Lecture 6

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Other Index Restriction Estimators

Ichimura: $E[y_i \mid x_i] = G(x_i'\beta_0)$, $G(\cdot)$ smooth

$$\hat{\beta}_{SIR} = \underset{\beta \in B}{\operatorname{argmin}} \sum_{i=1}^{N} \left(y_i - \hat{G}(x_i'\beta)\right)^2 \cdot t_{iN}$$

$\hat{G}(x_i'\beta)$ = kernel regression of y_i on $x_i'\beta$

Klein-Spady: $\Pr\{y_i = 1 \mid x_i\} = G(x_i'\beta)$

$$\hat{\beta}_{KS} = \underset{\beta \in B}{\operatorname{argmax}} \sum_{i=1}^{N} \{y_i \log \hat{G}(x_i'\beta) + (1-y_i)\log(1-\hat{G}(x_i'\beta))\} \cdot t_{iN}$$

$t_{iN} \to 0$ as $\|x_i'\beta\| \to \infty$

$$\sqrt{N}(\hat{\beta} - \beta_0) \xrightarrow{d} \mathcal{N}(0, \tilde{V})$$

Robinson's Semilinear Model: $y_i = x_i'\beta_0 + \theta(w_i) + \varepsilon_i$

$$E[\varepsilon_i \mid w_i, x_i] = 0$$

$$E[y_i \mid w_i] = E(x_i \mid w_i)'\beta_0 + \theta(w_i)$$

$$\tilde{y}_i = y_i - E[y_i \mid w_i] = \tilde{x}_i'\beta_0 + \varepsilon_i$$

$$\tilde{\beta} = \left[\sum \tilde{x}_i \tilde{x}_i'\right]^{-1} \tilde{x}_i \tilde{y}_i$$

or $\hat{\tilde{\beta}} = \left[\sum \hat{\tilde{x}}_i \hat{\tilde{x}}_i'\right]^{-1} \hat{\tilde{x}}_i y_i$

or $\hat{\tilde{\beta}} = \left[\sum_i \hat{\tilde{x}}_i x_i\right]^{-1} \hat{\tilde{x}}_i y_i$

$$\sqrt{N}(\tilde{\beta}-\beta_0) \xrightarrow{d} \mathcal{N}(0,\ D^{-1} V D^{-1})$$

$$V = E[\varepsilon_i^2 \tilde{x}_i \tilde{x}_i']$$

$$D = E[\tilde{x}_i \tilde{x}_i']$$

Same true for index estimators

$\tilde{V}$ with unknown G same form as known $G(\cdot)$, replacing "x_i" with "$x_i - E[x_i \mid x_i'\beta_0]$"

Klein-Spady: $\beta = \begin{pmatrix} 1 \\ \gamma \end{pmatrix}$ $\quad x_i = \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix}$

$$\tilde{V}_\gamma = \left\{ E\left[\frac{[G'(x_i'\beta_0)]^2}{G(x_i'\beta_0)(1-G(x_i'\beta_0))} (x_{i2} - E(x_{i2} \mid x_i'\beta_0)) \cdot (x_{i2} - E(x_{i2} \mid x_i'\beta))' \right] \right\}$$

Semilinear estimator uses $x_i - \hat{E}(x_i \mid w_i) \equiv \hat{\tilde{x}}_i$

$\hat{E}(x_i \mid w_i)$ from kernel regression

$$\hat{\beta}_{SL} = \left[\sum_{i=1}^N \hat{\tilde{x}}_i \hat{\tilde{x}}_i' \hat{t}_{iN} \right]^{-1} \sum_{i=1}^N \hat{\tilde{x}}_i \hat{\tilde{y}}_i \hat{t}_{iN}$$

$$\sqrt{N}(\hat{\beta}_{SL} - \beta_0) \xrightarrow{d} \mathcal{N}(0, D^{-1} V D^{-1})$$

<u>Censored Selection Model</u> ("Type II"):

$$d_i = 1(w_i'\delta_0 + \eta_i \geq 0) \qquad \begin{pmatrix} \varepsilon_i \\ \eta_i \end{pmatrix} \perp \begin{pmatrix} w_i \\ x_i \end{pmatrix}$$

$$y_i = d_i \cdot [x_i'\beta_0 + \varepsilon_i]$$

$$y_i = d_i \cdot [x_i'\beta_0 + \varepsilon_i] \qquad (\eta_i) \perp (x_i)$$

$$E[y_i | x_i, w_i, d_i = 1] = x_i'\beta_0 + E[\varepsilon_i | w_i, x_i, d_i = 1]$$
$$= x_i'\beta_0 + \theta(w_i'\delta_0)$$

$$E[d_i | w_i, x_i] = G(w_i'\delta_0)$$

get $\hat{\delta}$ from d_i, w_i

get $\hat{\beta}$ from $y_i - E[y_i | w_i'\delta_0]$, $x_i - E[x_i | w_i'\delta_0]$

$$V[x_i - E[x_i | w_i'\delta_0]] > V[x_i - E[x_i | w_i]]$$

Identification helped if x_i subset of w_i

need exclusion rest'n – no $w_i = x_i$

Estimators: Cosslett, Newey

$$y_i \cong x_i'\beta_0 + \sum_{j=1}^{J} \alpha_j \rho_j(w_i'\hat{\delta}) + \varepsilon_i$$

(Trimmed) LS of y_i on x_i, $\rho_1(w_i'\hat{\delta}), \ldots, \rho_J(w_i'\hat{\delta})$

Let $J \to \infty$ as $N \to \infty$

$\hat{\beta} \xrightarrow{p} \beta_0$, maybe $\sqrt{N}(\hat{\beta} - \beta_0) \xrightarrow{d} \mathcal{N}(0, ?)$

Ahn-Powell:
$$d_i = 1(g(w_i) + \eta_i \geq 0)$$
$$y_i = d_i \cdot [x_i'\beta_0 + \varepsilon_i]$$

$$\begin{pmatrix} \eta_i \\ \varepsilon_i \end{pmatrix} \perp \begin{pmatrix} w_i \\ x_i \end{pmatrix}, \quad p(w_i) \equiv E[d_i | w_i]$$
$$= G(g(w_i)) \quad -\eta_i \sim G(\cdot)$$

$$E[y_i \mid x_i, w_i, d_i = 1] = x_i'\beta_0 + \Theta(g(w_i))$$
$$= x_i'\beta_0 + \Theta(G^{-1}(p(w_i)))$$
$$= x_i'\beta_0 + \Theta^*(p(w_i))$$

<u>Step 1</u> : $\hat{p}(w_i) = \hat{E}[y_i \mid w_i]$ np regression

$(d_i = d_j = 1)$ $\quad y_i - y_j = (x_i - x_j)'\beta_0 + (\Theta^*(p_i) - \Theta^*(p_j)) + u_i - u_j$

$$E[u_i - u_j \mid x_i, x_j, w_i, w_j, d_i = d_j = 1] = 0$$

if $p_i = p_j$ $\qquad E[y_i - y_j \mid \ldots] = (x_i - x_j)'\beta$

$\hat{p}_i \cong \hat{p}_j$

$$\hat{\beta} = \left[\sum_{i<j} K\left(\frac{\hat{p}_i - \hat{p}_j}{h} \right) (x_i - x_j)(x_i - x_j)' \right]^{-1}$$
$$\sum_{i<j} K\left(\frac{\hat{p}_i - \hat{p}_j}{h} \right)(x_i - x_j)(y_i - y_j)$$

$$\sqrt{N}(\hat{\beta} - \beta_0) \xrightarrow{d} \mathcal{N}(0, ?)$$

<u>Kyriazidou</u> : $\qquad d_{it} = 1(w_{it}'\delta_0 + \tau_i + \eta_{it} \geq 0)$
$$y_{it} = d_{it}[x_{it}'\beta_0 + \alpha_i + \varepsilon_{it}]$$

<u>Stage 1</u> - Manski's Max Score Panel Data $\hat{\delta}$

<u>Stage 2</u> : $\quad y_{i2} - y_{i1} = (x_{i2} - x_{i1})'\beta + \Theta(w_{i2}'\delta + \tau_i) - \Theta(w_{i1}\delta + \tau_i)$
$$+ u_{i2} - u_{i1}$$

for $d_{i1} = d_{i2} = 1$

match using $w_{i2}'\hat{\delta} \cong w_{i3}'\hat{\delta}$

$\hat{\beta}$ consistent, not $\sqrt{N}$ consistent

Control Function: $y = H(x_i, \varepsilon)$ $\quad \varepsilon \not\perp x$

$\varepsilon \mid x \not\sim \varepsilon$

Average Structural Function $\quad G(x) = \int H(x,\varepsilon)\, dF_\varepsilon$

if $\quad y = g(x) + \varepsilon_i \quad , E[\varepsilon_i] = 0$

$$G(x) = g(x)$$

$$G(x) \neq E[y \mid x] = \int H(x,\varepsilon)\, dF_{\varepsilon \mid x}$$

ASSUME INSTRUMENTAL variables z_i, unrelated to ε_i

NPIV: $\quad H(x,\varepsilon) = g(x) + \varepsilon = y$

$$E[\varepsilon / z] = 0$$

Control Function : $\quad y = H(x,\varepsilon)$

$\varepsilon \mid x, z \sim \varepsilon \mid V$, V identifiable "control variable"

e.g., $\quad V = x - E[x \mid z]$ $\quad$ (Blundell-Powell)

or $\quad V = F_{x \mid z}[x \mid z]$ $\quad$ (Imbens-Newey)

$x = L(z, \eta)$ $\quad$ (x scalar, η scalar)

$z \perp (\varepsilon, \eta)$

$\eta = L^{-1}(z, x)$

(for selection bias, $v = p(w_i) = \Pr\{d_i = 1 \mid w_i\}$)

$\varepsilon \mid x, z \sim \varepsilon \mid v \qquad v_i = v(x_i, z_i)$

$$E[y_i \mid x_i, v_i] \equiv H^*(x_i, v_i)$$

$$= \int H(x, \varepsilon)\, dF_{\varepsilon \mid v}$$

$$\int H^*(x, v)\, dF_v = \int H(x, \varepsilon)\, dF_\varepsilon = G(x)$$

$$G'(x) = \int \frac{\partial H(x, \varepsilon)}{\partial x}\, dF_\varepsilon$$

Blundell-Powell - Binary Response w/ endogeneity

Newey-Powell-Vella - $y = g(x) + \varepsilon$

$= g(x) + \lambda(v) + u$

Das-Newey-Vella - Selection with endogeneity

Given $d_i = 1$, $E[y \mid \cdots] = g(x) + \lambda(v, p)$

Chesher - Quantiles of $y \mid x, z$

NPIV: $\pi(z) = E[y \mid z] = \int g(x)\, dF_{x \mid z}$

$(y_i = g(x_i) + \varepsilon_i, \; E[\varepsilon \mid z] = 0)$

$$\pi(z) = T_{x \mid z}(g(x))$$

$$T^{-1}(\pi(z)) = g(x)$$

$T^{-1}(\cdot)$ not cts in Π

restrict $g \in G$, compact

$\Rightarrow$ SMOOTHNESS OR MONOTONICITY

<u>Estimation</u>: $g(x) \tilde{=} \sum_{j=1}^{J} \alpha_j \rho_j(x)$

$$\hat{\alpha} = \text{argmin} \sum \left(y_i - \sum_{j=1}^{J} \alpha_j \hat{E}[\rho_j(x) | z] \right)^2 + \lambda \left[\alpha' S \alpha - \beta \right]$$

$$\hat{g}(x) = \sum_j \hat{\alpha}_j \rho_j(x)$$