“New Developments in Econometrics”

Lecture 5

Instrumental Variables with Treatment Effect Heterogeneity: Local Average Treatment Effects

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1. Introduction

1. Instrumental variables estimate average treatment effects, with the average depending on the instruments.

2. Population averages are only estimable under unrealistically strong assumptions ("identification at infinity", or under the constant effect).

3. Compliers (for whom we can identify effects) are not necessarily the subpopulations that are *ex ante* the most interesting subpopulations, but need extrapolation for others.

4. The set up here allows the researcher to sharply separate the extrapolation to the (sub-)population of interest from exploration of the information in the data.

2. Basics

Linear IV with Constant Coefficients. Standard set up:

$$Y_i = \beta_0 + \beta_1 \cdot W_i + \varepsilon_i.$$

There is concern that the regressor W_i is endogenous, correlated with ε_i. Suppose that we have an instrument Z_i that is both uncorrelated with ε_i and correlated with W_i.

In the single instrument / single endogenous regressor, we end up with the ratio of covariances

$$\hat{\beta}_1^{\text{IV}} = \frac{\frac{1}{N}\sum_{i=1}^{N}(Y_i - \bar{Y}) \cdot (Z_i - \bar{Z})}{\frac{1}{N}\sum_{i=1}^{N}(W_i - \bar{W}) \cdot (Z_i - \bar{Z})}.$$

Using a central limit theorem for all the moments and the delta method we can infer the large sample distribution without additional assumptions.

Potential Outcome Set Up

Let $Y_i(0)$ and $Y_i(1)$ be two potential outcomes for unit i, one for each value of the endogenous regressor or treatment. Let W_i be the realized value of the endogenous regressor, equal to zero or one. We observe W_i and

$$Y_i = Y_i(W_i) = \begin{cases} Y_i(1) & \text{if } W_i = 1 \\ Y_i(0) & \text{if } W_i = 0. \end{cases}$$

Define two potential outcomes $W_i(0)$ and $W_i(1)$, representing the value of the endogenous regressor given the two values for the instrument Z_i. The actual or realized value of the endogenous variable is

$$W_i = W_i(Z_i) = \begin{cases} W_i(1) & \text{if } Z_i = 1 \\ W_i(0) & \text{if } Z_i = 0. \end{cases}$$

So we observe the triple Z_i, $W_i = W_i(Z_i)$ and $Y_i = Y_i(W_i(Z_i))$.

3. Local Average Treatment Effects

The key instrumental variables assumption is

Assumption 1 (Independence)

$$Z_i \perp\!\!\!\perp (Y_i(0), Y_i(1), W_i(0), W_i(1)).$$

It requires that the instrument is as good as randomly assigned, and that it does not directly affect the outcome. The assumption is formulated in a nonparametric way, without definitions of residuals that are tied to functional forms.

Assumptions (ctd)

Alternatively, we separate the assumption by postulating the existence of four potential outcomes, $Y_i(z, w)$, corresponding to the outcome that would be observed if the instrument was $Z_i = z$ and the treatment was $W_i = w$.

Assumption 2 (Random Assignment)

$$Z_i \perp\!\!\!\perp (Y_i(0,0), Y_i(0,1), Y_i(1,0), Y_i(1,1), W_i(0), W_i(1)).$$

and

Assumption 3 (Exclusion Restriction)

$$Y_i(z, w) = Y_i(z', w), \qquad \text{for all } z, z', w.$$

The first of these two assumptions is implied by random assignment of Z_i, but the second is substantive, and randomization has no bearing on it.

Compliance Types

It is useful for our approach to think about the compliance behavior of the different units

		$W_i(0)$ 0	1
$W_i(1)$	0	never-taker	defier
	1	complier	always-taker

We cannot directly establish the type of a unit based on what we observe for them since we only see the pair (Z_i, W_i), not the pair $(W_i(0), W_i(1))$. Nevertheless, we can rule out some possibilities.

		Z_i 0	1
W_i	0	complier/never-taker	never-taker/defier
	1	always-taker/defier	complier/always-taker

Monotonicity

Assumption 4 (Monotonicity/No-Defiers)

$$W_i(1) \geq W_i(0).$$

This assumption makes sense in a lot of applications. It is implied directly by many (constant coefficient) latent index models of the type:

$$W_i(z) = 1\{\pi_0 + \pi_1 \cdot z + \varepsilon_i > 0\},$$

but it is much weaker than that.

Implications for Compliance types:

		Z_i	
		0	1
W_i	0	complier/never-taker	never-taker
	1	always-taker	complier/always-taker

For individuals with $(Z_i = 0, W_i = 1)$ and for $(Z_i = 1, W_i = 0)$ we can now infer the compliance type.

Distribution of Compliance Types

Under random assignment and monotonicity we can estimate the distribution of compliance types:

$$\pi_a = \mathrm{Pr}(W_i(0) = W_i(1) = 1) = \mathbb{E}[W_i|Z_i = 0]$$

$$\pi_c = \mathrm{Pr}(W_i(0) = 0, W_i(1) = 1) = \mathbb{E}[W_i|Z_i = 1] - \mathbb{E}[W_i|Z_i = 0]$$

$$\pi_n = \mathrm{Pr}(W_i(0) = W_i(1) = 0) = 1 - \mathbb{E}[W_i|Z_i = 1]$$

Now consider average outcomes by instrument and treatment:

$$\mathbb{E}[Y_i|W_i = 0, Z_i = 0] =$$

$$\frac{\pi_c}{\pi_c + \pi_n} \cdot \mathbb{E}[Y_i(0)|\text{complier}] + \frac{\pi_n}{\pi_c + \pi_n} \cdot \mathbb{E}[Y_i(0)|\text{never} - \text{taker}],$$

$$\mathbb{E}[Y_i|W_i = 0, Z_i = 1] = \mathbb{E}[Y_i(0)|\text{never} - \text{taker}],$$

$$\mathbb{E}[Y_i|W_i = 1, Z_i = 0] = \mathbb{E}[Y_i(1)|\text{always} - \text{taker}],$$

$$\mathbb{E}[Y_i|W_i = 1, Z_i = 1] =$$

$$\frac{\pi_c}{\pi_c + \pi_a} \cdot \mathbb{E}[Y_i(1)|\text{complier}] + \frac{\pi_a}{\pi_c + \pi_a} \cdot \mathbb{E}[Y_i(1)|\text{always} - \text{taker}].$$

From this we can infer the average outcome for compliers,

$$\mathbb{E}[Y_i(0)|\text{complier}], \quad \text{and} \quad \mathbb{E}[Y_i(1)|\text{complier}],$$

Local Average Treatment Effect Hence the instrumental variables estimand, the ratio of these two reduced form estimands, is equal to the local average treatment effect

$$\beta^{\text{IV}} = \frac{\mathbb{E}[Y_i|Z_i = 1] - \mathbb{E}[Y_i|Z_i = 0]}{\mathbb{E}[W_i|Z_i = 1] - \mathbb{E}[W_i|Z_i = 0]}$$

$$= \mathbb{E}[Y_i(1) - Y_i(0)|\text{complier}].$$

4. Extrapolating to the Full Population

We can estimate

$$\mathbb{E}\left[Y_i(0)|\text{never}-\text{taker}\right], \quad \text{and} \quad \mathbb{E}\left[Y_i(1)|\text{always}-\text{taker}\right]$$

We can learn from these averages whether there is any evidence of heterogeneity in outcomes by compliance status, by comparing the pair of average outcomes of $Y_i(0)$;

$$\mathbb{E}\left[Y_i(0)|\text{never}-\text{taker}\right], \quad \text{and} \quad \mathbb{E}\left[Y_i(0)|\text{complier}\right],$$

and the pair of average outcomes of $Y_i(1)$:

$$\mathbb{E}\left[Y_i(1)|\text{always}-\text{taker}\right], \quad \text{and} \quad \mathbb{E}\left[Y_i(1)|\text{complier}\right].$$

If compliers, never-takers and always-takers are found to be substantially different in levels, then it appears much less plausible that the average effect for compliers is indicative of average effects for other compliance types.

5. Covariates

Traditionally the TSLS set up is used with the covariates entering in the outcome equation linearly and additively, as

$$Y_i = \beta_0 + \beta_1 \cdot W_i + \beta_2' X_i + \varepsilon_i,$$

with the covariates added to the set of instruments. Given the potential outcome set up with general heterogeneity in the effects of the treatment, one may also wish to allow for more heterogeneity in the correlations between treatment effects and covariates.

Here we describe a general way of doing so. Unlike TSLS type approaches, this involves modelling both the dependence of the outcome and the treatment on the covariates.

Heckman Selection Model

A traditional parametric model with a dummy endogenous variables might have the form (translated to the potential outcome set up used here):

$$W_i(z) = 1\{\pi_0 + \pi_1 \cdot z + \pi_2' X_i + \eta_i \geq 0\},$$

$$Y_i(w) = \beta_0 + \beta_1 \cdot w + \beta_2' X_i + \varepsilon_i,$$

with (η_i, ε_i) jointly normally distributed (e.g., Heckman, 1978). Such a model impose restrictions on the relation between compliance types, covariates and outcomes:

$$i \text{ is a } \begin{cases} \text{never} - \text{taker} & \text{if} \quad \eta_i < -\pi_0 - \pi_1 - \pi_2' X_i \\ \text{complier} & \text{if} \quad -\pi_0 - \pi_1 - \pi_2' X_i \leq \eta_i < -\pi_0 - \pi_1 - \pi_2' X \\ \text{always} - \text{taker} & \text{if} \quad -\pi_0 - \pi_2' X_i \leq \eta_i, \end{cases}$$

which imposes strong restrictions, e.g., if $\mathbb{E}[Y_i(0)|n, X_i] < \mathbb{E}[Y_i(0)|c, X_i]$, then $\mathbb{E}[Y_i(1)|c, X_i] < \mathbb{E}[Y_i(1)|a, X_i]$

Flexible Alternative Model

Specify

$$f_{Y(w)|X,T}(y|x,t) = f(y|x;\theta_{wt}),$$

for $(w,t) = (0,n),(0,c),(1,c),(1,a)$. A natural model for the distribution of type is a trinomial logit model:

$$\Pr(T_i = \text{complier}|X_i) = \frac{1}{1+\exp(\pi_n' X_i)+\exp(\pi_a' X_i)},$$

$$\Pr(T_i = \text{never}-\text{taker}|X_i) = \frac{\exp(\pi_n' X_i)}{1+\exp(\pi_n' X_i)+\exp(\pi_a' X_i)},$$

$$\Pr(T_i = \text{always}-\text{taker}|X_i) =$$

$$1 - \Pr(T_i = \text{complier}|X_i) - \Pr(T_i = \text{never}-\text{taker}|X_i).$$

The log likelihood function is then, factored in terms of the contribution by observed (W_i, Z_i) values:

$$\mathcal{L}(\pi_n, \pi_a, \theta_{0n}, \theta_{0c}, \theta_{1c}, \theta_{1a}) =$$

$$\times \prod_{i|W_i=0,Z_i=1} \frac{\exp(\pi_n' X_i)}{1+\exp(\pi_n' X_i)+\exp(\pi_a' X_i)} \cdot f(Y_i|X_i;\theta_{0n})$$

$$\times \prod_{i|W_i=0,Z_i=0} \left(\frac{\exp(\pi_n' X_i)}{1+\exp(\pi_n' X_i)} \cdot f(Y_i|X_i;\theta_{0n}) + \frac{1}{1+\exp(\pi_n' X_i)} \cdot f(Y$$

$$\times \prod_{i|W_i=1,Z_i=1} \left(\frac{\exp(\pi_a' X_i)}{1+\exp(\pi_a' X_i)} \cdot f(Y_i|X_i;\theta_{1a}) + \frac{1}{1+\exp(\pi_a' X_i)} \cdot f(Y$$

$$\times \prod_{i|W_i=1,Z_i=0} \frac{\exp(\pi_a' X_i)}{1+\exp(\pi_n' X_i)+\exp(\pi_a' X_i)} \cdot f(Y_i|X_i;\theta_{1a}).$$

Application: Angrist (1990) effect of military service

The simple ols regression leads to:

$$\log(\widehat{\text{earnings}})_i = \underset{(0079)}{5.4364} - \underset{(0.0167)}{0.0205} \cdot \widehat{\text{veteran}}_i$$

In Table we present population sizes of the four treatmen/instrument samples. For example, with a low lottery number 5,948 individuals do not, and 1,372 individuals do serve in the military.

		Z_i 0	1
W_i	0	5,948	1,915
	1	1,372	865

Using these data we get the following proportions of the various compliance types, given in Table , under the non-defiers assumption. For example, the proportion of nevertakers is estimated as the conditional probability of $W_i = 0$ given $Z_i = 1$:

$$\Pr(\text{nevertaker}) = \frac{1915}{1915 + 865}.$$

		$W_i(0)$: 0	$W_i(0)$: 1
$W_i(1)$	0	never-taker (0.6888)	defier (0)
	1	complier (0.1237)	always-taker (0.1875)

Estimated Average Outcomes by Treatment and Instrument

		Z_i = 0	Z_i = 1
W_i	0	$\widehat{\mathbb{E}[Y]} = 5.4472$	$\widehat{\mathbb{E}[Y]} = 5.4028$
	1	$\widehat{\mathbb{E}[Y]} = 5.4076,$	$\widehat{\mathbb{E}[Y]} = 5.4289$

Not much variation by treatment status given instrument, but these comparisons are not causal under IV assumptions.

		$W_i(0)$ 0	1
	0	$\widehat{\mathbb{E}[Y_i(0)]} = 5.4028$	defier (NA)
$W_i(1)$	1	$\widehat{\mathbb{E}[Y_i(0)]} = 5.6948$, $\widehat{\mathbb{E}[Y_i(1)]} = 5.4612$	$\widehat{\mathbb{E}[Y_i(1)]} = 5.4076$

The local average treatment effect is -0.2336, a 23% drop in earnings as a result of serving in the military.

Simply doing IV or TSLS would give you the same numerical results:

$$\widehat{\log(\text{earnings})}_i = \underset{(0.0289)}{5.4836} - \underset{(0.1266)}{0.2336} \cdot \widehat{\text{veteran}}_i$$

It is interesting in this application to inspect the average outcome for different compliance groups. Average log earnings for never-takers are 5.40, lower by 29% than average earnings for compliers who do not serve in the military.

This suggests that never-takers are substantially different than compliers, and that the average effect of 23% for compliers need not be informative never-takers.

Note that
$\mathbb{E}[Y_i(0)|n, X_i] < \mathbb{E}[Y_i(0)|c, X_i]$,
but also $\mathbb{E}[Y_i(1)|c, X_i] > \mathbb{E}[Y_i(1)|a, X_i]$

Compliers earn more than nevertakers when not serving, and more than always-takers when serving. Does not fit standard gaussian selection model.

6. Multivalued Instruments

For any two values of the instrument z_0 and z_1 satisfying the local average treatment effect assumptions we can define the corresponding local average treatment effect:

$$\tau_{z_1,z_0} = \mathbb{E}[Y_i(1) - Y_i(0)|W_i(z_1) = 1, W_i(z_0) = 0].$$

Note that these local average treatment effects need not be the same for different pairs of instrument values (z_0, z_1).

Comparisons of estimates based on different instruments underly conventional tests of overidentifying restrictions in TSLS settings. An alternative interpretation of rejections in such testing procedures is therefore treatment effect heterogeneity.

Interpretation of IV Estimand

Suppose that monotonicity holds for all (z, z'), and suppose that the instruments are ordered in such a way that $p(z_{k-1}) \leq p(z_k)$, where $p(z) = \mathbb{E}[W_i|Z_i = z]$. Also suppose that the instrument is relevant, $\mathbb{E}[g(Z_i) \cdot W_i] \neq 0$. Then the instrumental variables estimator based on using $g(Z)$ as an instrument for W estimates a weighted average of local average treatment effects:

$$\tau_{g(\cdot)} = \frac{\text{Cov}(Y_i, g(Z_i))}{\text{Cov}(W_i, g(Z_i))} = \sum_{k=1}^{K} \lambda_k \cdot \tau_{z_k, z_{k-1}},$$

$$\lambda_k = \frac{(p(z_k) - p(z_{k-1})) \cdot \sum_{l=k}^{K} \pi_l (g(z_l) - \mathbb{E}[g(Z_i)]}{\sum_{k=1}^{K} p(z_k) - p(z_{k-1})) \cdot \sum_{l=k}^{K} \pi_l (g(z_l) - \mathbb{E}[g(Z_i)]},$$

$$\pi_k = \Pr(Z_i = z_k).$$

These weights are nonnegative and sum up to one.

Marginal Treatment Effect

If the instrument is continuous, and $p(z)$ is continuous in z, we can define the limit of the local average treatment effects

$$\tau_z = \lim_{z' \downarrow z, z'' \uparrow z} \tau_{z',z''}.$$

Suppose we have a latent index model for the receipt of treatment:

$$W_i(z) = 1\{h(z) + \eta_i \geq 0\},$$

with the scalar unobserved component η_i independent of the instrument Z_i. Then we can define the marginal treatment effect $\tau(\eta)$ (Heckman and Vytlacil, 2005) as

$$\tau(\eta) = \mathbb{E}\left[Y_i(1) - Y_i(0) \right| \eta_i = \eta \left. \right].$$

This marginal treatment effect relates directly to the limit of the local average treatment effects

$$\tau(\eta) = \tau_z, \quad \text{with} \quad \eta = -h(z)).$$

Note that we can only define this for values of η for which there is a z such that $\tau = -h(z)$.

Normalizing the marginal distribution of η to be uniform on $[0, 1]$, this restricts η to be in the interval $[\inf_z p(z), \sup_z p(z)]$, where $p(z) = \Pr(W_i = 1|Z_i = z)$.

Now we can characterize various average treatment effects in terms of this limit. E.g.:

$$\tau = \int_\eta \tau(\eta) dF_\eta(\eta).$$

7. Multivalued Endogenous Variables

$$\tau = \frac{\mathrm{Cov}(Y_i, Z_i)}{\mathrm{Cov}(W_i, Z_i)} = \frac{\mathbb{E}[Y_i|Z_i = 1] - \mathbb{E}[Y_i|Z_i = 0]}{\mathbb{E}[W_i|Z_i = 1] - \mathbb{E}[W_i|Z_i = 0]}.$$

Exclusion restriction and monotonicity:

$$Y_i(w)\ W_i(z) \perp\!\!\!\perp Z_i, \qquad W_i(1) \geq W_i(0),$$

Then

$$\tau = \sum_{j=1}^{J} \lambda_j \cdot \mathbb{E}[Y_i(j) - Y_i(j-1)|W_i(1) \geq j > W_i(0)],$$

$$\lambda_j = \frac{\Pr(W_i(1) \geq j > W_i(0))}{\sum_{i=1}^{J} \Pr(W_i(1) \geq i > W_i(0))}.$$

with the weights λ_j estimable.

Illustration: Angrist-Krueger (1991) Returns to Educ.

$$\widehat{\text{educ}_i} \;=\; \underset{(0.006)}{12.797} \;-\; \underset{(0.013)}{0.109} \cdot \text{qob}_i$$

$$\log(\widehat{\text{earnings}})_i \;=\; \underset{(0.001)}{5.903} \;-\; \underset{(0.003)}{0.011} \cdot \text{qob}_i$$

The instrumental variables estimate is the ratio

$$\hat{\beta}^{\mathrm{IV}} = \frac{-0.1019}{-0.011} = 0.1020.$$

Weights $\gamma_j = \Pr(W_i(1) \geq j > W_i(0)$ can be estimated as

$$\hat{\gamma}_j = \frac{1}{N_1} \sum_{i|Z_i=1} 1\{W_i \geq j\} - \frac{1}{N_0} \sum_{i|Z_i=0} 1\{W_i \geq j\}.$$

Figure 1: histogram estimate of density of years of education

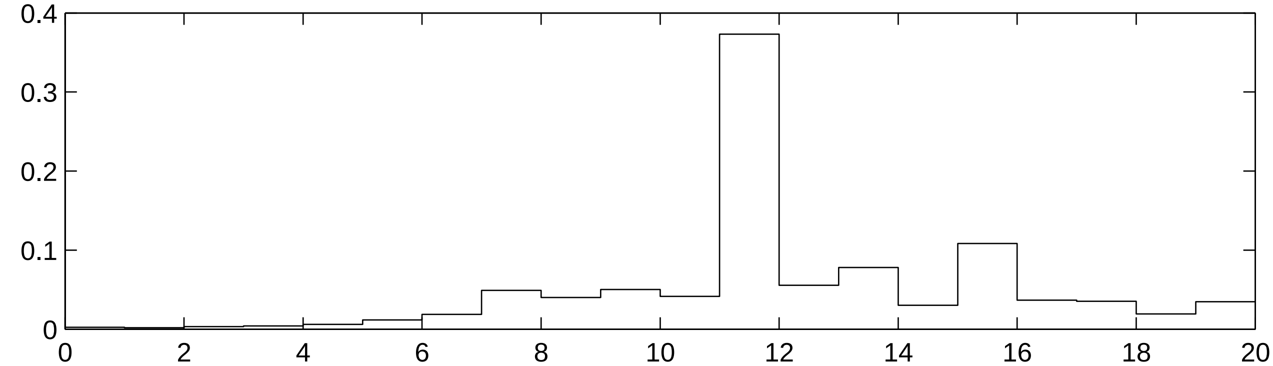

Figure 2: Normalized Weight Function for Instrumental Variables Estimand

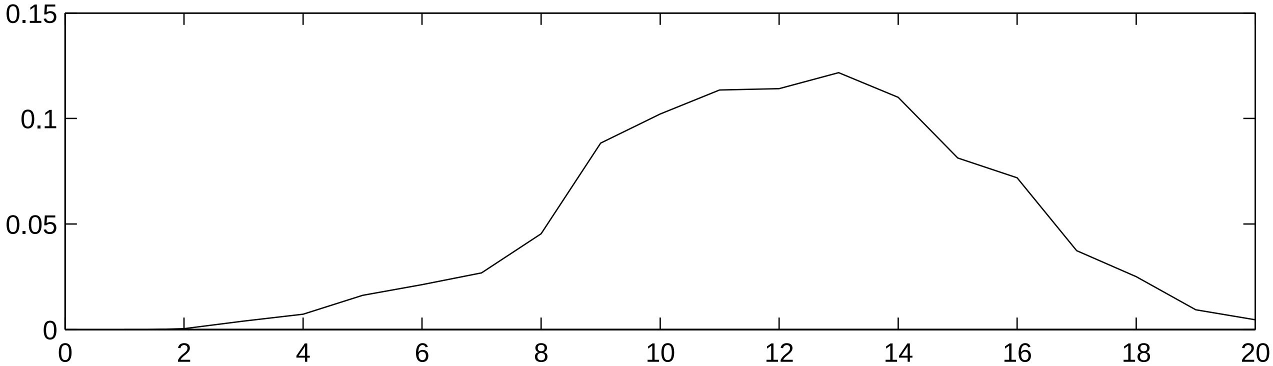

Figure 3: Unnormalized Weight Function for Instrumental Variables Estimand

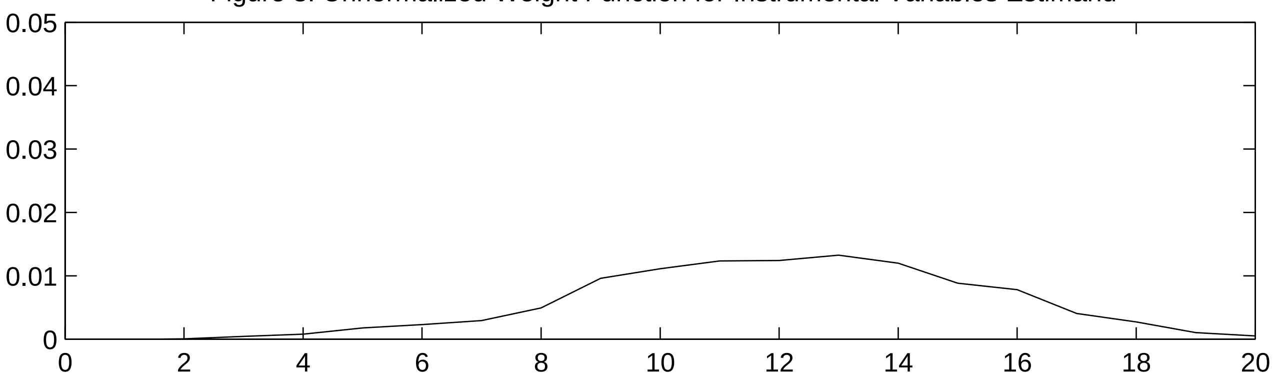

Figure 3: Education Distribution Function by Quarter