DOES IT PAY TO GET A REVERSE MORTGAGE?

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Many of America's elderly are considering reverse mortgages as a way to relieve their financial pressures. These financial instruments let homeowners remain in their homes for as long as possible and borrow against their home equity at terms that include large up-front costs and high interest rates. Repayment is triggered by moving, made out of home sale proceeds, and capped by the value of those proceeds. Reverse mortgagees who borrow sums that are large relative to their home's value and remain in their home for an extended period win this gamble. They enjoy both the use of their home and the borrowed money and simply hand over the keys when they exit their home for the last time. Reverse mortgagees who borrow large sums and exit early lose this gamble. Thus, a reverse mortgage constitutes the purchase of a no-exit annuity: one that pays off in the form of housing services provided one does not permanently exit the home. Since not exiting is partly conditioned on not dying, the no-exit annuity encompasses some longevity insurance. But it also introduces extra risk associated with exiting the home prior to death.

This paper uses data on single households from the Health and Retirement Study (HRS) to study the economic gains or losses associated with reverse mortgages. These data are examined within a dynamic structural life-cycle model featuring consumption, housing, and mobility decisions with uncertainty about both life span and mobility. I develop and apply new methods for solution and estimation based on a combination of four state-of-the-art mathematical programming tools. I find that reverse mortgages are likely to impose large losses on house-rich but cash-poor households. For such households, taking out the standard reverse mortgage and borrowing the maximum amount permitted reduces expected utility, on average, to the same degree as a 14 percent loss in financial wealth.

KEYWORDS: housing, elderly mobility, dynamic discrete and continuous choices, constrained optimization approach

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1 Introduction

Reverse mortgages are federally insured private loans specifically designed for house-rich but cash-poor homeowners, as safe instruments able to relieve their financial pressure. However, few homeowners have used them. Although 86 percent of seniors know what a reverse mortgage is (Harris Interactive, 2007), in 2007 only 1 percent of the 30.8 million Americans 62 and over closed a reverse mortgage contract. The reverse mortgage market was created in 1987 by the Department of Housing and Urban Development (HUD) and after 20 years is still at 1 percent of its estimated potential (Redfoot et al., 2007).

This paper explains the low usage of reverse mortgages by showing that they are risky to the borrower, with risks arising both from the probability of earlier-than-expected death and from the probability of moving prior to death. For most retirees, their home is their most valuable asset, and potentially their major source of finance for retirement. More than 80 percent of retirees own their home (Munnell et al., 2007), for a total value of approximately $4 trillion. According to the traditional life-cycle model, developed first by Modigliani and Brumberg (1954), Ando and Modigliani (1963), and Friedman (1957), individuals make their saving choices so as to smooth consumption over their lifetime. By that theory, households build savings during their working years and divest those savings to meet their consumption needs in retirement. Empirically, however, saving over the life cycle does not follow that pattern. In particular, most seniors do not cash in their home equity to finance consumption in retirement. Instead, homeownership rates remain stable until later in life. One reason for this pattern in the past may have been that, before the advent of the reverse mortgage, liquidating home equity effectively required selling the home and moving out.

Reverse mortgages under the HUD program differ from conventional home loans in several respects. These loans are federally insured and regulated by the Federal Housing Administration (FHA). In addition, there are no income or other credit requirements, and there is no risk of default or foreclosure. Nevertheless, reverse mortgages are characterized by large up-front costs and high interest rates. The main reason for the high up-front costs is the mortgage insurance premium paid to the FHA. By charging this premium, the FHA is able to insure the borrower against the risk of the lender's default. A reverse mortgage accrues interest beginning with the first payment to the borrower. Although there are no interest payments for the duration of the loan, moving triggers the repayment of the loan plus accumulated interest. The balance due is repaid out of the proceeds from the sale of the home and is capped by the value of those proceeds.

Several economists have advocated strong public policy support for reverse mortgages. However, the relative weakness of the demand for such mortgages suggests that they do not meet retirees' needs and wants. The focus of this paper is on the systemic reasons that prevent reverse mortgages from becoming a common tool to finance consumption in retirement. In particular, why do house-rich but cash-poor homeowners choose not to cash in the savings locked in their home through a HUD reverse mortgage but instead prefer to maintain low levels of consumption?

There are many psychological reasons why older homeowners may be reluctant to tap their home equity, such as aversion to debt and a desire to keep the home debt free. This paper provides a rational explanation for their behavior, namely the risk of having to move. Since moving triggers repayment of the loan, exogenous and unpredictable events that force the retiree to move out cannot be disregarded. Therefore, assessing the potential for reverse mortgages requires jointly analyzing consumption, housing, and moving decisions.

The degree of risk aversion and the preference for housing over consumption are not observable; hence, I use a structural model to estimate these preference parameters. The estimated structural model is sufficiently rich that it can be used to perform policy experiments and to evaluate the welfare gain from reverse mortgages under different conditions. The model features consumption, liquid saving, and illiquid homes. Homes can be owned or rented. Moving is costly. Households are subject to life span uncertainty and to housing preference shocks that could force them to move. Some simplifications are introduced to keep the model tractable and to highlight the novel computational contribution. One of these is that home prices are assumed to be non stochastic. This assumption is consistent with the sample data, given that movements in home prices for the segment of population analyzed are relatively small.

Financial, demographic, and consumption data on reverse mortgagees are not publicly available. Consequently, I select a subsample of single retirees from the Health and Retirement Study (HRS) that could represent a potential target segment for reverse mortgages. These retirees are homeowners 62 years old or older, with little or no debt, who support their consumption mainly with Social Security income. Typically, their non housing financial wealth is a fraction of their home value. The sample data include both discrete and continuous data; therefore the paper extends the literature on discrete choice by including continuous choices.

I develop and apply new methods for solution and estimation based on a combination of four state-of-the-art mathematical programming tools.The empirical model is estimated using a recently developed set of mathematical programming tools. The past decade has seen a significant increase in computing speed and technological progress in the algorithms and software used to solve large-scale problems. Although many economic applications involve nonlinear large-scale and optimization problems, very few economic problems have been examined using mathematical programming approaches. Specifically, the set of tools used in this paper includes mathematical programming with equilibrium constraints (MPEC), flexible polynomial approximation, shape preservation, and the imposition of the envelope theorem. By formulating the structural estimation of a life-cycle dynamic model as a constrained optimization problem, I avoid having to repetitively solve the structural model. Instead, only one equilibrium is computed, that associated with the optimal structural parameters and the optimal economic variables. Moreover, using an L1 approximation makes the functional form more flexible and easily adaptable to changing requirements. Finally, by imposing shape preservation and the envelope theorem, in addition to a high-order polynomial to approximate the continuous state value function, I obtain an accurate approximation, which is crucial for structural estimation. This is the first paper to use the four tools in combination.

This analysis yields two main results. First, I obtain reasonable estimates of the structural parameters that are within the range of estimates found in the related literature. The coefficient of relative risk aversion is 1.4. Retirees receive equal utility from housing services and from consumption. Retirees' degree of altruism is zero. This final estimate is consistent with the characteristics of my sample, most of whom are relatively poor and value their own consumption more than leaving a bequest.

Second, the model explains why house-rich but cash-poor homeowners have not bought reverse mortgages, by invoking issues related to moving risk. Reverse mortgages provide liquidity and a form of longevity insurance; however, moving becomes a risky proposition. A homeowner who moves out must repay the lesser of the home's value and the outstanding debt, and the up-front costs are lost. Both consumption and housing profiles are affected in the periods following the move. I find that reverse mortgages are a very bad option for house-rich but cash-poor homeowners. For such homeowners, taking out the standard reverse mortgage and borrowing the maximum amount permitted reduces expected utility, on average, to the same degree as a 14 percent loss in financial wealth. On the other hand, cash-rich homeowners benefit from the contract. Moreover, even though home prices are assumed to be non stochastic in the model because of the small variation in home prices in the sample data, the risk of moving could be even larger if, after closing a reverse mortgage, the value of the home declines. In this scenario, the resources available for consumption after moving would be lower, and therefore the retiree's welfare loss would increase.

The structure of the paper is as follows. Section 2 reviews the literature. Section 3 explains the features of a reverse mortgage contract, and provides some empirical evidence about reverse mortgagees. Section 4 presents the household's life-cycle model. Section 5 describes the solution method. Section 6 illustrates the HRS data. Section 7 reports the results and the welfare analysis. Section 8 presents some policy experiments. Section 9 concludes.

2 Literature Review

This paper draws on three main sources of economic literature: on life-cycle and precautionary saving, on housing and portfolio choice, and on discrete choice.

I build on the studies of life-cycle behavior in Kotlikoff and Summers (1981), Carroll and Summers (1991), and Kotlikoff et al. (2001). Hubbard et al. (1994) and Carroll (1997) parameterize and simulate life-cycle consumption models with uncertainty. Attanasio et al. (1999), Gourinchas and Parker (2002), Cagetti (2003), French (2005), and De Nardi et al. (2009) structurally estimate life-cycle models. Hubbard et al. (1994), Palumbo (1999), and Hurd (1989) represent good attempts at modeling consumer behavior after retirement. However, these papers do not take housing into account. Given the empirical evidence that for most retirees their home is their most valuable asset, I extend this literature by examining the optimal consumption and housing choice for older homeowners.

Specifically, I follow Cocco (2004) and Yao and Zhang (2005) by explicitly modeling the housing decision and allowing households to derive utility from both housing and other consumption goods. Meyer and Speare (1985) study types and determinants of senior mobility.

Additionally, I follow the literature on discrete choice. Keane and Wolpin (1997) structurally estimate a discrete-choice, finite-horizon dynamic model of schooling, work, and occupational decisions. They use a simulation and interpolation method. First, multiple integrals are simulated at a subset of the state space points using Monte Carlo integration. Then, their value is interpolated at every other point using a regression function that fits the points in the initial subset. Even though this interpolating function works well, the arguments can be costly to compute, and the entire state space has to be repeatedly spanned in each simulation or interpolation. I use instead a flexible polynomial approximation in the state variable, which is much cheaper because only the coefficients of the polynomial have to be computed to evaluate the approximated value function over the entire state space. Rust (1987,1988) introduced the literature on infinite-horizon, discrete-choice models. This framework was further extended in Hotz and Miller (1993) and Aguirregabiria and Mira (2002). Judd and Su (2008) applied the MPEC approach to estimate the Zurcher bus model (Rust, 1987) and showed that the direct optimization approach to the problem is significantly faster than the nested fixed-point approach. However, most of the theoretical papers and the empirical applications focus only on discrete choice. Given that my sample involves both discrete and continuous data, I extend this literature by including continuous choices. Moreover, I present the first application of the MPEC approach to an empirical structural model with finite-horizon dynamic programming, where continuous states are introduced using a flexible functional form.

3 Characteristics of Reverse Mortgages

The reverse mortgage market in the United States was created in 1987 with the HUD program called Home Equity Conversion Mortgage (HECM). The Congress passed the FHA reverse mortgage legislation, the Housing and Community Development Act of 1987 (S. 825), on December 22, 1987, and President Ronald Reagan signed it into law on February 5, 1988. In 1996 the Federal National Mortgage Association (Fannie Mae) created the Home Keeper reverse mortgage to address needs unsatisfied by the HECM program, for example those of individuals with higher property values, condominium owners, and seniors wishing to use a reverse mortgage to purchase a new home.[1] Together the two programs allow nearly every senior citizen to access the equity in her home without moving out or taking out a conventional mortgage.

3.1 Features and Requirements of the HECM

A reverse mortgage is a particular kind of home equity loan that allows the owner to cash in some of the equity in her home. The loan does not have to be repaid so long as the borrower lives in the home. To be eligible for a federally insured reverse mortgage, a borrower must be 62 years of age or older, own the home outright (or have a low loan balance), and have no other liens against the home. The borrower does not have to satisfy any credit or income requirements. She can receive the proceeds in one of the following ways: as a lump sum at the beginning, as monthly payments for a fixed term or as a life long annuity, as a line of credit with or without accrual of interest on the credit balance, or some combination of the above. There are no monthly or other payments to be made during the term of the loan. However, a reverse mortgage accrues interest charges, beginning when the first payment is made to the borrower. When she dies or relocates, the repayment is capped at value of the home: the loan is thus a nonrecourse loan. The amount that may be borrowed is a function of the age of the borrower and any co-applicant, the current value of the property and its expected appreciation rate, the current interest rate, and interest rate volatility.

A reverse mortgage is just one of several financial instruments that allow a homeowner to secure liquid funds against the equity in her home. In general, home equity conversion products could be useful to all who are house-rich but cash-poor. Conventional home equity loans are different from reverse mortgages in four respects. First, they require the periodic payment of interest and some principal before moving. Second, because borrowers promise to make those periodic payments, their ability to make them is an issue, and thus the maximum amount that can be borrowed is determined by factors including the borrower's credit history and income. Third, failure to repay the loan or meet loan requirements may result in foreclosure. Fourth, the up-front costs are generally lower.

In the early 1990s, projections of the potential demand for reverse mortgages among older households varied between 800,000 (Merrill et al., 1994) and more than 11 million (Rasmussen et al., 1995). A more recent study (Stucki, 2005) estimated the potential market at 13.2 million. Yet in fact, only 265,234 federally insured reverse mortgages had been issued at the end of 2007 (HUD, 2007). This represents about 1 percent of the 30.8 million households with at least one member aged 62 and older in 2006 (U.S. Census Bureau, 2006) and about 2 percent of the potential market as estimated by Stucki.

[1] A few lending institutions offer non-HECM reverse mortgages. Such loans may exceed the HECM limit, are not federally insured, and usually have a higher interest rate.

3.2 Empirical Evidence on Reverse Mortgage

Financial, demographic, and consumption data for reverse mortgagees are not publicly available. However, in December 2006 AARP conducted the first national survey of reverse mortgage borrowers. According to that survey, more than half of reverse mortgage borrowers (54 percent) reported having less than \$25,000 in financial savings. For a third of borrowers (33 percent), self-reported annual income was less than \$20,000, and for nearly two-thirds (62 percent) it was less than \$30,000. Reverse mortgagees tend to be house-richer than typical older homeowners. Nearly half of reverse mortgage borrowers (46 percent) had homes worth \$100,000 to \$200,000, compared with only about one-third of general homeowners (34 percent). The average property value among borrowers was \$142,000 in 2000, whereas the median home value among non borrowers was \$65,624. More than half (57 percent) of reverse mortgagees in 2000 were single women.

Bishop and Shan (2008), using all 18 years of HECM loan data, present the first systematic evidence on loan characteristics over time. The termination hazard rate is low immediately after the closure of the contract and then increases with time. However, if the reverse mortgage contract has not been terminated within 10 years, the borrower is expected to remain in the home for a very long time (see Figure 1).

Davidoff and Welke (2007) show that reverse mortgage borrowers have exited their homes surprisingly quickly. Only 66 percent of male and 62 percent of female loan terminations are ascribed to death as opposed to payoff while alive.

4 The Model

This section describes a model of post retirement decision making. I consider optimal consumption, housing, and moving decisions for a single retiree. When the retiree decides to move out of her house, transaction costs are incurred.

4.1 Preferences

Individual i's plan is to maximize her expected lifetime utility at age t, where $t = 64, ..., T$. T is set exogenously and equals 95. In each period she receives utility U from nondurables consumption and housing services.

The within-period retiree's preference over consumption and housing services is represented by the Cobb-Douglas utility function:

$$U(C_{i,t}, d_{i,t}) = \frac{(C_{i,t}^{1-\omega}(\psi^{rent} H_{i,t}^{\omega}))^{1-\gamma}}{1-\gamma} + \varepsilon_{i,t}(d_{i,t}), \tag{1}$$

where $C_{i,t}$ denotes consumption, $H_{i,t}$ home value, $\psi^{rent} H_{i,t}$ housing services (implicit rent), ω the relative importance of housing services versus the numeraire nondurable consumption good, and γ the coefficient of relative risk aversion. Let $d_{i,t}$ be the discrete housing choice, described in the next subsection.

Individuals move out of their homes for several reasons, which are explained in detail in the HRS survey. They may move out for financial reasons, looking for a smaller or less expensive home; because they desire to live near or with their children or other relatives; because of health problems; for climate or weather reasons; for reasons related to leisure activities or public transportation; or because of changes in marital status. I model this unobserved utility from moving as a housing preference shock, $\varepsilon_{i,t}(d_{i,t})$. It is extreme value type I distributed and independent across individuals and time.

When the individual dies, her terminal wealth $TW_{i,t}$ is bequeathed according to a bequest function $b(TW_{i,t})$:

$$b(TW_{i,t}) = \theta_B \frac{TW_{i,t}^{1-\gamma}}{1-\gamma}. \tag{2}$$

The degree of altruism is given by the parameter θ_B. Carroll (2000) employs a similar bequest function.

4.2 Choice Set

In each discrete period t, the household makes two joint and simultaneous choices, a discrete housing choice and a continuous consumption choice.

Housing is a discrete multistage choice: each household chooses whether to move or stay in the home; households that move out choose whether to own or to rent, and the value of the new home. Consistent with the sample of HRS data, I assume that homeowners that move could not afford a larger home and that renters can only rent a new home (of any value).

First, the household makes the discrete choice of staying or moving out in period t:

$$d_{i,t}^1 = \begin{cases} D_{i,t}^M = 1 & \text{if household } i \text{ moves out of her home in period } t \\ D_{i,t}^M = 0 & \text{otherwise.} \end{cases}$$

Second, if she moves out, she makes the binary choice of owning or renting a new home:

$$d_{i,t}^2 | d_{i,t}^1 = \begin{cases} D_{i,t}^O = 1 & \text{if household } i \text{ owns her home in period } t \\ D_{i,t}^O = 0 & \text{if household } i \text{ rents her home in period } t. \end{cases}$$

Third, if she chooses to own or rent a new home, she chooses the value of the new home. To simplify the computation, I discretize home values:

$$d_{i,t}^3 | d_{i,t}^1, d_{i,t}^2 = H_{i,t}.$$

Therefore, the discrete choice set $d_{i,t}$ is

$$d_{i,t} = \{d^1_{i,t}, d^2_{i,t}, d^3_{i,t}\}.$$

Let $C_{i,t}$ be the continuous choice of consumption.

4.3 Housing Expenses

Per-period housing expenses ψ are assumed to be a fraction of the home's value; these are deterministic and constant over time. For homeowners, they correspond to a maintenance cost, incurred to keep the house at a constant quality level. For renters, they represent the rental cost. These expenses are denoted by ψ^{own} and ψ^{rent}, respectively, for homeowners and for renters:

$$\psi_{i,t} = [D^O_{i,t}\psi^{own} + (1 - D^O_{i,t})\psi^{rent}]H^*_{i,t}, \tag{3}$$

where $H^*_{i,t} = D^M_{i,t}H_{i,t} + (1 - D^M_{i,t})H_{i,t-1}$.

If the retiree decides to sell her home at time t and move to another, she pays or receives the difference in owner-occupied housing wealth, depending on whether the new home's value is greater or smaller than that of the previous home. In addition, she sustains a one-time transaction cost $\phi(D^O_{i,t})$. The cost of moving is

$$M_{i,t} = D^M_{i,t}D^O_{i,t-1}[D^O_{i,t}H_{i,t} - H_{i,t-1} + H_{i,t}\phi(D^O_{i,t})] + D^M_{i,t}(1 - D^O_{i,t-1})(1 - D^O_{i,t})H_{i,t}\phi^{rent}. \tag{4}$$

The transaction cost equals a fraction ϕ^{own}or ϕ^{rent} of the value of the new home:

$$\phi(D^O_{i,t}) = [D^O_{i,t}\phi^{own} + (1 - D^O_{i,t})\phi^{rent}]. \tag{5}$$

Generally, the transaction cost is larger when buying a new home than when renting it; that is, $\phi^{own} > \phi^{rent}$.

4.4 The Household's Problem

The state space in period t consists of variables $X_{i,t}$ that are observed by the agent and the econometrician and by variables $\varepsilon_{i,t}(d_{i,t})$ observed only by the agent:

$$X_{i,t} = \{W_{i,t}, H_{i,t-1}, D^O_{i,t-1}, Age_{i,t}\},$$

where $W_{i,t}$ is household i's nonhousing financial wealth at time t, $H_{i,t-1}$ the previous-period home value, and $D^O_{i,t-1}$ the previous-period housing tenure.

The term $\varepsilon_{i,t}(d_{i,t})$ refers to a vector of unobserved utility components determined by the discrete alternative. Let $\varepsilon_{i,t}$ mean $\varepsilon_{i,t}(d_{i,t})$.

The household maximizes expected lifetime utility over consumption $C_{i,t}$ and housing $d_{i,t}$:

$$V_{i,t}(X_{i,t}, \varepsilon_{i,t}) = \max_{d_{i,t}, C_{i,t}} E_t \left[\sum_{t=64}^{T} \beta^{t-64} (N(t-1,t)\eta_{it} U(C_{i,t}, d_{i,t}) | X_{i,t}, \varepsilon_{i,t}) + b(TW_{i,t}) \right], \tag{6}$$

subject to

$$W_{i,t+1} = RW_{i,t} + y - C_{i,t} - \psi_{i,t} - M_{i,t} \tag{7}$$

$$C_{i,t} \geq C_{MIN}, \tag{8}$$

where $\eta_{i,t}$ denotes the probability of being alive at age t conditional on being alive at age $(t-1)$. Let $N(t,j) = (1/\eta_j) \prod_{k=1}^{t} \eta_k$ denote the probability of living to age t, conditional on being alive at age j.

Eq. (7) represents retiree i's budget constraint in period t. Let y denote the retiree's income, which includes Social Security, pension, and other retiree benefits.

The value function for period t is given by the following expression:

$$V_{i,t}(X_{i,t}, \varepsilon_{i,t}) = \max_{d_{i,t}, C_{i,t}} U(C_{i,t}, d_{i,t}) + \varepsilon_{i,t} + \beta \eta_{i,t+1} EV_{i,t+1}(W_{i,t+1}, H^*_{i,t}, D^O_{i,t}, \varepsilon_{i,t+1} | X_{i,t}, C_{i,t}), \tag{9}$$

subject to

$$\begin{aligned} W_{i,t+1} &= RW_{i,t} + y - C_{i,t} - \psi_{i,t} - M_{i,t} \\ H^*_{i,t} &= D^M_{i,t} H_{i,t} + (1 - D^M_{i,t}) H_{i,t-1} \\ C_{i,t} &\geq C_{MIN}. \end{aligned}$$

The computation of the optimal policy functions is complicated by the presence of the vector $\varepsilon_{i,t}$. It enters nonlinearly in the unknown value function $EV_{i,t+1}$. Following Rust (1987), I introduce the additivity and the conditional independence assumptions. Thus, $EV_{i,t+1}$ does not depend on $\varepsilon_{i,t}$.

Therefore the Bellman equation can be rewritten as

$$\begin{aligned} V_{i,t}(X_{i,t}, \varepsilon_{i,t}) &= \max_{d_{i,t}, C_{i,t}} [U(C_{i,t}, d_{i,t}) + \varepsilon_{i,t} + \beta \eta_{i,t+1} EV_{i,t+1}(X_{i,t+1})] \qquad (10) \\ &= \max_{d_{i,t}} \left\{ \left[\max_{C_{i,t}} \{ U(C_{i,t}, d_{i,t}) + \beta \eta_{i,t+1} V_{i,t+1}(X_{i,t+1}) | d_{i,t} \} \right] + \varepsilon_{i,t} \right\}. \end{aligned}$$

The solution of period t's problem can be divided in two parts. There is an inner maximization with respect to the continuous choice conditional on the discrete housing choice, and an outer maximization with respect to the multistage discrete choice.

I assume that there is a measurement error in consumption, distributed normally with mean 0 and unknown variance σ^2. Given the observed realization of household choices and states $\{C_{i,t}, d_{i,t}, X_{i,t}\}$, the objective is to estimate the preferences denoted as $\theta = \{\gamma, \omega, \sigma, \theta_B\}$. I allow for heterogeneity in the state variables $X_{i,t}$ and $\varepsilon_{i,t}$, but not in the preferences θ.

4.5 Inner Maximization

Let $r(X_{i,t}, d_{i,t})$ represent the indirect utility function associated with the discrete choice $d_{i,t}$:

$$r(X_{i,t}, d_{i,t}) = \max_{C_{i,t}} \{U(C_{i,t}, d_{i,t}) + \beta \eta_{i,t+1} V_{i,t+1}(X_{i,t+1}) | d_{i,t}\}. \tag{11}$$

This function has to be computed for each possible $d_{i,t}$, subject to the contemporary budget constraint and the constraint on consumption.

4.6 Outer Maximization

Under the assumption that $\varepsilon_{i,t}$ is distributed as an extreme value type I error, the conditional choice probabilities are given by the following formula:

$$P(j|X_{i,t}, \theta) = \frac{\exp\{r(X_{i,t}, j)\}}{\sum_{k \in d_{i,t}(X_{i,t})} \exp\{r_{i,t}(X_{i,t}, k)\}} \tag{12}$$

and $V_{i,t+1}(X_{i,t+1})$ is given by:

$$V_{i,t+1}(X_{i,t+1}) = \ln \left[\sum_{k \in d_{i,t}(X_{i,t})} \exp\{r(X_{i,t}, k)\} \right].$$

5 Solution Method

I use a recently developed set of mathematical programming tools to estimate an empirical model. Specifically, this set includes the mathematical programming with equilibrium constraints (MPEC) approach, a flexible polynomial approximation, shape preservation, and the imposition of the envelope theorem for calculating the value functions. This is the first paper to use the envelope theorem in this way, and the first to use the four tools in combination. Moreover, this is the first example of employing the MPEC approach to solve an empirical structural model with finite-horizon dynamic programming.

I illustrate the approach for a simple life-cycle model, underlining its novelty with respect to the conventional approach. The use of a mathematical programming language allows me to rewrite the dynamic programming and estimation problems as a constrained optimization problem that involves the optimization of an objective function subject to equality and inequality constraints. I present the details for the full model in the Appendix.

5.1 Simple Life-Cycle Model

For ease of exposition, I assume that there is only one continuous state variable (wealth) and one continuous choice variable (consumption).

The backward solution from time T for true value functions is described as follows. The last-period value function is known and equal to $V_T(W)$.

In periods $t = 1...(T-1)$ the Bellman equation is

$$V_t(W) = \max_c \ u(c) + \beta V_{t+1}(RW - c).$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three equations that determine optimal consumption, c^*, $V_t(W)$, and $V_t'(W)$:

Euler equation:

$$u_t'(c^*) - \beta V_{t+1}'(RW - c^*) = 0$$

Bellman equation:

$$V_t(W) = u(c^*) + \beta V_{t+1}(RW - c^*)$$

Envelope condition:

$$V_t'(W) = \beta R V_{t+1}'(RW - c^*).$$

The backward solution from time T for approximate value functions requires several steps.

I choose a functional form and a finite grid of wealth levels. Let $W_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. I will use these grid points for approximating the value functions. Let $\Phi(W; a)$ be the function that I use to approximate the value functions, $V(W)$. If I assume that it is a seventh-order polynomial centered at $\overline{W}$, then

$$\Phi(W; a, \overline{W}) = \sum_{k=0}^{7} a_k (W - \overline{W})^k.$$

The time t value function is approximated by

$$V_t\,(W) = \Phi(W; a_t, \overline{W}_t) = \sum_{k=0}^{7} a_{k+1,t} (W - \overline{W}_t)^k, \tag{13}$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{W}$ on time. I choose $\overline{W}_t = (W_t^{\max} + W_t^{\min})/2$, average wealth in period t. Note that $\overline{W}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, I drop it as an explicit argument of Φ. So, $\Phi(W; a_t)$ will mean $\Phi(W; a_t, \overline{W}_t)$.

I would like to find coefficients a_t such that at each time t the Bellman equation, along with the Euler equation and the envelope condition, holds with the Φ approximation; that is, for each time $t < T-2$, I want to find coefficients a_t such that for all W,

$$\Phi(W; a_t) = \max_c \; u(c) + \beta\Phi(RW - c; a_{t+1}),$$

and for time $t = T-1$, I want to find coefficients a_t such that for all W,

$$\Phi(W; a_t) = \max_c \; u(c) + \beta V_T(RW - c).$$

I need to solve the Bellman equation approximately. To this end, I need to specify the various errors that may arise in the approximation. I will consider three errors and one side condition.

First, at each time t and for each $W_{i,t}$, the absolute value of the Euler equation if consumption is $c_{i,t}$, which I denote as $\lambda_{i,t}^e \geq 0$, satisfies the inequality

$$-\lambda_{i,t}^e \leq u'(c_{i,t}) - \beta\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_{i,t}^e, \tag{14}$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $W_{i,t}$ with consumption $c_{i,t}$ is denoted by λ_t^b and satisfies

$$-\lambda_t^b \leq \Phi(W_{i,t}; a_t) - [u(c_{i,t}) + \beta\Phi(RW_{i,t} - c_{i,t}; a_{t+1})] \leq \lambda_t^b. \tag{15}$$

Third, the envelope condition error, λ_t^{env}, satisfies

$$-\lambda_t^{env} \leq \Phi'(W_{i,t}; a_t) - \beta R\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_t^{env}, \tag{16}$$

where $\Phi'(x; a_t)$ is the derivative of $\Phi(x; a_t)$ with respect to x..

Fourth, because the true value functions are concave, I want the approximate value functions to also be concave. Sometimes I will impose concavity of the approximate value functions on the $W_{i,t}$ grid with the secant condition

$$\Phi(W_{i,t}; a_t) \geq \Phi(W_{i-1,t}; a_t) + \frac{\Phi(W_{i+1,t}; a_t) - \Phi(W_{i-1,t}; a_t)}{(W_{i+1,t} - W_{i-1,t})}(W_{i,t} - W_{i-1,t}). \tag{17}$$

With these definitions, the constrained optimization approach to a life-cycle dynamic programming problem can be rewritten as

$$\min_{a,c,\lambda} \sum_t \sum_i \lambda_{i,t}^e + \sum_t \lambda_t^b + \sum_t \lambda_t^{env} \tag{18}$$

subject to

$$-\lambda^e_{i,t} \leq u'(c_{i,t}) - \beta\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda^e_{i,t}$$

$$-\lambda^b_t \leq \Phi(W_{i,t}; a_t) - [u(c_{i,t}) + \beta\Phi(RW_{i,t} - c_{i,t}; a_{t+1})] \leq \lambda^b_t$$

$$-\lambda^{env}_t \leq \Phi'(W_{i,t}; a_t) - \beta R\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda^{env}_t,$$

where I choose the value function approximation parameters, a, the consumption choices on the wealth grid, c, and the errors, $\lambda \geq 0$, so as to minimize the sum of errors. I may also add the concavity constraint if necessary to attain a concave value function approximation.

There are many variations on this theme. Standard value function iteration ignores the $\sum_t \lambda^{env}_t$ term and imposes $\lambda^e_{i,t} = 0$, both of which I could do here. A more general specification would be

$$\min_{a,c,\lambda} P^e \left(\sum_t \sum_i \lambda^e_{i,t} \right) + P^b \left(\sum_t \lambda^b_t \right) + P^{env} \left(\sum_t \lambda^{env}_t \right),$$

where the P^j parameters are penalty terms. Conventional value function iteration is $P^{env} = 0$ and P^e being "infinitely" larger than P^b.

This setup can be easily extended by also including discrete state variables. This would require redefining both the a coefficients and the errors λ over the grid points of the discrete state variables.

In sum, given the last-period value function, I find simultaneously consumption, saving, and the other endogenous variables in each period. Hence, creating a link between past, current, and future economic variables, I obtain the only equilibrium that is associated with the optimal consumption and saving decisions in each period. Given the enormous increase in computer speed and progress in algorithms and software for large-scale problems in the last decade, this technique offers certain advantages. It allows to keep track of the grid of possible values of the state variables, and it is adequate for solving any consumption-saving problem of reasonable complexity.

Given a solution for the dynamic programming problem, I can now consider the empirical analysis. The sample includes continuous data on wealth and consumption. I assume that the measurement error in consumption is normally distributed with mean 0 and unknown variance σ^2. I can use the Euler equation to recover the predicted value of consumption, denoted as c^{pred}. The probability that household n chooses consumption $c_{n,tp}$ in period tp is

$$\Pr(c_{n,tp}|W^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c^{data}_{n,tp} - c^{pred}_{n,tp})^2}{2\sigma^2}}.$$

Therefore the log-likelihood is given by

$$L(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \Pr(c_{n,tp} \mid W^{data}_{n,tp}, \theta). \tag{19}$$

The constrained optimization approach to structural estimation with finite-horizon dynamic programming is

$$Max \qquad L(\theta) - P \cdot \Lambda \tag{20}$$

subject to:

Euler error
Bellman error
envelope error,

where $\Lambda = \sum_t \sum_i \lambda^e_{i,t} + \sum_t \lambda^b_t + \sum_t \lambda^{env}_t$.

The traditional approach to estimating finite-horizon dynamic structural models consists in taking a guess of the structural parameters, solving the dynamic programming problem, calculating the log-likelihood, and repeating these steps until the log-likelihood is maximized. This can be computationally very demanding. Instead, I use the MPEC approach to solve the empirical model. The structural estimation of the life-cycle dynamic model then simply becomes a problem of optimizing an objective of many variables subject to a set of constraints. The structural parameters and endogenous economic variables are chosen simultaneously and symmetrically. The MPEC approach relies on ideas and methods developed in the statistical and econometric literatures; nevertheless, the current econometric literature seems to consider this approach infeasible. Judd and Su (2008) show that it is feasible if one uses conventional techniques in the mathematical programming literature. I extend their approach presenting the MPEC with finite-horizon dynamic programming. The penalty parameter approach introduced in this paper is an example of a nonsmooth exact penalty method. Using an exact penalty function implies that, for certain values of the penalty parameter, a single minimization with respect to the choice variables produces the exact solution of the nonlinear programming problem. For a proof and further reading see Theorem 17.3 in Nocedal and Wright (2000).

Furthermore, I present an example of using a flexible polynomial approximation in empirical work. The continuous state value function is approximated by a seventh-order polynomial, as this functional form appears adequate to the analysis. Since this approach could be applied to a wide range of economic problems, the functional form is easily adaptable to new, different, or changing requirements. For example, if necessary for the accuracy of the solution, the functional form could include a specific basis function in addition to the polynomial.

Next, I introduce shape preservation in approximating the value functions. Under the standard assumption of risk-averse utility, the value function has two main shape features: concavity and monotonicity. If these shape features are deformed by the approximation methods, the approximation errors propagate as the number of computations increases, making the approximate solution inaccurate. This fact motivates me to introduce additional constraints that guarantee the preservation of the shape characteristics of the value function. Specifically, I introduce the secant condition (Eq. 17). This condition is not always necessary; however, if needed, it can be easily added to the set of constraints. For further reading on shape preservation methods see Judd (1998).

The fourth innovative aspect is the imposition of the envelope condition in the set of constraints. In optimization problems, the envelope theorem provides the solution via differentiability techniques; in dynamic programming problems, it is key for characterization, analysis, and computation of the optimal value function from its derivative. By imposing the envelope condition, I obtain both a precise characterization of the optimal solution that is appropriate for computation and an explicit expression for the derivative of the value function. Fernandez-Villaverde et al. (2006) show that dynamic economic models typically lack a closed-form solution; hence, economists approximate the policy functions numerically. It follows that only an approximated likelihood associated with the approximated policy function, instead of the exact likelihood, can be evaluated. Fernandez-Villaverde et al. argue that as the approximated policy function converges to the exact policy function, the approximated likelihood also converges to the exact likelihood. To obtain an accurate approximation of the policy function, a high-order polynomial is required. By introducing both a high-order polynomial approximation and the envelope condition, my approach generates an accurate approximation for the policy function, which is crucial for structural estimation.

The inequality approach I use is formulated as constraints in a nonlinear programming problem and, to my knowledge, is the only stable method for dynamic programming problems of this kind.

Finally, the mathematical programming language I use, AMPL, presents several advantages. AMPL is an extremely easy to use modeling language for linear and nonlinear optimization problems involving discrete or continuous variables. It allows the user to easily access the best algorithm on hand for the specific problem. By using the increasing number of solvers for which AMPL interfaces are available, the researcher can compare alternative optimization methods for any application. In this study I use KNITRO, a solver designed for large nonlinear optimization problems, which is highly valued for its robustness and efficiency. In addition, when mathematical programming problems are expressed in AMPL, the true analytic derivatives are efficiently computed, invisibly to the user, through automatic differentiation. This significantly improves the speed without any additional cost for the user. Moreover, frequently in economic models, Jacobians and Hessians are sparse. That is, even though they could be large in terms of number of elements, most entries equal zero. The major algorithms and software for constrained optimization problems are based on sparse-matrix methods.

6 The Data

The Health and Retirement Study is a U.S. panel survey that covers a wide range of topics. The survey questions on family structure, employment status, demographic characteristics, housing, stocks, bonds, Individual Retirement Accounts (IRAs), other financial assets, income, and pension and Social Security benefits are relevant to the present analysis. Questionnaires assessing individual activities and household patterns of consumption are mailed to a subsample of the HRS. The Consumption and Activities Mail Survey (CAMS), the survey including this information on consumption, was first conducted in 2001. The survey is carried out every two years.

I select a group of households that is a potential target segment for a reverse mortgage. This sample includes single retired homeowners 62 years old or older. I eliminate all households with incomplete records or missing information. After these cuts are made, a sample of 165 single households observed for three consecutive periods between 2000 and 2005 remains.

The data used to estimate the model are nonhousing financial wealth, consumption, housing, income, and demographic data. Nonhousing financial wealth includes stocks, bonds, saving accounts, mutual funds, IRAs, and other assets. It does not include the value of any real estate or business. Given that the target segment has almost no debt, focusing on total nonhousing financial wealth gives nearly the same results as focusing on nonhousing financial assets. Consumption includes vehicles, washing machines, dryers, dishwashers, televisions, computers, telephones, cable, Internet, vehicle finance charges, vehicle insurance, health insurance, food and beverages, dining and drinking out, clothing and apparel, gasoline, prescription and nonprescription medications, health care services, medical supplies, trips and vacations, tickets to movies, sporting events and performing arts, hobbies, contributions to religious, educational, charitable, or political organizations, and cash or gifts to family and friends. Housing expenses for homeowners represent the maintenance cost incurred to keep the home at a constant quality, and for renters represent the rental cost. Social Security is the homeowners' main source of income. Pensions and earned interest on financial assets contribute much less.

Table 1 shows the descriptive statistics for home value, financial wealth, consumption, Social Security income, and age for the first year in the panel. Housing represents a significant proportion of the retirees' total wealth. The median home value is \$70,000. Consumption seems to parallel Social Security income. The average per-period income is \$20,000.

Table 2 presents the composition of the financial portfolio. For almost all the retirees in the sample, the financial portfolio does not contain risky assets. Retirees have most of their savings in checking and saving accounts and transportation. About 40 percent of the retirees have certificates of deposit and approximately 25 percent have IRAs. Fewer than 10 percent have stocks and about 5 percent have bonds.

In each period, about 8 percent of households in the sample move out of their home. Among those who move, about 20 percent decide to rent a new home, while about 80 percent buy a new home. At the end of the three years of the panel, about 20 percent of the population have moved and about 5 percent have rented a new home. Table 3 shows that 67.5 percent of households that move choose to buy a home of equal value and 12.5 percent choose to rent a home of equal value. The moving decision does not appear to be strictly related to age. About 50 percent of the retirees move to live near or with children or other relatives or friends. About 25 percent move for financial reasons, and the remaining 25 percent move because of health problems, for weather or climate reasons, to have a better location, or for other reasons.

7 Calibration and Results

The subjective discount rate β is 0.96 and the real interest rate r is 4 percent. Following Yao and Zhang (2005), the rental rate is $\psi^{rent} = 6$ percent and maintenance cost is $\psi^{own} = 1.5$ percent. Transaction costs are $\phi^{own} = 6$ percent and $\phi^{rent} = 1$ percent, respectively, when moving to an owner-occupied home and to a rental home. Table 4 presents the calibration.

Home value is discretized into three states: \$40,000, \$80,000, and \$120,000. These states are chosen to match the empirical distribution of retiree home values. Even though home prices nationwide appreciated in the period analyzed, each individual's discretized home value does not change significantly in the sample.

I use the MPEC approach to estimate γ, ω, σ, and θ_B. Table 5 presents the estimation results. I find reasonable estimates of the preference parameters that are in the range of previous literature findings.

The coefficient of relative risk aversion γ is 1.419. This estimate is higher than the estimate obtained by Gourinchas and Parker (2002), who fit nonretiree consumption paths through the method of simulated moments. It is, however, lower than the estimate obtained by De Nardi et al. (2009), who match retiree assets at each age, conditional on cohort and income quintile, and lower than the estimate obtained by Cagetti (2003), who matches wealth profiles over the life cycle using the method of simulated moments. Nevertheless, it is similar to the estimate obtained by Attanasio et al.(1999), who fit nonretiree consumption paths through Euler equation estimation.

I obtain an estimate of the preference parameter over housing ω equal to 0.53. To my knowledge, there are no previous structural estimates of this parameter for retirees. This estimate of ω is consistent with the sample data, in which retiree consumption is about the same as implicit rent.

I estimate the degree of altruism θ_B equal to 0. Even though leaving an estate is an important reason to save for many retirees, in reality many households are neither able nor eager to leave an estate. Consistent with their low overall level of financial wealth, I find that retirees do not receive any utility from leaving an estate and prefer to consume all their assets while alive. A value of θ_B equal to 0 implies that any bequest is accidental, generated by the fact that the life span is uncertain. This result is consistent with Hurd (1989), who finds no evidence of a bequest motive.

I compute the standard errors using a bootstrap procedure. Resampling was conducted by sampling with replacement across households, as is standard practice in panel models. In total, the standard errors are calculated with 100 bootstraps.

7.1 Do Reverse Mortgages Pay?

I assume that the reverse mortgage borrower i chooses to receive the proceeds as a lump sum upon closure of the contract at time j.

The maximum amount that can be borrowed initially $V_{i,j}$ is assumed to be a fraction of the home's value and a function of the borrower's age:

$$V_{i,j} = \kappa_i H_{i,j}. \tag{21}$$

In general, the older the borrower, the larger the amount that can be borrowed. At closing, the retiree has to pay some up-front costs, which I denote as $F_{i,j}$. These are assumed to be a fraction λ of the home's value plus closing costs f. Specifically, they include an origination fee that covers the lender's operating expenses (2 percent of the home's value), an up-front mortgage insurance premium (MIP, 2 percent of the home's value), and an appraisal fee and certain other standard closing costs (about \$ 4000).

$$F_{i,j} = \lambda H_{i,j} + f. \tag{22}$$

The up-front costs of reverse mortgages have been significantly larger than those for conventional home loans. This fact has often been cited as one of the main motives for the relative weakness in demand. The main reason for the high up-front costs is the MIP charged by the FHA. In addition to the initial MIP, the FHA charges an ongoing 0.5 percent annual premium on the loan balance. By charging MIPs, the FHA is able to insure the borrower against the risk of the lender's default. Additionally, it insures the lender against the risk that the outstanding debt will exceed the home's value at loan termination. Thus, in this contract the FHA bears the risk of default, and this explains the higher insurance premium compared with those on conventional loans. Until 2008, because of home price growth and borrowers' mobility, the FHA has experienced small losses and retained substantial reserves.

Let $\overline{B}_{i,j}$ denote the cash available to borrower i at time j, after payment of up-front costs. $\overline{B}_{i,j}$ is the lender's initial cost. A reverse mortgage accrues interest charges, beginning when the first payment is made. Thereafter, the interest is compounded annually. Let $G_{i,t}$ be the outstanding debt at time t:

$$G^{RM}_{i,t} = \overline{B}_{i,j} \sum_{j=1..t} (1 + i_D)^{t-j}, \tag{23}$$

where i_D is the nominal interest rate on a reverse mortgage.

If the retiree decides to move out, she has to repay the lesser of the value of the home and the accumulated debt, plus a one-time transaction cost $\phi(D^O_{i,t})$. The cost of moving is

$$M_{i,t} = D^O_{i,t-1} D^M_{i,t} [D^O_{i,t} H_{i,t} - \max(0, H_{i,t-1} - G^{RM}_{i,t}) + H_{i,t}\phi(D^O_{i,t})]. \quad (24)$$

The welfare gain from a reverse mortgage is calculated as the percentage increase in the initial financial wealth that makes the household as well off in expected utility terms without a reverse mortgage as with one. For each household in the sample, I calculate the expected lifetime utility from closing the reverse mortgage contract in 2000, the first year of the panel. Then, I compute the percentage increase in financial wealth that generates the same lifetime utility without a reverse mortgage as with one. I explain the simulation results and assess the validity of the model in predicting the retirees' behavior in light of the empirical evidence on reverse mortgagees.

I first introduce some notation. I define as "cash-poor" those households with initial nonhousing financial wealth less than \$40,000, and all others as "cash-rich". I define as "house-poor" those households with discretized home value equal to \$40,000, and "house-rich" those with discretized home value equal to \$80,000 or \$120,000. The groups are defined so as to have about the same number of households in each group.

Table 6 displays the median nonhousing financial wealth for each group. Table 7 and table 8 display the median welfare gain from taking a reverse mortgage as a percentage and as a dollar value. The common belief is that a reverse mortgage benefits households with resources tied up in home equity, that is those defined as house-rich but cash-poor. This simulation shows otherwise. Specifically, house-rich but cash-poor homeowners experience the largest welfare *loss* from a reverse mortgage, equal to a 14 percent decrease in their initial wealth. Indeed, all cash-poor households experience a welfare loss, whereas all cash-rich households experience a welfare gain.

This simulation, highlighting the pros and the cons of the contract, may help explain why the reverse mortgage is still a niche product after about 20 years. A reverse mortgage provides liquidity and a form of longevity insurance. The retiree can cash in some of the savings locked in her house and thus experience higher consumption than otherwise possible. Furthermore, she can live in the same house while alive regardless of the amount of outstanding debt. A reverse mortgage constitutes the purchase of a no-exit annuity, one that pays off in the form of the housing services of the current home (implicit rent) provided that the retiree does not permanently exit her home. Since not exiting is partly conditioned on not dying, the no-exit annuity encompasses some longevity insurance. However, closing this contract imposes very high up-front costs, which contribute significantly to the welfare loss for house-poor homeowners. For example, a 62-year-old homeowner with a \$40,000 house can borrow about \$20,000. But the cash available at closing, after the payment of about \$10,000 in up-front costs, is only about \$10,000.

Moreover, a reverse mortgage is a financial instrument that incorporates an unusual risk, the risk of moving and having to repay the accumulated debt[2]. Empirical evidence supports this finding. Reverse mortgages should be appealing to homeowners who plan to remain in their home for a long time. In fact, however, reverse mortgagees have exited their homes surprisingly rapidly, suggesting that an unexpected event forced them to move out. If homeowners have to move for any exogenous reason, their future well-being, ability to meet unforeseen costs, consumption profile, and housing choices could be significantly affected. This is especially true for homeowners with initially low financial wealth. The cash-poor homeowner has most of her life savings locked in the home. If she closes a reverse mortgage contract, she reallocates all her savings into a risky financial instrument, which pays little in the worst case, namely, when she has to move out. Moreover, the homeowner has to consider how much money she can comfortably afford to lose in the worst-case scenario. By closing a reverse mortgage, a cash-poor homeowner takes on a high-risk investment from which she cannot escape if she has to move out. Some of the choices over consumption and housing available before closing the reverse mortgage contract are no longer affordable after. For cash-poor homeowners, the cost associated with moving risk exceeds the benefit from longevity insurance, so they experience an overall welfare loss from taking out a reverse mortgage. Hence, a precautionary motive appears to be mostly concentrated among cash-poor homeowners.

On the other side, if they move out while still alive, cash-rich homeowners have enough financial resources to repay the loan without limiting their future consumption and housing choices, and if they remain in their home until death, they benefit from a higher level of consumption and from the longevity insurance. For cash-rich homeowners, the benefit from longevity insurance exceeds the cost associated with the moving risk, so they experience an overall welfare gain from taking out a reverse mortgage.

8 Counterfactual Experiments

The framework presented above allows for many possible counterfactual experiments. In this section I choose the following three: no moving risk, no up-front costs, and a reduction in current income. These experiments allow me to better identify the risk-expanding and the risk-mitigating aspects of a reverse mortgage.

8.1 No Moving Risk

Assume that the retiree faces no moving risk and remains in her house while alive. A reverse mortgage then becomes a safe financial instrument. After clos-

[2] In this study, moving risk is associated with the decrease in initial financial wealth that generates the same lifetime utility without a reverse mortgage as with one.

ing, the retiree receives the proceeds as a lump sum net of the initial closing costs, accumulates financial assets, consumes, pays the per-period housing expenses, faces longevity risk, and pays off the lesser of the loan and the home value when she dies. Table 9 and table 10 present the results. All retirees in this scenario experience a significant welfare gain. The gain for house-rich but cash-poor homeowners is equal to a 430 percent increase in their initial financial wealth, or $45,662 on average. This result supports the rationale behind reverse mortgage contracts. House-rich but cash-poor homeowners can greatly benefit from the contract if they do not move out of their home.

8.2 No Up-Front Costs

According to the AARP survey, many possible reasons could explain the reluctance of older homeowners to tap their home equity: aversion to debt, a desire to leave an estate, or a desire to use home equity as a last resort for economic or health emergencies (Fisher et al., 2007). However, among homeowners who went through counseling but ultimately chose not to apply for a reverse mortgage, high costs were the most frequently cited reason for not applying (by 63 percent of nonapplicants).

In this subsection, I assume zero up-front costs. The retiree receives the proceeds as a lump sum at closing, accumulates financial assets, consumes, pays the per-period housing costs, faces longevity and moving risk, and repays the lesser of the loan and the home value when she moves out. Table 11 and table 12 show the simulation results. Compared with the baseline case, the welfare gain is larger, given the larger portion of liquid funds accessible at closing. Nonetheless, reverse mortgages remain risky financial instruments that are unappealing to house-rich but cash-poor homeowners. The welfare loss comes from the fact that the interest rate on the loan exceeds the interest on savings.

8.3 Reduction in Current Income

Reverse mortgages were originally introduced as a means of relieving retirees from financial pressure. In this subsection, I investigate the case of a 10 percent reduction in current income, to assess the importance of the liquidity insurance aspect of these loans. Increases in living costs and in health care costs and cutbacks in Social Security or in other employee benefits can expose retirees to reductions in their per-period resources available for consumption. Consequently, they might have to adjust to a decreased standard of living in their older years.

After closing, the retiree receives the proceeds as a lump sum net of the initial closing costs, accumulates financial assets, consumes, pays the per-period housing expenses, faces longevity and moving risk, and pays the lesser of the home value and the loan when she moves out.

In this model, retirees are not allowed to borrow, and current consumption is limited by current resources. Thus, a reduction in current income causes a decrease in current consumption. Moreover, the resources available after moving out are lower than in the baseline scenario. Therefore, the moving risk is even more pronounced than in the baseline scenario, and the welfare loss is larger for cash-poor homeowners.

9 Conclusion

This paper examines retiree consumption and housing and mobility decisions and provides a plausible explanation for the limited popularity of reverse mortgages. Using a structural dynamic life-cycle model, I find that reverse mortgages provide liquidity and a form of longevity insurance but introduce a new risk, the moving risk. Closing this contract is especially risky for house-rich but cash-poor homeowners. If they have drawn on their home equity through a reverse mortgage, their ability to meet unforeseen costs or move into alternative housing may be limited. Intuitively, a reverse mortgage can be seen as a gamble. Gambling involves risking a small stake for a large prize. The small stake is the up-front cost that the retiree has to pay to participate in the "reverse mortgage game." The big prize is the use of her own home and the higher consumption that she could enjoy if she "wins", by not moving out. If the retiree moves out while alive, she loses the gamble and incurs a significant welfare loss. Gambling can allow someone who is poor to become rich. However, luck plays an important role. These results underline the urgency for further policy analysis directed at designing safe and appealing financial instruments for the elderly that let them liquidate some of their home equity without incurring major risks. Specifically, the introduction of some form of insurance against the risk of moving would make this financial instrument more attractive for house-rich but cash-poor homeowners.

This paper presents a novel application of four mathematical programming tools. This application is valuable both for solving life-cycle models and for estimating them. It avoids repetitive solutions of the model for estimation purposes and at the same time permits to have more flexible functional forms to approximate continuous state variables. To explain the main features of the methodology, the model is kept sufficiently simple. Nevertheless, this methodology could be very useful in many contexts and is quite adaptable to changing requirements.

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Technical Appendix

MPEC with Dynamic Programming: Discrete and Continuous Choices

The panel data used in this study involve 3 years and 165 individuals. The available data are both continuous and discrete. The continuous data include consumption and nonhousing financial wealth. The discrete (or discretized) data are the individual's housing tenure (own versus rent), her moving decision, and her home value. I have additional data on the individuals' demographics, including age.

The MPEC with dynamic programming (DP) approach simultaneously solves the dynamic programming problem and the maximum likelihood estimation of the preference parameters.

Dynamic Programming with Approximation of the Value Function

Life-Cycle Model

One continuous state variable: financial wealth.

Two discrete state variables: previous-period housing tenure and previous-period house value.

One continuous choice variable: consumption.

Many discrete choices: Not move(N), Move to home with value h with housing tenure q (Mhq), where $q =\{\text{own, rent}\}$.

Backward Solution from Time T for True Value Functions

In each period, the household chooses whether to stay in the home or move out. If she moves out, she can either buy or rent a new home and can choose its value. Let the subscripts d^N, d^{Mhq} denote, respectively, the decision not to move and the decision to move to a home valued at h with housing tenure q. Housing tenure is a binary variable that takes a value of 1 if the household owns the home.

The last-period value function is known and equal to $V_T(W, H, Q)$, where W is the individual's nonhousing financial wealth, H her previous-period house value, and Q her previous-period housing tenure.

In periods $t = 1...(T-1)$ I define

$$
\begin{aligned}
V_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q) + \varepsilon_t^N \\
V_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q) + \varepsilon_t^{Mqh},
\end{aligned}
$$

where M is the transaction cost:

$$M = Q[qh - H + \phi^{own}qh + \phi^{rent}(1-q)h] + (1-Q)(1-q)\phi^{rent}h$$

and ψ is the per period housing expense:

$$\psi = [Q\psi^{own} + (1-Q)\psi^{rent}]H + [q\psi^{own} + (1-q)\psi^{rent}]h.$$

c_{d^N} and $c_{d^{Mqh}}$ are the consumption levels, respectively, if the individual does not move and if she moves to home value h choosing housing tenure q. y is the household's per-period income. η_{t+1} is her survival probability. ε_t^N and ε_t^{Mqh} are extreme value type I errors.

Following Rust (1987), I assume that the additivity and the conditional independence assumptions hold.

To simplify the notation, I introduce the following expressions, which are evaluated at the optimal consumption level:

$$\begin{aligned}
\widehat{V}_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q) \\
\widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q).
\end{aligned}$$

The extreme value assumption on ε_t implies that I can reduce the dimensionality of the dynamic programming problem. The Bellman equation is given by the following closed-form solution:

$$\begin{aligned}
V_t(W,H,Q) &= \Pr(N|W,H,Q)\cdot\widehat{V}_{d^N,t} + E(\varepsilon_t^N|N) \\
&\quad + \sum_h\sum_q\{\Pr(Mhq|W,H,Q)\cdot\widehat{V}_{d^{Mhq},t} + E(\varepsilon_t^{Mhq}|Mhq)\} \\
&= \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h\sum_q\exp(\widehat{V}_{d^{Mhq},t})\right\}
\end{aligned}$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three sets of equations that determine optimal consumption, $c^*_{d^N}$, $c^*_{d^{Mhq}}$, $V_t(W,H,Q)$, and $V'_t(W,H,Q)$:

Euler equations:

$$\begin{aligned}
u'(c^*_{d^N}, H) - \beta\eta_{t+1}V'_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q) &= 0 \\
u'(c^*_{d^{Mhq}}, h) - \beta\eta_{t+1}V'_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q) &= 0
\end{aligned}$$

Envelope condition:

$$V'_t(W,H,Q) = \Pr(N|W,H,Q)\cdot\widehat{V}'_{d^N,t} + \sum_h\sum_q\Pr(Mhq|W,H,Q)\cdot\widehat{V}'_{d^{Mhq},t}$$

Bellman equation:

$$V_t(W,H,Q) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}.$$

The time $t = 1...(T-1)$ probabilities of not moving and moving to home value h with housing tenure q are:

$$\Pr(N|W,H,Q) = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(V_t(W,H,Q))}$$

$$\Pr(Mhq|W,H,Q) = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(V_t(W,H,Q))}.$$

Backward Solution from Time T for Approximate Value Functions

Let $\Phi(W,H,Q;a)$ and $\Phi_d(W,H,Q;b)$ be the functions that I use to approximate, respectively, the value functions $V(W,H,Q)$ and the policy functions $c_d^*(W,H,Q)$, with $d = \{d^N, d^{Mhq}\}$. If I assume that they are seventh-order polynomials centered at $\overline{W}$, then

$$\Phi(W,H,Q;a,\overline{W}) = \sum_{k=0}^{7} a_{k,H,Q}(W-\overline{W})^k.$$

The time t value function is approximated by

$$V_t\,(W,H,Q) = \Phi(W,H,Q;a_t,\overline{W}_t) = \sum_{k=0}^{7} a_{k+1,H,Q,t}(W-\overline{W}_t)^k.$$

The time t policy functions are approximated by

$$c_{d,t}^*\,(W,H,Q) = \Phi(W,H,Q;b_{d,t},\overline{W}_t) = \sum_{k=0}^{7} b_{k+1,H,Q,d,t}(W-\overline{W}_t)^k,$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{W}$ on time, and the dependence of the policy functions on time is represented by the dependence of the b coefficients and the center $\overline{W}$.

I choose $\overline{W}_t = (W_t^{\max} + W_t^{\min})/2$, the period t average level of wealth. Note that $\overline{W}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, I drop it as an explicit argument of Φ. So, $\Phi(W,H,Q;a_t)$ will mean $\Phi(W,H,Q;a_t,\overline{W}_t)$.

I would like to find coefficients a_t and $b_{d,t}$ such that each time t Bellman equation, along with the Euler and envelope conditions, holds with the Φ approximation; that is, for each time $t < T-2$, I want to find coefficients a_t such that for all W

$$\Phi(W, H, Q; a_t) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\},$$

where

$$\begin{aligned}\widehat{V}_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}\Phi_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q; a_{t+1}) \\ \widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}\Phi_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q; a_{t+1}),\end{aligned}$$

and for time $t = T-1$, I want to find coefficients a_t given that

$$\begin{aligned}\widehat{V}_{d^N,T-1} &= u(c^*_{d^N}, H) + \beta\eta_T V_T(RW - c^*_{d^N} - \psi + y; H, Q) \\ \widehat{V}_{d^{Mhq},T-1} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_T V_T(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q).\end{aligned}$$

I need to solve the Bellman equation approximately. To this end, I define various errors.

First, I create a finite grid of asset levels I will use for approximating the value functions. Let $W_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. Then I create a grid of home values. Let $H_{j,t}$ be grid point j in the time t grid. Housing tenure is a binary variable. Let $Q_{z,t}$ be grid point z in the time t grid.

Next I need to specify the various errors that may arise in the approximation. I will consider three errors and one side condition.

First, at each time t and for each $W_{i,t}$ and each previous-period home value $H_{j,t-1}$ and housing tenure $Q_{z,t-1}$, the absolute value of the Euler equations if consumption is, respectively, $c^*_{i,j,z,d^N,t}$ and $c^*_{i,d^{Mhq},t}$, which I denote as $\lambda^e_{i,j,z,t} \geq 0$, satisfies the inequality

$$-\lambda^e_{i,j,z,t} \leq u'(c^*_{i,j,z,d^N,t}, H_{j,t-1}) - \beta\eta_{t+1}\Phi'(RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \leq \lambda^e_{i,j,z,t}$$

$$-\lambda^e_{i,j,z,t} \leq u'(c^*_{i,d^{Mhq},t}, h_t)$$

$$-\beta\eta_{t+1}\Phi'(RW_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1}) \leq \lambda^e_{i,j,z,t},$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $W_{i,t}$ with consumption $c_{i,j,z,d^N,t}$ and $c_{i,d^{Mhq},t}$ is denoted by $\lambda^b_{j,z,t}$ and satisfies

$$-\lambda^b_{j,z,t} \leq \Phi(W_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) - \ln\left\{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^b_{j,z,t},$$

where

$$\begin{aligned}\widehat{V}_{i,j,z,d^N,t} &= u(c^*_{i,j,z,d^N,t}, H_{j,t-1}) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \\ \widehat{V}_{i,d^{Mhq},t} &= u(c^*_{i,d^{Mhq},t}, h_t) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1})\end{aligned}$$

Third, the envelope condition errors, $\lambda^{env}_{j,z,t}$, satisfy

$-\lambda^{env}_{j,z,t} \leq \Phi'(W_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) - \{f_{i,j,z,d^N,t} \cdot \Phi'(RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t-1}, Q_{z,t-1}; a_{t+1})$

$+ \sum_h \sum_q [f_{i,d^{Mhq},t} \cdot \Phi'(RW_{i,t} - c_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1})]\} \leq \lambda^{env}_{j,z,t},$

where

$$f_{i,j,z,d,t} = \Pr(d|W_{i,t}, H_{j,t}, Q_{z,t}) = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}.$$

Fourth, I introduce the policy function errors:

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_{z,t}; b_t) - c^*_{i,j,z,d,t}(W_{i,t}, H_{j,t}, Q_{z,t}) \leq \lambda^{cons}_{i,j,z,d,t}.$$

Empirical Part

In the theoretical DP part I obtain the coefficients used in the approximation of the value function. In this part, for any individual data on financial wealth, previous-period home value and age, I calculate predicted consumption and the probability of moving. The individual makes the housing decision $d_{n,tp}$ and the consumption decision simultaneously.

Let $c^{pred}_{n,tp}$ and $c^{data}_{n,tp}$ denote, respectively, the predicted and the true value of consumption for household n at time tp. For any given discrete choice on housing $d_{n,tp}$, using the real data on consumption, I calculate the measurement error:

$$\Pr(c_{n,tp}|d_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c^{data}_{n,tp} - c^{pred}_{n,tp})^2}{2\sigma^2}}.$$

The probability for the discrete choice on housing is given by

$$\Pr(d_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) = \frac{e^{\widehat{V}_{d,n,tp}}}{\sum_m e^{\widehat{V}m,n,tp}}.$$

Therefore the joint probability of making the discrete housing choice $d_{n,tp}$ and the continuous consumption choice $c_{n,tp}$ is given by

$$\Pr(d_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) =$$

$$\Pr(d_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) \cdot \Pr(c_{n,t}|d_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}).$$

The log-likelihood is given by:

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, \theta),$$

where N denotes the number of individuals in the sample and TP the number of time periods in the panel data.

MPEC

With these definitions, let

$$\Lambda = \sum_t \sum_i \sum_j \sum_z \lambda^e_{i,j,z,t} + \sum_t \sum_j \sum_z \lambda^b_{j,z,t} + \sum_t \sum_j \sum_z \lambda^{env}_{j,z,t} + \sum_t \sum_i \sum_j \sum_z \sum_d \lambda^{cons}_{i,j,z,d,t},$$

and let P be a penalty parameter.

The MPEC approach to the estimation of the preference parameters is

$$\underset{\theta,a,c}{Max}\ \mathcal{L}(\theta) - P \cdot \Lambda$$

subject to
Bellman error

$$-\lambda^b_{j,z,t} \leq \Phi(W_{i,t}; a_t) - \ln \left\{ \exp(\widehat{V}_{i,i,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t}) \right\} \leq \lambda^b_{j,z,t}$$

Euler errors

$$\begin{aligned} -\lambda^e_{i,j,z,t} &\leq u_{c;i,j,z,d^N,t} + \beta \Phi^+_{W;i,j,z,d^N,t} \leq \lambda^e_{i,j,z,t} \\ -\lambda^e_{i,j,z,t} &\leq u_{c;i,d^{Mhq},t} + \beta \Phi^+_{W;i,d^{Mhq},t} \leq \lambda^e_{i,j,z,t} \end{aligned}$$

envelope error

$$-\lambda^{env}_{j,z,t} \leq \Phi_{W;i,z,t} - \{ f_{i,j,z,d^N,t} \cdot \Phi^+_{W;i,j,z,d^N,t} + \sum_h \sum_q [f_{i,d^{Mhq},t} \cdot \Phi^+_{W;i,d^{Mhq},t}] \} \leq \lambda^{env}_{j,z,t}$$

policy function error

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_{z,t}; b_{d,t}) - c^*_{i,j,z,d,t}(W_{i,t}, H_{j,t}, Q_{z,t}) \leq \lambda^{cons}_{i,j,z,d,t}$$

The probability of decision d is

$$f_{i,j,z,d,t} = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})},$$

where Φ^+ denotes the approximation for the next-period value function, as described in the next subsection.

AMPL

Backward Solution from Time T for Approximate Value Functions in AMPL

In order to formulate this problem in AMPL, I need to list every quantity that is computed. The time-specific asset grids $W_{i,t}$ are fixed. The parameters are

$$W_{i,t}, \beta, \eta_{i,t}, R, \psi^{own}, \psi^{rent}, \phi^{own}, \phi^{rent}, \theta_B.$$

The basic variables of interest are

$$\begin{array}{l} c_{i,j,z,d^N,t},\ c_{i,d^{Mhq},t} \\ a_{k,j,z,t},\ b_{k,j,z,d,t} \\ \lambda^e_{i,j,z,t},\ \lambda^b_{j,z,t},\ \lambda^{env}_{j,z,t},\ \lambda^{cons}_{i,j,z,d,t}. \end{array}$$

AMPL does not allow procedure programming; therefore, I need to define other variables to represent quantities defined in terms of other variables. I first need

$$\begin{aligned} u_{i,j,z,d^N,t} &\equiv u\left(c^*_{i,j,z,d^N,t}, H_{j,t-1}\right) \\ u_{c;i,j,z,d^N,t} &\equiv u'\left(c^*_{i,j,z,d^N,t}, H_{j,t-1}\right) \\ W^+_{i,j,z,d^N,t} &\equiv RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y \\ f_{i,j,z,d^N,t} &= \Pr(N|W_{i,t}, H_{j,t-1}, Q_{z,t-1}) \\ u_{i,d^{Mhq},t} &\equiv u\left(c^*_{i,d^{Mhq},t}, h_t\right) \\ u_{c;i,d^{Mhq},t} &\equiv u'\left(c^*_{i,d^{Mhq},t}, h_t\right) \\ W^+_{i,d^{Mhq},t} &\equiv RW_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{hq} - M_{j,z,d^{Mhq}} + y \\ f_{i,d^{Mhq},t} &= \Pr(Mhq|W_{i,t}, H_{j,t-1}, Q_{z,t-1}). \end{aligned}$$

I next use these variables to construct more variables:

$$\begin{aligned} \Phi_{i,j,z,t} &\equiv \Phi(W_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) \\ \Phi_{W;i,j,z,t} &\equiv \Phi'(W_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) \\ \Phi^+_{i,j,z,d^N,t} &\equiv \Phi(W^+_{i,j,z,d^N,t}, H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \\ \Phi^+_{W;i,j,z,d^N,t} &\equiv \Phi'(W^+_{i,j,z,d^N,t}, H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \\ \Phi^+_{i,d^{Mhq},t} &\equiv \Phi(W^+_{i,d^{Mhq},t}, h_t, q_t; a_{t+1}) \\ \Phi^+_{W;i,d^{Mhq},t} &\equiv \Phi'(W^+_{i,d^{Mhq},t}, h_t, q_t; a_{t+1}) \\ \Psi_{i,j,z,d,t} &\equiv \Phi(W_{i,t}, H_{j,t-1}, Q_{z,t-1}; b_{d,t}) \end{aligned}$$

With these variables defined, the Bellman equation error inequality becomes

$$-\lambda^b_{j,z,t} \le \Phi_{i,j,z,t} - \ln\left\{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})\right\} \le \lambda^b_{j,z,t},$$

where

$$\begin{aligned}\widehat{V}_{i,j,z,d^N,t} &= u_{i,j,z,d^N,t} + \beta\eta_{t+1}\Phi^+_{i,j,z,d^N,t} \\ \widehat{V}_{i,d^{Mhq},t} &= u_{i,d^{Mhq},t} + \beta\eta_{t+1}\Phi^+_{i,d^{Mhq},t}\end{aligned}$$

the Euler equation error inequalities become

$$\begin{aligned}-\lambda^e_{i,j,z,t} &\le u_{c;i,j,z,d^N,t} + \beta\Phi^+_{W;i,j,z,d^N,t} \le \lambda^e_{i,j,z,t} \\ -\lambda^e_{i,j,z,t} &\le u_{c;i,d^{Mhq},t} + \beta\Phi^+_{W;i,d^{Mhq},t} \le \lambda^e_{i,j,z,,t},\end{aligned}$$

and the envelope error inequality becomes

$$-\lambda^{env}_{j,z,t} \le \Phi_{W;i,j,z,t} - \{f_{i,j,z,d^N,t}\cdot\Phi^+_{W;i,j,z,d^N,t} + \sum_h \sum_q [f_{i,d^{Mhq},t}\cdot\Phi^+_{W;i,d^{Mhq},t}]\} \le \lambda^{env}_{j,z,t}.$$

The probability of decision d is then

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}.$$

The policy function errors are

$$-\lambda^{cons}_{i,j,z,d,t} \le \Psi_{i,j,z,d,t} - c^*_{i,j,z,d,t} \le \lambda^{cons}_{i,j,z,d,t}.$$

Empirical Part in AMPL

I consider individuals in our sample such that $Age^{data}_{n,tp} = 1...(T-2)$.

Let $W^{data}_{n,tp}$, $Age^{data}_{n,tp}$, $H^{data}_{n,tp-1}$ and $Q^{data}_{n,tp-1}$ denote the data on nonhousing financial wealth, age, previous-period home value and previous-period housing tenure for household n in year tp in the panel data. Given these data, the variables of interest are

$$c^{pred}_{d^N,n,tp} = \Psi_{d^N}(W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; b_{Age^{data}_{n,tp}})$$

$$c^{pred}_{d^{Mhq},n,tp} = \Psi_{d^{Mhq}}(W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; b_{Age^{data}_{n,tp}})$$

$$u^{pred}_{d^N,n,tp} \equiv u\left(c^{pred}_{d^N,n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}\right)$$

$u_{c;d^N,n,tp} \equiv u'\left(c_{d^N,n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}\right)$

$W^+_{d^N,n,tp} \equiv RW^{data}_{n,tp} - c^{pred}_{d^N,n,tp} - \psi(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) + y$

$f^{pred}_{d^N,n,tp} = \Pr(N|W^{data}_{n,t}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp})$

$u^{pred}_{d^{Mhq},n,tp} \equiv u\left(c^{pred}_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}\right)$

$u_{c;d^{Mhq},n,tp} \equiv u'\left(c_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}\right)$

$W^+_{d^{Mhq},n,tp} \equiv RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) +$
y

$f^{pred}_{d^{Mhq},n,tp} = \Pr(Mhq|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp})$.

I next use these variables to construct more variables

$\Phi^{data}_{n,tp} \equiv \Phi(W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}})$

$\Phi^{data}_{W;n,tp} \equiv (\Phi^{data})'(W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}})$

$\Phi^+_{d^N,n,tp} \equiv \Phi^{data}(W^+_{d^N,n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})$

$\Phi^+_{W,d^N,n,tp} \equiv \Phi'(W^+_{d^N,n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})$

$\Phi^+_{d^{Mhq},n,tp} \equiv \Phi(W^+_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})$

$\Phi^+_{W;d^{Mhq},n,tp} \equiv \Phi'(W^+_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})$

$\widehat{V}^{pred}_{d^N,n,tp} = u(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp}) + \beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{NM},n,tp} - \psi(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) +$
$y; H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})$

$$\widehat{V}^{pred}_{d^{Mhq},n,tp} = u(c^{pred}_{d^{Mhq},n,tp}, H^{choice}_{n,tp}) + \beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) \\ - M(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + y; H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1}).$$

The probabilities of not moving and moving are

$$f^{pred}_{d^N,n,tp} = \Pr(H_{d^N,n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^N,n,tp})}{\exp(\widehat{V}^{pred}_{d^N,n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(H_{d^{Mhq},n,tp} | W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}{\exp(\widehat{V}^{pred}_{d^{N},n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}.$$

The measurement error in consumption is normally distributed with mean 0 and variance σ^2:

$$\Pr(c_{n,tp} | d_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(c^{data}_{n,tp} - c^{pred}_{d,n,tp})^2}{2\sigma^2}).$$

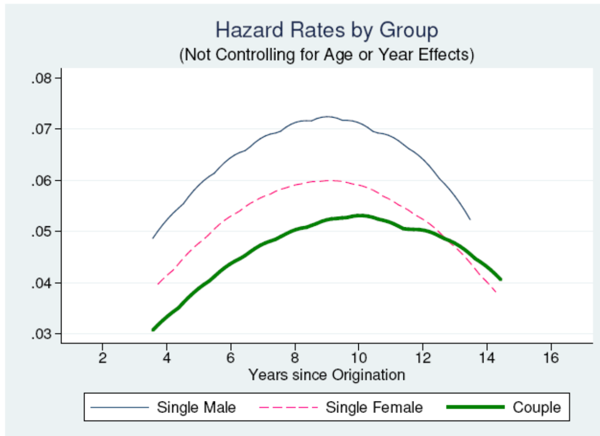

Figure 1: Termination Hazard Rates of HECM Loans for Single Males, Single Females, and Couples (Bishop and Shan,2008)

Table 1. Descriptive Statistics

	Percentiles			Min	Max
	25%	**50%**	**75%**		
H	$40,000	$70,000	$90,000	$ 3,000	$170,000
W	$6,000	$25,000	$ 69,500	$0	$ 276,548
C	$ 6,347	$ 9,774	$ 15,409	$650	$84,380
ss	$7,200	$ 9,600	$11,748	$0	$ 18,907
Age	69	75	80	66	86

Table 2 Financial Portfolio Composition

	Percentiles			Min	Max
	25%	**50%**	**75%**		
Stocks	$0	$0	$0	$0	$125,000
Chck	$750	$3,600	$10,000	$0	$100,000
CDs	$0	$0	$5,300	$0	$273,548
Tran	$1000	$4,000	$ 8,000	$0	$30,000
Bonds	$0	$0	$0	$0	$80,000
IRA	$0	$0	$2,500	$0	$137,000
Debt	$0	$0	$0	$0	$12,000

Table 3 Housing Choices when Moving

			Housing Choices		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Percentage of Households	67.5%	12.5%	12.5%	2.5%	5%

Housing Choices:
(1) Buy a house of equal value
(2) Rent a house of equal value
(3) Buy a smaller house
(4) Rent a smaller house
(5) Rent a larger house

Table 4. Calibration

Parameter	Variable	
β	Subjective discount rate	0.96
ψ^{rent}	Rental rate	6%
ψ^{own}	Maintenance costs	1.5%
ϕ^{rent}	Transaction costs	1%
ϕ^{own}	Transaction costs	6%
r	Real interest rate	4%
ϑ^{RM}	Fixed reverse mortgage margin plus ongoing premium	4%
r_D	Fixed interest rate on a reverse mortgage	8%
λ	Origination fees plus MIP	4%
f	Standard closing costs	$ 4000

Table 5. Structural Estimation Results

Parameter	Variable	Estimate
γ	Coefficient of relative risk aversion	1.4196 (0.013)
ω	Preference parameter over housing	0.5325 (0.032)
σ	s.d. of measurement error in consumption	1.206 (0.640)
θ_B	Degree of altruism	0.000 (0.001)

Table 6 Median Nonhousing Financial wealth

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	$8,500	$ 10,600
Cash-Rich	$107,800	$ 90,200

Table 7 Median Welfare Gain, Baseline Case (Percentage)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	-9%	-14%
Cash-Rich	50%	41%

Table 8 Median Welfare Gain, Baseline Case (Dollars)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	- $ 767	- $ 1,525
Cash-Rich	$ 53,302	$ 36,863

Table 9. Median Welfare Gain, No Moving Risk (Percentage)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	207%	430%
Cash-Rich	18%	54%

Table 10. Median Welfare Gain, No Moving Risk (Dollars)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	$ 17,600	$ 45,622
Cash-Rich	$ 19,054	$ 48,883

Table 11. Median Welfare Gain, No Up-Front Costs (Percentage)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	2%	-12%
Cash-Rich	56%	49%

Table 12. Median Welfare Gain, No Up-Front Costs (Dollars)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	$ 146	- $ 1,374
Cash-Rich	$ 60,021	$ 43,784

Table 13. Median Welfare Gain, 10% Cut in Current Income (Percentage)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	-20%	-16%
Cash-Rich	49%	46%

Table 14. Median Welfare Gain, 10% Cut in Current Income (Dollars)

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	- $ 1,657	- $ 1,654
Cash-Rich	$ 53,298	$ 41,728