

COMPETITIVE LENDING WITH PARTIAL KNOWLEDGE OF LOAN REPAYMENT

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Abstract

We study a competitive credit market in which lenders, having partial knowledge of loan repayment, use a Bayesian, maximin, or minimax-regret criterion to make lending decisions. Lenders allocate endowments between loans and a safe investment, while borrowers demand loans to undertake investments. Borrowers may incompletely repay their loans when investment productivity turns out to be low *ex post*. We characterize market equilibrium, the contracted repayment rate being the price variable that equilibrates loan supply and demand. We explore market dynamics when a credit market that is initially in steady state experiences an unanticipated shock that temporarily lowers the productivity of borrower investments. The shock reduces loan repayment and lenders, not knowing whether the shock is temporary, then reduce loan supply. We study two forms of government intervention to restore the steady state. One policy manipulates the return on the safe investment and the other guarantees a minimum loan return to lenders. We conclude that the minimum-return guarantee is preferable. This policy directly reduces lender ambiguity in a transparent manner.

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1. Introduction

Consider a loan contract specifying that a borrower will receive one dollar today and repay r dollars tomorrow, $r - 1$ being the interest rate. A common informational problem is partial knowledge today of the amount that will actually be repaid tomorrow. A borrower may repay in full, in part, or not at all. At the time that loans are transacted, the lender may not know the amount that will be repaid.

The standard economic perspective on credit markets assumes that lenders place subjective probability distributions on loan repayment and maximize expected utility. The presumption that lenders and other agents maximize expected utility is so strong among most economists as to hardly warrant mention. Indeed, the norm is to go further and assume that expectations are rational, in the sense that subjective distributions are objectively correct conditional on available information. Moreover, economists generally presume that agents use Bayes Rule to update their expectations when new information becomes available.

A subjective probability distribution is a form of knowledge. There are realistic circumstances in which an agent may have no credible basis for asserting one at all, never mind one that is objectively correct. These circumstances pose problems of decision making under ambiguity (aka Knightian uncertainty).

Ambiguity may be particularly prevalent when a previously stable market experiences a significant unanticipated shock. Market participants may be unsure how to interpret the shock. It may perhaps have been temporary, but it may indicate a regime change. The standard practice of economists has been to assume that agents use Bayes Rule to update expectations after experiencing shocks. However, this assumption may not be accurate. It may be that agents have traditional probabilistic expectations until the shock occurs, but are unsure how to update them afterwards.

Consider, for example, the American credit crisis of 2007-2008. The origin of the crisis is still poorly understood. However, it is clear that lenders experienced an unanticipated shock when mortgage loan repayment rates fell well below recent norms. Moreover, it appears that lenders have subsequently been unsure how to predict future repayment rates. This paper does not claim to explain the credit crisis, but we

discuss one aspect of it in Section 5.

We study the operation of a competitive credit market when lenders have partial knowledge of loan repayment. We suppose that lenders may make loans that maximize expected utility or they may use one of two criteria for choice under ambiguity, the maximin or minimax-regret criterion. Our modeling of behavior under ambiguity builds in part on our earlier work. Brock (2006) considered the behavior of an isolated lender facing repayment ambiguity. Manski (2005, 2006, 2007, 2008) analyzed various problems of social planning under ambiguity. Whereas our earlier work studied the behavior of a single agent, be it a lender or planner, here we analyze a competitive market in which agents may have to cope with ambiguity.

Section 2 develops the formal foundation for our analysis. We model a competitive credit market in which lenders and borrowers have predetermined partial knowledge of the return to lending, which is the product of the contracted repayment rate and the rate of loan repayment. Loan supply is determined by lenders who allocate monetary endowments between loans and a safe investment. Borrowers demand loans to enable them to undertake potentially productive investments. Incomplete repayment occurs when persons borrow with partial knowledge of the productivity of their investments. When productivity turns out to be low ex post, they sometimes lack the resources to fully repay their loans. Bankruptcy law then limits their liability for repayment.

Placing weak qualitative assumptions on loan demand, we pose alternative specific assumptions about loan supply, considering Bayesian, maximin, and minimax-regret lending behavior. Whereas a Bayesian lender asserts a subjective distribution on loan repayment and maximizes expected utility, maximin and minimax-regret lenders choose allocations that, in different senses, perform uniformly well over all states of nature. These are, of course, not the only criteria that have been suggested for decision making with partial knowledge. However, they cover a range of possibilities, they have long histories of study in decision analysis, and they are all reasonably tractable in the setting we study. We characterize equilibrium under these alternative assumptions on loan supply, an equilibrium occurring when the contracted repayment rate

equates loan supply and demand.

Section 3 explores market dynamics following an unanticipated productivity shock. We consider a market that is initially in steady state, with lenders knowing the loan return. We suppose that an unanticipated shock temporarily lowers the productivity of the investments that borrowers make, with a consequent reduction in loan repayment. Assuming for simplicity that the shock does not affect subsequent loan demand, we consider the response of lenders.

Not knowing how to interpret the shock, lenders may use Bayesian, maximin, or minimax-regret criteria to determine loan supply. In each case, the equilibrium contracted repayment rate will rise immediately following the shock, to a degree that depends on the decision criteria that lenders use. Some fairly realistic numerical calculations show a potentially large increase, as investors adjust their portfolios towards the safe asset in a “flight to liquidity.” The longer run market dynamic depends on how lenders interpret the shock, what decision criteria they use, and how they revise their beliefs as new empirical evidence accumulates. There are many plausible possibilities, so we are unable to make sharp predictions.

Section 3 suggests that a temporary unanticipated productivity shock may disturb or even shut down a credit market. Section 4 asks how a social planner who knows that a productivity shock is temporary might use policy instruments to restore the steady state. We suppose that the welfare objective is to maximize the aggregate return to the investments financed by lender’s endowments. Considering a market whose steady state has a known loan return, we show that two policies can restore the steady state immediately. One policy manipulates the return on the safe investment and the other guarantees a minimum loan return to lenders.

We conclude that the loan guarantee policy is preferable to manipulation of the return on the safe investment. Effective manipulation of the safe return requires detailed knowledge of lender behavior, whereas setting an effective loan guarantee does not. Moreover, effective manipulation of the safe return may require setting the return at an infeasibly low value.

Sections 2 through 4 permit borrowers to have heterogeneous loan demands and repayment

prospects, but we assume that borrowers are observationally identical to lenders. Section 5 studies credit markets with borrowers who are observationally heterogeneous to lenders. We assume that lenders are able to price loans differentially, setting contracted repayment rates that vary with observed borrower covariates.

We first consider a setting in which lenders know loan returns. Analysis of such a market is a straightforward extension of our earlier work. Unsurprisingly, we find that classes of borrowers who yield high loan returns face relatively low contracted repayment rates in equilibrium, while those who yield lower returns face higher contracted rates.

We then extend our analysis of market dynamics and government intervention following temporary shocks. In addition to the productivity shock of Section 3, we consider a type of shock that may have occurred in the recent credit crisis, where deceptive securitization of mortgage loans may have enabled borrowers with low repayment rates to masquerade as ones with high repayment rates. In both cases, government guarantee of a minimum return to lenders can restore the steady state.

Although we will refer to the crisis in the American mortgage market in Section 5, we think it important to caution the reader right now that our model of a credit market is too simple to adequately represent what has occurred in the mortgage market. We call particular attention to our assumption that the return on the investment made by a borrower depends only on the magnitude of his own investment, not on the investments made by other borrowers. This assumption is seriously unrealistic in the mortgage market, where the return on a mortgage-financed home purchase depends on the future price of homes, which is determined by aggregate home purchase decisions.

We also call attention to the fact that the social welfare function we use to motivate government intervention is inappropriate for study of the mortgage market. We assume that the private and social returns to loan-financed investments coincide. This is not reasonable when evaluating returns to mortgage-financed home purchases. A home owner experiences a private return when housing prices rise and he realizes a capital gain by selling his home. This private return does not imply a commensurate increase in the assets

of the economy at large.

A reader concerned with the American credit crisis may be disappointed that this paper does not claim to explain or resolve the crisis. Nevertheless, we think that the paper makes contributions that advance the understanding of credit markets and that warrant the attention of policy makers. We advance understanding of credit markets by studying competitive equilibrium when lenders use various decision criteria to make lending decisions with partial knowledge of loan repayment. We characterize equilibrium in abstraction and we report simple analytical findings that hold in illustrative settings. Considering policy, we study instruments that the government may use to restore a steady state following a temporary shock that lenders are unable to interpret, thus preventing a reduction in loan supply that would reduce social welfare. In particular, we conclude that government guarantee of a minimum loan return is effective policy in the setting that we examine.

Credit markets are complex, and the theoretical literature studying them is diverse and vast, with different authors emphasizing different aspects of market operation. We are aware of a couple of other recent studies of financial markets that assume agents face some sort of ambiguity and then ask how government intervention might mitigate unpalatable market outcomes. Easley and O'Hara (2008) study the sub-optimal asset pricing that may occur when a subset of "ambiguity averse" investors choose not to participate in the market. They suggest a possible corrective role for regulation that limits the occurrence of extreme events. Caballero and Krishnamurthy (2008a) consider an environment in which agents face ambiguity about the timing of liquidity shocks. They find a salutary role for a Central Bank as a lender of last resort.

These precedent studies share our broad concern with the positive and normative analysis of competitive financial markets under ambiguity, but they differ greatly from our work in their specifics. Whereas they use the maximin expected utility model to express agent behavior under ambiguity, we study maximin and minimax-regret behavior. Whereas they pose relatively abstract general equilibrium models of financial markets, we develop a partial-equilibrium model of a credit market with relatively explicit

institutional features. In particular, we differentiate lenders who choose how to allocate asset endowments from borrowers who demand loans to make productive investments and who have limited liability for repayment. We locate the source of ambiguity as lender inability to interpret a productivity or other shock that reduces loan returns relative to an initial steady state. There also are differences across studies in the type of normative analysis performed. We pose an explicit social welfare function in which the objective is to maximize the aggregate return on investments funded by the assets that lenders hold.

There are many directions for potentially fruitful extension of our work, and we cite some of them as we go along. We think it particularly important to relax the assumption in our welfare analysis that the government knows when an unanticipated shock is temporary. This assumption is important to our conclusion that guarantee of a minimum loan return is effective policy. Yet we do not explain the informational asymmetry whereby the government has knowledge that lenders lack. We think it highly desirable to study intervention in settings where the government, like lenders, does not know how to interpret an unanticipated shock. Such analysis requires explicit consideration of how a social planner might cope with ambiguity.

2. Equilibrium in a Credit Market with Partial Knowledge of Loan Repayment

We consider a setting that is simplified in many respects, to enable a straightforward analysis of market equilibrium. The credit market under consideration is small relative to the economy as a whole and is competitive. Lenders interact only through this market. Thus, there exist no financial derivatives markets in which lenders with heterogeneous knowledge and decision criteria may trade with one another.

Loans are the one-period contracts described in the opening paragraph of the paper. Inflation is anticipated, so we denote all monetary quantities in real terms. Until Section 5, all borrowers in the market

under consideration are observationally identical to lenders and face the same contracted repayment rate. Loans are securitized (that is, pooled) rather than held by individual lenders, repayment to each lender being proportional to his share of the aggregate supply of loans. Lender knowledge of loan repayment is predetermined.

Formally, we consider a population J of borrowers and a set K of lenders who interact in period t . A loan contract specifies that a borrower receives one dollar at time t and repays r_t dollars at $t + 1$. The equilibrium contracted repayment rate equates the aggregate demand for and supply of loans. $D_t(r)$ and $S_t(r)$ denote the aggregate demand and supply of loans at contracted repayment rate r .

Section 2.1 gives maintained assumptions on loan demand and poses a model of borrower behavior that explains why borrowers may not fully repay their loans. Section 2.2 gives several alternative models of lender behavior with partial knowledge of loan repayment. Section 2.3 characterizes market equilibrium under these models of lender behavior.

2.1. The Demand for and Repayment of Loans

Our positive analysis of the credit market does not require an explicit model of borrower behavior. It is enough to assume that the aggregate demand function $D_t(\cdot)$ is continuous and strictly decreasing for r such that $D_t(r) > 0$, with $D_t(1) > 0$ and $\lim_{r \rightarrow \infty} D_t(r) = 0$. However, a model of borrower behavior is necessary to explain why persons borrow and why some borrowers may not repay their loans fully. Moreover, it is necessary to perform the welfare analysis of Section 4.

We pose a simple model in which persons borrow to enable them to undertake potentially productive investments. Incomplete repayment occurs when persons borrow with partial knowledge of the productivity of their investments. If they have the resources, borrowers repay their loans in full. However, when productivity turns out to be low ex post, they may lack the resources to repay in full.

Suppose that person j lives for two periods, say t and $t + 1$. Consider behavior in period t , with a given contracted repayment rate r . Person j receives an asset endowment $v_{jt} > 0$ at time t and will obtain a further endowment $v_{j(t+1)} > 0$ at $t + 1$. These endowments cannot be transferred across time; they must be consumed in the periods they are received. The person may choose to receive a loan of any positive magnitude x at time t , in which case he will be contractually obligated to repay rx at $t + 1$. Obtaining a loan allows the person to make an investment, yielding a return at $t + 1$ which is then consumed. For example, the person might invest in a college education or open a new business.

Suppose that person j wants to solve this two-period utility-maximization problem:

$$(1) \quad \max_{x \geq 0} u_{jt}(v_{jt}) + u_{j(t+1)} \{ \max [0, v_{j(t+1)} + g_{jt}(x) - rx] \}.$$

Here $u_{jt}(\cdot)$ and $u_{j(t+1)}(\cdot)$ are single-period utility functions that are strictly increasing in their arguments. Function $g_{jt}(\cdot)$ gives the return on the investment made using the loan. We assume that $g_{jt}(0) = 0$ and that $g_{jt}(\cdot)$ is increasing, differentiable, and concave.

If the borrower chooses loan magnitude x , then $v_{j(t+1)} + g_{jt}(x) - rx$ gives tentative period $t + 1$ consumption, which sums the period $t + 1$ endowment and investment return and subtracts the loan repayment. This quantity is actual period $t + 1$ consumption if it is non-negative. However, the borrower cannot fully repay the loan if this quantity is negative. In that event, the person declares bankruptcy, consumes zero, and repays $v_{j(t+1)} + g_{jt}(x)$. Thus, we assume that the borrower repays as much of the loan as he is able to, subject to the limitation on liability given by bankruptcy law. Another source of incomplete repayment, present in real credit markets but not in our model, is imperfect lender ability to enforce legal repayment claims.

A borrower with perfect foresight regarding $g_{jt}(\cdot)$ can solve problem (1). The optimization problem reduces to

$$(2) \quad \max_{x \geq 0} g_{jt}(x) - rx.$$

Given that $g_{jt}(0) = 0$, the maximum value of $g_{jt}(x) - rx$ over $x \geq 0$ is non-negative. Hence, the person repays his loan fully. Given that $g_{jt}(\cdot)$ is increasing, differentiable, and concave, the optimal loan magnitude is positive if $\partial g_{jt}(0)/\partial x > r$ and is zero otherwise.

A borrower with partial knowledge of $g_{jt}(\cdot)$ may not be able to solve problem (1). He may, perhaps, place a subjective probability distribution on $g_{jt}(\cdot)$ and choose x to maximize expected utility. Or he may perceive ambiguity about the investment return and use some criterion for decision making under ambiguity to choose x . In any case, borrowing $x > 0$ may be a reasonable decision ex ante but a poor one ex post. If the investment return turns out to be sufficiently low, a borrower may not be able to fully repay his loan.

Given our assumption that loans are securitized, lenders are concerned with the aggregate repayment of loans rather than with the repayment of a particular individual's loans. Let $x_{jt}(r)$ be loan demand by borrower j at contracted repayment rate r . If aggregate loan demand at r is positive, the loan return at this rate is

$$(3) \quad \lambda_t(r) \equiv \frac{\sum_{j \in J} \min\{rx_{jt}(r), v_{j(t+1)} + g_{jt}[x_{jt}(r)]\}}{\sum_{j \in J} x_{jt}(r)}.$$

The denominator is aggregate loan demand and the numerator is aggregate repayment. If aggregate demand is zero, the loan return is indeterminate. In this case, we find it convenient and harmless to set $\lambda_t(r) = 0$.

Some Simple Special Cases

A simple form for loan demand with perfect foresight emerges if the investment-return function $g_{jt}(\cdot)$ has the power-law form $g_{jt}(x) = v_{jt}x^{\beta_{jt}}$, where $v_{jt} \geq 0$ and $0 < \beta_{jt} < 1$. Loan demand given knowledge of the

parameters (v_{jt}, β_{jt}) solves problem (2). The result is $x_{jt}(r) = (v_{jt}\beta_{jt}/r)^{1/(1-\beta_{jt})}$.

Loan demand with partial knowledge of the parameters depends on what the borrower knows and the decision criterion he uses. A simple result emerges if the borrower knows that $v_{j(t+1)} = 0$, that $g_{jt}(\cdot)$ has the multiplicative form $g_{jt}(x) = v_{jt}h_{jt}(x)$, where $v_{jt} \geq 0$, and $h_{jt}(0) = 0$, and he perceives there to be two feasible values of v_{jt} , being $v_{0jt} = 0$ and $v_{1jt} > 0$. Suppose that the borrower place subjective probabilities (p_{0jt}, p_{1jt}) on these values and chooses x to maximize expected utility. With these assumptions, expected utility is

$$p_{0jt} \cdot u_{j(t+1)}(0) + p_{1jt} \cdot u_{j(t+1)} \{ \max[0, v_{1jt}h_{jt}(x) - rx] \}.$$

Hence, whatever shape $u_{j(t+1)}(\cdot)$ may have, the optimization problem reduces to the perfect foresight problem $\max_{x \geq 0} v_{1jt}h_{jt}(x) - rx$.

If $g_{jt}(\cdot)$ has the power law form, loan demand is $x_{jt}(r) = (v_{1jt}\beta_{jt}/r)^{1/(1-\beta_{jt})}$. More generally, loan demand is positive and downward sloping in r if $h_{jt}(\cdot)$ is strictly concave and differentiable, with $dh_{jt}(0)/dx = \infty$ and $\lim_{w \rightarrow \infty} dh_{jt}(w)/dx = 0$. Then loan demand at rate r is $x_{jt}(r) = (dh_{jt}/dx)^{-1}(r/v_{1jt})$.

Observe that loan demand with partial knowledge is the same as what demand would be if the borrower knew that $v_{jt} = v_{1jt}$. The reason is bankruptcy protection. The borrower knows that if he receives a bad draw on v , he will realize the utility of zero consumption regardless of what magnitude loan he demands. Hence, expected utility maximization ignores the possibility of a bad draw and optimizes for the case of a good draw.

It is of interest to compare loan demand with bankruptcy protection with what demand would be in the absence of such protection. In the latter case, expected utility would be

$$p_{0jt} \cdot u_{j(t+1)}(-rx) + p_{1jt} \cdot u_{j(t+1)}[v_{1jt}h_{jt}(x) - rx].$$

Now the shape of the utility function and the values of the subjective probabilities (p_{0jt} , p_{1jt}) affect demand. If the borrower is risk-neutral, expected utility is $p_{1jt} \cdot v_{1jt} h_{jt}(x) - rx$. Hence, loan demand at rate r in the absence of bankruptcy protection equals loan demand at rate r/p_{1jt} with bankruptcy protection.

The loan return function $\lambda_t(\cdot)$ has a simple form if borrowers have rational expectations with common probabilities (p_{0t} , p_{1t}), if realizations of the parameter values are statistically independent across borrowers, and if the population J of borrowers is “large,” in the sense of being an atomless probability space. Rational expectations implies that each borrower repays his loan in full with probability p_{1t} and repays nothing with probability p_{0t} . The assumption of a large borrower population with independent parameter realizations implies that equation (3) reduces to $\lambda_t(r) = p_{1t}r$.

2.2. The Supply of Loans

The supply of loans is determined by the aggregate portfolio choices of the set of lenders. Let lender k be endowed with m_{kt} dollars. The lender must allocate this asset between loans and an alternative safe investment. A dollar invested in the safe investment at time t returns a known value $\rho_t \geq 1$ at time $t + 1$, where ρ_t is predetermined to participants in the credit market. In Sections 2 and 3 we do not need to be explicit about the identity of the safe investment. However, in Section 4 we will take it to be a government-issued security with guaranteed rate of return ρ_t .

Lender k must choose a fraction $\delta \in [0, 1]$, implying that he allocates δm_{kt} dollars to loans and $(1 - \delta)m_{kt}$ dollars to the safe investment. An allocation is *singleton* if $\delta = 0$ or 1 and is *fractional* if $0 < \delta < 1$. Fractional allocations are also said to be *diversified*.

Suppose that the contracted repayment rate is r . Given that loans are securitized, if the lender chooses allocation δ , his asset endowment next period will be $[\delta\lambda_t(r) + (1 - \delta)\rho_t]m_{kt}$. The lender’s objective is to maximize some strictly increasing function of this quantity. Thus, lender k wants to solve the problem

$$(4) \quad \max_{\delta \in [0, 1]} f_k \{ [\delta \lambda_t(r) + (1 - \delta) \rho_t] m_{kt} \},$$

where $f_k(\cdot)$ is strictly increasing. The unique optimal allocation is $\delta = 1$ if $\lambda_t(r) > \rho_t$ and $\delta = 0$ if $\lambda_t(r) < \rho_t$. All allocations are optimal if $\lambda_t(r) = \rho_t$.

Our concern is asset allocation when problem (4) cannot be solved because the lender has partial knowledge of loan repayment. Let Γ_{kt} denote the states of nature that lender k thinks feasible for $t + 1$, given the information he has at t . Let $\lambda_{0kt}(r) \equiv \min_{\gamma \in \Gamma_{kt}} \lambda_\gamma(r)$ and $\lambda_{1kt} \equiv \max_{\gamma \in \Gamma_{kt}} \lambda_\gamma(r)$. The optimal allocation is indeterminate if $\lambda_{0kt}(r) < \rho_t < \lambda_{1kt}(r)$.

Although a lender may not know the optimal allocation, he must somehow choose one. We consider Bayesian, maximin, and minimax-regret lending. We do not argue that lenders “should” use a particular decision criterion. If the optimal allocation is determinate, all of the criteria considered here yield it. If it is indeterminate, there is no unique “right” way for lenders to make decisions.

To simplify the notation, we suppress the indices k and t below, as well as the contracted repayment rate r .

Bayesian Lending

A Bayesian lender places a subjective probability distribution on the states of nature, computes subjective expected utility under each allocation, and chooses an allocation that maximizes this quantity.

Let π denote the subjective distribution. Then the lender solves the optimization problem

$$(5) \quad \max_{\delta \in [0, 1]} \int f \{ [\delta \lambda_\gamma + (1 - \delta) \rho] m \} d\pi.$$

Let $\lambda_M \equiv \int \lambda_\gamma d\pi$ be the subjective mean of λ . If $f(\cdot)$ is convex, including the boundary case of linear utility, the solution to (5) is generically singleton, being $\delta = 0$ when $\lambda_M < \rho$ and $\delta = 1$ when $\lambda_M > \rho$. All $\delta \in$

$[0, 1]$ are solutions when $\lambda_M = \rho$.

The solution may be fractional if $f(\cdot)$ has strictly concave segments. Manski and Tetenov (2007, Proposition 5) show that the Bayes allocation is $\delta = 0$ if $f(\cdot)$ is concave and $\lambda_M < \rho$. It is fractional if $f(\cdot)$ is continuously differentiable, $\lambda_M > \rho$, and $\int f(\lambda_\gamma) d\pi < f(\rho)$.

The Maximin Criterion

To determine the maximin allocation, one first computes the minimum utility attained by each allocation across all states of nature. One then chooses an allocation that maximizes this minimum utility.

Thus, the criterion is

$$(6) \quad \max_{\delta \in [0, 1]} \min_{\gamma \in \Gamma} f\{\delta\lambda_\gamma + (1 - \delta)\rho\}m.$$

The solution is $\delta = 0$ if $\lambda_0 < \rho$ and $\delta = 1$ if $\lambda_0 > \rho$. All $\delta \in [0, 1]$ are solutions if $\lambda_0 = \rho$. All strictly increasing $f(\cdot)$ yield this solution.

The Minimax-Regret Criterion

By definition, the regret of allocation δ in state of nature γ is the difference between the maximum achievable utility and the utility achieved with this allocation. The maximum utility achievable in state of nature γ is $\max [f(\lambda_\gamma m), f(\rho m)]$. Hence, regret is $\max [f(\lambda_\gamma m), f(\rho m)] - f\{\delta\lambda_\gamma + (1 - \delta)\rho\}m$. The minimax-regret (MR) rule computes the maximum regret of each allocation over all states of nature and chooses an allocation to minimize maximum regret. Thus, the criterion is

$$(7) \quad \min_{\delta \in [0, 1]} \max_{\gamma \in \Gamma} \max [f(\lambda_\gamma m), f(\rho m)] - f\{\delta\lambda_\gamma + (1 - \delta)\rho\}m.$$

Manski (2007, 2008) show that the solution to problem (7) is always fractional when the lender faces ambiguity; that is, when $\lambda_0 < \rho < \lambda_1$. The minimax-regret allocation takes a very simple form if $f(\cdot)$ is linear or logarithmic. If $f(\cdot)$ is linear, the MR allocation is

$$(8) \quad \delta_{MR} = \min \left[\max \left(\frac{\lambda_1 - \rho}{\lambda_1 - \lambda_0}, 0 \right), 1 \right].$$

If $f(\cdot)$ is logarithmic, it is

$$(9) \quad \delta_{MR} = \frac{\rho(\lambda_1 - \rho)}{\rho(\lambda_1 - \rho) + \lambda_1(\rho - \lambda_0)}.$$

Hence, the fraction of assets invested in loans when utility is logarithmic is smaller than when it is linear.

2.3. Equilibrium Contracted Repayment Rates and Loan Transactions

Section 2.2 characterized the asset allocation of a competitive lender with partial knowledge of the loan return. A competitive lender takes the contracted repayment rate as predetermined. However, this rate must equate aggregate demand and supply. This section characterizes market equilibrium.

Let $\delta_{kt}(r)$ be the asset allocation that lender k would choose at time t if the contracted repayment rate were r . Section 2.2 showed that $\delta_{kt}(r)$ may be point or set-valued. Hence, the aggregate supply of loans at rate r is the possibly set-valued mapping

$$(10) \quad S_t(r) = \sum_{k \in K} \delta_{kt}(r) m_{kt}.$$

Rate r equilibrates supply with demand if

$$(11) \quad D_t(r) \in \sum_{k \in K} \delta_{kt}(r) m_{kt}.$$

The solutions to (11) depend on the lender asset allocation mappings $\delta_{kt}(\cdot)$, $k \in K$. These mappings are determined by lenders' information and decision criteria. The derivations below show the credit market equilibria that occur when lenders have various beliefs about the feasible states of nature and use various decision criteria.

To give a concrete sense of the possibilities, we repeatedly consider a class of tractable settings in which all lenders have the same information and use the same decision criterion. In particular, all lenders believe that the highest feasible loan return is r and the lowest feasible return is $\alpha_t r$, for some $\alpha_t \in [0, 1]$. Thus, each lender k sets $\lambda_{0kt}(r) = \alpha_t r$ and $\lambda_{1kt}(r) = r$ as the lowest and highest feasible loan returns at contracted repayment rate r . The boundary case $\alpha_t = 1$ denotes common knowledge that all loans will be repaid in full. The boundary case $\alpha_t = 0$ denotes a common belief among lenders that all logically possible loan returns are feasible. We note in passing that when lenders are homogeneous as assumed here, we need not assume the non-existence of financial derivatives markets. Such markets will not arise with homogeneous lenders.

Common Knowledge of the Loan Return

Suppose first that the loan return function $\lambda_t(\cdot)$ is common knowledge. Given a value of r , each lender k sets $\delta_{kt}(r) = 0$ if $\lambda_t(r) < \rho_t$, sets $\delta_{kt}(r) = 1$ if $\lambda_t(r) > \rho_t$, and is indifferent among all $\delta \in [0, 1]$ if $\lambda_t(r) = \rho_t$. Hence, an equilibrium value of r satisfies the inequality

$$(12) \quad 1[\lambda_t(r) > \rho_t] m_t \leq D_t(r) \leq 1[\lambda_t(r) \geq \rho_t] m_t,$$

where $m_t \equiv \sum_{k \in K} m_{kt}$ is the aggregate asset endowment of all lenders. Here and elsewhere, $1[\cdot]$ is the indicator function taking the value one if the logical condition in the brackets holds, and zero otherwise.

Three types of equilibria may occur, which we label *full-supply*, *indifferent-supply*, and *zero-supply* equilibria. Consider the set $[1, \infty)$ of all feasible values of r . Lenders supply their full asset endowment m_t to the credit market when r lies in the set $R_{ft} \equiv [r: \lambda_t(r) > \rho_t]$. They are indifferent among all loan supplies $[0, m_t]$ when r lies in $R_{it} \equiv [r: \lambda_t(r) = \rho_t]$. They supply nothing to the credit market when r is in $R_{zt} \equiv [r: \lambda_t(r) < \rho_t]$. Hence, a full-supply equilibrium occurs if $r \in R_{ft}$ and $D_t(r) = m_t$, an indifferent-supply equilibrium if $r \in R_{it}$ and $D_t(r) \in [0, m_t]$, and a zero-supply equilibrium if $r \in R_{zt}$ and $D_t(r) = 0$.

Our maintained assumptions on loan demand imply that a credit market has at most one full-supply equilibrium, the reason being that there exists at most one value of r such that $D_t(r) = m_t$. Co-existence of a full-supply equilibrium with one or more indifferent-supply and zero-supply equilibria can occur in principle. However, it is easy to show that the credit market has a unique equilibrium with positive loan transactions given mild restrictions on $\lambda_t(\cdot)$ and $D_t(\cdot)$.

Suppose that $\lambda_t(\cdot)$ satisfies the single-crossing property with respect to ρ_t ; that is, there exists a unique r_t^* such $\lambda_t(r_t^*) = \rho_t$, with $\lambda_t(r) < \rho_t$ for $r < r_t^*$ and $\lambda_t(r) > \rho_t$ for $r > r_t^*$. Suppose as well that $D_t(r_t^*) > 0$. If $D_t(r_t^*) \leq m_t$, then r_t^* is the unique equilibrium. If $D_t(r_t^*) > m_t$, then equilibrium occurs at the unique $r > r_t^*$ such that $D_t(r) = m_t$.

The simplest case satisfying the single-crossing property is common knowledge of full repayment. Here $\lambda_t(r) = r$ and $r_t^* = \rho_t$. The property also holds in the special case of loan demand and repayment presented at the end of Section 2.1. There $\lambda_t(r) = p_{1t}r$ and $r_t^* = \rho_t/p_{1t}$.

Bayesian Decision Making with Linear Utility

Analysis of equilibrium with Bayesian lending is straightforward if all lenders have linear or, more generally, convex utility functions. Let $\lambda_{\text{Mkt}}(\cdot)$ describe how lender k 's subjective mean for λ varies with r .

Lender k sets $\delta_{kt}(r) = 0$ if $\lambda_{Mkt}(r) < \rho_t$, sets $\delta_{kt}(r) = 1$ if $\lambda_{Mkt}(r) > \rho_t$, and is indifferent among all $\delta \in [0, 1]$ if $\lambda_{Mkt}(r) = \rho_t$. Hence, an equilibrium value of r satisfies the inequality

$$(13) \quad \sum_{k \in K} 1[\lambda_{Mkt}(r) > \rho_t] m_{kt} \leq D_t(r) \leq \sum_{k \in K} 1[\lambda_{Mkt}(r) \geq \rho_t] m_{kt}.$$

A simple special case occurs if every lender gives λ a uniform distribution on an interval $[\alpha_t r, r]$, where $0 \leq \alpha_t \leq 1$. Then $\lambda_{Mkt}(r) = (\alpha_t + 1)r/2$ for all lenders and (13) takes the form

$$(14) \quad 1[r > 2\rho_t/(\alpha_t + 1)] m_t \leq D_t(r) \leq 1[r \geq 2\rho_t/(\alpha_t + 1)] m_t.$$

The equilibrium value of r is $2\rho_t/(\alpha_t + 1)$ if $0 < D_t[2\rho_t/(\alpha_t + 1)] \leq m_t$ and is r such that $D_t(r) = m_t$ if $D_t[2\rho_t/(\alpha_t + 1)] > m_t$. No loans are transacted if $D_t[2\rho_t/(\alpha_t + 1)] = 0$.

It is of interest to compare the uniform-prior Bayesian equilibrium with the one that occurs if lenders have common knowledge of full repayment. The two equilibria coincide if $D_t[2\rho_t/(\alpha_t + 1)] > m_t$ or if $D_t(\rho_t) = 0$. However, they differ otherwise. The difference is transparent when $0 < D_t[2\rho_t/(\alpha_t + 1)] < D_t(\rho_t) \leq m_t$. Then the contracted repayment rate with common knowledge of full repayment is ρ_t and the Bayesian rate is $2\rho_t/(\alpha_t + 1)$.

Maximin Decision Making

Suppose that all lenders use the maximin criterion. The maximin allocation for lender k at repayment rate r is $\delta_{kt}(r) = 0$ if $\lambda_{0kt}(r) < \rho_t$ and $\delta_{kt}(r) = 1$ if $\lambda_{0kt}(r) > \rho_t$. All $\delta \in [0, 1]$ are maximin solutions if $\lambda_{0kt}(r) = \rho_t$. An equilibrium value of r satisfies the inequality

$$(15) \quad \sum_{k \in K} 1[\lambda_{0kt}(r) > \rho_t] m_{kt} \leq D_t(r) \leq \sum_{k \in K} 1[\lambda_{0kt}(r) \geq \rho_t] m_{kt}.$$

A simple special case occurs if each lender sets $\lambda_{0kt}(r) = \alpha_t r$, where $0 \leq \alpha_t \leq 1$. Then (15) becomes

$$(16) \quad 1[r > \rho_t/\alpha_t]m_t \leq D_t(r) \leq 1[r \geq \rho_t/\alpha_t]m_t.$$

The equilibrium repayment rate is ρ_t/α_t if $0 < D_t(\rho_t/\alpha_t) \leq m_t$ and is r such that $D_t(r) = m_t$ if $D_t(\rho_t/\alpha_t) > m_t$. No loans are transacted if $D_t(\rho_t/\alpha_t) = 0$.

Maximin lending yields the same equilibrium as occurs with knowledge of full repayment if $D_t(\rho_t/\alpha_t) > m_t$ or if $D_t(\rho_t) = 0$. The equilibria differ otherwise. The difference is transparent when $0 < D_t(\rho_t/\alpha_t) < D_t(\rho_t) \leq m_t$. Then the equilibrium rate with knowledge of full repayment is ρ_t and the maximin rate is ρ_t/α_t .

Minimax-Regret Decision Making with Linear Utility

Suppose that all lenders use the minimax-regret criterion. Analysis of equilibrium is straightforward if all lenders have linear utility functions. The MR allocation for lender k at repayment rate r is

$$(17) \quad \delta_{MRkt}(r) = \min \left[\max \left(\frac{\lambda_{1kt}(r) - \rho_t}{\lambda_{1kt}(r) - \lambda_{0kt}(r)}, 0 \right), 1 \right].$$

Hence, an equilibrium value of r solves the equation

$$(18) \quad D_t(r) = \sum_{k \in K} \min \left[\max \left(\frac{\lambda_{1kt}(r) - \rho_t}{\lambda_{1kt}(r) - \lambda_{0kt}(r)}, 0 \right), 1 \right] m_{kt}.$$

A simple special case occurs if every lender k sets $\lambda_{0kt}(r) = \alpha_t r$ and $\lambda_{1kt}(r) = r$, where $0 \leq \alpha_t \leq 1$. Then (18) becomes

$$(19) \quad D_t(r) = \min \left[\max \left(\frac{r - \rho_t}{(1 - \alpha_t)r}, 0 \right), 1 \right] m_t.$$

The factor multiplying m_t on the right-hand side of (19) is an increasing continuous function of r , whose value increases strictly from zero at $r = \rho_t$ to one at $r = \rho_t/\alpha_t$. No loans are transacted if $D_t(\rho_t) = 0$. If $D_t(\rho_t/\alpha_t) > m_t$, the equilibrium repayment rate is r such that $D_t(r) = m_t$. If $D_t(\rho_t) > 0$ and $D_t(\rho_t/\alpha_t) \leq m_t$, the equilibrium rate is the $r \in (\rho_t, \rho_t/\alpha_t)$ that solves the equation

$$(20) \quad D_t(r) = \frac{r - \rho_t}{(1 - \alpha_t)r} m_t.$$

3. Market Dynamics Following a Temporary Productivity Shock

In Section 2 we did not explain why lenders may perceive repayment ambiguity. We simply took the informational situation as predetermined and studied the resulting equilibrium. This section identifies a possible source of ambiguity and explores the resulting market dynamics.

We consider a credit market that is initially in steady state, with time-invariant loan demand and supply, time-invariant returns on loans and the safe investment, and common knowledge of the loan return. We do not study how lenders achieved common knowledge of the loan return—some form of updating and convergence of beliefs using data on historically realized returns presumably took place. We suppose that an unanticipated aggregate shock upsets the steady state by temporarily lowering the productivity of borrowers' investments, with a consequent reduction in the realized loan return relative to the historical norm.

We investigate how the shock may affect subsequent market outcomes. In principle, the shock could

shake the beliefs of both borrowers and lenders, with implications for loan demand and supply. For simplicity, we suppose that the shock does not affect borrower behavior. This can be explained in either of two ways. It may be that borrowers, although initially surprised by the shock, quickly learn that they have experienced only a temporary reduction in the productivity of their investments. Or it may be that borrowers cannot distinguish idiosyncratic from aggregate shocks and, hence, never become aware that a temporary aggregate shock occurred.

We focus on how lenders respond to the shock. They may correctly interpret it as a one-time event, or they may incorrectly interpret it as the beginning of a new regime. Or they may view future loan repayment as uncertain/ambiguous and use Bayesian, maximin, or minimax-regret criteria to make lending decisions. The resulting market dynamics depend on how lenders interpret the shock, what decision criteria they use, and how they revise their beliefs as new empirical evidence accumulates.

To formalize the setting, let $D(\cdot)$, $\lambda(\cdot)$, ρ , and m be the steady-state demand function, loan-return function, return on the safe investment, and asset endowment. Let lenders have common knowledge of $\lambda(\cdot)$, which satisfies the single-crossing property of Section 2.3. Let the steady-state equilibrium contracted repayment rate be the unique r^* such that $\lambda(r^*) = \rho$. Thus, we assume that the asset endowment m is large enough to yield the indifferent-supply equilibrium.

Now suppose that a temporary productivity shock occurs at $t = 1$, yielding a reduced loan return $\lambda_1(r^*) < \rho$. Lenders learn this at $t = 2$, at which time they must form beliefs about $\lambda_2(\cdot)$ and choose allocations $\delta_{k_2}(\cdot)$, $k \in K$. Their beliefs about $\lambda_2(\cdot)$ depend on how they interpret the shock. Their supply behavior depends on the decision criteria they use to cope with partial knowledge of the loan return.

We do not think it reasonable to suggest a specific way that lenders “should” interpret the shock. After all, lenders find themselves in a situation that they did not anticipate. They do not know whether the shock is temporary, permanent, or something in between. All they know is that, ex ante, they believed that $\lambda_1(r^*)$ would equal the return ρ on the safe investment and, ex post, $\lambda_1(r^*)$ turned out to be smaller than ρ .

How lenders in this situation should update their beliefs on future loan returns is not transparent.

To better understand the operation of credit markets, we think it important to perform empirical research investigating how lenders actually interpret and respond to unanticipated shocks. In the absence of such research, we explore the market outcomes that would occur if lenders were to act in specified ways.

Section 3.1 considers lender beliefs and behavior at $t = 2$, immediately following the shock. Section 3.2 discusses how lenders might update beliefs and behave in subsequent periods.

3.1. The Immediate Response to the Shock

We think it reasonable to conjecture that a negative unanticipated shock to the loan return will induce lenders to revise downward their beliefs about future loan returns. Whatever their particular downward revisions may be, and whatever decision criteria lenders use (Bayes, maximin, or minimax-regret), the supply of loans will fall at $t = 2$. Assuming that the loan demand function $D(\cdot)$ does not change after the shock, the equilibrium contracted repayment rate r_2 will exceed the steady-state rate r^* and the volume $D(r_2)$ of loans transacted will be smaller than the steady-state volume $D(r^*)$.

To give a concrete sense of the possibilities, suppose that the steady state has common knowledge of full loan repayment; thus, $\lambda(r) = r$ and $r^* = \rho$. Suppose that, after the shock, all lenders believe that the highest feasible loan return at $t = 2$ is r and the lowest feasible return is $\alpha_2 r$, for some $\alpha_2 \in [0, 1]$. Then the analysis of Section 2.3 applies.

If all lenders have linear utility and are Bayesian with uniform subjective distributions on $\lambda_2(\cdot)$, then $r_2 = 2\rho/(\alpha_2 + 1)$. If all lenders use the maximin criterion, then $r_2 = \rho/\alpha_2$. If all lenders have linear utility and use the minimax-regret criterion, then r_2 solves the equation $D(r_2) = m_2(r_2 - \rho)/[(1 - \alpha_2)r_2]$. The minimax-regret depends on the shape of $D(\cdot)$ and the value of m_2 , but we showed in Section 2.3 that $r_2 \in (\rho, \rho/\alpha)$.

Discussions of outcomes in financial markets after unanticipated negative shocks sometimes refer

to a “flight to liquidity,” as investors adjust portfolios towards assets with relatively safe returns. Bayesian, maximin, and minimax-regret lending practices all generate this phenomenon to some degree, with the maximin criterion generating the largest reduction in lending. For example, suppose that $\rho = 1.02$, a realistic real rate of return on a safe investment. Suppose that lenders set $\alpha_2 = 0.9$ after the negative productivity shock; thus, lenders believe they could face a moderate reduction in the return to loans at $t = 2$. Then the steady-state contracted repayment rate is $r^* = 1.02$. The equilibrium rate at $t = 2$ is $r_2 = 2.04/1.9 \approx 1.07$ with uniform-prior Bayesian lending and $r_2 = 1.02/0.9 \approx 1.13$ with maximin lending. In the present-day context, these are quite substantial increases in contracted repayment rates.

3.2. Subsequent Market Outcomes

We are unable to make a sharp prediction on longer run market dynamics. There are many plausible ways in which lenders could revise beliefs and make decisions as new empirical evidence accumulates; hence, there are many plausible sequences of market outcomes. We first pose a favorable scenario and then consider less favorable ones.

A Favorable Scenario

Suppose that $D(r_2) > 0$. At $t = 3$, lenders who supplied positive loan quantities at $t = 2$ observe that their loans are fully repaid; that is, $\lambda_2(r_2) = r_2$. Lenders who supplied no loans at $t = 2$ do not directly observe the loan return. However, suppose they learn it indirectly through communication across lenders.

We think it reasonable to conjecture that full repayment of the loans transacted at $t = 2$ will be interpreted as evidence that the shock at $t = 1$ was temporary. Lenders may not go so far as to conclude that the steady state has been restored; after all, the evidence only reveals $\lambda_2(r_2)$, not $\lambda_2(r^*)$. Nevertheless, they presumably will revise their beliefs upward to some degree and, hence, increase the supply of loans relative

to $t = 2$. The result is $r^* < r_3 < r_2$ and, accordingly, $D(r_2) < D(r_3) < D(r^*)$.

At $t = 4$, lenders observe that $\lambda_3(r_3) = r_3$. This provides further evidence that the shock was temporary, making it reasonable for lenders to further revise their beliefs upward and increase the supply of loans relative to $t = 3$. The result is $r^* < r_4 < r_3 < r_2$ and, accordingly, $D(r_2) < D(r_3) < D(r_4) < D(r^*)$. Thus, from $t = 3$ on, we expect that the credit market moves towards the steady state.

Less Favorable Scenarios

The least favorable scenario occurs if lender beliefs and actions make r_2 so high that $D(r_2) = 0$, shutting down the credit market. If this happens, lenders do not observe a loan return at $t = 2$ and, hence, obtain no empirical evidence that the shock was temporary. The situation then repeats itself at $t = 3$. Thus, the credit market could disappear permanently unless some lenders decide to experiment, making loans that currently appear unprofitable in order to update their beliefs about the loan return.

A less dire but still problematic scenario occurs if only a few lenders make loans at $t = 2$ and if communication between these and non-active lenders is weak. Then updating of beliefs at $t \geq 3$ takes place slowly, implying a slow return to the steady state. The smaller the fraction of active lenders and the weaker the communication with non-active lenders, the worse the problem.

4. Government Intervention in the Credit Market

This section studies government intervention in the credit market. We have from the outset assumed one form of intervention, this being the bankruptcy law that limits the liability of borrowers. Here we take the bankruptcy law as given and consider other forms of intervention.

Section 4.1 poses the social welfare function that we use to evaluate interventions. We measure

welfare by the aggregate return to the investments financed by lenders' endowments.

Section 4.2 considers intervention to stabilize the credit market after a temporary productivity shock of the type studied in Section 3. We examine an ideal setting where the steady state has common knowledge of full loan repayment. The rationale for intervention in this setting is that the reduction in loan supply occurring after a temporary shock reduces borrower initiation of productive investments and, hence, reduces welfare. We suppose that the government acts through an entity, perhaps a central bank or regulatory agency, that we label the Authority. We show that two policies can restore the steady state immediately. One policy manipulates the return on the safe investment. The other guarantees a minimum loan return to lenders. We conclude that the latter policy is preferable to the former.

The analysis of Section 4.2 is instructive, but a steady state with common knowledge of full loan repayment is uncharacteristic of many credit markets. Section 4.3 considers intervention in a market where the steady state has common knowledge of partial repayment. As earlier, guaranteeing a minimum return to lenders works well to restore the steady state after a temporary productivity shock. However, it is much harder to determine how the government should set the steady state return on the safe investment.

We assume throughout this section that the government knows the productivity shock is temporary. We do not explain the informational asymmetry whereby the government has knowledge that lenders lack. An important topic for future research is to analyze intervention in settings where neither lenders nor the government know whether the shock is temporary.

4.1. The Social Welfare Function

We measure social welfare by the aggregate return to the investments financed by lenders' endowments m_t . Suppose that, in each period t , the Authority chooses a policy from a set C of feasible policies. Let $(x_{jt}, j \in J)$ be the loan allocation that would occur in credit market equilibrium if the Authority

were to implement policy c ; thus, $x_{jtc} \geq 0$ for $j \in J$ and $\sum_{j \in J} x_{jtc} \leq m_t$.

We have thus far not needed to specify the nature of the safe investment. However, we must do so here. We take the safe investment to be a government-issued security offered by the Authority to lenders, with guaranteed return ρ_t . The social rate of return ρ^* , which need not equal ρ_t , measures the productivity of public investments made by the government with the funds that the Authority borrows from lenders. Although ρ^* need not in principle be time-invariant, we assume this for simplicity. To eliminate macroeconomic concerns, we assume that the government-issued security is offered only to lenders in the credit market under consideration, not to investors more broadly.

We take the social welfare achieved under policy c to be

$$(21) \quad W_t(c) \equiv \sum_{j \in J} g_{jt}(x_{jtc}) + \rho^*(m_t - \sum_{j \in J} x_{jtc}) = \sum_{j \in J} [g_{jt}(x_{jtc}) - \rho^*x_{jtc}] + \rho^*m_t.$$

The expression $\sum_{j \in J} g_{jt}(x_{jtc})$ is the return on the investments made with loan financing, and $\rho^*(m_t - \sum_{j \in J} x_{jtc})$ is the social return on the assets that lenders allocate to the safe investment.

Two simplifying features of this social welfare function warrant attention. First, we assume that implementation of government policy is costless. Second, the welfare function only considers asset returns in period t , not in later periods. A policy chosen at t could affect lender beliefs about loan repayment in later periods and, hence, affect later market outcomes. The specified welfare function does not recognize this possibility.

Observe that the assumed social welfare function is not utilitarian; that is, the government does not maximize a weighted sum of the private utilities achieved by lenders and borrowers. Instead, the government aims to maximize economy-wide output. In particular, social welfare does not depend on the loan return $\lambda_t(r)$ that lenders obtain. When a borrower cannot repay his loan in full, a transfer occurs from the lender, who receives less than the contracted repayment, to the borrower, who does not fulfill the contract. We take this

zero-sum transfer to be entirely a private matter, not one of social import.

4.2. Restoration of a Steady State with Common Knowledge of Full Repayment

In this section we suppose that the credit market is initially in steady state with common knowledge of full loan repayment. The safe investment is a government-issued security offered at the social rate of return ρ^* . We assume that $0 < D(\rho^*) \leq m$. Hence, the steady-state contracted repayment rate is $r^* = \rho^*$.

A temporary unanticipated productive shock occurs at $t = 1$. As in Section 3, the shock does not affect borrower behavior but may affect lending. The Authority knows the shock to be temporary; thus, it knows that the future state of nature will be the same as the one that prevailed before the shock. However, for whatever reason, the Authority cannot credibly communicate this information to lenders. It can only attempt to affect the credit market by providing incentives to lenders.

In this setting, welfare function (21) implies that the Authority should choose a policy to restore the steady state equilibrium. The reason is that the steady state maximizes the aggregate return to lender assets. The aggregate social return with any vector $(x_j, j \in J)$ of borrower investments is $\sum_{j \in J} [g_j(x_j) - \rho^* x_j] + \rho^* m$. Given perfect foresight and $r^* = \rho^*$, each borrower j chooses x to maximize $g_j(x) - \rho^* x$. Thus, decentralized borrowing maximizes the aggregate return.

We show here that two policies can restore the steady state at $t = 2$, after the productivity shock.

Manipulation of the Return on the Safe Investment

Suppose that the Authority can manipulate the private return ρ_2 on the safe investment, making it deviate from the social return ρ^* . If the Authority knows how lenders behave, it can set ρ_2 to restore the credit market to the steady state. For concreteness, consider again the setting of Section 3.1 in which all lenders believe that the highest feasible loan return at $t = 2$ is r and the lowest feasible return is $\alpha_2 r$, where

$\alpha_2 \in [0, 1]$. Then the Authority can restore the steady state by setting ρ_2 as follows:

Uniform-Prior Bayesian Decision Making with Linear Utility: In the absence of intervention, the equilibrium contracted repayment rate is $r_2 = 2\rho_2/(\alpha_2 + 1)$. To make $r_2 = \rho^*$, the Authority should set $\rho_2 = \rho^*(\alpha_2 + 1)/2$. Thus, the Authority should reduce the return to the safe investment by the multiplicative factor $(\alpha_2 + 1)/2$. In the example of Section of 3.1, where $\rho^* = 1.02$ and $\alpha_2 = 0.9$, the Authority should set $\rho_2 = 0.969$.

Maximin Decision Making: In the absence of intervention, $r_2 = \rho_2/\alpha_1$. To make $r_2 = \rho^*$, the Authority should set $\rho_2 = \alpha_2\rho^*$. Thus, it should reduce the return to the safe investment by the multiplicative factor α_2 . Observe that $\alpha_2 < (\alpha_2 + 1)/2$. Hence, restoration of the steady state requires more drastic action when lenders use the maximin criterion than when they apply the uniform-prior Bayes criterion. If $\rho^* = 1.02$ and $\alpha_2 = 0.9$, the Authority should set $\rho_2 = 0.918$.

Minimax-Regret Decision Making with Linear Utility: In the absence of intervention, r_2 solves the equation $D(r_2) = m_2(r_2 - \rho_2)/[(1 - \alpha_2)r_2]$. To make $r_2 = \rho^*$, the Authority should set ρ_2 to solve the equation $D(\rho^*) = m_2(\rho^* - \rho_2)/[(1 - \alpha_2)\rho^*]$. Thus, the Authority should set $\rho_2 = \rho^* - [(1 - \alpha_2)\rho^*]D(\rho^*)/m_2$. Observe that the minimax-regret value of ρ_2 equals the uniform-prior Bayesian value if $D(\rho^*) = m_2/2$ and equals the maximin value if $D(\rho^*) = m_2$.

The above shows that manipulation of the return to the safe investment can restore steady state in the credit market after a temporary productivity shock. However, two caveats temper the appeal of this policy instrument.

First, successful manipulation of ρ_2 requires the Authority to know how lenders behave with partial knowledge. In the cases considered here, the Authority must know both α_2 and the decision criteria that

lenders use. The Authority may not have this information in practice.

Second, our example with $\rho^* = 1.02$ and $\alpha_2 = 0.9$ indicates that a large reduction in the return to the safe investment may be required to restore the steady state. In practice, the Authority may find it infeasible to set ρ_2 to such a low value. For example, if fiat money is available as a store of value, ρ_2 can be set no lower than one minus the inflation rate.

Guaranteeing a Minimum Loan Return

Suppose that the Authority is able to monitor loan repayment and, therefore, knows the return $\lambda_t(r_t)$ that lenders receive in each period t . Then an appealing way to restore the steady state is for the Authority to guarantee to lenders that their loan return will equal the contracted repayment rate. Given this guarantee, lenders would set aside their own beliefs about repayment and continue to behave as they did before the productivity shock. Borrowers are observationally identical, so the guarantee does not give lenders an incentive to make bad loans. Assuming that the productivity shock does not affect borrower behavior, the guarantee immediately restores the steady state. Moreover, borrowers fully repay their loans, so the Authority does not have to pay off on the guarantee.

Guaranteeing a minimum loan return is a remarkably simple and effective policy instrument. Successful implementation of this policy does not require the Authority to know how lenders behave with partial knowledge. This contrasts with manipulation of the return on the safe investment.

4.3. Intervention in a Credit Market with Partial Loan Repayment

In this section we suppose that the credit market is initially in steady state with common knowledge of the loan return, which yields less than full repayment. In steady state, the safe investment is a government-issued security offered at a private rate of return ρ , which now need not equal the social rate of return ρ^* .

Assume that $0 < D(\rho) \leq m$ and that the loan-return function $\lambda(\cdot)$ satisfies the single-crossing property with respect to ρ . Then, as shown in Section 2.3, the steady-state contracted repayment rate is the unique r^* such that $\lambda(r^*) = \rho$.

Guaranteeing a Minimum Loan Return

If the market experiences a temporary productive shock, the Authority can use a loan guarantee to restore the steady state. The argument here extends the one at the end of Section 4.2.

As earlier, suppose that the Authority knows the loan return that lenders actually receive each period. Suppose the Authority guarantees that the loan return at any contracted repayment rate r will at least equal the steady-state return $\lambda(r)$ at this rate. Given this guarantee, lenders would set aside their own beliefs about repayment and behave as they did before the productivity shock. The guarantee immediately restores the steady state. The realized repayment rate is $\lambda(r^*)$, so the Authority does not have to pay off on the guarantee.

Setting the above guarantee requires the Authority to know the steady-state loan return function $\lambda(\cdot)$. Observation of the pre-shock steady state reveals that $\lambda(r^*) = \rho$, but it does not reveal $\lambda(r)$ at other values of r . Fortunately, the Authority does not need to know the full structure of $\lambda(\cdot)$ to make an effective guarantee. Consider any guarantee function $\mu(\cdot)$ that satisfies the single-crossing property with respect to ρ ; that is, $\mu(r) < \rho$ for $r < r^*$, $\mu(r^*) = \rho$, and $\mu(r) > \rho$ for $r > r^*$. Any such guarantee induces steady state behavior by lenders.

Setting the Steady-State Return on the Safe Investment

Although the loan guarantee policy works just as well here as in the case of full repayment, intervention in a credit market with partial loan repayment is more complex than in a market with full repayment. The complication is how to choose the steady-state return on the safe investment. In the case of full repayment, a simple argument showed that it was optimal to set $\rho = \rho^*$. This is not necessarily optimal when borrowers lack the perfect foresight needed to ensure full repayment.

How should the Authority set ρ ? Welfare function (21) is maximized if the investment made by each borrower j solves the problem $\max_{x \geq 0} g_j(x) - \rho^* x$. A borrower with perfect foresight makes the socially optimal investment when $r^* = \rho^*$, but one with partial knowledge of $g_j(\cdot)$ cannot solve the optimization problem. Hence, decentralized borrowing need not be optimal. This opens the possibility that the Authority can improve welfare by setting ρ to a value other than ρ^* .

To formalize the policy problem, let C be the set of feasible values for ρ . Let $x_{j\rho}$ be borrower j 's loan demand in the credit market equilibrium that would occur if the Authority were to set the return on the safe investment equal to ρ . Then steady-state welfare under policy ρ is

$$(22) \quad W(\rho) \equiv \sum_{j \in J} g_j(x_{j\rho}) + \rho^*(m - \sum_{j \in J} x_{j\rho}) = \sum_{j \in J} [g_j(x_{j\rho}) - \rho^* x_{j\rho}] + \rho^* m.$$

If it has the requisite knowledge, the Authority should choose ρ to maximize $W(\cdot)$.

A Simple Special Case

In practice, it strains the imagination to suppose that the Authority would have the knowledge of $\{[x_{j\rho}, g_j(x_{j\rho})], j \in J\}$, $\rho \in C$ needed to solve the optimization problem. Nevertheless, it is instructive to work out the solution in a simple special case.

In Section 2.1, we derived loan demand and repayment when the investment-return function $g_j(\cdot)$ has the multiplicative form $g_{jt}(x) = v_{jt}h_{jt}(x)$. Given this and various other assumptions, we showed that loan demand at rate r is $x_{jt}(r) = (dh_{jt}/dx)^{-1}(r/v_{jt})$ and the loan-return function is $\lambda_t(r) = p_{1t}r$. We consider this special case again here, with the additional assumption that all borrowers have the same time-invariant parameter v_1 and return function $h(\cdot)$.

With these assumptions, steady-state welfare under policy ρ is

$$(23) \quad W(\rho) = p_1 v_1 h\{x[r(\rho)]\} - \rho^* x[r(\rho)] + \rho^* m,$$

where $r(\rho)$ is the equilibrium contracted repayment rate given ρ and $x[r(\rho)]$ is equilibrium demand. We have $x[r(\rho)] = q[r(\rho)/v_1]$, where $q(\cdot) \equiv (dh/dx)^{-1}(\cdot)$. We will assume that the equilibrium is of the indifferent-supply type for all values of ρ ; hence, $r(\rho) = \rho/p_1$. It follows that

$$(24) \quad W(\rho) = p_1 v_1 h\{q[\rho/(p_1 v_1)]\} - \rho^* q[\rho/(p_1 v_1)] + \rho^* m.$$

The first derivative of $W(\cdot)$ is

$$(25) \quad \begin{aligned} dW(\rho)/d\varphi &= p_1 v_1 dh\{q[\rho/(p_1 v_1)]\}/dx \cdot \partial q[\rho/(p_1 v_1)]/\partial \varphi - \rho^* \partial q[\rho/(p_1 v_1)]/\partial \varphi \\ &= (\rho - \rho^*) \partial q[\rho/(p_1 v_1)]/\partial \varphi, \end{aligned}$$

where φ indexes values of the safe rate of return. Equation (25) shows that $dW(\rho^*)/d\varphi = 0$. The maintained assumptions imply that $\partial q[\rho/(p_1 v_1)]/\partial \varphi$ decreases in ρ . Hence, it is optimal for the Authority to set $\rho = \rho^*$, just as it was with common knowledge of full repayment.

This finding is intriguing. However, the assumptions made here are special. We caution against extrapolating the finding to other settings with partial loan repayment.

5. A Credit Market with Observationally Heterogeneous Borrowers

Sections 2 through 4 permitted borrowers to have heterogeneous loan demands and repayment prospects. However, we assumed that borrowers are observationally identical to lenders. This section

studies credit markets in which borrowers have covariates that are observed by lenders.

Section 5.1 considers a steady state where lenders know the loan return functions of borrowers with different covariates. The analysis is a straightforward extension of our earlier work. Section 5.2 explores market dynamics and government intervention following temporary shocks. In addition to the productivity shock of Section 3, we consider a type of shock that may have occurred in the recent credit crisis. This shock results when deceptive securitization of loans enables borrowers with low repayment rates to masquerade as ones with high repayment rates.

Throughout this section we assume that when lenders observe borrower covariates, they are able to price loans differentially to classes of borrowers with different covariates. Although differential pricing of loans is widespread in practice, we should note that usury laws often place a ceiling on maximum allowable contracted repayment rates. When usury laws or other institutional constraints prevent lenders from setting profitable rates, they may profile borrowers, choosing to lend to some types and not to others. Then equilibrium in the credit market is achieved by the joint forces of pricing and profiling rather than by pricing alone. It would be useful to extend the analysis of this paper to such markets.

5.1. Steady State Equilibrium with Common Knowledge of the Loan Return

We use this notation. Lenders observe that borrower j has covariates $z_j \in Z$, where Z is a finite space of covariates. For $z \in Z$, J_z is the class of borrowers with covariates z , $D_z(\cdot)$ is its steady-state demand function, and $\lambda_z(\cdot)$ is its steady-state loan return function. Class J_z faces the z -specific contracted repayment rate r_z , and we define the vector $r_z \equiv (r_z, z \in Z)$. The population-wide demand function is $D(\cdot) = \sum_{z \in Z} D_z(\cdot)$.

Let there be common knowledge of $\lambda_z(\cdot)$, $z \in Z$. Extending the demand assumptions maintained earlier in the paper, assume that for all $z \in Z$, $D_z(\cdot)$ is continuous and strictly decreasing with $D_z(1) > 0$ and $\lim_{r \rightarrow \infty} D_z(r) = 0$. Extending a key loan repayment assumption made earlier in the paper, let $\lambda_z(\cdot)$ satisfy the

single-crossing property with respect to ρ ; that is, there exists a unique r_z^* such $\lambda_z(r_z^*) = \rho$, with $\lambda_z(r) < \rho$ for $r < r_z^*$ and $\lambda_z(r) > \rho$ for $r > r_z^*$. Also assume that $D_z(r_z^*) > 0$.

Suppose that $\sum_{z \in Z} D_z(r_z^*) \leq m$. Then there exists a unique indifferent-supply equilibrium with $r_z = (r_z^*, z \in Z)$. This vector of contracted repayment rates uniquely makes lenders indifferent between investing in the safe asset and lending to borrowers with different covariates. Lenders possess sufficient assets to meet all loan demand. Hence, $(r_z^*, z \in Z)$ is the unique equilibrium.

Suppose that $\sum_{z \in Z} D_z(r_z^*) > m$. Then lenders cannot meet all loan demand at $(r_z^*, z \in Z)$. Hence, there can only be a full-supply equilibrium, with $r_z > (r_z^*, z \in Z)$ such that $\sum_{z \in Z} D_z(r_z) = m$. The demand and loan repayment assumptions made thus far do not imply that such an equilibrium exists or is unique. However, strengthening the assumptions yields a unique equilibrium with positive lending to all classes of borrowers. Regarding demand, assume that for all $z \in Z$, $D_z(r) > 0$ for all r . Regarding repayment, assume that $\lambda_z(\cdot)$ is continuous and strictly increasing for $r > r_z^*$. Also assume that $\lim_{r \rightarrow \infty} \lambda_z(r) = \lim_{r \rightarrow \infty} \lambda_{z'}(r)$ for all $(z, z') \in Z \times Z$. These assumptions imply that there exists a unique $r_z > (r_z^*, z \in Z)$ such that $\sum_{z \in Z} D_z(r_z) = m$ and $\lambda_z(r_z) = \lambda_{z'}(r_{z'})$ for all $(z, z') \in Z \times Z$. Thus, demand equals supply and lenders are indifferent between lending to borrowers with different covariates.

Some Simple Special Cases

An equilibrium with uniform pricing of loans occurs when all borrowers fully repay their loans. Then $\lambda_z(r) = r$ and $r_z^* = \rho$ for all $z \in Z$. The equilibrium is of the indifferent-supply type if $\sum_{z \in Z} D_z(\rho) \leq m$.

An equilibrium with differential pricing occurs if each class of borrowers satisfies the special loan demand and repayment assumptions studied at the end of Section 2.1, with z -specific parameters. Then $\lambda_z(r) = p_{1z}r$ and $r_z^* = \rho/p_{1z}$. The equilibrium is of the indifferent-supply type if $\sum_{z \in Z} D_z(\rho/p_{1z}) \leq m$.

Observe that, with p_{1z} appearing in the denominator of the expression for r_z^* , what may seem small differences in repayment probabilities imply large differences in contracted repayment rates. Suppose, for

example, that $\rho = 1.02$, $p_{1z} = 0.99$, and $p_{1z'} = 0.95$. Then borrowers with covariates z face rate $r_z^* = 1.030$ and ones with covariates z' face rate $r_{z'}^* = 1.074$.

Comparison with a Market with Observationally Identical Borrowers

It is of interest to compare the present steady state with the one where borrowers are observationally identical. Given observation of z , a unique indifferent-supply equilibrium with contracted repayment rates $(r_z^*, z \in Z)$ exists if $\sum_{z \in Z} D_z(r_z^*) \leq m$, where $(r_z^*, z \in Z)$ solves $\lambda_z(r_z^*) = \rho$, $z \in Z$. When z is unobserved, an equilibrium with rate r^* exists if $D(r^*) \leq m$, where r^* solves the equation $\sum_{z \in Z} \lambda_z(r^*) d_z(r^*) = \rho$ and where $d_z(r^*) \equiv D_z(r^*) / \sum_{z' \in Z} D_{z'}(r^*)$. The single-crossing property implies that $\min_{z \in Z} r_z^* \leq r^* \leq \max_{z \in Z} r_z^*$.

Given observation of z , a unique full-supply equilibrium exists if $\sum_{z \in Z} D_z(r_z^*) > m$, in which case there exists a unique $r_z > (r_z^*, z \in Z)$ such that $\sum_{z \in Z} D_z(r_z) = m$ and $\lambda_z(r_z) = \lambda_{z'}(r_{z'})$ for all $(z, z') \in Z \times Z$. When z is unobserved, such an equilibrium exists if $D(r^*) > m$, in which case there exists a unique $r > r^*$ such that $\sum_{z \in Z} D_z(r) = m$. Given that demand functions slope downward, it follows that $\min_{z \in Z} r_z \leq r \leq \max_{z \in Z} r_z$.

These findings indicate that observation of borrower covariates benefits classes of borrowers with relatively high repayment rates, who pay lower contracted repayment rates than they would otherwise. Symmetrically, it harms classes of borrowers with relatively low repayment rates, who pay higher rates than they would otherwise. In the aggregate, observation of covariates may increase or decrease equilibrium loan volume compared to the case where borrowers are observationally identical.

5.2. Market Dynamics and Government Intervention after Temporary Shocks

We discuss two types of temporary shocks here. One is the productivity shock of Section 3. The other is a securitization shock that may have occurred in the recent credit crisis.

A Temporary Productivity Shock

Suppose as in Section 3 that the market experiences a temporary productivity shock, but lenders do not know how to interpret the shock. The broad features of our earlier analysis of market dynamics extend to the current setting. Not knowing how to interpret the shock, lenders may use Bayesian, maximin, or minimax-regret criteria to determine loan supply. In each case, equilibrium contracted repayment rates will rise immediately following the shock, to a degree that depends on the decision criteria that lenders use. The longer run market dynamic depends on how lenders interpret the shock, what decision criteria they use, and how they revise their beliefs as new empirical evidence accumulates.

We will not formally analyze Bayesian, maximin, and minimax-regret lending behavior here because the details are more complex than in Section 3. There lenders only had to form beliefs about the aggregate borrower population J and make a scalar lending decision. Now they must form joint beliefs about repayment prospects in the multiple class $(J_z, z \in Z)$ and make a corresponding vector of lending decisions.

Fortunately, the Authority can still use a loan guarantee to restore the steady state. The argument is as earlier, except that now the Authority makes z -specific guarantees. Borrowers are observationally identical conditional on z , so the guarantee does not give lenders an incentive to make bad loans. The Authority can use any guarantee function $\mu_z(\cdot)$ that satisfies the single-crossing property with respect to ρ ; that is, $\mu_z(r) < \rho$ for $r < r_z^*$, $\mu(r_z^*) = \rho$, and $\mu_z(r) > \rho$ for $r > r_z^*$. Any such guarantee induces steady state behavior by lenders.

A Temporary Securitization Shock

Discussions of the recent American credit crisis have sometimes cited deceptive securitization, or bundling, of mortgage loans made to different types of borrowers as an antecedent event. In the language of this paper, the idea is that the mortgage market was initially in the steady state of Section 5.1, with mortgages priced differentially to classes of borrowers with different observable characteristics. For

simplicity, it suffices to distinguish a class z_p of “prime” borrowers, with high repayment rates and low contracted repayment rates, from a class z_s of “subprime” borrowers, with low repayment rates and high contracted repayment rates.

An unanticipated shock may have occurred when loan originators began to offer prime-rate mortgages to subprime borrowers, deceptively bundling these loans with those made to prime borrowers. The loan originators sold the bundles to unsuspecting lenders, who believed that the bundles would have the high repayment rate characteristic of prime borrowers. Thus, securitization enabled subprime borrowers to masquerade as prime ones. Ex post, the loan bundles had lower repayment rates than lenders anticipated, reflecting the actual mix of prime and sub-prime loans. The deceptive bundling process was ended when regulators caught on to what was happening; hence, the shock was temporary. Lenders, however, did not know how to interpret the shock. As a consequence, they reduced loan supply in the manner of Section 3.

An Authority who understands the shock is temporary and who is able to prevent further deception can use a loan guarantee to restore the steady state. The argument is as earlier, so we do not repeat it here.

6. Conclusion

Ambiguity (aka Knightian uncertainty) about investment returns has increasingly been asserted to be a negative influence on the operation of financial markets, inducing investors to allocate more of their portfolios to safe assets than is socially optimal. Government intervention to reduce ambiguity has been recommended as a suitable treatment. For example, Greenspan (2004, p. 38) has written:

When confronted with uncertainty, especially Knightian uncertainty, human beings invariably attempt to disengage from medium- to long-term commitments in favor of safety and liquidity. . .

The immediate response on the part of the central bank to such financial implosions must be to inject

large quantities of liquidity.

Considering the recent credit crisis, Caballero and Krishnamurthy (2008b, p. 2) have written:

The heart of the recent crisis is a rise in *uncertainty*—that is, a rise in unknown and immeasurable risk rather than the measurable risk that the financial sector specializes in managing. . . . What should central banks do in this case? They must find a way to re-engage the *private sector's liquidity*. Re-engagement will only occur as agents' uncertainty over outcomes is reduced.

These assertions have intuitive appeal, but much theoretical and empirical analysis of financial markets will have to be performed to get to the bottom of the matter.

The main message of our theoretical work is to corroborate the conjecture that ambiguity can have a negative influence on financial markets and that government intervention can be a corrective. This adds evidence to the corroborative studies of Easley and O'Hara (2008) and Caballero and Krishnamurthy (2008a), whose positive and normative analyses differ considerably from ours. Indeed, our work strengthens the conjecture because it does not rest on the common assumption that agents facing ambiguity use the conservative maximin expected utility criterion to make decisions. We find that a reduction in loan supply may also occur if lenders use Bayesian or minimax-regret decision rules following an unanticipated shock.

From a policy perspective, our work suggests that guaranteeing a minimum loan return can be an effective way for the government to reduce lender ambiguity about loan repayment and, hence, to prevent loan supply from falling after an unanticipated shock. Although our formal analysis of a minimum-return guarantee rests on various idealized assumptions, this type of policy has an appealing feature that holds in generality. Whatever lenders may believe about loan repayment after a shock, a minimum-return guarantee immediately induces them to shed their concern about obtaining a return lower than the guarantee. Thus, a minimum-return guarantee directly reduces lender ambiguity in a transparent manner.

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