

Chapter 22

Optimal Unemployment Insurance

22.1. History-dependent unemployment insurance

This chapter applies the recursive contract machinery studied in chapters 20, 21, and 23 in contexts that are simple enough that we can go a long way toward computing optimal contracts by hand. The contracts encode history dependence by mapping an initial promised value and a random time t observation into a time t consumption allocation and a continuation value to bring into next period. We use recursive contracts to study good ways of providing consumption insurance when incentive problems come from the insurance authority's inability to observe the effort that an unemployed person exerts searching for a job. We begin by studying a setup of Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) that focuses on a single isolated spell of unemployment followed by permanent employment. Later we take up settings of Wang and Williamson (1996) and Zhao (2001) with alternating spells of employment and unemployment in which the planner has limited information about a worker's effort while he is on the job, in addition to not observing his search effort while he is unemployed. Here history dependence manifests itself in an optimal contract with intertemporal tie-ins across these spells. Zhao uses her model to rationalize unemployment compensation that replaces a fraction of a worker's earnings on his or her previous job.

22.2. A one-spell model

This section describes a model of optimal unemployment compensation along the lines of Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). We shall use the techniques of Hopenhayn and Nicolini to analyze a model closer to Shavell and Weiss's. An unemployed worker orders stochastic processes of consumption and search effort $\{c_t, a_t\}_{t=0}^{\infty}$ according to

$$E \sum_{t=0}^{\infty} \beta^t [u(c_t) - a_t] \quad (22.2.1)$$

where $\beta \in (0, 1)$ and $u(c)$ is strictly increasing, twice differentiable, and strictly concave. We assume that $u(0)$ is well defined. We require that $c_t \geq 0$ and $a_t \geq 0$. All jobs are alike and pay wage $w > 0$ units of the consumption good each period forever. An unemployed worker searches with effort a and with probability $p(a)$ receives a permanent job at the beginning of the next period. Once a worker has found a job, he is beyond the grasp of the unemployment insurance agency.¹ Furthermore, $a = 0$ when the worker is employed. The probability of finding a job is $p(a)$ where p is an increasing and strictly concave and twice differentiable function of a , satisfying $p(a) \in [0, 1]$ for $a \geq 0$, $p(0) = 0$. The consumption good is nonstorable. The unemployed worker has no savings and cannot borrow or lend. The insurance agency is the unemployed worker's only source of consumption smoothing over time and across states.

¹ This is Shavell and Weiss's assumption, but not Hopenhayn and Nicolini's. Hopenhayn and Nicolini allow the unemployment insurance agency to impose history-dependent taxes on previously unemployed workers. Since there is no incentive problem after the worker has found a job, it is optimal for the agency to provide an employed worker with a constant level of consumption, and hence, the agency imposes a permanent per-period history-dependent tax on a previously unemployed worker. See exercise 22.2.

22.2.1. The autarky problem

As a benchmark, we first study the fate of the unemployed worker who has no access to unemployment insurance. Because employment is an absorbing state for the worker, we work backward from that state. Let V^e be the expected sum of discounted one-period utilities of an employed worker. Once the worker is employed, $a = 0$, making his period utility be $u(c) - a = u(w)$ forever. Therefore,

$$V^e = \frac{u(w)}{(1 - \beta)}. \quad (22.2.2)$$

Now let V^u be the expected present value of utility for an unemployed worker who chooses the current period pair (c, a) optimally. The Bellman equation for V^u is

$$V^u = \max_{a \geq 0} \left\{ u(0) - a + \beta [p(a)V^e + (1 - p(a))V^u] \right\}. \quad (22.2.3)$$

The first-order condition for this problem is

$$\beta p'(a) [V^e - V^u] \leq 1, \quad (22.2.4)$$

with equality if $a > 0$. Since there is no state variable in this infinite horizon problem, there is a time-invariant optimal search intensity a and an associated value of being unemployed V^u . Let $V_{\text{aut}} = V^u$ denote the solution of Bellman equation (22.2.3).

Equations (22.2.3) and (22.2.4) form the basis for an iterative algorithm for computing $V^u = V_{\text{aut}}$. Let V_j^u be the estimate of V_{aut} at the j th iteration. Use this value in equation (22.2.4) and solve for an estimate of effort a_j . Use this value in a version of equation (22.2.3) with V_j^u on the right side to compute V_{j+1}^u . Iterate to convergence.

22.2.2. Unemployment insurance with full information

As another benchmark, we study the provision of insurance with full information. An insurance agency can observe and control the unemployed person's consumption and search effort. The agency wants to design an unemployment insurance contract to give the unemployed worker expected discounted utility $V > V_{\text{aut}}$. The planner wants to deliver value V in the most efficient way, meaning the way that minimizes expected discounted cost, using β as the discount factor. We formulate the optimal insurance problem recursively. Let $C(V)$ be the expected discounted cost of giving the worker expected discounted utility V . The cost function is strictly convex because a higher V implies a lower marginal utility of the worker; that is, additional expected "utils" can be awarded to the worker only at an increasing marginal cost in terms of the consumption good. Given V , the planner assigns first-period pair (c, a) and promised continuation value V^u , should the worker be unlucky and not find a job; (c, a, V^u) will all be chosen to be functions of V and to satisfy the Bellman equation

$$C(V) = \min_{c, a, V^u} \left\{ c + \beta[1 - p(a)]C(V^u) \right\}, \quad (22.2.5)$$

where the minimization is subject to the promise-keeping constraint

$$V \leq u(c) - a + \beta \{ p(a)V^e + [1 - p(a)]V^u \}. \quad (22.2.6)$$

Here V^e is given by equation (22.2.2), which reflects the assumption that once the worker is employed, he is beyond the reach of the unemployment insurance agency. The right side of Bellman equation (22.2.5) is attained by policy functions $c = c(V)$, $a = a(V)$, and $V^u = V^u(V)$. The promise-keeping constraint, equation (22.2.6), asserts that the 3-tuple (c, a, V^u) attains at least V . Let θ be the Lagrange multiplier on constraint (22.2.6). At an interior solution, the first-order conditions with respect to c , a , and V^u , respectively, are

$$\theta = \frac{1}{u'(c)}, \quad (22.2.7a)$$

$$C(V^u) = \theta \left[\frac{1}{\beta p'(a)} - (V^e - V^u) \right], \quad (22.2.7b)$$

$$C'(V^u) = \theta. \quad (22.2.7c)$$

The envelope condition $C'(V) = \theta$ and equation (22.2.7c) imply that $C'(V^u) = C'(V)$. Strict convexity of C then implies that $V^u = V$. Applied

repeatedly over time, $V^u = V$ makes the continuation value remain constant during the entire spell of unemployment. Equation (22.2.7a) determines c , and equation (22.2.7b) determines a , both as functions of the promised V . That $V^u = V$ then implies that c and a are held constant during the unemployment spell. Thus, the unemployed worker's consumption c and search effort a are both "fully smoothed" during the unemployment spell. But the worker's consumption is not smoothed across states of employment and unemployment unless $V = V^e$.

22.2.3. The incentive problem

The preceding efficient insurance scheme requires that the insurance agency control both c and a . It will not do for the insurance agency simply to announce c and then allow the worker to choose a . Here is why. The agency delivers a value V^u higher than the autarky value V_{aut} by doing two things. It *increases* the unemployed worker's consumption c and *decreases* his search effort a . But the prescribed search effort is *higher* than what the worker would choose if he were to be guaranteed consumption level c while he remains unemployed. This follows from equations (22.2.7a) and (22.2.7b) and the fact that the insurance scheme is costly, $C(V^u) > 0$, which imply $[\beta p'(a)]^{-1} > (V^e - V^u)$. But look at the worker's first-order condition (22.2.4) under autarky. It implies that if search effort $a > 0$, then $[\beta p(a)]^{-1} = [V^e - V^u]$, which is inconsistent with the preceding inequality $[\beta p'(a)]^{-1} > (V^e - V^u)$ that prevails when $a > 0$ under the social insurance arrangement. If he were free to choose a , the worker would therefore want to fulfill (22.2.4), either at equality so long as $a > 0$, or by setting $a = 0$ otherwise. Starting from the a associated with the social insurance scheme, he would establish the desired equality in (22.2.4) by *lowering* a , thereby decreasing the term $[\beta p'(a)]^{-1}$ (which also lowers $(V^e - V^u)$ when the value of being unemployed V^u increases). If an equality can be established before a reaches zero, this would be the worker's preferred search effort; otherwise the worker would find it optimal to accept the insurance payment, set $a = 0$, and never work again. Thus, since the worker does not take the cost of the insurance scheme into account, he would choose a search effort below the socially optimal one. The efficient contract exploits the agency's ability to control *both* the unemployed worker's consumption *and* his search effort.

22.2.4. Unemployment insurance with asymmetric information

Following Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), now assume that the unemployment insurance agency cannot observe or enforce a , though it can observe and control c . The worker is free to choose a , which puts expression (22.2.4), the worker's first-order condition under autarky, back in the picture.² Given any contract, the individual will choose search effort according to the first-order condition (22.2.4). This fact leads the insurance agency to design the unemployment insurance contract to respect this restriction. Thus, the recursive contract design problem is now to minimize the right side of equation (22.2.5) subject to expression (22.2.6) and the incentive constraint (22.2.4).

Since the restrictions (22.2.4) and (22.2.6) are not linear and generally do not define a convex set, it becomes difficult to provide conditions under which the solution to the dynamic programming problem results in a convex function $C(V)$. As discussed in Appendix A of chapter 20, this complication can be handled by convexifying the constraint set through the introduction of lotteries. However, a common finding is that optimal plans do not involve lotteries, because convexity of the constraint set is a sufficient but not necessary condition for convexity of the cost function. Following Hopenhayn and Nicolini (1997), we therefore proceed under the assumption that $C(V)$ is strictly convex in order to characterize the optimal solution.

Let η be the multiplier on constraint (22.2.4), while θ continues to denote the multiplier on constraint (22.2.6). But now we replace the weak inequality in (22.2.6) by an equality. The unemployment insurance agency cannot award a higher utility than V because that might violate an incentive-compatibility constraint for exerting the proper search effort in earlier periods. At an interior solution, the first-order conditions with respect to c, a , and V^u , respectively, are³

$$\theta = \frac{1}{u'(c)}, \tag{22.2.8a}$$

$$C(V^u) = \theta \left[\frac{1}{\beta p'(a)} - (V^e - V^u) \right] - \eta \frac{p''(a)}{p'(a)} (V^e - V^u)$$

² We are assuming that the worker's best response to the unemployment insurance arrangement is completely characterized by the first-order condition (22.2.4), an instance of the so-called "first-order" approach to incentive problems.

³ Hopenhayn and Nicolini let the insurance agency also choose V^e , the continuation value from V , if the worker finds a job. This approach reflects their assumption that the agency can tax a previously unemployed worker after he becomes employed. See exercise 22.2.

$$= -\eta \frac{p''(a)}{p'(a)} (V^e - V^u), \quad (22.2.8b)$$

$$C'(V^u) = \theta - \eta \frac{p'(a)}{1-p(a)}, \quad (22.2.8c)$$

where the second equality in equation (22.2.8b) follows from strict equality of the incentive constraint (22.2.4) when $a > 0$. As long as the insurance scheme is associated with costs, so that $C(V^u) > 0$, first-order condition (22.2.8b) implies that the multiplier η is strictly positive. The first-order condition (22.2.8c) and the envelope condition $C'(V) = \theta$ together allow us to conclude that $C'(V^u) < C'(V)$. Convexity of C then implies that $V^u < V$. After we have also used equation (22.2.8a), it follows that in order to provide the proper incentives, the consumption of the unemployed worker must decrease as the duration of the unemployment spell lengthens. It also follows from (22.2.4) at equality that search effort a rises as V^u falls, i.e., it rises with the duration of unemployment.

The duration dependence of benefits is designed to provide incentives to search. To see this, from (22.2.8c), notice how the conclusion that consumption falls with the duration of unemployment depends on the assumption that more search effort raises the prospect of finding a job, i.e., that $p'(a) > 0$. If $p'(a) = 0$, then (22.2.8c) and the strict convexity of C imply that $V^u = V$. Thus, when $p'(a) = 0$, there is no reason for the planner to make consumption fall with the duration of unemployment.

22.2.5. Computed example

For parameters chosen by Hopenhayn and Nicolini, Figure 22.2.1 displays the replacement ratio c/w as a function of the duration of the unemployment spell.⁴ This schedule was computed by finding the optimal policy functions

$$\begin{aligned} V_{t+1}^u &= f(V_t^u) \\ c_t &= g(V_t^u). \end{aligned}$$

and iterating on them, starting from some initial $V_0^u > V_{\text{aut}}$, where V_{aut} is the autarky level for an unemployed worker. Notice how the replacement ratio

⁴ This figure was computed using the Matlab programs `hugo.m`, `hugo1a.m`, `hugofoc1.m`, `valhugo.m`. These are available in the subdirectory `hugo`, which contains a readme file. These programs were composed by various members of Economics 233 at Stanford in 1998, especially Eva Nagypal, Laura Veldkamp, and Chao Wei.

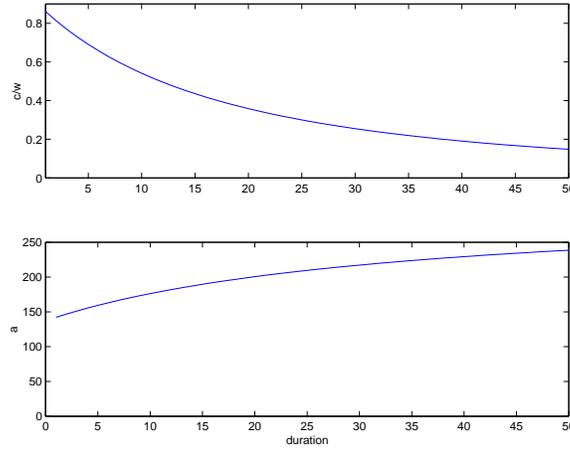


Figure 22.2.1: Top panel: replacement ratio c/w as a function of duration of unemployment in the Shavell-Weiss model. Bottom panel: effort a as a function of duration.

declines with duration. Figure 22.2.1 sets V_0^u at 16,942, a number that has to be interpreted in the context of Hopenhayn and Nicolini's parameter settings.

We computed these numbers using the parametric version studied by Hopenhayn and Nicolini.⁵ Hopenhayn and Nicolini chose parameterizations and parameters as follows: They interpreted one period as one week, which led them to set $\beta = .999$. They took $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and set $\sigma = .5$. They set the wage $w = 100$ and specified the hazard function to be $p(a) = 1 - \exp(-ra)$, with r chosen to give a hazard rate $p(a^*) = .1$, where a^* is the optimal search effort under autarky. To compute the numbers in Figure 22.2.1 we used these same settings.

⁵ In section 4.7.3, we described a computational strategy of iterating to convergence on the Bellman equation (22.2.5), subject to expressions (22.2.6) at equality, and (22.2.4).

22.2.6. Computational details

Exercise 22.1 asks the reader to solve the Bellman equation numerically. In doing so, it is useful to note that there are natural lower and upper bounds to the set of continuation values V^u . The lower bound is the expected lifetime utility in autarky, V_{aut} . To compute the upper bound, represent condition (22.2.4) as

$$V^u \geq V^e - [\beta p'(a)]^{-1},$$

with equality if $a > 0$. If there is zero search effort, then $V^u \geq V^e - [\beta p'(0)]^{-1}$. Therefore, to rule out zero search effort we require

$$V^u < V^e - [\beta p'(0)]^{-1}.$$

(Remember that $p''(a) < 0$.) This step gives our upper bound for V^u .

To formulate the Bellman equation numerically, we suggest using the constraints to eliminate c and a as choice variables, thereby reducing the Bellman equation to a minimization over the one choice variable V^u . First express the promise-keeping constraint (22.2.6) as $u(c) = V + a - \beta\{p(a)V^e + [1 - p(a)]V^u\}$. That is, consumption is equal to

$$c = u^{-1}(V + a - \beta[p(a)V^e + (1 - p(a))V^u]). \quad (22.2.9)$$

Similarly, solving the inequality (22.2.4) for a and using the assumed functional form for $p(a)$ leads to

$$a = \max \left\{ 0, \frac{\log[r\beta(V^e - V^u)]}{r} \right\}. \quad (22.2.10)$$

Formulas (22.2.9) and (22.2.10) express (c, a) as functions of V and the continuation value V^u . Using these functions allows us to write the Bellman equation in $C(V)$ as

$$C(V) = \min_{V^u} \{c + \beta[1 - p(a)]C(V^u)\} \quad (22.2.11)$$

where c and a are given by equations (22.2.9) and (22.2.10).

22.2.7. Interpretations

The substantial downward slope in the replacement ratio in Figure 22.2.1 comes entirely from the incentive constraints facing the planner. We saw earlier that without private information, the planner would smooth consumption over the unemployment spell by keeping the replacement ratio constant. In the situation depicted in Figure 22.2.1, the planner can't observe the worker's search effort and therefore makes the replacement ratio fall and search effort rise as the duration of unemployment increases, especially early in an unemployment spell. There is a "carrot-and-stick" aspect to the replacement rate and search effort schedules: the "carrot" occurs in the forms of high compensation and low search effort early in an unemployment spell. The "stick" occurs in the low compensation and high effort later in the spell. We shall see this carrot-and-stick feature in some of the credible government policies analyzed in chapters 23, 24, and 25.

The planner offers declining benefits and asks for increased search effort as the duration of an unemployment spell rises in order to provide unemployed workers with proper incentives, not to punish an unlucky worker who has been unemployed for a long time. The planner believes that a worker who has been unemployed a long time is unlucky, not that he has done anything wrong (i.e., not lived up to the contract). Indeed, the contract is designed to induce the unemployed workers to search in the way the planner expects. The falling consumption and rising search effort of the unlucky ones with long unemployment spells are simply the prices that have to be paid for the common good of providing proper incentives.

22.2.8. Extension: an on-the-job tax

Hopenhayn and Nicolini allow the planner to tax the worker *after* he becomes employed, and they let the tax depend on the duration of unemployment. Giving the planner this additional instrument substantially decreases the rate at which the replacement ratio falls during a spell of unemployment. Instead, the planner makes use of a more powerful tool: a *permanent* bonus or tax after the worker becomes employed. Because it endures, this tax or bonus is especially potent when the discount factor is high. In exercise 22.2, we ask the reader to set up the functional equation for Hopenhayn and Nicolini's model.

22.2.9. Extension: intermittent unemployment spells

In Hopenhayn and Nicolini's model, employment is an absorbing state and there are no incentive problems after a job is found. There are not multiple spells of unemployment. Wang and Williamson (1996) built a model in which there can be multiple unemployment spells, and in which there is also an incentive problem on the job. As in Hopenhayn and Nicolini's model, search effort affects the probability of finding a job. In addition, while on a job, effort affects the probability that the job ends and that the worker becomes unemployed again. Each job pays the same wage. In Wang and Williamson's setup, the promised value keeps track of the duration and number of spells of employment as well as of the number and duration of spells of unemployment. One contract transcends employment and unemployment.

22.3. A multiple-spell model with lifetime contracts

Rui Zhao (2001) modifies and extends features of Wang and Williamson's model. In her model, effort on the job affects output as well as the probability that the job will end. In Zhao's model, jobs randomly end, recurrently returning a worker to the state of unemployment. The probability that a job ends depends directly or indirectly on the effort that workers expend on the job. A planner observes the worker's output and employment status, but never his effort, and wants to insure the worker. Using recursive methods, Zhao designs a history-dependent assignment of unemployment benefits, if unemployed, and wages, if employed, that balance a planner's desire to insure the worker with the need to provide incentives to supply effort in work and search. The planner uses history dependence to tie compensation while unemployed (or employed) to earlier outcomes that partially inform the planner about the workers' efforts while employed (or unemployed). These intertemporal tie-ins give rise to what Zhao interprets broadly as a "replacement rate" feature that we seem to observe in unemployment compensation systems.

22.3.1. The setup

In a special case of Zhao's model, there are two effort levels. Where $a \in \{a_L, a_H\}$ is a worker's effort and $\bar{y}_i > \bar{y}_{i-1}$, an employed worker produces $y_t \in [\bar{y}_1, \dots, \bar{y}_n]$ with probability

$$\text{Prob}(y_t = \bar{y}_i) = p(\bar{y}_i; a).$$

Zhao assumes:

ASSUMPTION 1: $p(\bar{y}_i; a)$ satisfies the *monotone likelihood ratio* property: $\frac{p(\bar{y}_i; a_H)}{p(\bar{y}_i; a_L)}$ increases as \bar{y}_i increases.

At the end of each period, jobs end with probability π_{eu} . Zhao embraces one of two alternative assumptions about the job separation rate π_{eu} , allowing it to depend on either current output y or current work effort a . She assumes:

ASSUMPTION 2: Either $\pi_{eu}(y)$ decreases with y or $\pi_{eu}(a)$ decreases with a .

Unemployed workers produce nothing and search for a job subject to the following assumption about the job finding rate $\pi_{ue}(a)$:

ASSUMPTION 3: $\pi_{ue}(a)$ increases with a .

The worker's one-period utility function is $U(c, a) = u(c) - \phi(a)$ where $u(\cdot)$ is continuously differentiable, strictly increasing and strictly concave, and $\phi(a)$ is continuous, strictly increasing, and strictly convex. The worker orders random $\{c_t, a_t\}_{t=0}^{\infty}$ sequences according to

$$E \sum_{t=0}^{\infty} \beta^t U(c_t, a_t), \quad \beta \in (0, 1). \quad (22.3.1)$$

We shall regard a planner as being a coalition of firms united with an unemployment insurance agency. The planner is risk neutral and can borrow and lend at a constant risk-free gross one-period interest rate of $R = \beta^{-1}$.

Let the worker's employment state be $s_t \in S = \{e, u\}$ where e denotes employed, u unemployed. The worker's output at t is

$$z_t = \begin{cases} 0 & \text{if } s_t = u, \\ y_t & \text{if } s_t = e. \end{cases}$$

For $t \geq 1$, the time t component of the publicly observed information is

$$x_t = (z_{t-1}, s_t),$$

and $x_0 = s_0$. At time t , the planner observes the history x^t and the worker observes (x^t, a^t) .

The transition probability for $x_{t+1} \equiv (z_t, s_{t+1})$ can be factored as follows:

$$\pi(x_{t+1}|s_t, a_t) = \pi_z(z_t; s_t, a_t)\pi_s(s_{t+1}; z_t, s_t, a_t) \quad (22.3.2)$$

where π_z is the distribution of output conditioned on the state and the action, and π_s encodes the transition probabilities of employment status conditional on output, current employment status, and effort. In particular, Zhao assumes that

$$\begin{aligned} \pi_s(u; 0, u, a) &= 1 - \pi_{ue}(a) \\ \pi_s(e; 0, u, a) &= \pi_{ue}(a) \\ \pi_s(u; y, e, a) &= \pi_{eu}(y, a) \\ \pi_s(e; y, e, a) &= 1 - \pi_{eu}(y, a). \end{aligned} \quad (22.3.3)$$

22.3.2. A recursive lifetime contract

Consider a worker with beginning-of-period employment status s and promised value v . For given (v, s) , let $w(z, s')$ be the continuation value of promised utility (22.3.1) for next period when today's output is z and tomorrow's employment state is s' . At the beginning of next period, (z, s') will be the labor market outcome most recently observed by the planner. Let $W = \{W_s\}_{s \in \{u, e\}}$ be two compact sets of continuation values, one set for $s = u$ and another for $s = e$. For each (v, s) , a *recursive contract* specifies a recommended effort level a today, an output-contingent consumption level $c(z)$ today, and continuation values $w(z, s')$ to be used to reset v tomorrow.

For each (v, s) , the contract $(a, c(z), w(z, s'))$ must satisfy:

$$\sum_z \pi_z(z; s, a) \left(u(c(z)) + \beta \sum_{s'} \pi_s(s'; z, s, a) w(z, s') \right) - \phi(a) = v \quad (22.3.4)$$

and

$$\begin{aligned} \sum_z \pi_z(z; s, a) \left(u(c(z)) + \beta \sum_{s'} \pi_s(s'; z, s, a) w(z, s') \right) - \phi(a) &\geq \\ \sum_z \pi_z(z; s, \tilde{a}) \left(u(c(z)) + \beta \sum_{s'} \pi_s(s'; z, s, \tilde{a}) w(z, s') \right) - \phi(\tilde{a}) &\quad \forall \tilde{a}. \end{aligned} \quad (22.3.5)$$

Constraint (22.3.4) entails *promise keeping*, while (22.3.5) are the incentive-compatibility or “effort-inducing” constraints. In addition, a contract has to satisfy $\underline{c} \leq c(z) \leq \bar{c}$ for all z and $w(z, s') \in W_{s'}$ for all (z, s') . A contract is said to be *incentive compatible* if it satisfies the incentive compatibility constraints (22.3.5).⁶

DEFINITION: A recursive contract $(a, c(z), w(z, s'))$ is said to be *feasible with respect to W* for a given (v, s) pair if it is incentive compatible in state s , delivers promised value v , and $w(z, s') \in W_{s'}$ for all (z, s') .

Let $C(v, s)$ be the minimum cost to the planner of delivering promised value v to a worker in employment state s . We can represent the Bellman equation for $C(v, s)$ in terms of the following two-part optimization:

$$\Psi(v, s, a) = \min_{c(z), w(z, s')} \left\{ \sum_z \pi_z(z; s, a) \left(-z + c(z) + \beta \sum_{s'} \pi_s(s'; z, s, a) C(w(z, s'), s') \right) \right\} \quad (22.3.6a)$$

subject to constraints (22.3.4) and (22.3.5), and

$$C(v, s) = \min_{a \in [a_L, a_H]} \Psi(v, s, a). \quad (22.3.6b)$$

The function $\Psi(v, s, a)$ assumes that the worker exerts effort level a . Later, we shall typically assume that parameters are such that $C(v, s) = \Psi(v, s, a_H)$, so that the planner finds it optimal always to induce high effort. Put a Lagrange multiplier $\lambda(v, s, a)$ on the promise-keeping constraint (22.3.4) and another multiplier $\nu(v, s, a)$ on the effort-inducing constraint (22.3.5) given a , and form the

⁶ We assume two-sided commitment to the contract and therefore ignore the participation constraints that Zhao imposes on the contract. She requires that continuation values $w(z, s')$ be at least as great as the autarky values $V_{s', \text{aut}}$ for each (z, s') .

Lagrangian:

$$L = \sum_z \pi_z(z; s, a) \left\{ -z + c(z) + \beta \sum_{s'} \pi_s(s'; z, s, a) C(w(z, s'), s') \right. \\ \left. - \lambda(v, s, a) \left[u(c(z)) + \beta \sum_{s'} \pi_s(s'; z, s, a) w(z, s') \right] - \phi(a) - v \right] \\ \left. - \nu(v, s, a) \left[u(c(z)) + \beta \sum_{s'} \pi_s(s'; z, s, a) w(z, s') - \phi(a) \right. \right. \\ \left. \left. - \frac{\pi_z(z; s, \tilde{a})}{\pi_z(z; s, a)} \left(u(c(z)) + \beta \sum_{s'} \pi_s(s'; z, s, \tilde{a}) w(z, s') - \phi(\tilde{a}) \right) \right] \right\},$$

where $\tilde{a} \in \{a_L, a_H\}$ and $\tilde{a} \neq a$. First-order conditions for $c(z)$ and $w(z, s')$, respectively, are

$$\frac{1}{u'(c(z))} = \lambda(v, s, a) + \nu(v, s, a) \left(1 - \frac{\pi_z(z; s, \tilde{a})}{\pi_z(z; s, a)} \right) \quad (22.3.7a)$$

$$C_v(w(z, s'), s') = \lambda(v, s, a) \\ + \nu(v, s, a) \left[1 - \frac{\pi_z(z; s, \tilde{a})}{\pi_z(z; s, a)} \frac{\pi_s(s'; s, z, \tilde{a})}{\pi_s(s'; z, s, a)} \right]. \quad (22.3.7b)$$

The envelope conditions are

$$\Psi_v(v, s, a) = \lambda(v, s, a) \quad (22.3.8a)$$

$$C_v(v, s) = \Psi_v(v, s, a^*) \quad (22.3.8b)$$

where a^* is the planner's optimal choice of a .

To deduce the dynamics of compensation, Zhao's strategy is to study the first-order conditions (22.3.7) and envelope conditions (22.3.8) under two cases, $s = u$ and $s = e$.

22.3.3. Compensation dynamics when unemployed

In the unemployed state ($s = u$), the first-order conditions become

$$\frac{1}{u'(c)} = \lambda(v, u, a) \quad (22.3.9a)$$

$$C_v(w(0, u), u) = \lambda(v, u, a) + \nu(v, u, a) \left[1 - \frac{1 - \pi_{ue}(\tilde{a})}{1 - \pi_{ue}(a)} \right] \quad (22.3.9b)$$

$$C_v(w(0, e), e) = \lambda(v, u, a) + \nu(v, u, a) \left[1 - \frac{\pi_{ue}(\tilde{a})}{\pi_{ue}(a)} \right]. \quad (22.3.9c)$$

The effort-inducing constraint (22.3.5) can be rearranged to become

$$\beta(\pi_{ue}(a) - \pi_{ue}(\tilde{a}))(w(0, e) - w(0, u)) \geq \phi(a) - \phi(\tilde{a}).$$

Like Hopenhayn and Nicolini, Zhao describes how compensation and effort depend on the duration of unemployment:

PROPOSITION: To induce high search effort, unemployment benefits must *fall* over an unemployment spell.

PROOF: When search effort is high, the effort-inducing constraint binds. By assumption 3,

$$\frac{1 - \pi_{ue}(a_L)}{1 - \pi_{ue}(a_H)} > 1 > \frac{\pi_{ue}(a_L)}{\pi_{ue}(a_H)}.$$

These inequalities and the first-order condition (22.3.9) then imply

$$C_v(w(0, e), e) > \Psi_v(v, u, a_H) > C_v(w(0, u), u). \quad (22.3.10)$$

Let $c_u(t)$, $v_u(t)$, respectively, be consumption and the continuation value for an unemployed worker. Equations (22.3.9) and the envelope conditions imply

$$\frac{1}{u'(c_u(t))} = \Psi_v(v_u(t), u, a_H) > C_v(v_u(t+1), u) = \frac{1}{u'(c_u(t+1))}. \quad (22.3.11)$$

Concavity of u then implies that $c_u(t) > c_u(t+1)$. In addition, notice that

$$C_v(w(0, u), u) - C_v(v, u) = \nu(v, u, a_H) \left(1 - \frac{1 - \pi_{ue}(a_L)}{1 - \pi_{ue}(a_H)} \right), \quad (22.3.12)$$

which follows from the first-order conditions (22.3.9) and the envelope conditions. Equation (22.3.12) implies that continuation values fall with the duration of unemployment. ■

22.3.4. Compensation dynamics while employed

When the worker is employed, for each promised value v , the contract specifies output-contingent consumption and continuation values a $c(y), w(y, s')$. When $s = e$, the first-order conditions (22.3.7) become

$$\frac{1}{u'(c(y))} = \lambda(v, e, a) + \nu(v, e, a) \left(1 - \frac{p(y; \tilde{a})}{p(y; a)} \right) \tag{22.3.13a}$$

$$C_v(w(y, u), u) = \lambda(v, e, a) + \nu(v, e, a) \left(1 - \frac{p(y; \tilde{a})}{p(y; a)} \frac{\pi_{eu}(y, \tilde{a})}{\pi_{eu}(y, a)} \right) \tag{22.3.13b}$$

$$C_v(w(y, e), e) = \lambda(v, e, a) + \nu(v, e, a) \left(1 - \frac{p(y; \tilde{a})}{p(y; a)} \frac{1 - \pi_{eu}(y, \tilde{a})}{1 - \pi_{eu}(y, a)} \right). \tag{22.3.13c}$$

Zhao uses these first-order conditions to characterize how compensation depends on output:

PROPOSITION: To induce high work effort, wages and continuation values increase with current output.

PROOF: For any $y > \tilde{y}$, let $d = \frac{p(\tilde{y}; a_L)}{p(\tilde{y}; a_H)} - \frac{p(y; a_L)}{p(y; a_H)}$. Assumption 1 about $p(y; a)$ implies that $d > 0$. The first-order conditions (22.3.13) imply that

$$\frac{1}{u'(c(y))} - \frac{1}{u'(c(\tilde{y}))} = \nu(v, e, a)d > 0, \tag{22.3.14a}$$

$$C_v(w(y, u), u) - C_v(w(\tilde{y}, u), u) \propto \nu(v, e, a)d > 0, \tag{22.3.14b}$$

$$C_v(w(y, e), e) - C_v(w(\tilde{y}, e), e) \propto \nu(v, e, a)d > 0. \tag{22.3.14c}$$

Concavity of u and convexity of C give the result. ■

In the following proposition, Zhao shows how continuation values at the start of unemployment spells should depend on the history of the worker's outcomes during previous employment and unemployment spells.

PROPOSITION: If the job separation rate depends on current output, then the replacement rate immediately after a worker loses a job is 100%. If the job separation rate depends on work effort, then the replacement rate is less than 100%.

PROOF: If the job separation rate depends on *output*, the first-order conditions (22.3.13) imply

$$\frac{1}{u'(c(y))} = C_v(w(y, u), u) = C_v(w(y, e), e). \tag{22.3.15}$$

This is because $\pi_{eu}(y, \tilde{a}) = \pi_{eu}(y, a)$ when the job separation rate depends on output. Let $c_e(t), c_u(t)$ be consumption of employed and unemployed workers, and let $v_e(t), v_u(t)$ be the assigned promised values at t . Then

$$\frac{1}{u'(c_e(t))} = C_v(v_{u,t+1}, u) = \frac{1}{c_u(t+1)}$$

where the first equality follows from (22.3.15) and the second from the envelope condition. If the job separation rate depends on *work effort*, then the first-order conditions (22.3.13) imply

$$\frac{1}{u'(c(y))} - C_v(w(y, u), u) = \nu(v, e, a) \frac{p(y; a_L)}{p(y; a_H)} \left(\frac{\pi_{eu}(a_L)}{\pi_{eu}(a_H)} - 1 \right). \quad (22.3.16)$$

Assumption 2 implies that the right side of (22.3.16) is positive, which implies that

$$\frac{1}{u'(c_e(t))} > C_v(v_u(t+1), u) = \frac{1}{u'(c_u(t+1))}.$$

■

22.3.5. Summary

A worker in Zhao's model enters a lifetime contract that makes compensation respond to the history of outputs on the current and past jobs, as well as on the durations of all previous spells of unemployment.⁷ Her model has the outcome that compensation at the beginning of an unemployment spell varies directly with the compensation attained on the previous job. This aspect of her model offers a possible explanation for why unemployment insurance systems often feature a "replacement rate" that gives more unemployment insurance payments to workers who had higher wages in their prior jobs.

⁷ We have analyzed a version of Zhao's model in which the worker is committed to obey the contract. Zhao incorporates an enforcement problem in her model by allowing the worker to accept an outside option each period.

22.4. Concluding remarks

The models that we have studied in this chapter isolate the worker from capital markets so that the worker cannot transfer consumption across time or states except by adhering to the contract offered by the planner. If the worker in the models of this chapter were allowed to save or issue a risk-free asset bearing a gross one-period rate of return approaching β^{-1} , it would interfere substantially with the planner's ability to provide incentives by manipulating the worker's continuation value in response to observed current outcomes. In particular, forces identical to those analyzed in the Cole and Kocherlakota setup that we analyzed at length in chapter 20 would circumscribe the planner's ability to supply insurance. In the context of unemployment insurance models like that of this chapter, this point has been studied in detail in papers by Ivan Werning (2002) and Kocherlakota (2004).

Exercises

Exercise 22.1 Optimal unemployment compensation

- a. Write a program to compute the autarky solution, and use it to reproduce Hopenhayn and Nicolini's calibration of r , as described in text.
- b. Use your calibration from part a. Write a program to compute the optimum value function $C(V)$ for the insurance design problem with incomplete information. Use the program to form versions of Hopenhayn and Nicolini's table 1, column 4 for three different initial values of V , chosen by you to belong to the set (V_{aut}, V^e) .

Exercise 22.2 Taxation after employment

Show how the functional equation (22.2.5), (22.2.6) would be modified if the planner were permitted to tax workers after they became employed.

Exercise 22.3 Optimal unemployment compensation with unobservable wage offers

Consider an unemployed person with preferences given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0, 1)$ is a subjective discount factor, $c_t \geq 0$ is consumption at time t , and the utility function $u(c)$ is strictly increasing, twice differentiable, and strictly concave. Each period the worker draws one offer w from a uniform wage distribution on the domain $[w_L, w_H]$ with $0 \leq w_L < w_H < \infty$. Let the cumulative density function be denoted $F(x) = \text{Prob}\{w \leq x\}$, and denote its density by f , which is constant on the domain $[w_L, w_H]$. After the worker has accepted a wage offer w , he receives the wage w per period forever. He is then beyond the grasp of the unemployment insurance agency. During the unemployment spell, any consumption smoothing has to be done through the unemployment insurance agency because the worker holds no assets and cannot borrow or lend.

- a. Characterize the worker's optimal reservation wage when he is entitled to a time-invariant unemployment compensation b of indefinite duration.
- b. Characterize the optimal unemployment compensation scheme under full information. That is, we assume that the insurance agency can observe and control the unemployed worker's consumption and reservation wage.
- c. Characterize the optimal unemployment compensation scheme under asymmetric information where the insurance agency cannot observe wage offers, though it can observe and control the unemployed worker's consumption. Discuss the optimal time profile of the unemployed worker's consumption level.

Exercise 22.4 **Full unemployment insurance**

An unemployed worker orders stochastic processes of consumption, search effort $\{c_t, a_t\}_{t=0}^{\infty}$ according to

$$E \sum_{t=0}^{\infty} \beta^t [u(c_t) - a_t]$$

where $\beta \in (0, 1)$ and $u(c)$ is strictly increasing, twice differentiable, and strictly concave. It is required that $c_t \geq 0$ and $a_t \geq 0$. All jobs are alike and pay wage $w > 0$ units of the consumption good each period forever. After a worker has found a job, the unemployment insurance agency can tax the employed worker at a rate τ consumption goods per period. The unemployment agency can make τ depend on the worker's unemployment history. The probability of finding a job is $p(a)$, where p is an increasing and strictly concave and twice differentiable function of a , satisfying $p(a) \in [0, 1]$ for $a \geq 0$, $p(0) = 0$. The consumption good is nonstorable. The unemployed person cannot borrow or lend and holds

no assets. If the unemployed worker is to do any consumption smoothing, it has to be through the unemployment insurance agency. The insurance agency can observe the worker's search effort and can control his consumption. An employed worker's consumption is $w - \tau$ per period.

a. Let V_{aut} be the value of an unemployed worker's expected discounted utility when he has no access to unemployment insurance. An unemployment insurance agency wants to insure unemployed workers and to deliver expected discounted utility $V > V_{\text{aut}}$ at minimum expected discounted cost $C(V)$. The insurance agency also uses the discount factor β . The insurance agency controls c, a, τ , where c is consumption of an unemployed worker. The worker pays the tax τ only after he becomes employed. Formulate the Bellman equation for $C(V)$.

Exercise 22.5 **Two effort levels**

An unemployment insurance agency wants to insure unemployed workers in the most efficient way. An unemployed worker receives no income and chooses a sequence of search intensities $a_t \in \{0, a\}$ to maximize the utility functional

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) - a_t\}, \quad \beta \in (0, 1)$$

where $u(c)$ is an increasing, strictly concave, and twice continuously differentiable function of consumption of a single good. There are two values of the search intensity, 0 and a . The probability of finding a job at the beginning of period $t + 1$ is

$$(2) \quad \pi(a_t) = \begin{cases} \pi(a), & \text{if } a_t = a; \\ \pi(0) < \pi(a), & \text{if } a_t = 0, \end{cases}$$

where we assume that $a > 0$. Note that the worker exerts search effort in period t and possibly receives a job at the beginning of period $t + 1$. Once the worker finds a job, he receives a fixed wage w forever, sets $a = 0$, and has continuation utility $V_e = \frac{u(w)}{1-\beta}$. The consumption good is not storable and workers can neither borrow nor lend. The unemployment agency can borrow and lend at a constant one-period risk-free gross interest rate of $R = \beta^{-1}$. The unemployment agency cannot observe the worker's effort level.

Subproblem A

- a.** Let V be the value of (1) that the unemployment agency has promised an unemployed worker at the start of a period (before he has made his search decision). Let $C(V)$ be the minimum cost to the unemployment insurance agency of delivering promised value V . Assume that the unemployment insurance agency wants the unemployed worker to set $a_t = a$ for as long as he is unemployed (i.e., it wants to promote high search effort). Formulate a Bellman equation for $C(V)$, being careful to specify any promise-keeping and incentive constraints. (Assume that there are no participation constraints: the unemployed worker must participate in the program.)
- b.** Show that if the incentive constraint binds, then the unemployment agency offers the worker benefits that decline as the duration of unemployment grows.
- c.** Now alter assumption (2) so that $\pi(a) = \pi(0)$. Do benefits still decline with increases in the duration of unemployment? Explain.

Subproblem B

- d.** Now assume that the unemployment insurance agency can tax the worker after he has found a job, so that his continuation utility upon entering a state of employment is $\frac{u(w-\tau)}{1-\beta}$, where τ is a tax that is permitted to depend on the duration of the unemployment spell. Defining V as above, formulate the Bellman equation for $C(V)$.
- e.** Show how the tax τ responds to the duration of unemployment.

Exercise 22.6 **Partially observed search effort**

Consider the following modification of a model of Hopenhayn and Nicolini. An insurance agency wants to insure an infinitely lived unemployed worker against the risk that he will not find a job. With probability $p(a)$, an unemployed worker who searches with effort a this period will find a job that earns wage w in consumption units per period. That job will start next period, last forever, and the worker will never quit it. With probability $1 - p(a)$ he will find himself unemployed again at the beginning of next period. We assume that $p(a)$ is an increasing and strictly concave and twice differentiable function of a with $p(a) \in [0, 1]$ for $a \geq 0$ and $p(0) = 0$. The insurance agency is the worker's only source of consumption (there is no storage or saving available to the worker). The worker values consumption according to a twice continuously differentiable and

strictly concave utility function $u(c)$ where $u(0)$ is finite. While unemployed, the worker's utility is $u(c) - a$; when he is employed it is $u(w)$ (no effort a need be applied when he is working).

With exogenous probability $d \in (0, 1)$ the insurance agency observes the search effort of a worker who searched last period but did not find a job. With probability $1 - d$, the insurance agency does not observe the last-period search intensity of an unemployed worker who was not successful in finding a job period.

Let V be the expected discounted utility of an unemployed worker who is searching for work this period. Let $C(V)$ be the minimum cost to the unemployment insurance agency of delivering V to the unemployed worker.

- a.** Formulate a Bellman equation for $C(V)$.
- b.** Get as far as you can in analyzing how the unemployment compensation contract offered to the worker depends on the duration of unemployment and the history of observed search efforts that are detected by the UI agency.

Hint: you might want to allow the continuation value when unemployed to depend on last period's search effort when it is observed.