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Raffaella Giacomini

The Institute for Fiscal Studies
Department of Economics, UCL

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Institute for Fiscal Studies, 7 Ridgmount Street, London WC1E 7AE **Tel:** 44 (0)20 7291 4800 **Email:** info@cemmap.ac.uk **www.cemmap.ac.uk**

The relationship between DSGE and VAR models*

Raffaella Giacomini

University College London, CeMMAP

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Abstract

This chapter reviews the literature on the econometric relationship between DSGE and VAR models from the point of view of estimation and model validation. The mapping between DSGE and VAR models is broken down into three stages: 1) from DSGE to state-space model; 2) from state-space model to VAR(∞); 3) from VAR(∞) to finite order VAR. The focus is on discussing what can go wrong at each step of this mapping and on critically highlighting the hidden assumptions. I also point out some open research questions and interesting new research directions in the literature on the econometrics of DSGE models. These include, in no particular order: understanding the effects of log-linearization on estimation and identification; dealing with multiplicity of equilibria; estimating nonlinear DSGE models; incorporating into DSGE models information from atheoretical models and from survey data; adopting flexible modelling approaches that combine the theoretical rigor of DSGE models and the econometric model's ability to fit the data.

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1 Introduction

This chapter reviews the literature on the econometric relationship between DSGE models and VAR models. The main focus is on formal statistical methods for estimation and validation of DSGE and VAR models, in particular via the use of impulse-response analysis. This focus brings up different issues from those involved in using DSGE and VAR models for forecasting, which are discussed in Giacomini (2013) and in the chapter by Gurkaynak and Rossi.

Understanding if a DSGE model can be represented as a reduced-form VAR is important for both estimation and model validation. The topic is relevant for empirical practice as it answers the question of whether the structural shocks ε_t can be recovered from a VAR analysis, that is, whether the following equivalence holds

$$\varepsilon_t = \Omega \left(Y_t - E \left(Y_t | Y^{t-1} \right) \right), \qquad (1)$$

where Y_t is a vector of observable macroeconomic variables, Y^{t-1} is the history of Y_t up to time $t-1$, $Y_t - E\left(Y_t|Y^{t-1}\right)$ the forecast error from a VAR and Ω a rotation matrix. Uncovering such a relationship by imposing identifying restrictions on Ω is the objective of Structural VAR (SVAR) analysis, but it also has important implications for both estimation of DSGE models (when it is carried out by matching impulse-response functions from the VAR) and for their validation, which often involves assessing whether the impulse responses from the DSGE model can replicate those from a SVAR. If the model shocks cannot be recovered from the SVAR shocks, model estimation and validation become meaningless. This issue has been hotly debated in the literature, with Chari et al. (2005) in one camp arguing that SVAR models are not suitable for model validation and estimation and Christiano et al. (2006) in the opposite camp defending SVAR models as a useful tool but cautioning against their incorrect use.

The main goal of this chapter is to present a selective review of the literature in order to clarify how and when a mapping between DSGE and VAR models can be obtained. I will show that the mapping consists of three stages. In the first stage, the equilibrium conditions from a DSGE model are mapped into a linear state-space model; in the second, the state-space model is represented as a VAR with an infinite number of lags; in the last stage, the VAR(∞) is either shown to be a VAR with a finite number of lags if the model satisfies some testable conditions, or the VAR(∞) is approximated by a finite order VAR.

I will show in detail how the mapping is obtained in the context of the prototypical DSGE model of An and Schorfheide (2007), whose log-linearized version turns out to have an exact VAR(1) representation.

My focus throughout the paper will be on highlighting what can go wrong at each stage and on drawing attention to the many assumptions that underlie the analysis. This will bring up some interesting open questions that are discussed at the end of the chapter, and inevitably will end up pointing to my ongoing work on the econometrics of DSGE models.

A notable omission in this chapter is a discussion of the literature on identification of structural parameters in DSGE models, which is fast growing and will soon require a separate survey article. A partial, must-read list includes: Canova and Sala (2009), Komunjer and Ng (2011), Iskrev (2010), Guerron-Quintana, Inoue and Kilian (forthcoming).

2 Stage 1. From DSGE to state-space model

As a first step towards understanding the relationship between DSGE and VAR models, I will begin by discussing the mapping between the equilibrium conditions of a DSGE model and a linear state-space model written in the ABCD form, which is the starting point for most of the analyses in the literature

$$\begin{aligned} X_t &= AX_{t-1} + B\varepsilon_t \\ Y_t &= CX_{t-1} + D\varepsilon_t. \end{aligned} \tag{2}$$

Here X_t are the state variables of the model, Y_t the observable variables and ε_t the structural shocks.

The discussion will assume that the parameters are either known (for example if the purpose of the exercise is to simulate a calibrated DSGE model to see if it matches moments implied by a VAR) or are fixed at a particular value, for cases in which stage 1 is part of an optimization routine whose ultimate goal is to estimate the model's parameters.

The chapter uses as an example the prototypical DSGE model of An and Schorfheide (2007, henceforth AS), which is a representative of the class of models currently used in the analysis of monetary policy at most central banks and policy institutions. The model is a simplified version of Smets and Wouters (2003) and Christiano et al. (2005), and is characterized by six equilibrium conditions:

$$1 = \beta E_t\big[e^{-\tau\widehat{c}_{t+1}+\tau\widehat{c}_t+\widehat{R}_t-\widehat{z}_{t+1}-\widehat{\pi}_{t+1}}\big] \tag{3}$$

$$\begin{aligned}\frac{1-\nu}{\nu\phi\pi^2}(e^{\tau\widehat{c}_t}-1) &= (e^{\widehat{\pi}_t}-1)\big\{\big[1-\frac{1}{2\nu}\big]e^{\widehat{\pi}_t}+\frac{1}{2\nu}\big\} \\ &\quad -\beta E_t\big[(e^{\widehat{\pi}_{t+1}}-1)e^{-\tau\widehat{c}_{t+1}+\tau\widehat{c}_t+\widehat{y}_{t+1}-\hat{y}_t+\widehat{\pi}_{t+1}}\big] \end{aligned} \tag{4}$$

$$e^{\widehat{c}_t-\widehat{y}_t} = e^{-\widehat{g}_t}-\frac{\phi\pi^2 g}{2}(e^{\widehat{\pi}_t}-1)^2 \tag{5}$$

$$\widehat{r}_t = \rho_R\widehat{r}_{t-1}+(1-\rho_r)\psi_1\widehat{\pi}_t+(1-\rho_r)\psi_2(\widehat{y}_t-\widehat{g}_t)+\sigma_r\varepsilon_{r,t} \tag{6}$$

$$\widehat{z}_t = \rho_z\widehat{z}_{t-1}+\sigma_z\varepsilon_{z,t} \tag{7}$$

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1}+\sigma_g\varepsilon_{g,t}. \tag{8}$$

The model has six state variables $X_t = (\widehat{c}_t, \widehat{z}_t, \widehat{g}_t, \widehat{\pi}_t, \widehat{r}_t, \widehat{y}_t)$ where

- $\widehat{c}_t = \ln(c_t/c)$, $c_t = C_t/A_t$ is detrended consumption relative to aggregate productivity A_t, assumed to evolve as $\ln A_t = \ln\gamma + \ln A_{t-1} + \ln z_t$, and $c = (1-\upsilon)^{1/\tau}$ is the steady-state of detrended consumption

- $\widehat{z}_t = \ln z_t$ is the innovation to the process governing aggregate productivity

- $\widehat{g}_t = \ln(g_t/g)$, where g_t is government spending, assumed to evolve as $\ln g_t = (1-\rho_g)\ln g + \rho_t \ln g_{t-1} + \sigma_g\varepsilon_{g,t}$

- $\widehat{\pi}_t = \ln(\pi_t/\pi)$, where π_t is inflation and π the steady-state inflation rate

- $\widehat{r}_t = \ln(r_t/r)$, where r_t is gross interest rate and $r = \frac{\gamma}{\beta}\pi^*$, with $\frac{\gamma}{\beta}$ the steady-state real interest rate and π^* the target inflation rate, which in equilibrium equals π

- $\widehat{y}_t = \ln(y_t/y)$, where $y_t = Y_t/A_t$ is detrended output and $y = g(1-\upsilon)^{1/\tau}$ is the steady-state of detrended output.

The model is driven by three independent exogenous shocks: $\varepsilon_t = (\varepsilon_{r,t}, \varepsilon_{z,t}, \varepsilon_{g,t})$, assumed to be i.i.d. N(0,1).

Equations (3) to (8) represent a system of six non-linear expectational equations for six state variables X_t and three shocks ε_t whose solution is an equation of the form

$$\text{State transition equation: } X_t = f(X_{t-1}, \varepsilon_t, \theta), \tag{9}$$

where $\theta = (\beta, \nu, \phi, \tau, \pi, \psi_1, \psi_2, \rho_r, \rho_z, \rho_g, \sigma_r, \sigma_z, \sigma_g)$ are the structural parameters of the model.

Once the state transition equation (9) is pinned down either by finding a closed-form expression for $f(\cdot)$ or by numerical approximation, the parameters of the model can be calibrated and one can simulate paths of the state variables. This would be sufficient is one's sole objective were to understand the model's internal propagation mechanism or to derive qualitative implications about the dynamics and the moments of some of the variables in the system.

Increasingly in recent years, the focus in the literature has shifted towards the adoption of formal econometric methods in the analysis of DSGE models. An early example is the literature that tries to formalize the calibration approach, such as Watson (1993), Canova (1994), Diebold, Ohanian and Berkowitz (1998), De Jong, Ingram and Whiteman (2000) and Dridi, Guay and Renault (2007). More recently, the development of models that are rich enough (and embed enough frictions) to generate realistic time series behavior has generated a demand for formal econometric methods for the estimation and evaluation of DSGE models.

The literature has considered several approaches to estimation, ranging from limited information methods such as GMM and minimum-distance estimation based on matching impulse-response functions (see the survey by Ruge-Murcia, 2007) to full-information likelihood-based methods. Perhaps because Smets and Wouters (2003) made the case that a DSGE model estimated by Bayesian methods could fit the data as well as a VAR model, the literature on the Bayesian estimation of DSGE models has grown tremendously in recent years and these models have been widely adopted by central banks around the world (see, respectively, the reviews by An and Schorfheide, 2007 and Tovar, 2009). In this article, I will accordingly focus on likelihood-based estimation of DSGE models.

In order to derive a likelihood for the DSGE model (3) - (8), one needs to first choose a vector of observable variables Y_t and then specify a measurement equation which links the observables to the state variables, which typically takes the following form:

$$\text{Measurement equation: } Y_t = HX_t. \tag{10}$$

Equations (9) and (10) form a state-space model which, under suitable conditions, gives a likelihood that can be computed using filtering methods. There are filtering methods for non-linear state-space models such as the particle filter (e.g., Pitt and Shephard, 1999) and they have been adopted by some authors (e.g., Fernandez-Villaverde and Rubio-Ramirez, 2007, Aruoba, Fernandez-Villaverde and Rubio-Ramirez, 2006) to perform Bayesian estimation of nonlinear DSGE models. The vast majority of the literature, however, resorts to log-linearization of the model around the steady state, in order to obtain a linear version of the state transition equation:

$$\text{Linearized state transition equation: } X_t = AX_{t-1} + B\varepsilon_t, \tag{11}$$

where the coefficient matrices A and B implicitly depend on the structural parameters θ. Assuming normality of ε_t, the linear state space model (10) and (11) can be estimated using the Kalman filter.

In the case of the AS model, log-linearization of (1) - (6) around the steady state gives the following system of linear expectational equations

$$\widehat{y}_t = E_t\widehat{y}_{t+1} + \widehat{g}_t - E_t\widehat{g}_{t+1} - \frac{1}{\tau}(\widehat{r}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{z}_{t+1}) \tag{12}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{\tau(1-\upsilon)}{\upsilon\pi^2\phi}(\widehat{y}_t - \widehat{g}_t) \tag{13}$$

$$\widehat{c}_t = \widehat{y}_t - \widehat{g}_t \tag{14}$$

$$\widehat{r}_t = \rho_r\widehat{r}_{t-1} + (1-\rho_r)\psi_1\widehat{\pi}_t + (1-\rho_r)\psi_2(\widehat{y}_t - \widehat{g}_t) + \sigma_r\varepsilon_{r,t} \tag{15}$$

$$\widehat{z}_t = \rho_z\widehat{z}_{t-1} + \sigma_z\varepsilon_{z,t} \tag{16}$$

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1} + \sigma_g\varepsilon_{g,t}. \tag{17}$$

which can be solved using, for example, the algorithms of Blanchard and Kahn (1980), Uhlig (1999) or Sims (2002) to obtain an equation of the form (11).

The next step involves choosing the set of observables, which in the AS model are

$$Y_t = (\widehat{r}_t, \widehat{y}_t, \widehat{\pi}_t). \tag{18}$$

Putting together (11) and (10) one can obtain the ABCD representation (2) of the log-linearized DSGE model, which is a useful starting point for discussing the relationship between DSGE and VAR models.

In the case of the AS model, for example, as shown by Komunjer and Ng (2011) and Morris (2012), the minimal state representation of the model (i.e., the representation of the model with the smallest number of state variables) for parameters calibrated as

$$\begin{aligned} \theta &= (\beta, \nu, \phi, \tau, \pi, \psi_1, \psi_2, \rho_r, \rho_z, \rho_g, \sigma_r, \sigma_z, \sigma_g) = \\ &\quad (0.995, 0.1, 53.68, 2, 1.01, 1.5, 0.5, 0.75, 0.9, 0.95, 0.002, 0.003, 0.006) \end{aligned} \tag{19}$$

is given by:

$$\begin{aligned} \underbrace{\begin{bmatrix} \widehat{z}_t \\ \widehat{g}_t \\ \widehat{r}_t \end{bmatrix}}_{X_t} &= \underbrace{\begin{bmatrix} 0.90 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0.55 & 0 & 0.51 \end{bmatrix}}_{A} \begin{bmatrix} \widehat{z}_{t-1} \\ \widehat{g}_{t-1} \\ \widehat{r}_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.61 & 0 & 0.69 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \varepsilon_{zt} \\ \varepsilon_{gt} \\ \varepsilon_{rt} \end{bmatrix}}_{\varepsilon_t} \\ \underbrace{\begin{bmatrix} \widehat{r}_t \\ \widehat{y}_t \\ \widehat{\pi}_t \end{bmatrix}}_{Y_t} &= \underbrace{\begin{bmatrix} 0.90 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0.55 & 0 & 0.51 \end{bmatrix}}_{C} \begin{bmatrix} \widehat{z}_{t-1} \\ \widehat{g}_{t-1} \\ \widehat{r}_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.61 & 0 & 0.69 \end{bmatrix}}_{D} \begin{bmatrix} \varepsilon_{zt} \\ \varepsilon_{gt} \\ \varepsilon_{rt} \end{bmatrix}. \end{aligned} \tag{20}$$

with error covariance matrix

$$\Sigma = \begin{bmatrix} 0.003^2 & 0 & 0 \\ 0 & 0.006^2 & 0 \\ 0 & 0 & 0.002^2 \end{bmatrix}. \tag{21}$$

2.1 Summary of stage 1 and discussion

To summarize, stage 1 consists of the following steps: a) start with the equilibrium conditions of a nonlinear DSGE model; b) linearize the equations around the non-stochastic steady-state; c) use a solution algorithm to find the log-linearized state transition equation (11); d) choose a vector Y_t of observable variables and write down the measurement equation (10); e) put c) and d) together and find matrices A,B,C,D in the representation (2).

There are several issues that can arise at each of the above steps, as well as some implicit assumptions that one needs to make when considering the ABCD representation as the starting point of the analysis.

The first issue is that a meaningful discussion of the relationship between DSGE and VAR models can only be carried out in the context of log-linearized DSGE models, taking for granted the adequacy of the linear approximation and ignoring possible nonlinear dynamics that cannot be replicated by linear VAR models.

Perhaps due to the high computational costs of using nonlinear filters in the likelihood evaluation, estimation of nonlinear DSGE models is still a small proportion of the literature, but it is nonetheless growing fast (e.g., Fernandez-Villaverde and Rubio-Ramirez, 2007, Amisano and Tristani, 2011, Chen , Petralia and Lopes, 2010). This makes it possible that in a few years nonlinear models will be the standard and thus much of the discussion contained in this chapter will become obsolete. At the time of writing, however, it is fair to say that the profession at large is not yet fully convinced of the need for nonlinear solution methods for DSGE models and routinely continues to rely on log-linearized models that are estimated by the Kalman filter. Nevertheless, the development of nonlinear methods continues at a rapid pace. For example, in ongoing research with Ron Gallant and Giuseppe Ragusa (Gallant et al, 2013), we develop new estimation methods for nonlinear DSGE models that do not require approximating the model in order to obtain a likelihood. Our limited-information state-space methods will allow one to estimate nonlinear DSGE models directly from the equilibrium conditions.

In the context of linear vs. nonlinear DSGE models, a further important issue that has been scarcely investigated is the effect of log-linearization on the identification and estimation of the structural parameters of the model. Much of the discussion in the literature that I report in the rest of the chapter will be about whether and when the state-space model (2) can be written as or approximated by a finite-order VAR, and about trying to understand the economic significance of the possible approximation error. Perhaps the same effort should be dedicated to understanding the importance of the log-linearization error - as it is possible that the latter might dominate the former in applications. See Fernandez-Villaverde et al. (2006) and den Haan and de Wind (2009) for some notable contributions on this topic.

The second issue that arises when finding a solution to the log-linearized DSGE model (12)-(17) is the possible existence of multiple solutions, what is commonly referred to in the literature as indeterminacy (see McCallum, 2003 for a differing view on why it might be helpful to keep the two concepts distinct). The issue of multiplicity of solutions in dynamic rational expectation models is well known in the literature, and several different criteria have been suggested for selecting one model among alternative model solutions, for example, the expectational stability criterion of Evans (1986), the minimum state variable criterion of McCallum (1983) and the stability criterion advocated e.g., by Sargent (1987) and Blanchard and Kahn (1980). As discussed by McCallum (1999), even though it is often unclear which of the selection criteria is adopted in a particular study, the most popular approach in the literature is to utilize the stability criterion and rule out unstable solutions by restricting the parameter space. For notable exceptions, see for example Benhabib and Farmer (2000) and Lubik and Schorfheide (2004), who show that indeterminacy might be empirically relevant.

The issue of the multiplicity of solutions is relevant for our discussion because it means that in many cases which variables enter X_t and what the matrices A and B are is not uniquely determined by the model, and the choices made by the researcher about whether and how to deal with multiple solutions will result in different state-space representations. This in turn may have an impact on the analysis in the following sections since a great part of the discussion will be about the relative sizes of X_t and Y_t and the values of the matrices A, B, C and D. To give a concrete example, the local identification analysis of Komunjer and Ng (2011) requires one to start from the minimal state-variable representation of the state-space system (2), and in their application to the AS model the authors rule out indeterminacy. See Qu and Tkachenko (2012) for an analysis of the interplay between indeterminacy and identification in DSGE models.

The third, related, issue is the choice of observable variables Y_t, which is even more arbitrary than the choice of how to deal with multiple solutions discussed above. Again, the interaction between the agent's information set (i.e., what is contained in X_t) and the econometrician's information set (what is in Y_t) will be a key factor in discussing the relationship between the state-space model (2) and a VAR, so it is important to pay attention to issues of data selection. Fortunately, this is an area where the econometrician has both more control (as she can decide which data to use) and a better understanding of the consequences of her choices (as there is some literature on the topic, which I discuss in the next section).

3 Stage 2. From state-space model to VAR(∞)

This section discusses the mapping between the state-space model written in the ABCD form (2) and a VAR(∞), and is the starting point of most of the literature on the topic.

Let us suppose that the model has been mapped into the state-space representation (2) where X_t is $n_x \times 1$, Y_t is $n_y \times 1$ and ε_t is $n_\varepsilon \times 1$, so that A is $n_x \times n_x$, B is $n_x \times n_\varepsilon$, C is $n_y \times n_x$ and D is $n_y \times n_\varepsilon$. The general representation of the log-linearized DSGE model is thus a VARMA (see, e.g., Aoki, 1990) so the question of whether the DSGE can be written as a VAR is equivalent to asking whether the VARMA model can be inverted and written as a VAR(∞). For the case in which there are as many observables as shocks (and thus D is non-singular), the invertibility condition can be formulated in terms of the matrices A, B, C and D (Hannan and Deistler, 1988, Fernandez-Villaverde et al., 2007) and is easy to check. We have the following result:

Proposition 1. If the eigenvalues of $A - BD^{-1}C$ are strictly less than one in absolute value the state-space model in (2) Y_t can be written as a VAR(∞):

$$Y_t = C \sum_{j=1}^{\infty} \left(A - BD^{-1}C\right)^j BD^{-1} Y_{t-j} + D\varepsilon_t. \qquad (22)$$

Fernandez-Villaverde et al. (2007) show that the condition in Proposition 1 does not hold in important classes of models, such as the permanent income consumption model (Sargent 1987, chapter XII). The literature on non-fundamentalness - which we will see in the next section is related to the same condition - has also pointed out several cases of empirical interest in which the mapping between model shocks and VAR shocks breaks down. We discuss this literature below.

3.1 Relationship with fundamentalness

Another way to state the condition in Proposition 1 is to relate the invertibility of VARMA models to the issue of fundamentalness of MA representations (see the reviews by Sims, 2012 and Alessi, Barigozzi and Capasso, 2008). As in the previous section, a discussion of fundamentalness is carried out in the context of square systems, in which there are as many observable variables as shocks. We have the following definition from Alessi et al. (2011):

Definition (Fundamentalness). A covariance stationary process Y_t has a fundamental MA representation $Y_t = \theta(L)\varepsilon_t$ if $\theta(L)$ has no poles inside the unit circle and det($\theta(L)$) has no roots inside the unit circle.

As discussed by Alessi et al. (2011), in practice detecting nonfundamentalness in the case of DSGE models is the same as checking the invertibility condition in Proposition 1. To understand the implications of nonfundamentalness, note that if an MA representation is fundamental the inverse of the MA lag polynomial $\theta(L)$ depends only on nonnegative powers of L, and thus the model can be equivalently written as a VAR(∞). It is important to keep the issue of fundamentalness separate from invertibility as it can happen (when det($\theta(L)$) has roots inside the unit circle) that the MA polynomial can still be inverted, but $\theta(L)^{-1}$ depends on negative powers of L . In this case, we say that there is invertibility in the future, which means that a VAR(∞) representation does not exist and thus one cannot recover nonfundamental shocks from (S)VAR analysis.

The problem of nonfundamentalness has been known and discussed in the macro literature for some time, at least since Hansen and Sargent (1980, 1991) and Lippi and Reichlin (1993, 1994). These authors showed that nonfundamental representations matter empirically, and can arise either as a feature of the model - for example in the context of permanent income models (Blanchard and Quah, 1993) or rational expectations (Hansens and Sargent, 1980) models - or because of the way in which the exogenous variables are modelled (Lippi and Reichlin, 1993). In the former case nonfundamentalness is often caused by a situation in which the information set of the agents is larger than the information set of the econometrician. An example of the latter case is considered by Lippi and Reichlin (1993), who show that a simple and plausible extension of the model considered by Blanchard and Quah (1993) with dynamics in productivity gives rise to nonfundamental representations and to radically different conclusions from impulse-response analysis and forecast error variance decompositions.

A second example for how the problem of nonfundamentalness may be addressed within a VAR model is Kilian and Murphy (forthcoming), who observe that nonfundamentalness is a problem for oil market VAR models when traders act on information about the future not available to the econometrician. They show that this problem may be overcome by the inclusion of data on oil inventories in the VAR model combined with suitable identifying restrictions derived from economic theory.

Finally, a field in which the issue of nonfundamentalness has come recently to the forefront of the academic discussion is that of VAR analysis aimed at understanding the effect of fiscal shocks, where nonfundamentalness can arise because of the failure to account for the effects of fiscal news (Leeper, Walker and Yang, 2008; Mertens and Ravn, 2010).

When a model does not satisfy the fundamentalness condition, a natural question to ask is: what should one do? One possibility is to estimate nonfundamental representations of the econometric model and see whether they yield plausible impulse-response functions that can be used for estimation or model validation. This is the route taken by Lippi and Reichlin (1994), who show how nonfundamental representations can be obtained applying Blaschke matrices to MA processes. See also Lanne and Saikkonen (2011) for an approach to estimating noncausal VAR models that are implied by nonfundamentalness. A second way to deal (at least in part) with nonfundamentalness that is caused by a model in which the agents have access to future information that the econometrician does not observe is to try to enlarge the econometrician's information set, for example by adding variables that capture information used by market participants (Kilian and Murphy, forthcoming) or by exploiting the cross-sectional dimension via factor models (Giannone et al, 2006, Boivin and Giannoni, 2006).

3.2 Summary of stage 2 and discussion

Once the DSGE has been log-linearized and written in the state-space form (2), which is equivalent to a VARMA, the question of whether the impulse-response functions implied by the DSGE can be meaningfully compared to the impulse-response functions obtained by a VAR(∞) crucially depends on whether a VAR(∞) representation of the model exists. The condition to check is that the eigenvalues $A - BD^{-1}C$ are strictly less than one in absolute value. If the condition is satisfied, the MA representation of the system is fundamental, and it is thus possible to recover the structural shocks and their associated impulse-responses from the innovations and the impulse-responses computed from a VAR, by imposing suitable identifying restrictions. This in turn means that it is meaningful to use SVAR impulse responses for DSGE model estimation and validation.

An issue that arises in stage 2 is that it cannot be performed when the model implies stochastic singularity, which occurs when there are fewer shocks than observables, i.e., $n_\varepsilon < n_y$. This is a common occurrence in DSGE models, where a small number of structural shocks typically drive a larger set of variables, linked by identities that might not hold in the data. The literature has dealt with stochastic singularity in three ways: 1) by only using a subset of the observable variables Y_t in the estimation of the VAR model; 2) by increasing the number of structural shocks in the DSGE model; and 3) by using all available observables but adding measurement errors in the DSGE model (e.g., Ireland, 2004). Each approach has drawbacks and involves arbitrary decisions on the part of the econometrician which may have important effects on the results of the analysis. Dropping variables from the estimation results in a loss of information and opens up the possibility that the results will differ depending on which subset of variables is used. Shocks that are added to the DSGE model may lack a structural interpretation and, from an econometric perspective, increase the number of parameters and the chance of misspecification. For example, it is common to add preference shocks which are merely violations of first order conditions. Like measurement error, they lack microfoundations and undermine the perceived advantage of DSGE models over VAR models, namely the advantage of being rigorously derived from micro foundations. That in turns raises the question of why we would take such semi-micro founded models as our starting point for the analysis. Measurement errors can similarly be viewed as an ad-hoc econometric device that lacks economic interpretation. In general, any arbitrary modification of the original model whose sole purpose is to make the model match empirical features can make the parameters lose structural interpretability and at the very least cast doubts on the model's predictions.

Canova et al. (2013) propose to deal with the problems of nonsingularity by selecting observables in a way that either optimizes parameter identification or that minimizes the distance between singular and non-singular models. King, Plosser, Stock and Watson (1991) advocate using reduced-rank models as a remedy to non-singularity, but the issue of identification should be better investigated in this context.

4 Stage 3. From VAR(∞) to VAR(p)

In this final stage of the analysis, let us suppose that all the previous conditions for the mapping between a DSGE and a VAR(∞) have been met. Even though a VAR(∞) representation exists, in practice one typically estimates a finite order VAR(p) and it is thus important to understand the impact of this approximation error on the resulting analysis. This problem is investigated in Ravenna (2007) and Morris (2012). Morris (2012) analyzes the question in the context of the representation (2) of the log-linearized DSGE and first of all points out that one trivial case in which an exact (restricted) VAR(1) representation exists is when X_t and Y_t are observable and thus the state-space system can be equivalently written as:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} \varepsilon_t. \tag{23}$$

In the more general and realistic case in which some state variables are unobservable, Morris (2012) further gives the following two conditions that can be used to assess whether a DSGE written in the ABCD form (2) can be expressed respectively as a VAR(1) or as a VAR(p), with $p > 1$ but finite.

Proposition 2 (VAR(1)). If Y_t is a linear function of X_t, $n_y \geq n_x$ and A and C are full column rank, then the DSGE model (2) has a VAR(1) representation:

$$Y_t = \Phi Y_{t-1} + D\varepsilon_t \tag{24}$$

with

$$\Phi = CA\left(C'C\right)^{-1} C'. \tag{25}$$

Proposition 3 (VAR(p)). The DSGE model (2) has a VAR(p) representation if and only if

$$\left[\gamma_2 \cdots \gamma_{p+1}\right] = \left[\gamma_1 \dots \gamma_p\right] \begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{p-1} \\ \gamma'_1 & \gamma_0 & \cdots & \gamma_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma'_{p-1} & \gamma'_{p-2} & \cdots & \gamma_0 \end{bmatrix}^{-1} \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_p \\ \gamma'_0 & \gamma_1 & \cdots & \gamma_{p-1} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma'_{p-2} & \gamma'_{p-3} & \cdots & \gamma_1 \end{bmatrix} \tag{26}$$

where $\gamma_i = E\left(Y_t Y'_{t-i}\right)$. In this case, the VAR(p) representation is

$$Y_t = \Phi\left[Y'_{t-1} \dots Y'_{t-p}\right]' + u_t \tag{27}$$

with

$$\Phi = \left[\gamma_1 \dots \gamma_p\right] \begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{p-1} \\ \gamma'_1 & \gamma_0 & \cdots & \gamma_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma'_{p-1} & \gamma'_{p-2} & \cdots & \gamma_0 \end{bmatrix}^{-1} \tag{28}$$

and

$$E\left(u_t u'_t\right) = \gamma_0 - \left[\gamma_1 \dots \gamma_p\right] \begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{p-1} \\ \gamma'_1 & \gamma_0 & \cdots & \gamma_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma'_{p-1} & \gamma'_{p-2} & \cdots & \gamma_0 \end{bmatrix}^{-1} \begin{bmatrix} \gamma'_1 \\ \gamma'_2 \\ \vdots \\ \gamma'_p \end{bmatrix}. \tag{29}$$

Ravenna (2007) similarly derives conditions for the existence of a finite order VAR representation of a DSGE model written in a different state-space form, which makes the distinction between endogenous state variables (W_t) and exogenous state variables (Z_t) explicit:

$$\begin{aligned} W_t &= RW_{t-1} + SZ_t \\ Y_t &= PW_{t-1} + QZ_t \\ Z_t &= \Phi Z_{t-1} + \varepsilon_t. \end{aligned} \tag{30}$$

For the AS model, for example, Z_t contains the technology and government spending processes $Z_t = (\widehat{z}_t, \widehat{g}_t)$ and W_t includes all remaining variables in the model.

Let W_t be $n \times 1$, Z_t be $m \times 1$ and Y_t be $r \times 1$. Under the assumptions that all elements of W_t and Y_t are observable and that $m = n + r$, the DSGE model (30) has a restricted VAR(2) representation

$$\begin{aligned} \widetilde{Y}_t &= (A + B\Phi B^{-1})\widetilde{Y}_{t-1} - (B\Phi B^{-1}A)\widetilde{Y}_{t-2} + B\varepsilon_t, \\ \widetilde{Y}_t &= \begin{bmatrix} W_t \\ Y_t \end{bmatrix}, \; A = \begin{bmatrix} R & 0 \\ P & 0 \end{bmatrix}, \; B = \begin{bmatrix} S \\ Q \end{bmatrix}. \end{aligned}$$

In the most realistic case in which some of the the state variables W_t are unobservable, we have the following result.

Proposition 4 (VAR(p)). A necessary and sufficient condition for a DSGE model written in the form (30) to have a finite order VAR representation is that the determinant of $\left[I - \left(R - SQ^{-1}P\right) L\right]$ is of degree zero in L.

When some endogenous state variables are unobservable and the condition in Proposition 4 does not hold but the condition in Proposition 1 is valid, it is important to understand the impact of approximation error when one estimates (as one does in practice) a finite order VAR when the true data-generating process is a VAR(∞). Starting from the DSGE representation (30), when the condition of fundamentalness is satisfied, the VAR(∞) has the form

$$Y_t = Q\Phi Q^{-1}Y_{t-1} - \left[Q\Phi Q^{-1}PL - P\right]\sum_{j=0}^{\infty}\left(R - SQ^{-1}P\right)^j L^{j+1}SQ^{-1}Y_t + Q\varepsilon_t$$

and thus a finite order VAR may still be a good approximation if the second term is small, which occurs when $Q\Phi Q^{-1}PL - P$ is small and/or $\left(R - SQ^{-1}P\right)^j$ converges to zero fast enough.

How important the approximation error is in practice can only be assessed on a case by case basis, but some general conclusions can still be reached about whether truncation affects the approximating VAR performance only via a *pure truncation bias* channel, or if in addition it induces *identification bias.* The former refers to the bias in estimated coefficients and impulse responses due to the omission of higher order lags; the latter can occur when these biased VAR coefficients are also used to identify structural shocks from the reduced form innovations.

Note that the identification bias in this case is not a result of incorrect identifying assumptions, nor is it only a matter of small sample bias. Using a calibrated RBC model with a VAR(∞) representation, Ravenna (2007) shows that truncation can have sizable effects on the impulse-response analysis based on the model.

A further consideration is that the choice of asymptotic or bootstrap inference for impulse-response functions matters when it is made in the context of VAR models that are an approximation of a true infinite order VAR. This issue is considered by Inoue and Kilian (2002) who show that the residual-based bootstrap is still valid in this situation.

4.1 Summary of stage 3 and discussion

In this final stage of the mapping between a DSGE and a VAR, we saw that a finite order VAR(p) representation of a DSGE model exists when either all endogenous state variables are observable or when the coefficients in the state space representations (2) or (30) satisfy the conditions in Propositions 2 or 3 or 4, from Ravenna (2007) and Morris (2012).

Given a particular calibration or estimation of a DSGE model expressed in form (2) or (30) one can check the conditions for the existence of a finite-order VAR representation of the model. For example, for the minimal state representation of the AS model calibrated as in (20), we have that $n_y = n_x$ and the coefficient matrices A and C are full rank, which means that Proposition 2 holds and the AS model has the following VAR(1) representation:

$$\begin{bmatrix} \widehat{r}_t \\ \widehat{y}_t \\ \widehat{\pi}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0.79 & 0 & 0.25 \\ 0.19 & 0.95 & -0.46 \\ 0.12 & 0 & .62 \end{bmatrix}}_{\Phi = CAC^{-1}} \begin{bmatrix} \widehat{r}_{t-1} \\ \widehat{y}_{t-1} \\ \widehat{\pi}_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} 0.61 & 0 & 0.69 \\ 1.49 & 1 & -1.10 \\ 1.49 & 0 & -0.75 \end{bmatrix}}_{D} \begin{bmatrix} \varepsilon_{zt} \\ \varepsilon_{gt} \\ \varepsilon_{rt} \end{bmatrix}$$

with variance $D \Sigma D'$.

As discussed by Morris (2012), one can similarly show that the model of Christiano et al. (2005) has a VAR(1) representation, whereas the popular model of Smets and Wouters (2007) does not, and caution should therefore be exercised when using impulse responses from estimated VAR models to validate Smets and Wouters' (2007) model.

Even if a finite-order VAR representation of a DSGE model exists, there is still the possibility that the VAR model one estimates in practice does not provide a representation of the data that is consistent with the DSGE model. I am referring in particular to the issue of data transformations and their effect on model estimation and validation. Gorodnichenko and Ng (2010) point out the fact that DSGE models implicitly embed assumption about whether the model variables have deterministic or stochastic trends and that even mild violations of these assumptions can severely affect estimation. They then propose estimation methods that do not require a researcher to take a stand on the nature of the persistence found in the data.

One crucial feature of the robust procedure proposed by Gorodnichenko and Ng (2010) is the fact that they apply the same transformations to both the data and the model variables. This is an important consideration to take into account when comparing DSGE and VAR models, as pointed out by Sims (2003) in a comment to Smets and Wouters (2003). Sims (2003) expresses the suspicion that the pre-processing of the data utilized by Smets and Wouters (2003) implicitly favours the DSGE over the VAR and Giacomini and Rossi (2010) confirm the correctness of this conjecture by applying their test for time-varying comparison of model fit to test the relative performance of Smets and Wouters' (2003) model and a Bayesian VAR (BVAR). They find that the way the data is detrended severely affects the conclusion of which model fit the data best at different time periods, and show that Smets and Wouters' (2003) model is outperformed by the BVAR once the detrending is done in a way that correctly takes into account the information that was available at the time of interest.

Canova (2012) similarly cautions against applying preliminary data transformations when estimating DSGE models, as they may introduce bias and distort a model's policy implications. He proposes an alternative approach to estimation which does not use some cross equations restrictions from the model and instead builds a flexible link between model and data that captures some features of the data, such as trends, which no longer need to be removed before estimation.

One further issue is that the identifying restrictions in the structural VAR model whose responses we compare to the DSGE model's responses have to be consistent with the DSGE model. That can be difficult especially when using delay restrictions for identification, as exemplified by Rotemberg and Woodford (1998), who tweaked the timing in their DSGE model to make it conform with their structural VAR model.

Consolo, Favero and Paccagnini (2009) add to the discussion on the suitability of reduced form VAR as benchmarks for evaluating DSGE models by resorting to the notion of statistical identification (Spanos, 1990). Spanos (1990) advocates validating structural models using reduced form models that are themselves valid, in the sense that they pass a battery of tests. Consolo et al. (2009) point out that in the literature on DSGE validation, instead, the choice of the reduced form model is solely driven by that of the structural model, without considerations about how accurately the reduced form model describes the data. The critique is applied in particular to the model evaluation approach of Del Negro and Schorfheide (2004, 2006), who obtain the reduced form model by relaxing the cross-equation restrictions imposed by the DSGE, but the point Consolo et al. (2009) make is relevant in our discussion as well. If the mapping between DSGE and VAR is done in the rigorous way mapped out in this chapter, the benchmark VAR will be low-dimensional, it will omit potentially relevant variables, it will not have time-varying parameters, and as a result it may not be a good representation of the data. Consolo et al. (2009) propose to overcome this limitation by comparing DSGE models to factor-agumented VAR, but the mapping from the theoretical to the reduced form model will in this case be lost and it will be difficult to understand whether the model is rejected because of misspecified restrictions or because of omitted variables.

5 What next?

The work of Canova (2012), Consolo et al. (2009) and Del Negro and Schorfheide (2004, 2006) share the concern that DSGE models are too stylized and that the restrictions they impose on their reduced form counterparts are too restrictive for the outcomes of their estimation and validation to be taken too seriously. A more radical view is expressed by Howitt et. al (2008) and Pesaran and Smith (2011), who call for the profession to move beyond the narrow and ad hoc modelling framework of DSGE models, echoing the widely expressed concern that these models ignored important financial, housing and foreign channels that were crucial during the

2007 crisis. Howitt et al. (2008) advocate a return to atheoretical econometric methods, and Pesaran and Smith (2011) suggest a flexible approach where theory guides the choice of long-run relationships among variables while the short run dynamics are less restricted, where trade and financial variables are incorporated via the use of global VAR models and where theoretical restrictions are used to discipline estimation in these necessarily high-dimensional models. As it is not clear how to identify structural shocks in this framework, Pesaran and Smith's (2011) approach has a "non-structural" flavour.

Caldara et al. (2012) focus on the missing channels in DSGE models and provide a similarly "semi-structural" approach to formalize what they argue is the current tendency at central banks to expand DSGE models to include missing channels and mechanisms of interest. They advocate augmenting the DSGE by including proxies for the missing channels (such as house prices) which are typically derived from auxiliary econometric models, such as VAR.

An almost opposite, "back to basics" approach is adopted by Giacomini and Ragusa (2012), who advocate starting from an econometric model which is known to provide a good description of the data and forcing it to satisfy some of the (nonlinear) equilibrium conditions implied by theory using exponential tilting. The approach is flexible and general, and can be used to incorporate only a subset of the equations from a DSGE model (e.g., the Euler equation), thus addressing the concern that not all the equations in the DSGE model may have the same "theoretical content" and that the researcher does not have equal faith in all aspects of the DSGE model.

6 Conclusion

Now that the dust has settled on the debate about the usefulness of reduced form time series methods for DSGE model estimation and validation, the emerging picture from the literature is that VAR analysis is a useful tool for estimation and validation of DSGE models, but of course as any tool it should be used with caution. As this chapter made clear, a rigorous mapping between the structural model and the reduced form model can only be established under very stringent assumptions. The literature offers some insight into whether such a mapping is possible for a given model and parameterization, but there are many situations in which the formal methods discussed here are not applicable.

The running theme throughout the chapter has been to advocate moving beyond the narrow question of whether a DSGE model can be written as a VAR, and to focus on understanding the impact of the many assumptions and arbitrary decisions that underlie the current practice of estimating and validating DSGE models.

First, the DSGE model has to be log-linearized, but the effects of log-linearization have been scarcely investigated, and it might well be that this approximation error will end up dominating the approximation error from using a finite order VAR on which the literature solely concentrates. More research is needed on this topic.

Second, the log-linearized model is solved and multiple solutions are typically ruled out, again without clear motivation or an understanding of the effects of this choice on the model estimates. This is another area that would benefit from deeper investigation.

Third, the conclusions depend on which arbitrary choices the researcher makes when bringing the log-linearized model to the data: the choice of observable variables, whether to drop observables, add shocks or measurement errors to avoid stochastic singularity, whether to rule out nonfundamental representations.

Fourth, even if a finite order VAR representation exists, the outcome of estimation and validation could be severely affected by preliminary data transformations such as demeaning and detrending and by the assumptions made about whether the trends are deterministic or stochastic.

Finally, a choice of benchmark VAR model that is purely driven by the initial DSGE model and by the selected observables, and not by the VAR model's ability to fit the data, casts doubts on the validity of the VAR's predictions. It is also not clear why a specific DSGE model should be the starting point of the analysis and why VAR models should be adapted to fit a given DSGE specification. DSGE models rely on many arbitrary choices about functional form or market structure, about the exogeneity of driving processes (such as government spending, productivity or monetary aggregates) and about the dynamic specification of the latent driving shocks. DSGE models might be rigorous, but they are not necessarily realistic. For example, changing the dynamic specification of the technology process in a real business cycle model from an AR(1) process to an MA(2) process changes the appropriate lag order specification of the implied VAR.

As a partial response to these concerns, the chapter concluded by listing recent contributions to the literature that advocate moving beyond a dogmatic belief in the DSGE model specification and restrictions, and drawing instead on the vast and better understood experience of fitting the data with atheoretical models. Examples are the idea of combining the DSGE model and its reduced form VAR model as a way of relaxing all theoretical restrictions in the DSGE model (Del Negro and Schorfheide, 2004, 2006); the idea of relaxing only some theoretical restrictions and letting the data "speak" about some features of the model such as trends (Canova, 2012) or short-run dynamics (Pesaran and Smith, 2011); the idea of starting from a DSGE model and adding information from econometric models to capture missing channels (Caldara et al., 2012) or the idea of starting from an atheoretical model that fits the data well and forcing it to satisfy some of the the equilibrium conditions implied by theory (Giacomini and Ragusa, 2012).

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