Econometrica, Vol. 68, No. 1 (January, 2000), 53–84

LATENT SEPARABILITY: GROUPING GOODS WITHOUT WEAK SEPARABILITY

BY RICHARD BLUNDELL AND JEAN-MARC ROBIN[1]

This paper develops a new concept of separability with overlapping groups—latent separability. This is shown to provide a useful empirical and theoretical framework for investigating the grouping of goods and prices. It is a generalization of weak separability in which goods are allowed to enter more than one group and where the composition of groups is identified by the choice of group specific exclusive goods. Latent separability is shown to be equivalent to weak separability in latent rather than purchased goods and provides a relationship between separability and household production theory. For the popular class of linear, almost ideal and translog demand models and their generalizations, we provide a method for choosing the number of homothetic separable groups. A detailed method for exploring the composition of the separable groups is also presented. These methods are applied to a long time series of British individual household data on the consumption of twenty two nondurable and service goods.

KEYWORDS: Consumer behavior, separability, price aggregation.

1. INTRODUCTION

SEPARABILITY HAS BEEN INTRODUCED into the literature on household behavior so as to make tractable the consumer's allocation problem when faced by a large number of goods characterized by different relative prices. The idea behind separability in consumer preferences is that there exists certain "natural" groupings of related commodities that reflect the budgeting decisions of consumers. In a classic paper Gorman (1959)[2] showed that weak separability was equivalent to a two-stage budgeting rule in which consumers allocate first to a set of broad commodity groups and then, conditional on this first-stage allocation, allocate expenditures across commodities within the groups. Two-stage budgeting is also used to cover the more restrictive situation in which a *single* price index, independent of outlay for *each* subgroup, is sufficient to determine intergroup allocations. This requires homothetic within group preferences unless cross-group preferences are additive, in which case a unique class of generalized Gorman polar form preferences for within group substitution can be derived.

[1]Acknowledgements: We are grateful to Tim Besley, Charles Blackorby, Martin Browning, Pierre-André Chiappori, Ian Crawford, Terence Gorman, Arthur Lewbel, Enrique Sentana, a co-editor, and two anonymous referees for helpful comments. This research is part of the program of the ESRC Centre for the Micro-Economic Analysis of Fiscal Policy at IFS. The financial support of the ESRC is gratefully acknowledged. Household data from the FES made available by the CSO through the ESRC Data Archive has been used by permission of the HMSO. Neither the CSO nor the ESRC Data Archive bear responsibility for the analysis or the interpretation of the data reported here. The usual disclaimer applies.

[2]Blackorby and Russell (1997) provide a complete proof encompassing the two-group case that is assumed away in Gorman (1959).

The advantage of weak separability is in the reduction of the allocation decision problem to a recursive sequence of manageable choice problems. The drawback of such separability assumptions is well known and stems primarily from the strong restrictions placed on substitution possibilities between commodities occupying different groups (see Blackorby, Primont, and Russell (1978) for a comprehensive survey). Without such separability assumptions, however, it is possible for *all* relative prices to enter the demand for each individual commodity, the only general restriction from consumer theory being the symmetry and the negativity of the substitution matrix.

The aim in this paper is to examine whether there exist restrictions on preferences that preserve some of the attractive features of weak separability without the strong assumption of *mutually exclusive* commodity groupings. We develop the concept of "latent separability" in which there may exist overlapping groups of commodities. Overlapping goods across groups are ruled out under weak separability. We show that for many popular classes of demand models, the number of underlying latent groups is related to the rank of an (estimated) parameter matrix. We are therefore able to present an econometric method for choosing the number of latent separable groups. This can provide important efficiency gains in empirical models of consumer demand without the imposition of weak separability.

The composition of the groups is reflected by a particular basis of the vector space spanned by the columns of this parameter matrix and as such its identification will depend on a normalization assumption. We suggest a normalization that can be interpreted as choosing at least one exclusive good per group. Each group may, for example, represent an intermediate technology for producing utility or a subutility corresponding to a family member. Although there are many observationally equivalent sets of exclusive goods, it is possible to test whether a particular choice is acceptable. Choosing among acceptable normalizations is purely a matter of interpretation and has no consequence on the consistency or the precision of elasticity estimates for observed commodities.

Latent separability will be shown to be equivalent to weak separability in *latent* rather than *purchased* goods. We can therefore draw on standard commodity aggregation and decentralization theorems in consumer theory. Since only purchased goods are directly observable and since these can be used by the consumer in more than one group, the latent allocations by the consumer are not directly observable to the econometrician. We derive a scheme for the identification and estimation of the composition of each group and therefore the full allocation process under latent separability.

The conditions for latent separability are used to formalize the identification of household production technologies in which intermediate goods are produced using combinations of purchased commodities. The idea of overlapping groups can be found in the theory of aggregation for capital inputs developed by Gorman (1978). In this framework homothetic latent separability is shown to have considerably more flexibility than homothetic weak separability since the demand relationship for any nonexclusive good runs through more than one channel. This is a feature shared with the characteristics model of Gorman (1956) and we highlight the relationship between latent separability and the characteristics model.

An alternative method of grouping goods follows from the Hicks-Leontief composite commodity theorem (Hicks (1946)). This reflects some underlying collinearity in relative prices. Hildenbrand (1994) shows that if subsets of prices are perfectly collinear then it is only possible to identify the structure of preferences over a smaller number of Hicksian composite commodities. The reduced demand system satisfies the hypothesis of utility maximization if the original demand system satisfies this property.[3] Under latent separability, prices can be aggregated even if they are not collinear as latent separability represents a property of consumer preferences rather than a feature of price data. However, where prices are collinear, latent separability could prove useful in providing a set of restrictions with which to improve the precision of estimated price effects without having to resort to the imposition of stronger forms of separability.

The implications of latent separability for commodity price aggregation are by no means a theoretical curiosity and will be shown to have practical significance in the empirical analysis of consumer behavior. They provide convenient restrictions on the form of the price parameters in many popular demand systems, thereby reducing the number of independent own and cross price parameters to be estimated. In an application to British Family Expenditure Survey data, we find that the demand for twenty two goods can be reduced to around eleven latent separable groups. For the resulting system there is found to be a considerable improvement in the precision of the estimated price elasticities. Exploring the composition of the latent separable structure reveals which goods are used across groups and, consequently, which goods imply a rejection of weak separability.

The structure of the paper is as follows. Section 2 describes the general concept of latent separability and its relationship to weak separability, household production, and characteristics models. The implications for price aggregation and commodity substitution are presented in Section 3. As a by-product, homothetic weak separability is placed in a parametrically testable framework. Alternative families of expenditure functions and price indices for some popular demand models under latent separability are examined in Section 4 and a method for choosing the number of latently separable groups is developed. Latent separability is applied to two popular classes of demand models. First, quasi-homothetic demand models including the linear expenditure system and, second piglog demand models that include the translog model of Jorgensen, Lau, and Stoker (1982) and the almost ideal model of Deaton and Muellbauer (1980) as well as polynomial generalizations of these models. Section 5 considers the identification and estimation for the almost ideal class of demand systems. Section 6 is an empirical application using the British Family Expenditure

[3]See Lewbel (1993, 1996) for a useful empirical exposition.

Survey. This application investigates the demand for twenty two nondurable and service goods recorded in the repeated cross-section data spanning some 20 years using quarterly price variation. The results show that a reasonably small number of latently separable groups are consistent with the expenditure patterns in the data even though homothetic weak separability is rejected. There are found to be large gains in precision for the estimation of own and cross price elasticities. Finally, in Section 7, we provide a summary and conclusions.

2. LATENT SEPARABILITY, WEAK SEPARABILITY AND CHARACTERISTICS

Latent separability is closely akin to a number of concepts that seek to generalize separability in consumer and producer theory, most clearly with household production theory and also with the demand for 'related goods' or the characteristics model introduced by Gorman (1956). In the characteristics model, related goods are those that could be combined to generate different amounts of certain basic intermediate characteristics. In Gorman's example the purchase of different types of eggs is used by the consumer to yield a desired amount of different proteins. In contrast, the grouping of goods under latent separability can be thought of as reflecting a household production technology in which certain goods are used to produce a number of different intermediate utilities. As a consequence, latent separability is more closely related to the commodity grouping that is presented in the Gorman (1968b, 1978) papers on capital aggregation in which he asked: Under which circumstance can the short run technology set of the economy be written as a function of a reduced set of capital aggregates?

In the context of consumer behavior, latent separability can be interpreted as the case where goods are utilized in the production of more than one intermediate good; e.g., electricity may be used for cooking and for lighting. Intermediate goods may also reflect the preferences of individual household members who share out the consumption of many purchased commodities. Although the number of latent separable groups is shown to be identified without further restrictions, the composition of the separable groups can only be identified after normalization. The individual "latent" elements of each group are not directly distinguishable since they are not separately observed and attract the same market price. In this paper the composition of the individual latent elements is determined by the choice of at least one exclusive good per group.

The idea of exclusive goods is found explicitly in the work on collective models of household behavior by Chiappori (1988, 1992). In that work, exclusive goods for each household member are required to separately identify household member preferences and the income sharing rule. The (maximum) number of groups is equal to the number of household members. Each groupwise subutility is then that of an individual household member. However, the *collective rationality* model of Chiappori does not require the maximization of a joint utility function and therefore does not impose the Slutsky condition. Collective rationality and the Slutsky condition imply latent separability.

In latent separability, as in household production theory, the value of each intermediate utility $U_k(\tilde{\mathbf{q}}^k)$ produced depends on the part of the vector of total consumption goods that is devoted to the kth intermediate utility production process. Each of the n purchased goods is shared out across the production of $m(<n)$ intermediate goods, $q_i \equiv \tilde{q}_i^1 + \tilde{q}_i^2 + \cdots + \tilde{q}_i^m$ for $i = 1, \ldots, n$.

To be more precise: Suppose utility is defined over goods q_i for $i = 1, \ldots, n$ and suppose that these can be arranged into $m(<n)$ groups. In this case *weak separability* would imply that preferences can be expressed as

$$\mathscr{U}(q_1, q_2, \ldots, q_n) = F\big(U_1(\mathbf{q}^1), \ldots, U_m(\mathbf{q}^m)\big) \tag{2.1}$$

in which the $\mathbf{q}^k$ vectors of group specific purchased goods describe a mutually exclusive and exhaustive separation:

$$\mathbf{q}^T = (q_1, q_2, \ldots, q_n) = (\mathbf{q}^{1T}, \ldots, \mathbf{q}^{mT}),$$

where ^{T} is the transpose operator. *Latent separability* relaxes this partition by allowing some goods to enter more than one group. As these separate inputs cannot be directly observed we denote the latent input of good i in group k as $\tilde{q}_i^k$ with $\tilde{\mathbf{q}}^k = (\tilde{q}_1^k, \ldots, \tilde{q}_n^k)$.

Definition 1: A direct utility function $\mathscr{U} : \mathbf{q} \in \mathbb{R}_+^n \mapsto \mathscr{U}(\mathbf{q}) \in \mathbb{R}_+$ is said to satisfy the property of *latent separability* if

$$\mathscr{U}(\mathbf{q}) = \max_{\tilde{\mathbf{q}}^1, \ldots, \tilde{\mathbf{q}}^m \in \mathbb{R}_+^n} \left\{ F\big(U_1(\tilde{\mathbf{q}}^1), \ldots, U_m(\tilde{\mathbf{q}}^m)\big) \,\middle|\, \sum_{k=1}^{m} \tilde{\mathbf{q}}^k = \mathbf{q} \right\} \tag{2.2}$$

where $F, U_1, \ldots, U_m$ are regular utility functions.[4]

The form of the overall utility function under latent separability $F(U_1(\tilde{\mathbf{q}}^1), \ldots, U_m(\tilde{\mathbf{q}}^m))$ shows that latent separability corresponds to weak separability in latent inputs $\tilde{\mathbf{q}}^k$. Latent separability only makes sense if $m < n$. There is no further restriction in assuming the aggregators $U_k(\tilde{\mathbf{q}}^k)$ are functionally independent, and to identify the number of groups this is all that is required. To identify the group composition a specific choice of normalization is required. In latent separability this is achieved by assuming at least one exclusive good per group. Therefore each aggregate is defined by at least one exclusive good. Although this can only be achieved by prior choice of a set of m exclusive goods, it is a much weaker requirement than is necessary under weak separability in which there are no overlapping groups.

The allocation problem faced by a consumer with such preferences is then to find an allocation vector $\mathbf{q} = (q_1, \ldots, q_n)^T$ such that there exists suballocation

[4] We use "regular" to mean "satisfying the usual regularity or smoothness conditions of consumer theory." We here always require strong smoothness conditions in order to avoid any marginal complications; i.e., direct utility functions must be increasing and differentiably strictly quasi-concave, and cost functions must be increasing and differentiably strictly concave in prices and increasing in the utility index.

vectors $\tilde{\mathbf{q}}^1, \dots, \tilde{\mathbf{q}}^m$, with $\Sigma_{k=1}^m \tilde{\mathbf{q}}^k = \mathbf{q}$, that maximize $F(U_1(\tilde{\mathbf{q}}^1), \dots, U_m(\tilde{\mathbf{q}}^m))$ subject to the budget constraint $\mathbf{p}^T\mathbf{q} = x$, where $\mathbf{p}$ is an $n \times 1$ price vector and x represents total expenditure on all goods $q_1, \dots, q_n$. As a consequence latent separability can be viewed as weak separability over latent rather than purchased goods.

Our focus will be on the case of homothetic subutilities or constant-return-to-scale household production technologies: $U_k(\lambda\tilde{\mathbf{q}}^k) = \lambda U_k(\tilde{\mathbf{q}}^k)$ for all λ.[5] With linearly homogenous subutilities, we show that a single exclusive good per group is sufficient to identify inter-group as well as intra-group preferences.

It is interesting to note the similarity of program (2.2) with program (P′) considered by Chiappori (1988, p. 74). In Chiappori's model of household behavior, $m = 2$ and the two subutilities are the utility functions of the two household members and $F(.)$ is a fixed welfare function. The exclusive goods are the leisure time of both members.[6]

Identification of the composition of each subtechnology $U_k(\tilde{\mathbf{q}}^k)$ is achieved through the choice of an exclusive good.

DEFINITION 2: Good i, $i \in \{1, \dots, m\}$, is *exclusive to group* k if $\forall l \in \{1, \dots, m\}$, $l \neq k$, $\forall(\tilde{q}_1^l, \dots, \tilde{q}_{i-1}^l, \tilde{q}_{i+1}^l, \dots, \tilde{q}_n^l)$, $\forall \tilde{q}_i^l, \tilde{q}_i^{l\prime}$,

$$U_l\left(\tilde{q}_1^l, \dots, \tilde{q}_{i-1}^l, \tilde{q}_i^l, \tilde{q}_{i+1}^l, \dots, \tilde{q}_n^l\right) = U_l\left(\tilde{q}_1^l, \dots, \tilde{q}_{i-1}^l, \tilde{q}_i^{l\prime}, \tilde{q}_{i+1}^l, \dots, \tilde{q}_n^l\right).$$

If good i is not an input for intermediate utility l, then the optimal choice for $\tilde{q}_i^l$ is 0. If good i is exclusive to group k, then $\tilde{q}_i^k = q_i$.

Unless each $\tilde{q}_i^k$ is exclusive to one particular group, $\tilde{q}_i^k$ and the outlay to the kth group given by $\Sigma_{i=1}^n p_i \tilde{q}_i^k = \tilde{x}^k$ are not directly observed. If there are no overlapping groups, all goods are exclusive to specific groups and all inputs are directly observed. The structure of preferences in this case reduces to that of weak separability in observed goods. In general the assumption of one exclusive good per group is shown to completely identify the composition of each group and therefore the full allocation process between and across groups.

Although latent separability and the theory of characteristics seem close, they are different concepts. In contrast to latent separability, in characteristics theory the value of each subutility $U_k(\mathbf{q})$ depends on the vector of total consumption $\mathbf{q}$. In characteristics theory it also is generally assumed that the subutility functions $U_k(\cdot)$ are linear (see Gorman (1976) for a discussion), although Gorman considers more general functions.[7]

The distinctions between these alternative concepts are summarized in Table I where $\mathbf{z}$ is included in the overall utility function to represent other goods and

[5]As we also note below, the restrictiveness of homothetic separability is considerably reduced in the latent separability case.

[6]The analysis of estimation and identification by Chiappori does not directly apply here since he considers a single commodity that combined with leisure produces individual satisfaction.

[7]An example of a nonlinear characteristics model is the CAPM in which the volatility of a portfolio is a nonlinear function of shares.

TABLE I

COMMODITY AGGREGATION CONCEPTS

Overall Utility	Total Consumption
Weak Separability	
$F(U_1(\mathbf{q}^1),\dots,U_m(\mathbf{q}^m),z)$	$\mathbf{q}=(\mathbf{q}^{1T},\dots,\mathbf{q}^{mT})^T$
Latent Separability	
Overall Utility	Total Consumption
$F(U_1(\tilde{\mathbf{q}}^1),\dots,U_m(\tilde{\mathbf{q}}^m),z)$	$\mathbf{q}=\tilde{\mathbf{q}}^1+\cdots+\mathbf{q}^m$
Characteristics Theory	
Overall Utility	Total Consumption
$F(U_1(\mathbf{q}),\dots,U_m(\mathbf{q}),z)$	$\mathbf{q}$

conditioning variables not included in the specific allocation problem under consideration.

3. PRICE AGGREGATION AND HOMOTHETIC LATENT SEPARABILITY

The aim of this paper is to derive testable implications on empirical demand functions in which there may be many groups with overlapping goods. It is natural therefore to look for implications of latent separability on the grouping of prices. For weak separability, following Gorman (1959), the dual implications are price aggregation and decentralization. Decentralization occurs when optimal allocations of expenditures to individual commodities within groups depends on group total expenditure and group prices only. Latent separability corresponds to a decentralization in which individual *latent* inputs are allocated optimally within groups according to group prices and total group outlay. Price aggregation is satisfied if optimal expenditure allocations to the m groups are functions of total expenditure and m group-price indices. Under latent separability, these price aggregates are shown to depend on overlapping prices.

3.1. *Characterizing the Grouping of Prices under Latent Separability*

To describe price aggregation under latent separability we make use of the duality between the consumer's expenditure function and the distance function.[8] Assuming, sufficient regularity conditions (see Blackorby, Primont, and Russell (1978)), the expenditure function dual to distance function

$$d(u,\mathbf{q})=\max_{\lambda}\{\lambda>0|U(\mathbf{q}\lambda^{-1})\geq u\}$$

is given by

$$e(\mathbf{p},u)=\min\{\mathbf{p}^T\mathbf{q}|U(\mathbf{q})\geq u\}.$$

[8]From Deaton (1979) or Gorman (1976) we can write the distance function, corresponding to direct utility $U(\mathbf{q})$ as $d(u,\mathbf{q})=\max_{\lambda}\{\lambda>0|U(\mathbf{q}\lambda^{-1})\geq u\}$.

Next, consider the Gorman (1976) definition of *implicit separability*, in which the expenditure function has the following form:

$$(3.1)\qquad e(\mathbf{p},u)=\tilde{e}(e^1(\mathbf{p}^1,u),\ldots,e^m(\mathbf{p}^m,u),u)$$

where $\mathbf{p}^T=(\mathbf{p}^{1T},\ldots,\mathbf{p}^{mT})$ provides a partition of the price vector $\mathbf{p}$ into $m(\leq n)$ subsets. McFadden (1978, Lemma 10) extends Gorman's characterization of implicit separability by showing that the distance function takes the form

$$(3.2)\qquad d(u,\mathbf{q})=\max\left\{\check{d}(u,d^1(u,\mathbf{q}^1),\ldots,d^m(u,\mathbf{q}^m))\,\middle|\,\sum_{k=1}^{m}\mathbf{q}^k=\mathbf{q}\right\},$$

if and only if the expenditure function has the form (3.1).

By writing the aggregators $d^k(u,\mathbf{q}^k)$ as functions of latent inputs $\tilde{\mathbf{q}}^k$, the distance function (3.2) can be generalized to allow for overlapping goods by writing

$$(3.3)\qquad d(u,\mathbf{q})=\max\left\{\check{d}(u,d^1(u,\tilde{\mathbf{q}}^1),\ldots,d^m(u,\tilde{\mathbf{q}}^m))\,\middle|\,\sum_{k=1}^{m}\tilde{\mathbf{q}}^k=\mathbf{q}\right\}.$$

Applying McFadden's Lemma, the distance function takes the form (3.3) if and only if the expenditure function has the implicit latent separable form

$$(3.4)\qquad e(\mathbf{p},u)=\tilde{e}(e^1(\mathbf{p},u),\ldots,e^m(\mathbf{p},u),u)$$

with $m\leq n$. Without normalization or exclusion restrictions on the aggregators *all* goods may enter as latent inputs in each group and consequently all prices may enter each group expenditure function $e^k(\mathbf{p},u)$. Implicit latent separability only makes sense if $m<n$. To identify the group composition a specific choice of normalization is required. This is achieved by assuming at least one exclusive good per group, where a group may correspond to an intermediate technology in the production of utility or a subutility corresponding to an individual family member. Exclusive goods correspond to exclusive prices in the expenditure function (3.4).

3.2. *Homothetic Latent Separability and the Form of Latent Demands*

Assuming that the aggregators in (3.3) or (3.4) are independent of the utility level u implies that

$$(3.5)\qquad d(u,\mathbf{q})=\max\left\{\check{d}(u,d^1(\tilde{\mathbf{q}}^1),\ldots,d^m(\tilde{\mathbf{q}}^m))\,\middle|\,\sum_{k=1}^{m}\tilde{\mathbf{q}}^k=\mathbf{q}\right\},$$

if and only

$$(3.6)\qquad e(\mathbf{p},u)=\tilde{e}(e^1(\mathbf{p}),\ldots,e^m(\mathbf{p}),u);$$

see McFadden (1978). Under (3.5) or (3.6) the utility function can be rewritten as

$$(3.7)\qquad \mathscr{U}(\mathbf{q})=\max_{\tilde{\mathbf{q}}^1,\ldots,\tilde{\mathbf{q}}^m\in\mathbb{R}^n_+}\left\{F\big(U_1(\tilde{\mathbf{q}}^1),\ldots,U_m(\tilde{\mathbf{q}}^m)\big)\,\middle|\,\sum_{k=1}^{m}\tilde{\mathbf{q}}^k=\mathbf{q}\right\}$$

with the subutilities $U_1(\tilde{\mathbf{q}}^1),\ldots,U_m(\tilde{\mathbf{q}}^m)$ linearly homogenous of degree one.

From (3.6) and (3.7) it can be seen that latent separability and implicit latent separability are equivalent dual concepts only when aggregators in the definition of implicit separability are independent of the utility level *and* when subutilities in the definition of latent separability are homothetic. We consider the following definition of *homothetic latent separability* of the expenditure function:

DEFINITION 3: A regular expenditure function $e(\mathbf{p}, u)$ is said to satisfy the conditions for *homothetic latent separability*, if it has the following form:

$$e(\mathbf{p}, u) = \tilde{e}(b^1(\mathbf{p}), \ldots, b^m(\mathbf{p}), u) \tag{3.8}$$

where $b^1(.), \ldots, b^m(.)$ are linearly homogeneous differentiable functions of prices, with the first derivative matrix $\mathbf{B}(\mathbf{p}) = [b_i^k(\mathbf{p})]_{i \times k}$ full column rank.

The equivalence result shown above is summarized in the following proposition:

PROPOSITION 1: *Homothetic latent separability of a regular direct utility function is equivalent to latent separability of the expenditure function with price aggregates that are homogeneous of degree one, increasing and strictly concave in their arguments and with a regular cross-group expenditure function.*

For the rest of our discussion we maintain the assumption of homothetic latent separability. Although each subgroup of latent demands is homothetic, there is a wide spectrum of possible income and substitution effects for purchased goods generated from the combination of different groups to which each nonexclusive good belongs. The homothetic restriction in the latent separability case is much less restrictive than in the weak separability case. Under latent separability the income and substitution possibilities for any nonexclusive good can work through more than one channel, thus relaxing the restrictions on income and substitution possibilities for goods in the same group under homothetic weak separability.

Under homothetic latent separability, compensated demands take the form

$$q_i = \frac{\partial e(\mathbf{p}, u)}{\partial p_i} \tag{3.9}$$

$$= \sum_{k=1}^{m} b_i^k(\mathbf{p}) \frac{\partial \tilde{e}(b^1(\mathbf{p}), \ldots, b^m(\mathbf{p}), u)}{\partial b^k} \tag{3.10}$$

$$= \sum_{k=1}^{m} b_i^k(\mathbf{p}) \phi^k(b^1(\mathbf{p}), \ldots, b^m(\mathbf{p}), u). \tag{3.11}$$

Latent separability in prices therefore implies restrictions on the range of the vector of demands when both prices and total expenditure or utility index vary. Notice that if $\tilde{e}$ is a regular expenditure function, then ϕ^k is positive for all k.

By linear homogeneity of price aggregates, (3.9)–(3.11) yield the following equivalent form for $e(\mathbf{p},u)$:

$$e(\mathbf{p},u)=\sum_{k=1}^{m} b^k(\mathbf{p})\frac{\partial \tilde{e}(b^1(\mathbf{p}),\dots,b^m(\mathbf{p}),u)}{\partial b^k} \tag{3.12}$$

$$=\sum_{k=1}^{m} b^k(\mathbf{p})\phi^k(b^1(\mathbf{p}),\dots,b^m(\mathbf{p}),u). \tag{3.13}$$

This expression was obtained by Gorman (1978) for the case where $e(\mathbf{p},u)$ is a profit function and u is a vector of capital stocks, as the form that implies the existence of a reduced set of capital aggregates.[9] This remains the most general commodity aggregation theorem available.

Although "intermediate demands" ϕ^k are not directly observed, knowledge of the input coefficients $b_i^k(\cdot)$ allows an interpretation for each intermediate good through the inputs of observed commodities. These decentralized latent demands are given by

$$\tilde{q}_i^k=\frac{b_i^k(\mathbf{p})}{b^k(\mathbf{p})}\tilde{x}_k \tag{3.14}$$

where $\tilde{x}_k$ is the total outlay on group k. Summing over all the latent demands (or inputs) for good i, the amount of good i purchased is given by

$$q_i=\sum_{k=1}^{m} b_i^k(\mathbf{p})\frac{\tilde{x}_k}{b^k(\mathbf{p})}. \tag{3.15}$$

3.3. *Gorman's Rank Theorem, Latent Separability and the Number of Groups*

The rank theorem of Gorman (1981) considers the conditions placed on the preferences of rational individuals whose uncompensated demands are of the form

$$q_i=\sum_{k=1}^{r} \theta_i^k(\mathbf{p})\psi^k(x) \qquad \text{for} \quad i=1,\dots,n. \tag{3.16}$$

Gorman shows that the rank of the matrix $[\theta_i^k(\mathbf{p})]$ for fixed $\mathbf{p}$ cannot be greater than three.[10]

This rank condition relates directly to price aggregation since a demand system has rank r if and only if r is the smallest integer such that the expenditure function is of the form

$$e(\mathbf{p},u)=h(\theta^1(\mathbf{p}),\dots,\theta^r(\mathbf{p}),u), \tag{3.17}$$

[9] We thank Charles Blackorby for bringing this to our attention.

[10] This is used by Gorman to derive the conditions for exact aggregation, that is the necessary and sufficient conditions for aggregate demands to be functions of r additive aggregate functions of total expenditure x.

for some linear homogeneous functions $\theta^1, \ldots, \theta^r$, and some function h. Moreover, if h is concave in prices, Gorman (1981) shows that this implies a structure for the direct utility function akin to latent separability.[11] For example, if $r = 3$, then there exist at least three linearly homogeneous subutility functions a, b, and c, such that utility has the form

$$u = \mathscr{U}(\mathbf{q}) = \max_{y,z,w} \{F(q(\mathbf{y}), b(\mathbf{z}), c(\mathbf{w})) | \mathbf{y} + \mathbf{z} + \mathbf{w} = \mathbf{q}\}.$$

Whereas the rank of demand system (3.16) can be uncovered from Engel curve estimation alone (see Lewbel (1991), for example), latent separability exploits variation in relative prices to allow the identification of a finer structure for grouping prices. That is there may exist homothetic functions $b^1, \ldots, b^m$ of prices such that

$$\theta^k(\mathbf{p}) = \zeta^k(b^1(\mathbf{p}), \ldots, b^m(\mathbf{p})). \tag{3.18}$$

This, in turn, allows additional commodity groupings in the direct utility function. Notice that the linear homogeneity of both sets of functions $\theta^k(\mathbf{p})$ and $b^k(\mathbf{p})$ implies the linear homogeneity of functions $\mathbf{b} \mapsto \zeta^k(\mathbf{b})$.

If we let $\nabla\theta^k(\mathbf{p})$ define the gradient of $\theta^k(\mathbf{p})$ and $\nabla\zeta^k(\mathbf{b})$ the gradient of $\zeta^k(\mathbf{b})$, then

$$\nabla\theta^k(\mathbf{p}) = \mathbf{B}(\mathbf{p}) \nabla\zeta^k(b^1(\mathbf{p}), \ldots, b^m(\mathbf{p})) \tag{3.19}$$

where $\mathbf{B}(\mathbf{p})$ is the $n \times m$ matrix $[b_i^1(\mathbf{p}), \ldots, b_i^m(\mathbf{p})]$ of partial derivatives of functions $b^k(\cdot)$. Gorman's rank theorem places a restriction on the rank of matrix $[\nabla\theta^1(\mathbf{p}), \ldots, \nabla\theta^r(\mathbf{p})]$ independently of $\mathbf{p}$. Latent separability places a restriction on the dimension of the manifold spanned by each vector $\nabla\theta^k(\mathbf{p})$ when prices vary. Thus, whereas it is possible to identify and estimate the rank of a demand system from a single cross-section, the identification and estimation of the number of latent groups requires price variation and the estimation of a complete demand system. In addition, to identify the composition of the groups, additional prior restrictions on group price indices $b^1(\mathbf{p}), \ldots, b^m(\mathbf{p})$ are necessary. Clearly, otherwise, one could just take $b^k(\mathbf{p}) = \theta^k(\mathbf{p})$ and $m = r$. Hence we have the following corollary of Gorman's rank theorem:

PROPOSITION 2: *The number of latent groups m is at least equal to the rank r of the demand system.*

In Section 4 some useful price indices and preference specifications that guarantee identification are developed. Section 4 also draws the connection between latent separability and Hicks-Leontief price aggregation. Before that we turn briefly to alternative approaches to assessing separability structures.

[11] See also Blackorby and Shorrocks (1995, page 353).

3.4. *Weak Separability Tests and Latent Separability*

The analysis of homothetic weak separability has been the subject of many studies. The most recent and complete is provided by Diewert and Wales (1995) in their extension of their earlier paper (Diewert and Wales (1987)). They provide a method for testing weak separability that can also be extended to the case of latent separability. The problem they consider is the following. Let

$$\pi(\mathbf{p},\mathbf{r}) = \max_{\mathbf{x},\mathbf{y}}\{\mathbf{p}^T\mathbf{x} + \mathbf{r}^T\mathbf{y} | F(\mathbf{x},\mathbf{y}) \leq 1\}$$

be a unit profit function (see McFadden (1978)) where $\mathbf{x}$ and $\mathbf{y}$ are two subsets of production input and output factors and $\mathbf{p}$ and $\mathbf{r}$ are the corresponding prices. They consider the problem of testing whether there exists a homogeneous weakly separable aggregate $g(\mathbf{x})$ such that $F(\mathbf{x},\mathbf{y})$ will have the following functional structure

$$F(\mathbf{x},\mathbf{y}) = f[g(\mathbf{x}),\mathbf{y}].$$

Let

$$b(\mathbf{p}) = \max_{\mathbf{x}}\{\mathbf{p}^T\mathbf{x} | g(\mathbf{x}) = 1\}$$

be the unit profit function that corresponds to $g(\mathbf{x})$ and let $P(p_0,\mathbf{r})$ be the aggregate unit profit function

$$P(p_0,\mathbf{r}) = \max_{y_0,y}\{p_0 y_0 + \mathbf{r}^T\mathbf{y} | f(y_0,\mathbf{y}) \leq 1\};$$

then homothetic weak separability implies that

$$\pi(\mathbf{p},\mathbf{r}) = P[b(\mathbf{p}),\mathbf{r}]. \tag{3.20}$$

The analysis in the previous section performed the analogous aggregation of prices for latent separability in terms of the consumer's expenditure function.

Diewert and Wales use normalized quadratic functional forms (or symmetric generalized McFadden; see Diewert and Wales (1987)) for the unit profit functions $b(\mathbf{p})$ and $P(p_0,\mathbf{r})$. For example,

$$b(\mathbf{p}) = \boldsymbol{\alpha}^T\mathbf{p} + \tfrac{1}{2}\mathbf{p}^T\mathbf{B}\mathbf{p}/\boldsymbol{\beta}^T\mathbf{p} \tag{3.21}$$

where $\boldsymbol{\alpha}$, $\mathbf{B}$, and $\boldsymbol{\beta}$ are appropriate parameters. The price aggregator function $b(\mathbf{p})$ is analogous to the group price aggregators $b^k(\mathbf{p})$ in the previous section for the case of only one group of interest.

The test for homogeneous weak separability they propose is the test of restriction (3.20) on this normalized quadratic specification. This is implemented by testing $\mathbf{c} = \mathbf{0}$ and $\mathbf{C} = 0$ in the following specification for $\pi(\mathbf{p},\mathbf{r})$:

$$\pi(\mathbf{p},\mathbf{r}) = P[r(\mathbf{p}),\mathbf{r}] + \mathbf{c}^T\mathbf{p} + \mathbf{p}^T\mathbf{C}\mathbf{r}/[\boldsymbol{\gamma}^T\mathbf{p} + \boldsymbol{\delta}^T\mathbf{r}]. \tag{3.22}$$

Testing exclusion restrictions in this way is a classical method of testing for weak separability. This method would generalize to the case of homothetic latent separability in which case the test would relate to coefficients on group exclusive prices only. The distinguishing feature of our approach is to allow for many groups with overlapping goods and to determine the number of groups. To implement this we simplify further the specification of the group price aggregators $b^k(\mathbf{p})$ but allow for many groups.

4. PRICE INDICES AND PREFERENCE SPECIFICATIONS

Before proceeding further, it is useful to sum up the principal definitions and results that we have so far established. *Latent separability* generalizes weak separability by allowing some overlapping in the grouping of commodities into different functions in the direct utility function. For example, a share of the total consumption of energy is required for cooking and another share for heating. *Homothetic latent separability* refers to latent separability when the subutility functions are homogenous of degree one. Using general results of duality theory, we have shown that the cost function in the latter case is such that the price vector can be grouped into m (if m is the number of latent groups) linearly homogenous price aggregators.

Homothetic latent separability has been shown to fit well with Gorman's theory of exact aggregation. Gorman's rank theorem shows that all exactly aggregable demand systems have the property that prices can be grouped into r price indices, with $r \leq 3$, where r, is the "rank" of the demand system. Yet this does not prevent further grouping of prices *within* the r price indices imposed by the rank theorem. Exactly aggregable demand systems therefore refer to a particular class of preferences satisfying the property of homothetic latent separability. Consequently, contrary to weak separability, homothetic latent separability should restrain very little the flexibility of demand systems when the number of latent groups is small. On the other hand, the rank theorem literature tells us that the only aspect of homothetic latent separability that can be identified from the analysis of Engel curves is the lower bound r. Recovering the true value for the number of latent groups m requires further variation in the price dimension. Given that m price indices suffice to sum up a price variation, demands will span a manifold of dimension m when prices vary.

The aim of this section is to find reasonable specifications for the price aggregates and for the cost function itself, that allow the identification of the number of latent groups and their composition from data on demands, prices, and total expenditure. We first define *quasi-linear* price indices as a class of price aggregators that have the property to linearize the manifold mentioned above. We then show that there exist natural parametric specifications of such quasi-linear price indices for the case of the linear expenditure system and the quadratic almost ideal demand system. In Section 6 we provide an empirical application of the latter specification.

4.1. *Group Price Indices and Exclusive Goods*

For price indices (price aggregators) $b^k(\mathbf{p})$, a particularly convenient assumption is the assumption of *quasi-linearity.*

DEFINITION 4: A function $b(\mathbf{p})$ of a n-vector $\mathbf{p}$ is quasi-linear if there exists a set of n scalars $\pi_1, \ldots, \pi_n$, such that $g[b(\mathbf{p})] = \Sigma_i \pi_i g(p_i)$ for some increasing function g.

This covers the price transformations found in all popular demand systems. In the translog and almost ideal class $g(p_i) = \ln p_i$ and in the generalized Leontief class $g(p_i) = \sqrt{p_i}$. More generally, any power function will work.

Suppose price indices $b^k(\mathbf{p})$ are associated to specific functions $g_k(\cdot)$ given by

$$g_k[b^k(\mathbf{p})] = \sum_i \pi_i^k g_k(p_i); \tag{4.1}$$

then equation (3.14) takes the form

$$q_i = \sum_{k=1}^{m} \pi_i^k \frac{g_k'(p_i)}{g_k'[b^k(\mathbf{p})]} \frac{\tilde{x}_k}{b^k(\mathbf{p})}, \tag{4.2}$$

where g_k' denotes the derivative of g_k and recall that $\tilde{x}_k$ is the (unobserved) outlay on group k. If all quasi-linear functions $b^k(\mathbf{p})$ have the same function $g(\cdot)$, then the relationship (4.3) between purchases and group (aggregate) demands takes the form

$$\frac{q_i}{g'(p_i)} = \sum_{k=1}^{m} \pi_i^k \frac{\tilde{x}_k}{g'[b^k(\mathbf{p})] b^k(\mathbf{p})}. \tag{4.3}$$

From (4.3) we have the following property:

PROPOSITION 3: *Let m^* be the dimension of the vector set spanned by vector $(\tilde{x}_1/g'[b^1(\mathbf{p})]b^1(\mathbf{p}), \ldots, \tilde{x}_m/g'[b^m(\mathbf{p})]b^m(\mathbf{p}))^T$ when prices and total outlay vary. An expenditure function that satisfies the conditions for homothetic latent separability with the same quasi-linear price aggregators, implies that the set of vectors $(q_1/g'(p_1), \ldots, q_n/g'(p_n))^T$ defines a linear space of dimension m^* when all prices $\mathbf{p}$ and total outlay vary.*

Proposition 3 allows the identification and estimation of the number of groups and will be shown to have general applicability to a number of popular demand systems below.[12] In general m^* will be equal to m. However, if prices, or some functions of them, are colinear, as in the context of the Hicks-Leontief composite commodity aggregation theorem, then m^* can be less than m. In this case

[12] Yet note that allowing for quasi-linear price aggregates with different and linearly independent functions g_k will generally make the vector of purchased demands span $\mathbb{R}_+^n$ in all its orthogonal directions (see Appendix A).

there may not be enough independent price variation to make aggregate demands vary in all directions.

Proposition 3 shows that, if $m^* = m$, the aggregate demand vector

$$\left(\frac{\tilde{x}_1}{g'[b^1(\mathbf{p})]b^1(\mathbf{p})}, \dots, \frac{\tilde{x}_m}{g'[b^m(\mathbf{p})]b^1(\mathbf{p})} \right)^T$$

is identified as a particular basis of the vector space spanned by commodity demands $(q_1/g'(p_1), \dots, q_n/g'(p_n))^T$. There are many (observationally equivalent) possible choices for a basis, yielding as many possible choices of $\boldsymbol{\Pi} = [\pi_j^k]$. The arbitrary normalization we choose to adopt is that there is at least one exclusive good per group. This is equivalent to choosing a subset of m linearly independent commodity demands (say $(q_1/g'(p_1), \dots, q_m/g'(p_m))^T$) as a particular basis. This completely identifies the allocation to groups and group composition.

For example, assuming a Cobb-Douglas form for indices (4.1) and assuming that the goods are ordered such that the kth of the first m prices is exclusive to the kth group, induces the following simple relationship between expenditures:

$$p_i q_i = \sum_{k=1}^{m} \frac{\pi_i^k}{\pi_i^k} p_k q_k \qquad \text{for all} \quad i = m+1, \dots, n.$$

Alternatively, in terms of expenditure shares

$$w_i = \sum_{k=1}^{m} \frac{\pi_j^k}{\pi_i^k} w_k \qquad \text{for all} \quad i = m+1, \dots, n, \tag{4.4}$$

where the share of total expenditure allocated to group k is

$$w_k = \pi_k^k \frac{\partial \ln \tilde{e}}{\partial \ln b^k} \qquad \text{for all} \quad k = 1, \dots, m. \tag{4.5}$$

The π_i^k terms measure the latent (unobserved) input of good i in group k. Linear homogeneity implies that

$$\sum_{i=1}^{n} \pi_i^k = 1 \qquad \text{for all} \quad k = 1, \dots, m,$$

and no additional normalization other than the exclusive good condition is necessary for identification.

4.2. *Preference Specifications and Parametric Demand Systems*

Latent price aggregation provides a natural way of grouping prices in many standard demand systems and coincides with an attractive representation of consumer preferences. In what follows we develop a methodology for exploring the presence of latent separability in two classes of popular models. The first class, quasi-homothetic preferences, includes a linear expenditure specification. The second class contains the translog model of Jorgensen, Lau, and Stoker (1982), the almost ideal demand model of Deaton and Muellbauer (1980), and polynomial generalizations of these models. We show how a judicious choice of price aggregators $b^k(\mathbf{p})$, $k = 1, \dots m$, in each case yields demand system specifications in which the number of separable groups m is given by the rank of a coefficient matrix. Moreover, for each class there exists a way of expressing demands (quantities, expenditures, budget shares, etc.) which gives the same structure to both group-level (aggregate) and good-level (individual) demands.

4.2.1. *Quasi-homothetic Demands*

The "aggregate" expenditure function in terms of group price indices (4.1), under quasi-homothetic preferences, takes the form

$$\tilde{e}(\mathbf{b}, u) = \tilde{a}(\mathbf{b}) + \tilde{c}(\mathbf{b})u$$

where $\mathbf{b} = \mathbf{b}(\mathbf{p}) = (b^1(\mathbf{p}), \dots, b^m(\mathbf{p}))^T$ is the vector of group price aggregators and where $\tilde{a}(\mathbf{b})$ and $\tilde{c}(\mathbf{b})$ are linear homogeneous functions of these group prices. We consider the case where these take the generalized Leontief form

$$\tilde{a}(\mathbf{b}) = \mathbf{b}^{\frac{1}{2}T} \tilde{\boldsymbol{\Gamma}}\, \mathbf{b}^{\frac{1}{2}},$$

$$\tilde{c}(\mathbf{b}) = \mathbf{b}^{\frac{1}{2}T} \tilde{\boldsymbol{\beta}}\, \mathbf{b}^{\frac{1}{2}},$$

in which $\tilde{\boldsymbol{\Gamma}}$ and $\tilde{\boldsymbol{\beta}}$ can be diagonal, yielding price functions that are linear in prices. Matrices $\tilde{\boldsymbol{\Gamma}}$ and $\tilde{\boldsymbol{\beta}}$ satisfy symmetry constraints

$$\tilde{\boldsymbol{\Gamma}}^T = \tilde{\boldsymbol{\Gamma}}, \qquad \tilde{\boldsymbol{\beta}}^T = \tilde{\boldsymbol{\beta}},$$

and are negative-definite. Aggregate demands are such that

$$\mathbf{b}^{-\frac{1}{2}} \odot \tilde{\mathbf{x}} = \left[\tilde{\boldsymbol{\Gamma}} + \tilde{\boldsymbol{\beta}} \frac{x - \tilde{a}(\mathbf{b})}{\tilde{c}(\mathbf{b})} \right] \mathbf{b}^{\frac{1}{2}},$$

where $\tilde{\mathbf{x}} = (\tilde{x}_1, \dots, \tilde{x}_m)^T$, $x = \tilde{x}_1 + \cdots + \tilde{x}_m$, and where $\odot$ denotes the row-wise vector multiplication operator.

The aggregate prices indices (4.1) are assumed to have the following form:

$$\mathbf{b}(\mathbf{p})^{\frac{1}{2}} = \boldsymbol{\Pi} \mathbf{p}^{\frac{1}{2}},$$

where $\boldsymbol{\Pi} = [\pi_i^k]$, a m by n matrix described in (4.1).

The overall expenditure function has the form

$$e(\mathbf{p}, u) = a(\mathbf{p}) + c(\mathbf{p})u$$

with

$$a(\mathbf{p}) = \mathbf{p}^{\frac{1}{2}T} \boldsymbol{\Gamma}\, \mathbf{p}^{\frac{1}{2}},$$

and

$$c(\mathbf{p}) = \mathbf{p}^{\frac{1}{2}T}\beta\mathbf{p}^{\frac{1}{2}},$$

where $\boldsymbol{\Gamma} = \boldsymbol{\Pi}^T\tilde{\boldsymbol{\Gamma}}\boldsymbol{\Pi}$ and $\boldsymbol{\beta} = \boldsymbol{\Pi}^T\tilde{\boldsymbol{\beta}}\boldsymbol{\Pi}$.

Demands for goods are given by

$$\mathbf{p}^{\frac{1}{2}} \odot \mathbf{q} = \boldsymbol{\Pi}^T(\mathbf{b}^{-\frac{1}{2}} \odot \tilde{\mathbf{x}}),$$

i.e.

$$\begin{aligned}\mathbf{p}^{\frac{1}{2}} \odot \mathbf{q} &= \boldsymbol{\Pi}^T\left[\tilde{\boldsymbol{\Gamma}} + \tilde{\boldsymbol{\beta}}\frac{x - \tilde{a}(\mathbf{b})}{\tilde{c}(\mathbf{b})}\right]\boldsymbol{\Pi}\mathbf{p}^{\frac{1}{2}} \\ &= \left[\boldsymbol{\Gamma} + \boldsymbol{\beta}\frac{x - \tilde{a}(\mathbf{b})}{\tilde{c}(\mathbf{b})}\right]\mathbf{p}^{\frac{1}{2}}.\end{aligned}$$

Note that the demand system for individual commodities takes the same form as the demand system for broad group demands, and parameters $\boldsymbol{\Gamma}$ and $\boldsymbol{\beta}$ have the same algebraic properties as $\tilde{\boldsymbol{\Gamma}}$ and $\tilde{\boldsymbol{\beta}}$.

4.2.2. *The Almost Ideal Class of Demand Systems*

Here we consider the quadratic almost ideal representation of preferences (Banks, Blundel, and Lewbel (1997)). The "aggregate" expenditure function in terms of the group price aggregators **b** takes the form

$$\ln \tilde{e}(\mathbf{b}, u) = \ln \tilde{a}(\mathbf{b}) + \tilde{c}(\mathbf{b})[\tilde{d}(\mathbf{b}) + u^{-1}]^{-1} \tag{4.6}$$

where

$$\begin{aligned}&\ln \tilde{a}(\mathbf{b}) = \tilde{\alpha}_0 + \tilde{\boldsymbol{\alpha}}^T \ln \mathbf{b} + \tfrac{1}{2}(\ln \mathbf{b})^T\tilde{\boldsymbol{\Gamma}}(\ln \mathbf{b}),\\ &\ln \tilde{c}(\mathbf{b}) = \tilde{\boldsymbol{\beta}}^T \ln \mathbf{b}, \qquad \text{and}\\ &\tilde{d}(p) = \tilde{\boldsymbol{\tau}}^T \ln \mathbf{b}.\end{aligned}$$

With $\tilde{\boldsymbol{\tau}} = 0$, (4.6) reduces to the familiar almost ideal form of piglog preferences due to Deaton and Muellbauer (1980). For (4.6) aggregate shares take the form

$$\tilde{\mathbf{w}} = \tilde{\boldsymbol{\alpha}} + \tilde{\boldsymbol{\Gamma}} \ln \mathbf{b} + \tilde{\boldsymbol{\beta}}[\ln x - \ln \tilde{a}(\mathbf{b})] + \frac{\tilde{\boldsymbol{\tau}}}{\tilde{c}(\mathbf{b})}[\ln x - \ln \tilde{a}(\mathbf{b})]^2, \tag{4.7}$$

where $\tilde{\mathbf{w}} = (\tilde{w}_1, \ldots, \tilde{w}_m)^T$ with $\tilde{w}_k = \tilde{x}_k/x$ and x is the total expenditure on all goods.

The parameters $\tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{\Gamma}}, \tilde{\boldsymbol{\beta}}, \tilde{\boldsymbol{\tau}}$ satisfy additivity and symmetry constraints.

$$\tilde{\boldsymbol{\alpha}}^T i_m = 1, \qquad \tilde{\boldsymbol{\Gamma}}^T i_m = \mathbf{0}_m, \qquad \tilde{\boldsymbol{\beta}}^T i_m = 0, \qquad \tilde{\boldsymbol{\tau}}^T i_m = 0, \qquad \tilde{\boldsymbol{\Gamma}}^T = \tilde{\boldsymbol{\Gamma}}, \tag{4.8}$$

where $\mathbf{0}_m$ is the m-vector of zeros.

The group price aggregates (4.1) are given by

$$\ln \mathbf{b}(\mathbf{p}) = \boldsymbol{\Pi} \ln \mathbf{p}, \qquad \text{where} \quad \boldsymbol{\Pi} i_n = i_m. \tag{4.9}$$

The overall expenditure function is therefore of the form

$$\ln e(\mathbf{p}, u) = \ln a(\mathbf{p}) + c(\mathbf{p})[d(\mathbf{p}) + u^{-1}]^{-1} \tag{4.10}$$

where

$$\ln a(\mathbf{p}) = \alpha_0 + \boldsymbol{\alpha}^T \ln \mathbf{p} + \tfrac{1}{2}(\ln \mathbf{p})^T \boldsymbol{\Gamma} (\ln \mathbf{p}), \tag{4.11}$$

$$\ln c(\mathbf{p}) = \boldsymbol{\beta}^T \ln \mathbf{p}, \qquad \text{and} \tag{4.12}$$

$$d(\mathbf{p}) = \boldsymbol{\tau}^T \ln \mathbf{p}, \tag{4.13}$$

with

$$\boldsymbol{\alpha} = \boldsymbol{\Pi}^T \tilde{\boldsymbol{\alpha}}, \qquad \boldsymbol{\beta} = \boldsymbol{\Pi}^T \tilde{\boldsymbol{\beta}}, \qquad \boldsymbol{\tau} = \boldsymbol{\Pi}^T \tilde{\boldsymbol{\tau}}, \tag{4.14}$$

and

$$\boldsymbol{\Gamma} = \boldsymbol{\Pi}^T \tilde{\boldsymbol{\Gamma}} \boldsymbol{\Pi}. \tag{4.15}$$

Expenditure shares of purchased goods are related to group expenditure shares by the equation

$$\mathbf{w} = \boldsymbol{\Pi}^T \tilde{\mathbf{w}}, \tag{4.16}$$

i.e.

$$\mathbf{w} = \boldsymbol{\alpha} + \boldsymbol{\Gamma} \ln \mathbf{p} + \boldsymbol{\beta}[\ln x - \ln \mathbf{a}(\mathbf{p})] + \frac{\boldsymbol{\tau}}{c(\mathbf{p})}[\ln x - \ln a(\mathbf{p})]^2. \tag{4.17}$$

5. IDENTIFICATION AND ESTIMATION IN THE ALMOST IDEAL CLASS

Here we consider the identification and estimation of the number of latent groups m for the almost ideal class of demand models introduced in Section 4. Equations (4.14) and (4.15) suggest that the estimation of the number of groups relates to a rank condition on the coefficient matrix $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\tau}, \boldsymbol{\Gamma})$. It is the estimation of the number of groups and the identification of $\tilde{\boldsymbol{\alpha}}$, $\tilde{\boldsymbol{\beta}}$, $\tilde{\boldsymbol{\tau}}$, and $\tilde{\boldsymbol{\Gamma}}$ from the unrestricted estimates of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\tau}$, and $\boldsymbol{\Gamma}$ in the quadratic almost ideal demand system (4.17) to which we now turn. We show precisely how the exclusivity restriction uniquely identifies the complete structure of latent separable preferences in this class of demand models. That is: assuming a minimum of m exclusive goods permits the unique recovery of $\tilde{\mathbf{B}} = (\tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{\beta}}, \tilde{\boldsymbol{\tau}})^T \in \mathbf{R}^{K \times m}$ and $\tilde{\boldsymbol{\Gamma}} \in \mathbf{R}^{m \times m}$ from $\mathbf{B} = (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\tau})^T \in \mathbf{R}^{K \times n}$ and $\boldsymbol{\Gamma} \in \mathbf{R}^{n \times n}$.[13]

[13]Note that $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\tau}$ would be matrices instead of vectors if they were modelled as $\boldsymbol{\alpha}\mathbf{z}$, $\boldsymbol{\beta}\mathbf{z}$, and $\boldsymbol{\tau}\mathbf{z}$ where $\mathbf{z}$ is a vector of exogenous variables capturing observed heterogeneity.

5.1. *Identification*

Let $\boldsymbol{\Theta}=(\mathbf{B}^T, \boldsymbol{\Gamma}^T)^T$ be the $(K+n)\times n$ matrix of all parameters. Latent separability implies that there exist $\tilde{\mathbf{B}}\in\mathbb{R}^{K\times m}$ and $\tilde{\boldsymbol{\Gamma}}\in\mathbb{R}^{m\times m}$ such that $\tilde{\boldsymbol{\Theta}}=(\tilde{\mathbf{B}}^T, \tilde{\boldsymbol{\Gamma}}^T)^T$ is full column rank and satisfies additivity and symmetry:

$$\tilde{\boldsymbol{\Theta}} i_m = \mathbf{1}_{m+k} \quad \text{(additivity)}, \tag{5.1}$$

$$\tilde{\boldsymbol{\Gamma}}^T = \tilde{\boldsymbol{\Gamma}} \quad \text{(symmetry)}, \tag{5.2}$$

where i_m is the m-vector of ones and $\mathbf{1}_{m+K}$ is the $(m+K)$-vector of zeroes except in the first row where there is a one.

The exclusivity assumption implies that there exists $\boldsymbol{\lambda}\in\mathbb{R}^m$, a vector of nonzero elements, and $\boldsymbol{\Psi}\in\mathbb{R}^{m\times n}$ such that

$$(\boldsymbol{\Lambda}:\boldsymbol{\Psi})\equiv\boldsymbol{\Pi}, \tag{5.3}$$

where $\boldsymbol{\Lambda}=\operatorname{diag}(\boldsymbol{\lambda})$. Moreover, identification requires an additional constraint on $\boldsymbol{\Lambda}$, and we require

$$\boldsymbol{\Pi} i_n = i_m, \tag{5.4}$$

which assures the homogeneity of degree one of price aggregates. Finally, both $\tilde{\boldsymbol{\Theta}}$ and $\boldsymbol{\Pi}$ must satisfy the property that

$$\mathbf{B}=\tilde{\mathbf{B}}(\boldsymbol{\Lambda}:\boldsymbol{\Psi}), \qquad \text{and} \tag{5.5}$$

$$\boldsymbol{\Gamma}=\begin{pmatrix}\boldsymbol{\Lambda}\\ \boldsymbol{\Psi}^T\end{pmatrix}\tilde{\boldsymbol{\Gamma}}(\boldsymbol{\Lambda}:\boldsymbol{\Psi}). \tag{5.6}$$

In the sequel we use the following matrix operators. Let $\mathbf{X}$ be a matrix; we denote as $\overline{\mathbf{X}}$, $\overline{\mathbf{X}}\rceil$, $\underline{\mathbf{X}}\rfloor$, and $\boxed{\mathbf{X}}$ the trimmed matrices in which the top row; top row and right column; bottom row and right column; and top row, right column, and bottom row are deleted.

PROPOSITION 4: *For the almost ideal class of demand systems*:

(i) *the number of latent groups is estimated as one plus the rank of the trimmed matrix* $\boxed{\boldsymbol{\Theta}}$;

(ii) *choice of m exclusive goods is sufficient to uniquely identify the complete structure of preferences*.

PROOF: See Appendix A.

It is important to note that, given that $\boldsymbol{\Theta}$ satisfies constraints (5.1) to (5.6) above, the rank restriction is equivalent to the following constraints on $\mathbf{B}$ and $\boldsymbol{\Gamma}$:

$$\overline{\mathbf{B}}\rceil=\mathbf{A}(\mathbf{I}_{m-1}:\boldsymbol{\Phi}), \tag{5.7}$$

$$\underline{\boldsymbol{\Gamma}}\rfloor=(\mathbf{I}_{m-1}:\boldsymbol{\Phi})^T\mathbf{G}(\mathbf{I}_{m-1}:\boldsymbol{\Phi}), \tag{5.8}$$

where $\mathbf{A}=\overline{\tilde{\mathbf{B}}}\rceil\,\underline{\boldsymbol{\Lambda}}\rfloor$ and $\mathbf{G}=\underline{\boldsymbol{\Lambda}}\rfloor\,\underline{\tilde{\boldsymbol{\Gamma}}}\rfloor\,\underline{\boldsymbol{\Lambda}}\rfloor$ are respectively the matrix formed by the first $m-1$ columns of $\overline{\mathbf{B}}\rceil$ and the first $m-1\times m-1$ block-diagonal part of $\underline{\boldsymbol{\Gamma}}\rfloor$.

It follows from the proposition that, without the exclusivity assumption, only the number of latent groups is identified by the data, not their composition. The choice of composition is only partially testable. That is, for any prior choice of m exclusive goods, the composition is identified and one can test whether it is an empirically acceptable choice.

5.2. *Estimation*

There are implicitly two stages in estimation: first, to estimate the number of groups and second to estimate the composition of the groups given some choice of exclusive goods. To estimate the number of groups, an estimate of the rank of $\boxed{\boldsymbol{\Theta}}$ is required. We adopt the method suggested by Cragg and Donald (1996) based on a minimum distance criterion, the minimization of which yields a consistent estimate of the group compositions for each choice of m.

The issue here is to estimate $\tilde{\mathbf{B}}$, $\tilde{\boldsymbol{\Gamma}}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Psi}$ under the null hypothesis that the first $m-1$ columns of $\boxed{\boldsymbol{\Theta}}$ are linearly independent. This problem conveniently reduces to two steps: first, find $m-1$ linearly independent columns under the hypothesis that rank $(\boxed{\boldsymbol{\Theta}}) \geq m-1$, and second, estimate the decomposition (5.5) and (5.6). For the first step, we can take the columns selected by the first $m-1$ steps of a LU decomposition with complete pivoting (as suggested by Cragg and Donald (1996); this provides a consistent method for choosing a set of linearly independent columns of a matrix), or any ad hoc selection. For the second step, we use a minimum distance estimation procedure.

To implement the second step we use quasi generalized nonlinear least squares (QGNLS) to estimate the decomposition (5.5) and (5.6). To obtain starting values for this estimator we consider a quasi generalized least squares estimator (QGLS) for an equivalent set of linear restrictions. Let

$$\overline{\mathbf{B}}\rceil = (\mathbf{B}_1 : \mathbf{B}_2) \qquad \text{and} \qquad \underline{\boldsymbol{\Gamma}}\rfloor = \begin{pmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} \\ \boldsymbol{\Gamma}_{21} & \boldsymbol{\Gamma}_{22} \end{pmatrix}$$

be partitions of $\overline{\mathbf{B}}\rceil$ and $\underline{\boldsymbol{\Gamma}}\rfloor$, where $\mathbf{B}_1$ is the matrix formed using the first $m-1$ columns of $\overline{\mathbf{B}}\rceil$ and $\boldsymbol{\Gamma}_{11}$ the square matrix formed using the first $m-1$ rows and $m-1$ columns of $\underline{\boldsymbol{\Gamma}}\rfloor$.[14] Then, from (5.7) and (5.8), latent separability implies the following set of implicit restrictions: there should exist matrices $\mathbf{A}$, $\mathbf{G}$, and $\boldsymbol{\Phi}$ such that

$$(5.9) \qquad \begin{cases} \mathbf{B}_1 = \mathbf{A}, \\ \mathbf{B}_2 = \mathbf{A}\boldsymbol{\Phi}, \\ \boldsymbol{\Gamma}_{11} = \mathbf{G}, \\ \boldsymbol{\Gamma}_{12} = \mathbf{G}\boldsymbol{\Phi}, \\ \boldsymbol{\Gamma}_{21} = \boldsymbol{\Phi}'\mathbf{G}, \\ \boldsymbol{\Gamma}_{22} = \boldsymbol{\Phi}'\mathbf{G}\boldsymbol{\Phi}. \end{cases}$$

[14]Any choice of exclusive goods can be achieved by reordering of the columns of $\mathbf{B}$ and the columns and rows of $\boldsymbol{\Gamma}$.

Replacing $\boldsymbol{\Phi}'\mathbf{G}$ by $\boldsymbol{\Gamma}_{21}$ in the last equation yields

$$\boldsymbol{\Gamma}_{22} = \boldsymbol{\Gamma}_{21}\boldsymbol{\Phi}$$

and the whole nonlinear system of restrictions (5.9) implies the following linear set of mixed restrictions identifying $\boldsymbol{\Phi}$:

$$(5.10) \qquad \begin{cases} \mathbf{B}_2 = \mathbf{B}_1\boldsymbol{\Phi}, \\ \boldsymbol{\Gamma}_{12} = \boldsymbol{\Gamma}_{11}\boldsymbol{\Phi}, \\ \boldsymbol{\Gamma}_{22} = \boldsymbol{\Gamma}_{21}\boldsymbol{\Phi}. \end{cases}$$

Restrictions (5.10) are written as

$$(5.11) \qquad \boldsymbol{\Theta}_2 = \boldsymbol{\Theta}_1\boldsymbol{\Phi}$$

where

$$\boldsymbol{\Theta}_2 = \begin{pmatrix} \mathbf{B}_2 \\ \boldsymbol{\Gamma}_{12} \\ \boldsymbol{\Gamma}_{22} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Theta}_1 = \begin{pmatrix} \mathbf{B}_1 \\ \boldsymbol{\Gamma}_{11} \\ \boldsymbol{\Gamma}_{21} \end{pmatrix}.$$

Using (5.11) and the estimated $\boldsymbol{\Theta}_2$ and $\boldsymbol{\Theta}_1$ parameters, $\boldsymbol{\Phi}$ can be estimated by quasi-generalized least squares applied to the system

$$(5.12) \qquad \mathbf{S}\,\mathrm{vec}\big(\hat{\boldsymbol{\Theta}}_2\big) = \mathbf{S}\big(\mathbf{I}_{n-m} \otimes \hat{\boldsymbol{\Theta}}_1\big)\mathrm{vec}(\boldsymbol{\Phi}) + \mathbf{u}$$

where $\mathbf{S}$ is the selection matrix which removes the redundant elements of vec $(\hat{\boldsymbol{\Theta}}_2)$ arising from the fact that $\hat{\boldsymbol{\Gamma}}_{22}$ is symmetry constrained. In (5.12) the vector $\mathbf{u}$ is a normal random vector with variance

$$\mathbf{V} = \mathbf{S}(\mathbf{S}_2 - (\boldsymbol{\Phi}' \otimes \mathbf{I}_{m-1})\mathbf{S}_1)\frac{\boldsymbol{\Omega}}{T}(\mathbf{S}_2 - (\boldsymbol{\Phi}' \otimes \mathbf{I}_{m-1})\mathbf{S}_1)'\mathbf{S}'$$

where $\boldsymbol{\Omega}$ is the asymptotic variance of $\sqrt{T}$ vec $(\hat{\boldsymbol{\Theta}})$ and where $\mathbf{S}_1$, and $\mathbf{S}_2$ are the selection matrices that select vec $(\boldsymbol{\Theta}_1)$ and vec $(\boldsymbol{\Theta}_2)$ from vec $(\boldsymbol{\Theta})$.

Finally, $\mathbf{A}$ and $\mathbf{G}$ are easily estimated by identifying $\mathbf{A} = \mathbf{B}_1$ and $\mathbf{G} = \boldsymbol{\Gamma}_{11}$.

Having estimated $\mathbf{A}$, $\mathbf{G}$, and $\boldsymbol{\Phi}$, the structural parameters $\tilde{\mathbf{B}}$, $\tilde{\boldsymbol{\Gamma}}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Psi}$ are identified according to the discussion in Section 5.1 above and can then be utilized as starting values for the QGNLS estimation of the decomposition (5.5) and (5.6).[15]

6. APPLICATION

6.1. *The Data*

The data used in this study are drawn from the detailed expenditure diaries of the British Family Expenditure Survey (FES) for the period 1974–1993. Prices are measured quarterly. We select a sample of 4786 married couples with two children living in the South East of England. This (reasonably) homogeneous selection is chosen so as to abstract from demographic and locational differ-

[15] We could not perform the QGNLS estimation and use the algorithm which provides the starting values. It happens that only QGNLS yields more efficient latent separability constrained parameters and elasticities.

ences and to focus on the price and income terms. We study the purchases of twenty two nondurable and service consumption goods comprising: wine, spirits, beer, six food categories, household fuel, clothing, household services, personal goods and services, leisure services, fares, tobacco, motoring and petrol. To avoid the problem of zero expenditures in the tobacco and petrol categories we only select car owning households that have at least one adult who smokes.[16] The definitions of all goods and their mean shares are presented in the first column of Table B.1 in Appendix B.

6.2. *Estimating the QUAIDS Model*

For each individual household we define w_{it} to be the expenditure share on commodity i for observation t with total budget x_t and the log price vector $\ln \mathbf{p}_t$. In the QUAIDS model (Banks, Blundell, and Lewbel (1997)), expenditure shares have the form (see 4.17):

$$w_{it} = \alpha_i + \gamma_i' \ln \mathbf{p}_t + \beta_i(\ln x_t - \ln a(\mathbf{p}_t)) + \tau_i \frac{(\ln x_t - \ln a(\mathbf{p}_t))^2}{c(\mathbf{p}_t)} + u_{it} \tag{6.1}$$

with the following nonlinear price aggregators:

$$\ln a(\mathbf{p}_t) = \boldsymbol{\alpha}^T \ln \mathbf{p}_t + \tfrac{1}{2} \ln \mathbf{p}_t^T \boldsymbol{\Gamma} \ln \mathbf{p}_t,$$

$$\ln c(\mathbf{p}_t) = \boldsymbol{\beta}^T \ln \mathbf{p}_t,$$

$$d(\mathbf{p}_t) = \boldsymbol{\tau}^T \ln \mathbf{p}_t,$$

and where $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)^T$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_N)^T$, $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_N)^T$, and

$$\boldsymbol{\Gamma} = \begin{pmatrix} \gamma_1' \\ \vdots \\ \gamma_N' \end{pmatrix}.$$

The familiar (linear) almost ideal demand system (LAIDS) simply sets $\tau_i = 0$ across all expenditure shares. The denominator $c(\mathbf{p}_t)$ in the quadratic term of the share equation (6.1) is required to maintain the integrability of the expenditure share system. The system is *conditionally linear* in price aggregators; therefore estimation using the iterated moment estimator developed in Blundell and Robin (1997) is straightforward.

Given the likely correlation between the error terms in each share equation u_{it} and the log total budget variable $\ln x_t$, we augment the QUAIDS specification (6.1) with the error v_t from a reduced form for $\ln x_t$. The error u_{it} is

[16] We do not attempt to account for possible endogenous selection, assuming smoking and car purchase decisions are predetermined for the expenditures on goods considered here.

TABLE II

ESTIMATED INCOME EFFECTS AND SPECIFICATION TESTS

Commodity	$\ln x$	$(\ln x)^2$	Exog. $\hat{v}\|t\|$	Ov.id. χ_2^2	Hom. $\|t\|$
Beer	−.0434 (.013)	−.0066 (.002)	.618	.659	.013
Wine	−.0046 (.011)	−.0029 (.002)	13.890	3.452	.065
Spirits	−.0060 (.009)	−.0034 (.001)	.921	3.901	.043
Bread	−.0117 (.006)	.0018 (.001)	.625	3.926	.098
Meat	−.0559 (.013)	−.0047 (.002)	2.201	.525	.010
Dairy	.0016 (.009)	.0006 (.001)	.603	1.492	.186
Fruit & Veg.	−.0143 (.014)	.0009 (.002)	.618	.225	.010
Other food	−.0206 (.011)	.0007 (.002)	1.616	1.009	.208
Food out	−.0506 (.013)	−.0128 (.002)	4.693	2.365	.136
Electricity	.0108 (.012)	.0071 (.002)	1.576	3.407	.059
Gas	−.0187 (.013)	−.0014 (.002)	2.359	.198	.031
Adult cloth.	−.0226 (.021)	−.0087 (.003)	2.246	1.001	.057
Child cloth.	−.0844 (.016)	−.0167 (.003)	3.585	1.588	.038
House. serv.	.1518 (.013)	.0189 (.002)	5.422	3.482	.194
Pers. goods	.0260 (.011)	.0007 (.002)	1.273	3.406	.428
Leis. goods	−.0330 (.012)	−.0066 (.002)	.096	.610	.198
Ent.	.2779 (.018)	.0321 (.003)	7.558	3.838	.059
Leis. serv.	−.0064 (.016)	.0008 (.002)	.390	.985	.016
Fares	.0019 (.012)	−.0026 (.002)	5.423	1.884	.022
Motoring	.0092 (.017)	.0030 (.003)	5.504	3.929	.071
Petrol	−.0840 (.045)	−.0104 (.007)	2.525	0.749	.010
Tobacco	−.0229 (.015)	.0047 (.002)	5.289	.403	.054

NOTES: Standard errors in parentheses.

written as the orthogonal decomposition

$$u_{it} = \rho_i v_t + \varepsilon_{it} \qquad \text{for goods} \quad i = 1, \dots, N,$$

and we assume $E(\varepsilon_{it}|x_t, \ln \mathbf{p}_t) = 0$ all i and t. The estimated reduced form equation for $\ln x$ is presented in Table B.2 of Appendix B. In addition to a linear trend, seasonal dummies and relative prices, income and income squared are used as additional instruments. These instruments are strongly significant.

Table II presents the estimated income coefficients for the QUAIDS model.[17] The importance of the quadratic logarithmic expenditure terms for many commodities is clearly established; for example, food consumed outside the home (Food out) and entertainment (Ent.). In contrast, for some other commodities—mainly food items, a specification that has budget shares linear in log expenditure is not strongly rejected. Fruit and vegetables (Fruit & Veg.), bread (Bread), and dairy products (Dairy) are such examples. For these commodities the Working-Leser specification, underlying the standard almost ideal model LAIDS, does seem to provide a reasonable description of Engel curve behavior.

[17] Full estimates are available on request from the authors as is the complete Gauss language subroutine for estimation of the QUAIDS model and the latent separability decomposition.

These results confirm the findings reported in the Blundell, Pashardes, and Weber (1993) study which considered broader aggregates.

The $|t|$ values for the significance of the $\hat{v}$ residual, reported in Table II, indicate that the exogeneity of log expenditure in the budget share equations is strongly rejected. The residual adjustment for endogeneity is therefore included in all results and all standard errors are adjusted for the estimated parameters in the price indices terms as well as the inclusion of the generated residual variable $\hat{v}_t$. The overidentification tests do not indicate any serious difficulties with the instrument set.

Homogeneity in prices and total expenditure is imposed throughout. There is no evidence of the rejection of homogeneity for any of the 22 commodity share equations as the final column of Table II shows. Symmetry was less easily acceptable with a χ^2 statistic of 253 and degrees of freedom 210.

In Table III we present the homogeneity and symmetry constrained own price and income elasticity estimates. The numbers reported refer to the 50th percentile points of the distribution of uncompensated own price elasticities ε^{50} and income elasticities η^{50}. Overall these median elasticity estimates appear to be reasonable although many of the price elasticities show a high degree of imprecision. The food items are largely price inelastic and, with the important exception of food purchased outside the home, are all clear necessities. At the

TABLE III

ESTIMATED BUDGET (η) AND OWN PRICE UNCOMPENSATED (ε) ELASTICITIES: MEDIAN VALUES FOR THE UNRESTRICTED MODEL

Commodity	η^{50}	ε^{50}
Beer	.992 (.078)	−2.181 (1.300)
Wine	2.398 (.267)	−.741 (.737)
Spirits	1.804 (.136)	−2.081 (.963)
Bread	.412 (.039)	−.317 (.609)
Meat	.701 (.036)	−.316 (.385)
Dairy	.405 (.029)	.050 (.192)
Fruit & Veg.	.568 (.035)	−.308 (.107)
Other food	.565 (.034)	−.606 (.224)
Food out	1.627 (.102)	−.364 (1.125)
Electricity	.234 (.043)	−.803 (.243)
Gas	.640 (.069)	−.168 (.289)
Adult cloth.	1.894 (.155)	−1.191 (1.179)
Child cloth.	1.392 (.093)	−1.213 (1.455)
House. serv.	1.732 (.280)	−2.634 (.985)
Pers. goods	1.400 (.068)	−2.751 (1.691)
Leis. goods	1.146 (.049)	−1.806 (.979)
Ent.	5.797 (.661)	−3.423 (.500)
Leis. serv.	.301 (.107)	−1.210 (.534)
Fares	2.410 (.285)	−1.598 (1.239)
Motoring	.802 (.088)	−1.331 (1.265)
Petrol	.805 (.080)	−.361 (.211)
Tobacco	.244 (.047)	−.469 (0.270)

Notes: Standard errrors in parentheses.

other extreme, entertainment is highly income and price elastic. Clothing items and household goods and services lie somewhere between.

6.3. *Efficiency Gains from Latent Separability*

Using the testing procedure described in Section 5.2 we were unable to reject a grouping of these twenty two goods into eleven (latent) separable groups. The value of the $\chi^2_{(132)}$ test statistic was 151.28 with a p value of .12. A specification with 10 or fewer groups could easily be rejected. Weak separability is also strongly rejected with a p value of zero. These results confirm our view that latent separability provides an interpretable and acceptable structure to place on consumer preferences.

An important consideration is whether this additional structure improves the precision of the estimated elasticities. Table IV presents the estimated price and income elasticities that correspond to Table III above. The gain in precision of the price elasticities is clear. The standard errors are typically more than halved and point estimates appear much more reasonable. Notice the alcoholic categories for example. These results are matched across the set of 22 commodities. Interestingly, but perhaps not surprisingly, there is little impact on the precision

TABLE IV

ESTIMATED BUDGET (η) AND OWN PRICE UNCOMPENSATED (ε) ELASTICITIES: MEDIAN VALUES WITH LATENT SEPARABILITY IMPOSED

Commodity	η^{50}	ε^{50}
Beer	1.000 (0.081)	−1.299 (0.314)
Wine	2.293 (0.202)	−0.909 (0.287)
Spirits	1.814 (0.132)	−1.393 (0.315)
Bread	0.412 (0.055)	−0.410 (0.104)
Meat	.692 (0.035)	−0.498 (0.205)
Dairy	0.403 (0.028)	−0.284 (0.135)
Fruit & Veg.	0.568 (0.052)	−0.303 (0.100)
Other food	0.552 (0.064)	−0.873 (0.126)
Food out	1.620 (0.089)	−0.572 (0.237)
Electricity	0.230 (0.090)	−0.797 (0.142)
Gas	0.649 (0.235)	−0.104 (0.616)
Adult cloth.	1.964 (0.149)	−1.317 (0.570)
Child cloth.	1.412 (0.081)	−1.984 (0.460)
House. serv.	1.571 (0.154)	−1.580 (0.501)
Pers. goods	1.414 (0.079)	−2.232 (0.617)
Leis. goods	1.160 (0.049)	−1.343 (0.460)
Ent.	4.056 (1.801)	−3.538 (0.468)
Leis. serv.	0.264 (0.107)	−1.318 (0.251)
Fares	2.548 (0.296)	−0.910 (0.351)
Motoring	0.935 (0.085)	−1.202 (0.450)
Petrol	0.851 (0.081)	−0.378 (0.177)
Tobacco	0.226 (0.134)	−0.841 (0.268)

Notes: Standard errors in parentheses.

or numerical value of the budget elasticities. This supports the argument that homothetic latent separability, while improving the precision of price elasticities, does not place the restrictive structure on income elasticities that would be the case under homothetic weak separability.

6.4. *Examining the Separability Structure and Decomposition*

To examine the composition of the groups we need a prior choice of exclusive goods. Typically there will be many row and column permutations that do not reject rank $m = 11$. We first need to eliminate those that correspond to a singular solution. There then remain a number of possible decompositions that give approximately the same value for the minimum chi-square criteria. A unique decomposition is achieved through the use of prior information on exclusive goods. In Table V we present such a decomposition. For this decomposition spirits, meat, bread, gas (used primarily for home heating), adult clothing, household goods and services, personal goods and services, leisure goods, entertainment, and petrol (gasoline) are chosen to be the exclusive goods.

TABLE V

AN ESTIMATED DECOMPOSITION: RESTRICTED $\hat{\Pi}$

Exclusive Goods	Wine	Beer	Wine	Dairy	Fruit & Veg.	Oth. Food.	Food Out	Elec.	Oth. Cloth.	L. Serv.	Fares	Oth. Cloth
Spirits	.554	.113	.333	.00	.00	.00	.00	.00	.00	.00	.00	.00
	(.041)	(.062)	(.032)									
Meat	.359	.00	.00	.00	.242	.273	.00	.126	.00	.00	.00	.00
	(.004)				(.034)	(.023)		(.034)				
Bread	.407	.00	.00	.593	.00	.00	.00	.00	.00	.00	.00	.00
	(.002)			(.003)								
Gas	.665	.00	.00	.00	.00	.00	.00	.335	.00	.00	.00	.00
	(.033)							(.042)				
Ad. Clth.	.365	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.636
	(.061))											(.034)
Hous.	1.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Sevr.	(—)											
Per.	1.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Goods	(—)											
Leis.	.748	.00	.00	.00	.00	.00	.00	.00	.00	.252	.00	.00
Goods	(.252)									(.090)		
Ent.	.537	.080	.139	.00	.00	.00	.209	.00	.00	.00	.080	.139
	(.064)	(.031)	(.045)				(.045)				(.035)	(.046)
Tob.	1.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	(—)											
Petrol	.337	.133	.00	.00	.00	.00	.250	.00	.00	.00	.046	.233
	(.045)	(.045)					(.120)				(.011)	(.091)

NOTES: Standard errors in parentheses.

Although illustrative, the results indicate some interesting combinations of goods. They also focus on the rejection of homothetic weak separability since there are a number of goods that enter more than one group with significant latent input coefficients. Some commodities, like tobacco, personal goods and services, and household goods and services, are exclusive to their own group. Given the "flexible" structure of the QUAIDS model at the aggregate or top stage, this allows for very general income and price elasticities for these goods. Notice, on the other hand, that some goods naturally group together. For example, alcohol—beer, wine, and spirits—forms one group. However, electricity is used together with meat and other foods in one group while it is used together with gas (mainly domestic heating) in another group. This is not surprising since, in most households, electricity is used for both cooking and heating.

Food consumed outside the house (food out) combines with the income elastic good "entertainment" and also with fares (for public transport), motoring expenditures, and beer. It also combines with a group of less income elastic travel inputs like petrol (gasoline for cars), a group that also includes fares (for public transport) and motoring expenditures. The low income elasticity food commodities: bread (including rice and cereal products) and diary products also group together.

Latent separability allows gains in the precision of estimated price elasticities without the strong restrictions placed on income elasticities that would be implied under homothetic weak separability. It also allows an interpretable decomposition across separable groups highlighting the importance of goods that overlap across groups.

7. CONCLUSIONS

This paper has introduced the new concept of "latent" separability in which there can be overlapping groups. Motivated by the large number of substitution effects that can enter disaggregated demand systems, latent separability has been shown to provide a useful theoretical and empirical framework for investigating the grouping of goods and of relative prices in demand analysis. Latent separability is a generalization of weak separability in which each group of goods requires at least one exclusive good. In contrast, traditional separability concepts require mutually exclusive groups. Latent separability has been shown to be equivalent to weak separability in latent inputs. It provides restrictions on the substitution matrix, thereby reducing the number of independent cross-price effects to be estimated.

For the translog and almost ideal models of consumer behavior and their generalizations, the number of latently separable groups has been shown to relate directly to the rank of an estimated price and income coefficient matrix. The identification of the composition of each group is achieved from the prior choice of at least one exclusive good per group. Homothetic latent separability has been shown to allow far more flexibility in income responses than the corresponding homothetic weak separability. Consequently gains in the precision of estimated price elasticities can be achieved without imposing the strong restrictions on estimated income elasticities implied by homothetic weak separability. A detailed method for exploring the presence of latent separability was presented and applied to the quasi-homothetic class and to the almost ideal class of demand models.

In an empirical application to the demand for 22 nondurables and services commodities using the British Family Expenditure Survey data, a grouping into 11 latent separable groups was found to be acceptable. There resulting estimates showed a considerable improvement in the precision of the price elasticities and a strong rejection of weak separability. An illustrative group decomposition was given showing certain goods, like electricity, to be used across a number of groups. These results suggest that latent separability can provide a powerful tool for improving the precision of substitution elasticities while imposing a structure on preferences that is considerably less restrictive than weak separability. It provides an interesting way of examining the decomposition of preferences into subutilities over groups of overlapping commodities.

Dept. of Economics, University College London, Gower St., London WCIE 63T, U.K., and Institute for Fiscal Studies; r.blundell@ucl.ac.uk; www.ifs.org.uk / staff / richard_b.shtml

and

INRA-LEA, 48 Bd. Jourdan, 75014 Paris, France and CREST-INSEE; robin@java.ens.fr

Manuscript received September, 1995; final revision received September, 1998.

APPENDIX A

EXTENDING DEFINITION 4: Were the price aggregates not quasilinear, but say quasiquadratic functions of prices, then it would be easy to see that the vector $(q_1/g'(p_1),\dots,q_n/g'(p_n))^T$ generally would also span the whole of $\mathbb{R}^n_+$.

One can define quasiquadratic function by extending quasilinearity as follows:

$$g(b(\mathbf{p})) = \sum_j \pi_j g(p_j) + \tfrac{1}{2}\sum_{ij} \gamma_{ij} g(p_i) g(p_j).$$

For example, quadratic translog or Leontief price indices are of this form (with $g(p) = \log p$ and $g(p) = \sqrt{p}$).

Specifically, assume that

$$g(b^k(\mathbf{p})) = \sum_j \pi_j^k g(p_j) + \tfrac{1}{2}\sum_{ij} \gamma_{ij}^k g(p_i) g(p_j) \qquad \text{for all } k.$$

Then

$$\text{(A.1)} \qquad \frac{q_j}{g'(p_j)} = \sum_{k=1}^{m} \left[\pi_j^k + \sum_{l=1}^{n} \frac{\gamma_{jl}^k + \gamma_{lj}^k}{2} g(p_l) \right] \frac{v_k}{g'(b^k(\mathbf{p}))}.$$

Hence$(q_j/g'(p_j))$ is a linear combination of $m(1+n)$ potentially linearly independent variables.

Proof of Proposition 5: First note that under the additivity and symmetry conditions (5.1) and (5.2) of $\tilde{\boldsymbol{\Theta}}$, additivity of $\boldsymbol{\Pi}$ (condition (5.4)) implies additivity of $\boldsymbol{\Theta}$ (i.e. $\boldsymbol{\Theta} i_n = \mathbf{1}_{n+K}$). Note also that symmetry of $\boldsymbol{\Gamma}$ is also automatically implied by symmetry of $\tilde{\boldsymbol{\Gamma}}$. It thus follows that constraints (5.5) and (5.6) imposed by the last column of $\boldsymbol{\Theta}$ and the last row of $\boldsymbol{\Gamma}$ are redundant.

We define $\mathbf{B}|$, $\tilde{\mathbf{B}}|$ and $\boldsymbol{\Pi}|$ as the matrices $\mathbf{B}$, $\tilde{\mathbf{B}}$, and $\boldsymbol{\Pi}$ without their last, right column, and $\boldsymbol{\Gamma}\rfloor$ and $\tilde{\boldsymbol{\Gamma}}\rfloor$ as matrices $\boldsymbol{\Gamma}$ and $\tilde{\boldsymbol{\Gamma}}$ without right column and bottom row, i.e., using the adding-up and symmetry conditions,

$$\tilde{\mathbf{B}} = (\tilde{\mathbf{B}}|, \mathbf{1}_K - \tilde{\mathbf{B}}| i_{m-1}),$$

$$\tilde{\boldsymbol{\Gamma}} = \begin{pmatrix} \tilde{\boldsymbol{\Gamma}}\rfloor & -\tilde{\boldsymbol{\Gamma}}\rfloor i_{m-1} \\ -i_{m-1}^T \tilde{\boldsymbol{\Gamma}}\rfloor & i_{m-1}^T \tilde{\boldsymbol{\Gamma}}\rfloor i_{m-1} \end{pmatrix},$$

with equivalent expressions for $\mathbf{B}$ and $\boldsymbol{\Gamma}$. We also write

$$\boldsymbol{\Pi}| = \begin{pmatrix} \boldsymbol{\Pi}\rfloor \\ \pi_m^T \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Lambda}\rfloor & 0 & \boldsymbol{\Psi}\rfloor \\ 0 & \lambda_m & \psi_m^T \end{pmatrix}$$

where $\boldsymbol{\Lambda}\rfloor = \operatorname{diag}(\lambda_1, \ldots, \lambda_{m-1})$.

Then equation (5.5) becomes

$$\text{(A.2)} \qquad \mathbf{B}| = \tilde{\mathbf{B}}\boldsymbol{\Pi}|$$

$$= \tilde{\mathbf{B}}|(\boldsymbol{\Pi}\rfloor - i_{m-1}\pi_m^T) + \mathbf{1}_K \boldsymbol{\pi}_m^T$$

$$\text{(A.3)} \qquad \equiv \tilde{\mathbf{B}}|\tilde{\boldsymbol{\Pi}} + \mathbf{1}_K \boldsymbol{\pi}_m^T,$$

with

$$\text{(A.4)} \qquad \tilde{\boldsymbol{\Pi}} = \boldsymbol{\Pi}\rfloor - i_{m-1}\boldsymbol{\pi}_m^T$$

$$= (\boldsymbol{\Lambda}\rfloor, -\lambda_m i_{m-1}, \boldsymbol{\Psi}\rfloor - i_{m-1}\psi_m^T)$$

and equation (5.6):

$$\text{(A.5)} \qquad \boldsymbol{\Gamma}\rfloor = \tilde{\boldsymbol{\Pi}}^T \tilde{\boldsymbol{\Gamma}}\rfloor \tilde{\boldsymbol{\Pi}}.$$

Isolating the first equation from the rest finally yields the following final form for constraint (A.3):

$$\text{(A.6)} \qquad \mathbf{b}_1 = \tilde{\boldsymbol{\Pi}}^T \tilde{\mathbf{b}}_1 + \boldsymbol{\pi}_m,$$

$$\text{(A.7)} \qquad \overline{\mathbf{B}} = \overline{\tilde{\mathbf{B}}}\rceil \tilde{\boldsymbol{\Pi}},$$

where

$$\mathbf{B}| = \begin{pmatrix} \mathbf{b}_1^T \\ \overline{\mathbf{B}}\rceil \end{pmatrix} \quad \text{and} \quad \tilde{\mathbf{B}}| = \begin{pmatrix} \tilde{\mathbf{b}}_1^T \\ \overline{\tilde{\mathbf{B}}}\rceil \end{pmatrix}.$$

Firstly, denoting by I_{m-1} the $(m-1) \times (m-1)$ identity matrix, remark that conditional on

$$\boldsymbol{\Lambda}\rfloor^{-1} \tilde{\boldsymbol{\Pi}} = \left[I_{m-1}, -\lambda_m \boldsymbol{\Lambda}\rfloor^{-1} i_{m-1}, \boldsymbol{\Lambda}\rfloor^{-1} (\boldsymbol{\Psi}\rfloor - i_{m-1}\psi_m^T) \right]$$

being *known*, equation (A.6) imposes no constraint on the first row of $\mathbf{B}\rfloor$. Parameterization $\mathbf{b}_1$ is simply changed into another equivalent parameterization,

$$\text{(A.8)} \qquad (\tilde{\mathbf{b}}_1^T \underline{\Lambda}\rfloor, \tilde{\mathbf{b}}_1^T \underline{\Lambda}\rfloor \boldsymbol{\Phi} + (\lambda_m, \psi_m^T))^T.$$

Hence, conditional on $\boldsymbol{\Phi}$, equation (A.6) unambiguously identifies $\tilde{\mathbf{b}}_1$, $\underline{\Lambda}\rfloor$, λ_m, and ψ_m.

Secondly, restrictions (A.5) and (A.7) can be reformulated in terms of $\boldsymbol{\Phi}$ as

$$\text{(A.9)} \qquad \overline{\mathbf{B}}\rceil = \mathbf{A}(\mathbf{I}_{m-1} : \boldsymbol{\Phi}),$$

$$\text{(A.10)} \qquad \underline{\boldsymbol{\Gamma}}\rfloor = (\mathbf{I}_{m-1} : \boldsymbol{\Phi})^T \mathbf{G}(\mathbf{I}_{m-1} : \boldsymbol{\Phi}),$$

where $\mathbf{A} = \overline{\tilde{\mathbf{B}}}\rceil \underline{\Lambda}\rfloor$ and $\mathbf{G} = \underline{\Lambda}\rfloor \underline{\tilde{\boldsymbol{\Gamma}}}\rfloor \underline{\Lambda}\rfloor$ are respectively the matrix formed by the first $m-1$ columns of $\overline{\mathbf{B}}\rceil$ and the first $(m-1)\times(m-1)$ block-diagonal part of $\underline{\boldsymbol{\Gamma}}\rfloor$. The problem of determining $\boldsymbol{\Phi}$ is then a standard rank reduction problem. Equations (A.9) and (A.10) unambiguously identify **A**, **G**, and $\boldsymbol{\Phi}$. Equation (A.6) unambiguously identifies λ_m and ψ_m. And $\boldsymbol{\Phi}$, therefore, unambiguously identifies $\underline{\Lambda}\rfloor$ and $\underline{\boldsymbol{\Psi}}\rfloor$. Finally, $\underline{\Lambda}\rfloor$ being identified, $\tilde{\mathbf{b}}_1 \underline{\Lambda}\rfloor$ unambiguously identifies $\tilde{\mathbf{b}}_1$.

APPENDIX B

DATA AND REDUCED FORM FOR LN x

TABLE B.1

DATA DESCRIPTION—THE 22 COMMODITIES

Commodity Group	Definition	$\bar{w}_i$	s.d.(w)
Beer	Beer, on and off license sales.	.0303	.0409
Wine	Wine, on and off license sales.	.0094	.0202
Spirits	Spirits, on and off license sales.	.0153	.0310
Bread	Bread, flour, rice & cereals.	.0407	.0215
Meat	All meat & fish.	.0774	.0480
Dairy	All dairy products.	.0586	.0311
Fruit and Vegetables	Fresh, tinned and dried vegetables & fruit.	.0521	.0260
Other food	Tea, coffee, drinks, sugar, jams & sweets.	.0591	.0310
Food consumed outside the home	Restaurants & canteen meals.	.0645	.0502
Electricity	Account & slot meter payments.	.0429	.0299
Gas	Account & slot meter payments.	.0320	.0301
Adult clothing	Adult clothing and footwear.	.0356	.0615
Children's clothing and footwear	Children's clothing & footwear.	.0629	.0637
Household services	Post, phone, domestic-services & fees.	.0578	.0585
Personal goods and services	Personal & chemist's goods & services.	.0482	0490
Leisure goods	Records, CDs, toys, books & gardenening.	.0527	.0514
Entertainment	Entertainment.	.0385	.0676
Leisure services	TV licenses & rentals.	.0177	.0167
Fares	Rail, bus & other fares.	.0147	.0324
Motoring	Maintenance, tax & insurance.	.0570	.0714
Petrol	Petrol & oil	.0678	.0577
Tobacco	Cigarettes, pipe tobacco & cigars.	.0649	0498

TABLE B.2

THE ESTIMATED REDUCED FORM FOR LN x

Variable	Coeff. (s.e.)
trend	−.0145 (.0216)
S_1	−.1395 (.0243)
S_2	−.0866 (.0209)
S_3	−.0725 (.0169)
income	−.5475 (.0461)
income^2	.0988 (.0048)
log prices	yes
R^2	.725
F (p value)	466.3 (.000)
T	4951

REFERENCES

BANKS, J., R. W. BLUNDELL, AND A. LEWBEL (1997): "Quadratic Engel Curves, Indirect Tax Reform and Welfare Measurement," *Review of Economics and Statistics*, 79, 527–539.

BLACKORBY, C., D. PRIMONT, AND R. R. RUSSELL (1978): *Duality, Separability and Functional Structure: Theory and Economic Applications*. New York: North-Holland.

BLACKORBY, C., R. R. RUSSELL (1997): "Two-stage Budgeting: An Extension of Gorman's Theorem," *Economic Theory*, 9, 185–193.

BLACKORBY, C., AND A. SHORROCKS, EDS. (1995): *Separability and Aggregation*, Vol. 1, Collected Works of W. M. Gorman. Oxford: Clarendon Press.

BLUNDELL, R. W., P. PASHARDES, AND G. WEBER (1993): "What Do We Learn About Consumer Demand Patterns from Micro Data?" *American Economic Review*, 83, 570–597.

BLUNDELL, R. W., AND J. M. ROBIN (1997): "Estimation in Large and Disaggregated Demand Systems: An Estimator for Conditionally Linear Systems," UCL Discussion Paper 97-2, forthcoming in *Journal of Applied Econometrics*.

CHIAPPORI, P.-A. (1988): "Rational Household Labor Supply," *Econometrica*, 56, 63–90.

——— (1992): "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100, 3, 437–467.

CRAGG, J., AND S. DONALD (1996): "Inferring the Rank of a Matrix," *Journal of Econometrics*, 76, 223–250.

DEATON, A. S. (1979): "The Distance Function in Consumer Behavior with Applications to Index Numbers and Optimal Taxation," *Review of Economic Studies*, 46, 391–406.

DEATON A. S., AND J. MUELLBAUER (1980): "An Almost Ideal Demand System," *American Economic Review*, 70, 312–336.

DIEWERT, W. E., AND T. J. WALES (1987): "Flexible Functional Forms and Global Curvature Conditions," *Econometrica*, 55, 43–68.

——— (1995): "Flexible Functional Forms and Tests for Homogeneous Separability," *Journal of Econometrics*, 67, 259–302.

GORMAN, W. M. (1956): "The Demand for Related Goods," Journal Paper No. J3129, Iowa Agricultural Experiment Station, November. Republished as "A Possible Procedure for Analyzing Quality Differentials in the Egg Market," *Review of Economic Studies*, 47, 843–856.

——— (1959): "Separable Utility and Aggregation," *Econometrica*, 21, 63–80.

——— (1968a): "The Structure of Utility Functions," *Review of Economic Studies*, 35, 369–390.

——— (1968b): "Measuring the Quantities of Fixed Factors," in *Value, Capital and Growth: Essays in Honour of Sir John Hicks*, ed. by J. N. Wolfe. Edinburgh: Edinburgh University Press, 141–172, reproduced with editorial summary in Blackorby and Shorrocks (1995).

——— (1976): "Tricks with Utility Functions," in *Essays in Economic Analysis*, ed. by M. Artis and R. Nobay. Cambridge: Cambridge University Press.

——— (1978): "More Measures of Fixed Factors," presented at the Nuffield Conference on Measurement, and Modelling, reproduced with editorial summary in Blackorby and Shorrocks (1995).

——— (1981): "Some Engel Curves," in *Essays in the Theory and Measurement of Consumer Behavior in Honor of Richard Stone*, ed. by Angus Deaton. Cambridge: Cambridge University Press.

HICKS, J. R. (1946): *Value and Capital*, 2nd. ed. London: Oxford University Press (Clarendon).

HILDENBRAND, W. (1994): *Market Demand: Theory and Empirical Evidence*. Princeton: Princeton University Press.

JORGENSON, D. W., L. LAU, AND T. L. STOKER (1982): "The Transcendental Logarithmic Model of Aggregate Consumer Behavior," in *Advances in Econometrics*, Vol. 1., ed. by R. L. Basmann and G. G. Rhodes. Greenwich: JAI Press, 97–238.

LEWBEL, A. (1991): "The Rank of Demand Systems: Theory and Nonparametric Estimation," *Econometrica*, 59, 711–730.

——— (1993): "Stochastic Hicksian Aggregation With an Application to Grouping Goods Without Separable Utility," *Annales d'Economie et de Statistiques*, 29, 17–42.

——— (1996): "Aggregation Without Separability: A Generalized Composite Commodity Theorem," *American Economic Review*, 86, 524–543.

MCFADDEN, D. (1978): "Cost, Revenue and Profit Functions," in *Production Economics: A Dual Approach to Theory and Applications*, ed. by M. Fuss and D. McFadden. New York: North Holland.