.502	0
Hours Full Time	.	1766	1929	2122.2	2144.1	2160.9	2192.4	2190.9	2182	2159	2166.5	.
	(.)	(.)	(.)	-372.4	-441.2	-369.9	-409.8	-404.5	-378.1	-351.6	-362.1	(.)
Hourly Wage	.	7.897	9.655	10.02	10.36	14.81	12.24	11.81	13.23	12.85	13.34	.
	(.)	(.)	(.)	-7.837	-5.806	-48.19	-11.06	-4.091	-8.899	-4.832	-5.355	(.)
Work Part Time	1	0.5	0.74	0.801	0.749	0.638	0.604	0.553	0.513	0.516	0.414	1
	(.)	-0.535	-0.443	-0.4	-0.434	-0.481	-0.489	-0.498	-0.501	-0.502	-0.496	0
Hours	100	287	692.8	779.8	903.8	965.8	945.7	987.8	1075.3	1017.2	1144.7	780
	(.)	-196.9	-405.5	-472.2	-462.9	-475.3	-473.2	-502.4	-489.5	-477.1	-394.6	-590.3
Hourly Wage	6.778	8.545	8.457	10.54	9.941	10.53	10.43	25.27	10.64	11.77	12.86	8.477
	(.)	-1.093	-4.878	-43.68	-14.6	-18.14	-11.29	-206.9	-6.528	-10	-7.926	-2.262

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Table 3: Proportion Working Full/Part time
Hours worked
Wages
Enrolled in 4 year college

	16	17	18	19	20	21	22	23	24	25	26
Work Full Time	0	0.0513	0.0347	0.0626	0.0831	0.11	0.133	0.224	0.313	0.326	0.25
	0	-0.223	-0.183	-0.242	-0.276	-0.313	-0.34	-0.417	-0.465	-0.471	-0.5
Hours	.	2990	2121.9	2238.8	2304.6	2255.8	2197.5	2232.7	2188.6	2353.9	2014
	(.)	-1654.6	-346.8	-518.5	-1095.9	-1141.5	-603.1	-482.3	-381.7	-510.1	(.)
Hourly Wage	.	9.377	10.09	12.67	10.22	20.64	12.07	13.32	14.55	14.48	15.02
	(.)	-0.513	-7.935	-31.88	-6.273	-96.73	-6.528	-10.28	-9.96	-6.713	(.)
Work Part Time	0.5	0.641	0.82	0.812	0.781	0.769	0.721	0.593	0.522	0.528	0.5
	-0.707	-0.486	-0.385	-0.391	-0.413	-0.422	-0.449	-0.492	-0.501	-0.502	-0.577
Hours	168	458.3	630.2	679.7	766.7	806.8	866.9	919.3	907.7	843.6	1253
	(.)	-377	-404.4	-408.3	-425.8	-445.1	-442.1	-416.6	-515.9	-427.5	-301.2
Hourly Wage	7.897	7.528	11.07	9.773	10.38	11.36	10.39	12.22	11.94	10.65	11.74
	(.)	-2.57	-67.21	-13.16	-19.74	-25.63	-7.576	-22.28	-8.408	-5.728	-2.113

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Table 4: College Financing
Enrolled in 2 year college

	15	16	17	18	19	20	21	22	23	24	25	26
Gift from parents	0	0	0.2	0.322	0.4	0.387	0.327	0.284	0.219	0.125	0.0714	0
	(.)	0	-0.404	-0.467	-0.49	-0.487	-0.469	-0.452	-0.414	-0.332	-0.259	0
Amount	.	.	3782.8	1464.9	1775.4	2001.8	1425.6	1548.2	1452.8	1020.3	1367.6	.
	(.)	(.)	-8914.7	-2798.6	-2812.7	-2666	-2173.3	-2157.3	-3003.4	-1154	-1752.3	(.)
Loan from parents	0	0	0.04	0.049	0.0691	0.0692	0.0566	0.0412	0.0226	0.0234	0	0
	(.)	0	-0.198	-0.216	-0.254	-0.254	-0.231	-0.199	-0.149	-0.152	0	0
Amount	.	.	196	1646.3	2448.3	1601	1339.4	1198.4	3387.9	258.9	.	.
	(.)	(.)	-95.08	-2013.5	-4341.5	-1981.7	-2576.6	-1373.3	-4521.9	-274.3	(.)	(.)
Gift from other	0	0.125	0.06	0.042	0.0495	0.0432	0.0334	0.0325	0.0453	0.00781	0.0429	0
	(.)	-0.354	-0.24	-0.201	-0.217	-0.204	-0.18	-0.178	-0.208	-0.0884	-0.204	0
Amount	.	342.2	1825.6	1556.3	1283.3	2036.6	1484.8	698.6	1383.5	329.5	659.1	.
	(.)	(.)	-1588.2	-1771.4	-1421.2	-3322.5	-2685.9	-588	-3048.9	(.)	-396.1	(.)
Loan from other	0	0	0.02	0.00699	0.0103	0.00541	0.0029	0.00434	0.00377	0	0	0
	(.)	0	-0.141	-0.0834	-0.101	-0.0734	-0.0538	-0.0658	-0.0614	0	0	0
Amount	.	.	658.1	1398.1	1675.3	1535.3	1218.1	178.8	908.6	.	.	.
	(.)	(.)	(.)	-674	-1956.5	-2265.8	-204.8	-137.4	(.)	(.)	(.)	(.)
Receive Grant	0	0	0.14	0.262	0.391	0.372	0.343	0.345	0.317	0.391	0.314	0.333
	(.)	0	-0.351	-0.44	-0.488	-0.484	-0.475	-0.476	-0.466	-0.49	-0.468	-0.577
Amount	.	.	1324.3	2815.9	2688.6	2766.6	2666.8	2079.1	2502.7	1638	1991.1	2197
	(.)	(.)	-1120.1	-4609.3	-3567.7	-4035.8	-4013.3	-1774.8	-3260.4	-1063.7	-1531.5	(.)
Any Subsidized Loan	0	0	0	0.0671	0.12	0.131	0.152	0.154	0.125	0.0859	0.186	0.333
	(.)	0	0	-0.25	-0.326	-0.337	-0.36	-0.361	-0.331	-0.281	-0.392	-0.577
Amount	.	.	.	4196.6	4622.3	2967.4	4669.1	4025.2	3337.5	2885	3802.5	10985
	(.)	(.)	(.)	-6413.8	-6585.7	-3126.3	-7596.7	-5121.7	-4419.4	-1836	-1817.5	(.)
Employer Aid	0	0	0	0.0042	0.00654	0.0151	0.016	0.0304	0.0528	0.0313	0.1	0
	(.)	0	0	-0.0647	-0.0806	-0.122	-0.125	-0.172	-0.224	-0.175	-0.302	0
Amount	.	.	.	1527.1	1343.2	1044.4	2200.6	5188.3	1040.5	353.4	1092.1	.
	(.)	(.)	(.)	-775.8	-670.5	-1033.3	-2865.6	-14861.4	-847	-150.2	-663.6	(.)

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Table 5: College Financing
Enrolled in 4 year college

	16	17	18	19	20	21	22	23	24	25	26
Gift from parents	0.5	0.179	0.433	0.581	0.604	0.591	0.507	0.37	0.249	0.281	0.25
	-0.707	-0.389	-0.496	-0.494	-0.489	-0.492	-0.5	-0.483	-0.433	-0.452	-0.5
Amount	329	7621.7	5591.2	6141	5730.6	5324.2	4531.5	2991.7	3140.5	2626.5	109.8
	(.)	-9771.7	-6510.5	-16621.8	-7698.9	-7563.7	-5764.3	-3295.4	-4525.1	-2215.8	(.)
Loan from parents	0	0.0256	0.0508	0.0713	0.0622	0.0683	0.0791	0.0389	0.0348	0.0562	0
	0	-0.16	-0.22	-0.257	-0.242	-0.252	-0.27	-0.194	-0.184	-0.232	0
Amount	.	131.6	5101.3	5273.3	5121.9	4916.1	3036.4	3105.8	3183.2	2262.9	.
	(.)	(.)	-6127.2	-6982.9	-8851.1	-8066.3	-3944.8	-2098.5	-3885.1	-3360.4	(.)
Gift from other	0	0.0256	0.06	0.0762	0.082	0.0743	0.0626	0.0545	0.0647	0.0337	0
	0	-0.16	-0.238	-0.265	-0.274	-0.262	-0.242	-0.227	-0.247	-0.181	0
Amount	.	3737.6	3486.6	2882	3519.2	3525.2	2219	2079	2069	1647.7	.
	(.)	(.)	-5381.4	-4162.7	-6265.3	-5303.9	-3419.3	-2136.2	-1440.4	-1318.2	(.)
Loan from other	0	0	0.00212	0.00586	0.00723	0.00848	0.0124	0.0117	0.00498	0.0112	0
	0	0	-0.046	-0.0763	-0.0848	-0.0917	-0.111	-0.108	-0.0705	-0.106	0
Amount	.	.	5117.9	1284	3959.2	3146.5	2209.1	2027	439.4	5492.5	.
	(.)	(.)	-4113.7	-1952.5	-5906.7	-4147.7	-2431.5	-2113.8	(.)	(.)	(.)
Receive Grant	0	0.205	0.444	0.527	0.54	0.53	0.51	0.444	0.453	0.326	0.75
	0	-0.409	-0.497	-0.499	-0.499	-0.499	-0.5	-0.497	-0.499	-0.471	-0.5
Amount	.	11072.7	6566.1	6689.4	6099.3	5867.4	5453.2	4317.9	4476.2	4305.9	3808.1
	(.)	-12679.7	-7151.1	-7869	-6561.3	-7321.5	-5960.7	-4564.2	-5754.1	-4389.1	-3368.5
Any Subsidized Loan	0.5	0.0513	0.237	0.318	0.361	0.401	0.447	0.438	0.398	0.393	0.5
	-0.707	-0.223	-0.425	-0.466	-0.48	-0.49	-0.497	-0.497	-0.491	-0.491	-0.577
Amount	13366.8	2582.4	4043.8	4067.4	4582.8	4255.1	4508.5	4336	4231.5	4686.4	18399.9
	(.)	-1890.1	-4168.4	-4382.7	-5220.1	-4215.7	-4581.9	-4086.7	-3338.3	-3675.1	-17477
Employer Aid	0	0	0.00917	0.0127	0.0121	0.0135	0.0206	0.0311	0.0697	0.0225	0
	0	0	-0.0954	-0.112	-0.109	-0.115	-0.142	-0.174	-0.255	-0.149	0
Amount	.	.	3043.5	2537.3	2373.2	1914.1	1217.7	2777.9	1627.7	3240.6	.
	(.)	(.)	-3644.6	-3377.9	-3156.2	-1517.9	-848.6	-5074	-914.1	-3495.4	(.)

Table 6: Probability of Completing 2 year College Conditional on Enrollment

	Quantiles of cognitive ability				
	1	2	3	4	Total
19	0	0.006173	0.010526	0.021739	0.010889
20	0.033613	0.01626	0.077491	0.08377	0.054414
21	0.063492	0.06278	0.083721	0.142857	0.087432
22	0.114286	0.090226	0.110294	0.127907	0.108235
23	0.083333	0.093333	0.078652	0.075758	0.082707
24	0.058824	0.128205	0	0.086957	0.072072

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Table 7: Probability of Completing 4 year College Conditional on Enrollment

	Quantiles of cognitive ability				
Age	1	2	3	4	Total
20	0	0.004464	0	0.003018	0.002261
21	0.018519	0.019901	0.014315	0.014957	0.015476
22	0.019608	0.082353	0.164103	0.228454	0.183309
23	0.2	0.153061	0.18	0.31361	0.244745
24	0	0.108696	0.119048	0.231707	0.148472
25	0	0.076923	0	0.034483	0.028169