Family Economics

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Introduction

The existence of a nuclear family is to a large extent dictated by nature. According to Aristotle (Politics, Book1 part 2) "there must be a union of those who cannot exist without each other; namely, of male and female, that the race may continue (and this is a union which is formed, not of deliberate purpose, but because, in common with other animals and with plants, mankind have a natural desire to leave behind them an image of themselves)". However, families are also economic units that share consumption, coordinate work activities, accumulate wealth and invest in children. Indeed, Aristotle adds that "The family is the association established by nature for the supply of men's everyday wants".

Economists' interest in the family dates back to Cantillon (1730), Smith (1776) and Malthus (1798). These authors investigated the connections between economic circumstances and the size of the population. In particular, they discussed the subsistence wage and family size that can support a stable work force over time, including the current workers and their descendants that will replace them.[1] The main economic decision discussed in this context was the timing of marriage as a means to control fertility.[2] Later writers, including Mill (1848) and Le Play (1855), have shifted attention to the impact of the family on the standard of living of its members, via self production, insurance and redistribution of family resources. An important issue in this context was the allocation of bequests among siblings, which can affect marriage patterns, the incentives of children (and parents) to work and save and the distribution of wealth in society.[3]

The role of the family has changed drastically in recent times. In modern societies, individuals can enter marriage and exit out of it almost at

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[1] "A man must always live by his work, and his wages must at least be sufficient to maintain him. They must even upon most occasions be somewhat more; otherwise it would be impossible for him to bring up a family, and the race of such workmen could not last beyond the first generation." (Smith, 1776, p. 67).

[2] "Universally, the practice of mankind on the subject of marriage has been much superior to their theories; and however frequent may have been the declamations on the duty of entering into this state, and the advantage of early unions to prevent vice, each individual has practically found it necessary to consider of the means of supporting a family, before he ventured to take so important a step." Malthus (1798, Book 4, Chapter 14).

[3] Based on his empirical study of family budgets, Le Play (1872) argued that the insurance role of the family, is better served by the British system of inheritance that allows a flexible allocation of bequests than by the French system that imposes equal division.

will, avoiding pregnancy is easy, child mortality is low, and both singles and married partners can choose whether or not to have children. Due to technological changes, the importance of the family as productive unit has declined sharply and it has become much more common for both husband and wife to work in the labor market. There is higher turnover and some individuals transit through several marriages, being single during a larger part of their life time. As marriages break and new marriages are formed, the traditional division of labor between husbands and wives, especially in taking care of the children, is put under pressure and transfers between ex-spouses and custody arrangements are required to maintain the welfare of children under variable family arrangements. Despite the higher turnover, and the changing household roles, marriage patterns in modern societies have some sustainable features, the most notable of which is the high correlation in the schooling attainments of husband and wives.

A unified approach to the family which is applicable to modern societies was first provided by Gary Becker (1973, 1974, 1991). This approach ties within family allocations of time and goods to the aggregate patterns of marriage and divorce. The important insight of Becker's approach is that when each man (woman) can choose among several alternative spouses, competition over spouses matters. Then, the options of each particular person willing to marry depend on whether individuals of the opposite sex are willing to marry him\her. Therefore, an equilibrium concept must be applied such that in existing marriages, no one wants to become single or wants and can replace the current spouse. This broader perspective can address the stability of alternative matching profiles in the society at large and ultimately explain the assortative matching patterns and high marital turnover that one observes in modern societies. Thus, assortative matching by schooling in the society can be linked to the fact that within households, the schooling of husband and wife complement each other.

Our book builds on Becker's work and the subsequent literature in empirical and theoretical family economics. There are two major strands to the recent economics literature on the family: what happens inside existing unions and who marries whom. Although the two strands of the literature have obvious mutual implications and sometimes meet, they are largely distinct (as can be seen from the largely disjoint set of contributors to the two strands). A principal aim of the book is to move forward the merging of the two strands (as well as providing a state of the art discussion of the two strands). Accordingly, we divide the book into two parts. The first part covers the decision making within families; the second part of the book examines the aggregate patterns in the marriage market and how the actions of different couples are interrelated.

Given the current, active state of the field, several different modeling strategies exist. Concerning the behavior of families, we explicitly recognize that spouses in marriage care about each other and their common children and yet may have conflicting interests. This situation allows for two distinct solution concepts; one is a non-cooperative self-enforcing outcome, the other is cooperative solution that is efficient and requires binding commitments, enforced by formal or informal agreements (Lundberg and Pollak, 1993). We discuss both alternatives, but the main message is that one can test which approach better fits the available data on consumption and work behavior of married couples. We show that even if the partners cooperate and act efficiently, the observed behavior, in terms of the consumption and work choices will generally differ from that of a single decision maker. The differences arises from the recognition that changes in prices or incomes that influence the family budget constraint can also influence the relative "power" of the partners. For instance, transfers of income between husband and wife (or parents to children) which do not affect total family income have no impact on family behavior according to he traditional "unitary" model of the family but have systematic testable effects under the "collective" model that we propose.

There are also two approaches to model the competition over spouses and the division of gains from marriage. One strategy emphasizes frictions such that one can meet only with few and random members of the opposite sex, before entering marriage (Mortensen, 1988). The other approach ignores frictions, assuming that it is relatively easy to meet many partners in a short period of time (Gale and Shapley, 1968, Shapley and Shubik, 1972, Roth and Sotomayor, 1990). In each of these cases, one can further distinguish between a 'no transfers' case in which partners must accept the characteristics of their spouse, good or bad, and the 'transfers' case in which spouses can effectively compensate (reward) for deficient (attractive) attributes. We discuss all these cases, showing their different implications for marriage patterns and for the division of the gains from marriage.

The economic approach to the family can be contrasted with that of biologists and sociologists. These two fields use different methods which may yield different testable predictions. For instance, in discussing sex roles economists often rely on the principle of comparative advantage. Thus, a mother will spend more time with child than her husband if the ratio of her productivity at home relative to her market wage exceeds that of father. The partners can then divide the gains in total output resulting from specialization. In biology, unequal division of labor is ascribed to the ability of women to have only few children from different men, while a man can have many children from different women. Thus, the mother is usually willing to invest more resources in the child than the father who can potentially free ride on her desire to invest in the children. Hence, men will compete for women who will select the most trustworthy man she can (based on some signals) but the end outcome is that men will spend less effort on each offspring. (Trivers, 1972). A significant difference between these two accounts is that the comparative advantage argument rests on transfers of resources across spouses, that is, exchange that makes both parties better off (Bergstrom, 1996 and Cox, 2007). As another example, sociologists often motivate assortative matching by inherent preferences to marry someone similar in terms of predefined attributes such as social status or ethnicity. In such a case, assortative mating is mainly constrained by groups size, and minorities are more likely to marry outside the group (Lewis and Oppenheimer, 2000). Economists obtain the a similar outcome, but the groups are formed in equilibrium as a consequence of optimal individual search and investment decisions (Burdett and Coles, 1999, Chiappori *et al.*, 2009). As these examples illustrate, economists bring to bear a large degree of free choice to individual agents subject to resource constraints and some aggregate consistency (an equilibrium) that makes all the individual choices mutually feasible and sustainable.

The economic approach to the family shares many features of the employment relationship that is widely discussed by economists. In both cases the issues of matching and the division of the surplus arise, as well investment and effort spent in search. However, there are important differences that originate from the *non* economic aspect of the marriage relationship. First, some initial blind trust in the form of love is required to undertake commitments between the two partners. Based on such commitments, the partners can coordinate work and investment decision that increase their gains from marriage and stabilize their marriage, *ex post*. Second, the presence of children, who are "public goods" for the parents, strongly influences entry into marriage and separation decisions. The partners cannot simply part and go their separate ways because they still care and are legally responsible for their children. These two differences make the analysis of the family radically different from the analysis of the employment relationship.

This book is intended for economists. It should be accessible to any graduate student in economics and has sufficient material for one or two semesters of lectures on family economics. Although somewhat technical, we verbalize and illustrate the main ideas so that the book can also be useful for scholars from other fields who wish to understand the economic approach, without necessarily agreeing with it. However, the book will be useful mainly for those interested in modern societies with high marital turnover. Important problems that face traditional societies are not covered in this book. We do not discuss intergenerational transfers and dynastic households. Nor do we discuss the important issues related to the demographic transition from a high to low population growth. For a recent account see Razin and Sadka (1995), Laitner (1997) and Hotz *et al* (1997) in the Handbook of Population Economics, edited by Stark and Rosenzweig.

The first chapter of the book presents some basic facts about the marriage market and the family. The chapter is intended to motivate the analysis that follows in the rest of the book by showing how marriage and fertility interact with economic variables such as work, wages and investment in schooling. We display data showing that married men work more and have higher wages than single men, while the opposite patterns hold for women. We also document the patterns of assortative matching and show how they were affected by the rising investments in schooling and the higher labor force participation of women. The subsequent chapters are then divided into two parts; the first part (chapters 2-6) provides a micro level analysis of family behavior and the second part (chapters 7-11) provides a macro level analysis of marriage patterns and their welfare implications.

Chapter 2, addresses the question ‘why marry’ and we discuss several broad sources of potential material gains from marriage, such as sharing consumption and coordination of work and investment decisions. Chapter 3 provides a basic theoretical framework for the analysis of family behavior. The framework is intentionally broad, including features such as altruism, public and private goods and interaction of several family members (including children) who may act independently or cooperatively. We compare the traditional "unitary" model that treats the family as if it is a single decision maker to alternative models that allow family members to have different views on the decisions that are to be made. We present both non-cooperative and cooperative variants of these non-unitary models. In particular, we discuss the "collective approach" which assumes efficiency and a stable rule for allocating family resources and provides a tractable way for predicting family behavior and its response to varying economic conditions (see Chiappori, 1982 and Browning and Chiappori, 1998). Chapter 4 discusses in detail the collective model and its testable implications. A particular emphasis is given to testing efficiency, an assumption embedded in all cooperative models of the family. We also discuss the normative implications of the collective assumption which replaces conventional analyzes of household welfare with an analysis of individual welfare. Chapter 5 discusses how to empirically recover individual preferences within the household and the associated decision rules implicit in the collective model. This chapter also summarizes the main empirical findings. It is shown that the unitary model is often rejected but efficiency is not rejected. Importantly, the rule for sharing the marital gains can be identified (up to a constant) and it is found to respond systematically to marriage market conditions such as sex ratios and divorce laws.

Chapter 6 extends the static framework and considers family choice over time and under conditions of uncertainty. We address the new strategic issues that arise in a dynamic setup and the important role of commitments. Partners anticipate upon marriage that a negative future shock in match quality may cause separation, which will reduce their benefits from collective goods, including children. Based on this anticipation, they choose how much to invest in children and how much to consume each period. To attain efficient investment and consumption outcomes, commitments made at the time of marriage are usually required. For instance, a binding contact, enforceable by law, can be signed at the time of marriage which determines the proportion of family assets that each partner would receive upon divorce.

Chapter 7 provides an extensive and integrated analysis of matching models. The main question here is ‘who marries whom’. To address this, we discuss models with and without frictions. Usually, there is less sorting when there are frictions or when utility is transferable within couples, but the reasons differ. With frictions, individuals are willing to compromise rather than wait for a more suitable match. With transferable utility, a less attractive spouse can bid for a more attractive spouse by giving up part of his\her share in the gains from marriage. Chapter 8 discusses in detail how the shares in the marital gains are determined jointly with the equilibrium matches when frictions are assumed away. The main insight is that the individual traits of two married partners, such as their schooling or income, are insufficient to determine the division. Rather, due to competitive forces and the endogeneity of the equilibrium matching, it is the distribution of traits in the population at large that determines the outcome. Chapter 9 uses the same friction-less approach to address premarital investments, such as schooling, whereby individuals can accumulate assets that will influence their prospects of marriage and their share in gains from marriage. We emphasize the contrast between inherited traits such as ethnicity and acquired traits, such as schooling. Both kinds of traits influence marriage patterns but acquired traits are also affected by these patterns. In this case, a rational expectations analysis is required to deal with the two way feedbacks that arise. We apply such equilibrium analysis to discuss the interesting reversal in the education attainments of men and women, whereby women who in the past invested less than men in schooling now invest more than men do.

Chapters 10 and 11 introduce search frictions to address turnover in the marriage market, allowing for divorce and remarriage. We examine the welfare implications of turnover for men, women and their children. We also discuss the role of different laws governing divorce, custody and child support. These chapters provide a less alarming perspective on divorce than is adopted by many observers. We recognize that the emotional components of a match are subject to unanticipated shocks and that divorce and remarriage allow the replacement of a bad match by a better one. Moreover, in a search environment, couples that received negative shocks can more easily find a new partner when many couples, rather than few choose to divorce.

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Part I

Models of Household Behavior

1

Facts

The purpose of this chapter is to present some basic and general facts about the marriage and the family. The chapter is intended to motivate the analysis that follows in the rest of the book by showing how presumably non-economic activities, such as marriage and fertility, interact with economic considerations such as work, wages and schooling.

1.1 Marriage, divorce and remarriage

1.1.1 Marital status

Marriage is a "natural state". Table 1.1 shows the proportions (the 'stocks') in each marital state for three different years for six high income countries. These figures show that most of the adult population (aged 20 or older) is married at any given time (about $55-70$ percent). However the proportion of the adult population that is married has declined in most countries in the last fifty years. This trend is accompanied by a larger proportion of never married and a higher proportion of divorced individuals, with little change in the proportion of widowed (because of the offsetting effects of reduced marriage and mortality). In all countries and at all times, the proportion of never married women is significantly lower than the proportion of never married men; this is partially attributable to the fact that men marry later. Explaining these cross-country regularities and trends is a major challenge for demographers and economists. Factors that may be related to the changes in marital status that we see in this table are: changes in the age structure; delays in marriage; the relative attractiveness of alternative household arrangements; higher turnover and longer life spans.

There are some notable differences among the countries, reflecting different social norms and legal regimes. As an obvious example, the low stock of divorcees in Italy reflects the fact that divorce was illegal until 1970. As another example, the increasing proportion of 'never married' in Denmark can be attributed to cohabitation, which has become common in Scandinavia.

Strictly, cohabitation should be seen as an alternative state and an extra column should be included in Table 1.1, but it has been relatively unimportant in most countries until recently. To give some idea of the level and changes in cohabitation we present numbers from the US and Denmark in Table 1.2; this gives the proportion of couples (classified by the head's age) who live together who are *not* formally married. As can be seen, there appear to be age, period and cohort effects. That is, cohabitation is more common amongst the young; at any given age cohabitation is more common amongst younger cohorts and cohabitation rates are higher now than twenty years ago. Dramatically, in Denmark, 80 percent of those aged $20-24$ who live together choose not to marry. Comparing the two countries we note that the rate of convergence between US rates and Danish rates, if any, is very slow.

The propensity to cohabit rather than marry is associated with having children. In Tables 1.3 and 1.4 we show the proportion of households with children, conditioned on whether the household head is single, married or cohabiting for Denmark and the US, respectively. We see that, in each age group, married couples have more children than cohabiting couples who in turn have more children than singles. Moreover, the proportion of cohabiting couples declines sharply with age. We can thus think of cohabitation as a "partial marriage" involving less investment in children and a lower commitment to a long term relationship.

1.1.2 Marital histories

Modern societies are characterized by marriage and divorce at will. Thus, although marriage appears to be the preferred state, one need not be married to the same person and, in fact, there is substantial *turnover*, especially among the young. Table 1.5 provides data on marriage histories by age in 2001. Among those who were $50-59$ years old in 2001, only 6 percent were never married, but about 31 percent of men and 26 percent of women had been married more than two times and about 40 percent of each gender divorced at least once. Widowhood at old ages is much more common among women and they are also more likely to be divorced when old.

A more refined picture of the marital histories is provided in Table 1.6 that records the marital history of the 1931-36 and 1937-41 US birth cohorts at different years as they age, separately for men and women. It is seen that the proportions of men and women in their first marriage tend to first rise and then decline, while the proportions in the second or third marriages and the proportion divorced rise. Women are more likely to be in a first marriage when young but less likely to be in a first marriage when old. In addition women are substantially more likely to be divorced when old, suggesting that women find it more difficult to maintain their first marriage and to remarry. For both cohorts we find an increase in the proportion divorced during the period 1970-1980, suggesting that the "divorce revolution" in the US that occurred in this period affected all couples and not only the newly married. However, in the more recent cohort individuals are more likely to be divorced at any given age.

Table 1.7 provides information form the NLS Panel that includes individuals who were aged $11-21$ years old in 1979 and then followed up until 2000. By age 35, most men (81 percent) and women (89 percent) were married at least once. However, the divorce rate has been substantial too and 35 percent of the women (26 percent of the men) had divorced at least once. By age 35, most men and women reported that they had finished their schooling but 21 percent of women and 16 percent of men have done so after marriage. About 16 percent of the women had a child prior to marriage.

1.1.3 Flows

The numbers presented so far refer to stocks but we are also interested in flows into and out of marriage. Figures 1.1 and 1.2 describe the crude marriage and divorce rates for a selection of high income countries. In contrast to Table 1.1 that provides information on the *stocks* in different marital states, these graphs describe the *flows* into the married and divorced states in a given year as proportions of the adult population. The picture is quite clear; starting in 1960 marriage rates have declined and divorce rates have risen in all the displayed countries. Divorce rates started to rise sharply in the late 1960's with a weak tendency for convergence around $3-4$ percent per year for some countries; but about 6 percent in the US and about 1 percent in Spain and Italy. The fact that divorced rates went up in many countries at about the same time suggest a common trigger, such as the anti-pregnancy pill (see Michael, 1988). Given that about 60 percent of the population is married in these late years, the implied probability that an average marriage will break up is roughly 2 percent per year (4 percent in the US).

Figures 1.3 and 1.4 provide a longer perspective of the marriage and divorce rates in the US (see also Stevenson and Wolfers (2007)). Figure 1.3 shows marriage and divorce rates per thousand. As can be seen, following a short episode of increase in the marriage rate after World War 2 (reflecting delayed marriages and divorces during the war) the marriage rate declined slightly from 1950 to 2000 with some ups and downs in between. In contrast, there is an abrupt change in the divorce rate starting at about 1965 with a doubling in the rate form 1965 to 1975. Although the crude marriage and divorce rates are informative, much more useful are hazard rates (that is, the proportion per the relevant groups at risk). Figure 1.4 shows hazards of marriage, divorce and re-marriage form 1922 to 1988 in the US.[1] This figure too shows the abrupt change in divorce rates after 1965. At about the same time the remarriage rate increased relative to the marriage rate, indicating a higher marital turnover. The presence of many divorcees raises the incentive of any given couple to divorce, because it would be easier to remarry following separation (see Chiappori and Weiss (2006)).

[1] Unfortunately it is not possible to extend the series beyond 1988.

1.1.4 Transitions

The most direct information on marital turnover within cohorts is given by the *transition rates* across marital states. To show these we use two different data sources. The first is the HRS which provides us with marital histories for a cohort born between 1931 and 1941 that reported (retrospectively) its marital status history in 2000. The second data source is the NLS(Y) which provides information on marital status up to age 40 for a younger cohort born between 1958 and 1965. Figures 1.5 and 1.6 present the annual transition rate from never married to first marriage of men and women for the two cohorts respectively. For both cohorts the entry rate into first marriage first rises and then declines as most individuals who wish or can marry have already married. The short phase of rising rates of entry indicates a delay associated with premarital investments and learning about one's potential spouse. However, women enter first marriage at a higher rate than men, suggesting that their gain from early marriage is higher.

Figure 1.7 presents the rate of dissolution of the first marriage by the duration of marriage for the same two cohorts. For each cohort we break up those who are married into those who married before the median age for that cohort ('early marriages') and those who marry latter than the median age ('late marriages'). These figures illustrate two important facts. First, the hazard of divorce is first rising and then declines and, second, the divorce hazard at any marriage duration is generally lower for later marriages. These two features are the consequence of the interplay between sorting and acquisition of information of match quality. The hazard of divorce is initially rising with duration of marriage because partners learn about each other. As new information arrives, some marriages break. However, with the passage of time, the weak matches are eliminated and the remaining marriages are increasingly stable. Similarly, the higher stability of late marriages can be ascribed to longer premarital search and courtship, which eliminates some of the potentially weak matches (see Becker, Landes and Michael, 1977 and Weiss and Willis, 1993, 1997). Although these features are common to the two cohorts, there is a very large difference in divorce rates between the two cohorts. For any duration of the first marriage, the younger cohort reports a divorce rate that is about twice as high. This reflects the general rise in the divorce rates during the period 1965-1975. All of the divorces of the younger cohort, born in 1958-65, happened after the divorce revolution, while most of the divorces of the cohort born between 1931 and 1941 happened before 1975.

Similarly to the first marriage rate, the remarriage rate of divorced individuals first rises with the time since divorce (indicating experimentation) and then declines sharply, because those divorcees who remained unmarried for a long period are less suitable or less willing to remarry (see Figure 1.8). The remarriage rate is much higher among the younger cohort, corresponding to their higher divorce rate. Thus, later cohorts are characterized by higher turnover which is reflected in both higher divorce rates and higher remarriage rates. The remarriage rates of men and women are similar at the early part of the 1958-1965 birth cohort. For earlier cohorts that are observed later in life, men remarry at substantial higher rates than women, especially at high ages. This reflects the fact that the ratio of eligible men to eligible women decreases because women marry earlier and live longer, so more of them are either divorced or widowed at late age. The remarriage options of men are further enhanced by the fact that the wage gap between female and male earning capacity is increasing with age because, on average, males had accumulated more work experience.

Comparing Figure 1.8 with Figures 1.5 and 1.6, we see that for both cohorts, the remarriage rates of those who remarry quickly exceed the rates of entry into first marriage. This suggests that some individuals are endowed with marital attributes that make them attractive in any marriage, whether it be the first or second.

1.1.5 Households

Marriage usually involves at least two people living together in the same household. This allows the sharing of housing and other consumption goods. The benefits from such sharing opportunities depend on the household size. Clearly, living in the same household does not require marriage and more than one family (or an extended family) can live in the same household. In Tables 1.8 and 1.9 we present some statistics on the prevalence of one person households. Table 1.8 shows that the proportion living alone ranges from 5 percent in Iberia to over 20 percent in Scandinavia. Given that there are significant material gains from living in many person households, the high level of people living alone in some countries represents a considerable potential loss of material well-being. Table 1.9 (for Denmark) shows that the latter high proportions are not simply a result of older people or younger people living alone, although the rates are higher for these groups. For example, the proportion of 40 year old living alone in Denmark is higher than the overall proportion for France.

Figures 1.9 and 1.10 give statistics on living arrangements over time for the US. We see that the proportion of households that are 'married with children' has declined from 40 percent in 1970 to 24 percent in 2000 and the proportion of 'married without children' has hardly changed. There have been sharp increases in the proportion of single person households, from 17.1 percent to 25.5 percent and in 'other' households (whether 'family' or 'non-family'), from 12.3 percent to 21.7 percent. Figure 1.10 shows the corresponding changes in household size. As can be seen, the proportion of large households (5+ members) has halved and the proportion of single person households has increased by about one half. Taken together, these figures suggest a substantial reduction in the gains from sharing consumption goods within households. One possible reason is that technological advances in home appliances allow singles to obtain household goods more cheaply, even without sharing with others (see Greenwood and Guner, 2005).

1.2 Marriage, work and wages

1.2.1 Time use

Marital status is strongly correlated with the allocation of work time and the market wages that individuals receive. Thus, compared with singles, married men work more in the market and have higher wages, while married women work less in the market, receiving lower market wages. This pattern may result from two different effects. First, the division of labor between the married partners, whereby, on average, wives takes a larger part of the household chores. Second, selection into marriage, whereby those willing and able to marry are high wage males with prospective strong market attachment and low wage females with prospective weak market attachment.

Time budget data allows a closer look at the relationship between marital status and the allocation of time. Such data is presented in Tables 1.10 (paid work and leisure) and 1.11 (for housework and some of its components) for four countries, where for each country we provide information for two time periods. A number of robust (if unsurprising) regularities can be seen. Most importantly, in all countries and for all marital states, men work more than women in the market and women do more housework than men. Over time, married women increase their market work (see Table 1.10) and reduce their non-market work (see Table 1.11), while married men increase non market work and reduce market work (although Canada provides some exceptions). However, this trend toward equalization is quite slow and by 2000, the gender gap in household roles remains large. When the children are less than 5 years old, women work in the market less than half the time that men do (2.8 versus 6.4 hours per day in the US) and about twice as much at home (2.7 versus 1.2 hours per day on child care and 2.6 versus. 1.4 hours per day on home production in the US).[2] Although technological advance has substantially reduced the time that women spend on household chores such as cooking and cleaning (from 3.7 hours a day to 2.6 hours a day in the US) the amount of time spent with children by both fathers and mothers has risen. Time spent on shopping has not changed much over time and women continue to spend about twice as much time on

[2] Note that child care is underreported since it is a residual category in time use diaries. Typically respondents record some other activity they are doing even when they are also looking after their children.

shopping than men, irrespective of their marital status. Note that gender differences in the allocation of time, whereby men work in the market and less at home, are also present among unmarried men and women, perhaps reflecting the higher average market wages of men. However, the difference in the allocation of time of married men and women are more pronounced, indicating an added role for the division of labor within couples. Another salient feature of these statistics is that although single men enjoy more leisure than single women, hours of leisure are about the same for married men and women, suggesting some coordination of leisure activities (see Aguiar and Hurst, 2006, and Burda *et al*, 2006). These averages, however, mask quite large differences across households; in some households we see one partner having twice as much leisure as the other (see Browning and Gørtz, 2006).

Similar patterns are observed in aggregate data. Table 1.12 presents statistics for ten countries on labor force participation. These statistics show clearly that historically men have participated more than women but this gap is narrowing as the participation of women rises (except in Japan) and the participation of men declines. In Figures 1.11 and 1.12, we report a more detailed examination of labor force participation for the US. These figures give the proportion of full time workers by age and marital status for two birth cohorts, 1945-54 and 1960-69.[3] We see a very clear pattern. At any age, married men are more likely to be fully employed than single men and married women are less likely to be fully employed than single women. Married men are substantially more likely to be fully employed than married women, suggesting a division of labor between married partners. This gap in labor market attachment initially rises with age (and time) and then declines within cohorts; it also declines across cohorts at given ages (compare Figure 1.12 to 1.11). These patterns can be related to the impact of children on the division of labor. When couples have young children, married women are more likely to reduce their labor force participation and, therefore, the participation gap between men and women is larger. Figures 1.13 and 1.14 compare the work patterns of married and divorced women and also show a strong impact of having children. Divorced women with children $0-18$ work more than married women with children $0-18$, suggesting that, due to the absence of partner and limited transfers, division of labor between parents is not feasible and divorced women with children are, therefore, "pushed" into the labor market. The higher participation rate of young married women in the younger cohort relative to the older cohort is associated with lower fertility, a delay in having children and a higher participation rate for mothers of young children in the younger cohort.

The gap in labor market attachment between married men and women

[3] In each subsample, we count the number of fully employed individuals and divide by the number of all individuals, including those who do not work.

may not capture the full extent of the division of labor within couples, because no control is made for the behavior of the spouse. In Figures 1.15 and 1.16 we display the work patterns of individuals who are married to each other for two age groups, women aged $40-60$ and $30-40$ respectively. As seen, the most common situation before 2000 was that the husband works full time and the wife works part time or does not work in the market at all. The differences between the age groups in the earlier years probably reflect the presence of children in the household. However, with time, the proportion of such couples has declined and the proportion of couples in which *both* partners work full time has risen sharply, reflecting the increase in the participation of married women into the labor force. On the other hand, the proportion of couples in which she is full-time and he is not remains small.

1.2.2 Wages

The gender differences in the employment of married individuals are closely related to the gender differences in market wages, because a wage gap may lead to different household choices for the husband and wife, based on comparative advantage. But, in parallel, differences in past and expected participation can cause different rates of investment in human capital that result in lower wages for married women compared with married men (see Mincer and Polacheck, 1974, and Weiss and Gronau, 1981).

Figures 1.17 and 1.18 display the development over time of weekly wages (in logs) of US full time workers by marital status for two birth cohorts, 1945-54 and 1960-69. The graphs show that married men have consistently the highest wage among men while never married women have the highest wage among women. In recent cohorts, divorced women are the lowest paid group, while in earlier cohorts the married women had the lowest pay. Within each cohort, these differences in log wages by marital status increase with age (and time), reflecting the cumulative effects of marital status on the acquisition of labor market experience. In contrast, the differences in wages by marital status decline with time as we move towards the more recent birth cohort, holding age constant. This reflects the stronger attachment of married women to the labor market noted above. As married women participate more, their wage becomes more similar to that of men and marital status becomes less important as a determinant of the wages.

1.2.3 The marriage premium

The proportional wage gap between married and single individuals is often (and somewhat misleadingly) referred to as the "marriage premium" which is positive for men and negative for women. In Figure 1.19, we compare married men to divorced and never married men and married women to divorced and never married women. We make these wage comparisons for individuals who are 30 to 39 old, using three year averages. We see that the marriage premium of both men and women has risen over time but the rise is sharper for women. The rise of the marriage premia is consistent with the notion that when fewer individuals marry, the quality of partners that do marry relative to those who do not rises. The sharper increase in the marriage premium for women in Figure 1.19 is a reflection of the rising participation of married women (see Figure 1.20) which is associated with higher wages and schooling (see Goldin, 2006). Because we report wage patterns only for women who work full time, an increase in the participation of married women can increase the marriage premium if the added workers are of relatively high ability (see Mulligan and Rubinstein, 2008).

1.3 Who Marries Whom?

Marriages are not formed randomly. Rather individuals sort themselves into marriage based on the attributes of both partners, because *interactions* in individual attributes generate mutual gains from marriage. For instance, an educated man may benefit more from marrying an educated woman than a less educated man, who may even resent having a wife who is more educated than him. Similarly, a marriage in which both partners are similar in age may create higher gains than a marriage with a large discrepancy in ages. Consequently, 'suitable marriages' are more likely to form and less likely to dissolve. This means that the observed attributes of married individuals may be quite different from the attributes of men and women in general. Additionally, assortative mating arises in which men and women with similar characteristics, such as age, race and education, marry each other.

Figure 1.21 records the distribution of age differences among married couples in the US. In most marriages, the husband is older than the wife but this proportion of such couples had declined from about 70 percent during 1968-78 to about 60 percent in 2000-2005. Among couples in which the wife is of the same age or older than her husband, the sharpest increase is in the proportion of couples in which the wife is older by 3 or more years than her husband, which has risen from about 5 percent in 1968-78 to about 13 percent in 2000-2005. Together, these trends suggest a moderate but steady reduction of age difference over time. This reduction in age differences is partially influenced by the changes in the age distributions of men and women (see Figures 1.22 and 1.23). Over time, the sex ratio of women to men has increased, especially at older ages because women live longer. This excess supply of older women raises, to some extent, the likelihood that men who are $30-40$ will marry older women, although as we have seen, an increasing proportion of the older women remain single.

Couples often sort based on schooling; see Lewis and Oppenheimer (2000).

This process is driven not only by the mutual gain from marriage, but also by the availability of partners with different levels of schooling in the population and the chance of meeting them in school or the work place (see Oppenheimer, 2000). The US (and other countries) has experienced a dramatic increase in the stock of educated women relative to educated men (see Figure 1.24). This change in relative supplies had a marked effect on the patterns of assortative mating by schooling (see Figure 1.25). While the proportion of couples in which the husband and wife have the same schooling has remained stable at about 50 percent, the past pattern whereby in 30 percent of the couples the husband is more educated has been replaced by the opposite pattern whereby in 30 percent of the couples the wife has a higher degree. Figures 1.26 and 1.27 show the distribution of the spouse's education for husbands and wives with different level of schooling, by cohort of birth. At lower levels of schooling (up to high school graduates), each gender mainly marries with individuals of the opposite sex with similar education. This was not the case for higher levels of education for earlier cohorts but becomes more common with time as the distributions of education among women and men become more similar. In particular, we see a large increase in the marriages in which husband and wife have some college education. Because the number of women with some college education has risen sharply relative to men, we see that husbands with some college have replaced wives with high school by wives with some college, while wives with some college replaced men with college and higher degree by men with some college. However, at higher levels of schooling, BA and more, where women are still relatively scarce we see that men of high education marry down while women with college education marry up. We should note that between the two periods, the proportion of couples in which both spouses are highly educated has risen while the proportion in which both are less educated declined. In this regard, the rise in education of men and women combined with assortative matching in schooling has contributed to the trend of rising inequality between households.

In contrast to other attributes, such as country of origin or race, schooling is an *acquired* attribute and investment in schooling is partially motivated by the prospect of marriage as well as enhanced market power (see Goldin, Katz, and Kuziemko, 2006 and Chiappori, Iyigun and Weiss, 2006). In Tables 1.13 and 1.14, we present some evidence on the interaction between marital status and investment in schooling from the NLS panel. As seen in Table 1.13, more educated men and women are more likely to be married and less likely to be separated or divorced at age 35 (after they have completed most of their schooling). The proportion of unmarried women at age 35 rises with schooling which is not the case for men. Table 1.14 presents mean cumulated schooling for men and women at marriage and at age 35. This Table shows, unsurprisingly, that most of the schooling acquired up to age 35 is taken prior to the first marriage. Those who married and never divorced acquired about 4 months of additional schooling during marriage out of 13.8 years, while those who married and divorced acquired about 6 months for men and 10 months for women after their first marriage, which is a relatively large effect given that these are means in which most women have no extra schooling after marriage.

Having considered schooling, it is natural to consider wages. Figure 1.28 provides a comparison of husband-wife correlations in wages and schooling (measured here in years). We examine the correlation in wages in two ways; wages (in logs) and wage residuals (in logs) netting out observable differences in schooling and age.[4] Thus the correlations in residuals represent correlations in unobservable factors that affect the wages of the two spouses.[5] The correlation by school years is relatively stable over time, at about 0.65. The correlations in wage residuals are also stable at a low level of about 0.1. However the correlations in wages rise from 0.2 to about 0.4.[6] The difference between the correlations for schooling and wages is striking. Some of the difference may be due to spurious factors such as higher measurement error for wages, the use of wages at the 'wrong' point in the life-cycle, the imputation of wages for non-participants etc.. However there may also be systematic reasons for the difference. For instance, the stronger sorting by education may be due to similar educations facilitating joint consumption and reducing conflicts on the choice of public good. In contrast, specialization within the household generates a *negative* correlation between the spouses' wages. The rise in the correlation for wages can then be attributed to a reduction in specialization within households associated with the rise in female labor force participation.

One reason for couples to sort based on schooling is that the schooling levels of the two spouses complement each other in generating marital surplus. Weiss and Willis (1977) found supporting evidence for this hypothesis showing that, among couples with the same schooling, divorce declines with schooling. We should then also expect that, as the proportion of couples in which *both* partners are highly educated rises, education will have a stronger impact in reducing the probability that a given man or woman will divorce. Figure 1.29 shows that this is indeed the case.

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[4] Wages were imputed for men and women who did not work at all or worked less than 10 hours a week.

[5] The wage correlation will be higher than for the residuals since the latter removes the correlation due to age and schooling.

[6] We have also calculated the correlations between the *percentiles* of husbands and wives in the respective (log) distributions of men and women each year. The correlation in wage percentiles is slightly higher than the correlation in wages but the trend over time is very similar. The correlation in residual percentiles is the same as the correlation in residuals.

1.4 Children

Children are the most important 'products' of the family. The decision about how many children to have, when to have them and how to care for them interacts importantly with a whole host of other decisions including schooling, marriage, divorce and re-marriage.

1.4.1 Fertility

As we saw, for marriage and divorce there is considerable heterogeneity across countries and time and this is even more true for fertility. Figure 1.30 presents the time path for completed fertility for cohorts of US women born between 1903 and 1956.[7] The most important feature of this figure is that there are significant variations across cohorts in the mean number of children per woman. Thus, women born early in the century had about 2.2 children, those born in the mid-1930's (the mothers of the 'baby-boom') had over three children and those born in the fifties had close to two. Table 1.15 shows the change in the distribution of children born for women born in the mid-1930's and in the late 1950's. As can be seen the change in the mean is partly a result of fewer women born in the 1930's being childless and partly a result of these women having larger families, conditional on having a child at all. Particularly striking is that the modal family size for the older cohort is 4+ but only 2 for the younger cohort. Figure 1.31 shows data on the number of children less than 18 of US women (married or single), aged $35-45$, at different periods of time.[8] As seen, the reduction in fertility and marriage rates during the second half of the 20'th century is associated with a decrease in the proportion of women with more than 3 children and an increase in the proportion of women with no children, while the proportion of women with 1 or 2 children remained unchanged at about half. By 2000-2005, the proportion of women with children is still high (67 percent) indicating that the natural desire to have children remains strong.

Figure 1.32 shows that the birth rate fluctuates dramatically over time. We see a large increase from the mid-1930's to the early 1960's and then

[7] Completed fertility is defined as the mean number of children born to women of a given generation at the end of their childbearing years. This is calculated by adding the fertility rates by age of the mother observed for successive years, when the cohort has reached the age in question (in general, only ages between 15 and 49 years are considered). In practice, the fertility rates for older women can be estimated using the rates observed for previous generations, without waiting for the cohort to reach the end of the reproductive period.

[8] Table 1.15 and Figure 20 provide different but complementary information. The table shows completed fertility whereas the figure shows the number of children less than 18 living with the mother. Therefore, the proportion of women who have no children living with them in the figure is larger than the proportion of women who never had children in the table.

a sharp decrease. This is consistent with Figure 1.30 which shows a peak in fertility for mothers born in the mid-1930's; this is the baby boom generation. The median age at first marriage has also increased at the same period suggesting fewer "forced marriages" (see Michael, 1988 and Goldin and Katz, 2002).

Figure 1.33 presents evidence on completed fertility for a cross-section of six western European countries for women born between 1931 and 1967. In common with the USA, all of theses countries display a falling pattern from the mid-1930's, although the US has a much higher value in the early years (3.1 as compared to 2.65 for the highest European values). Thus all these countries indicate a 'baby-bust' even though the trends show significant differences across countries. For example, Italy has the lowest values throughout this period with a steady decline from 2.3 to 1.5 children per women. In contrast, the Netherlands starts off with a high value of 2.6 and falls quickly by about 0.7 children in 1946 and then falls much more slowly over the next twenty years by about 0.2 children. Most dramatic is the case of Spain which has the highest value in the early 1940's (at 2.6 children per woman) and one of the lowest 25 years latest (at 1.6).

The timing of children is also of interest. In Figures 1.34 and 1.35 we show the timing of first marriage and first birth for the same countries as in Figure 1.33. There is a clear relationship between reduced fertility and the delay in marriage. On the average, age of first child is only two years after year of marriage (28 and 26, respectively for the latest cohort born in 1963). In these figures, marriage does not include cohabitation. In most countries the latter is low for women born before 1960 but for some countries there is considerable cohabitation. For example, the dramatic rise in the Danish age at first marriage largely reflects the fact that marriage before the birth of a child is increasingly rare amongst younger cohorts.

1.4.2 Children under different household arrangements

One consequence of the increasing marital turnover is the sharp rise in the number of children who live in single parent and step parent households (see Table 1.16). In the US, 2005, 68 percent of children less than 18 years old lived with two parents (including step parents), 23 percent lived only with their mother and 5 percent lived only with their father whilst the rest lived in households with neither parent present. The impact of living with single parents on the children depends on the amount of transfers between unmarried parents. Generally, such transfers are small with a substantial proportion of eligible mothers receiving no transfer at all. Only about half of eligible women receive any child support and when a transfer is received it is about 20 percent of the mother's income (see Table 1.17 and Figure 1.36). The consequence is that divorced mothers have less than half of the family income of married mothers and, therefore, children living with single mothers are often in poverty. The impact of marital turnover on children is a major policy concern and much research has been directed to the analysis and measurement of this effect (see Weiss and Willis, 1985, Chiappori and Weiss, 2006, Piketty, 2003, Gruber, 2004, and Bjorklund and Sundstrom, 2006).

1.5 Saving and life stages

Progression through life-stages has a major impact on consumption, saving and wealth. In the savings literature the traditional picture of the life-cycle is very circumscribed. Agents are born, they receive education, they work and then retire and finally they die. Within such an environment the natural emphasis is on financing schooling decisions, smoothing consumption in the presence of income fluctuations and saving for retirement and bequests. When we take account of leaving home, marrying, having children, divorce and remarriage, a much more nuanced pattern emerges.

The empirical evidence suggests that savings rates vary substantially across family types. The evidence for the US presented in Avery and Kennickel (1991), Bosworth *et al* (1991) and Lupton and Smith (2006) suggests that couples without children have the highest savings rate and lone parents have the lowest rate. Avery and Kennickel (1991) show that married couples have the highest wealth and the highest savings rate whereas divorced people dissave from substantial wealth holdings. Bosworth *et al* (1991) investigate more closely the variations with children and show that households with children present save less than those without. The latter group is largely split between younger couples, many of whom will have children later and those who have children who have left home. Lupton and Smith (2000) use three waves of the PSID and concentrate on changes in savings rates consequent on transitions between marital states. Finally, Zagorsky (2005) presents evidence based on the NLSY79 that suggest that the wealth of divorcees is much lower than the wealth of continuously married individuals and those who never married. Overall, the main finding is that transitions into being married raises savings rates[9] and transitions out of being married lowers them. Although all these studies present a consistent picture, much still remains to be found out about saving and marital status.

Measuring (or even defining) wealth and/or savings in surveys is fraught with difficulties. Using consumption we can illustrate some of the patterns associated with children more clearly. To do this we shall break the evolution of married life into four life stages: being a couple before having children; having young children in the household; having only older chil-

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[9] Blow, Browning and Ejrnæs (2009) find similar results for the transition into marriage using UK expenditure data.

dren in the household and living together after the children have left; see Apps and Rees (2009), chapter 5 for a similar analysis using Australian data. Unfortunately in cross-sections we do not observe whether younger households that do not currently have children will have them in the future. On the other side, for older households with no children present, we do not observe whether they have had children. Instead we take the earliest life stage to be being a couple with no children and the wife aged less than 41 and the fourth life stage to be having no children with the wife aged over 40.[10] Table 1.18 presents some facts on income, nondurable expenditures and budget shares for some goods. The data are drawn from the Canadian Family Expenditure Surveys (FAMEX) for 1986, 1990 and 1992. [11]We select out households in which the husband reports less than 35 hours of full-time work in the year to take account of long spells of unemployment and retirement. There is no selection on the wife's labor force participation. The top panel of Table 1.18 gives details of income and nondurable expenditure. Through the four life stages, expenditure is highest when there are older children present and drops significantly when they leave home. This is partially reflected in the evolution of income but changes in income are not the sole driving force, as can be seen from the expenditure/income ratio. To show this more clearly, Table 1.19 presents the results from regressing log nondurable expenditure on log income and dummies for the last three life-stages. As can be seen, even when we control for income the life-stage has a large and highly significant impact on nondurable expenditures. The bottom panel of Table 11.18 shows how patterns of demand, conditional on total expenditure, evolve through life stages. In the earliest period budget shares for restaurants and alcohol and tobacco are high. These fall on the arrival of the first child and budget shares for food at home rise. As children age, more is spent (relatively) on clothing. Interestingly, although the post-children life stage patterns show some reversion to the pre-children patterns the two are not the same, even though net income is similar.

The impact of children on consumption emerges even more clearly if we follow quasi-panels through time. To do this we use UK Family Expenditure Surveys from 1968 to 1995.[12] We consider only married or cohabiting couples. To construct quasi-panel data we first construct cohorts according to the wife's age and her level of education ('minimum' or 'more than minimum'). We then take cell means for each cohort and year. That is, we

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[10]Browning and Ejrnæs (2009) present a quasi-panel analysis on UK data that takes into account that some younger 'childless' households will never have children and some older 'childless' households have never had them.

[11]We use the FAMEX since it is the only large expenditure survey that collects information on annual expenditures. Most budget surveys employ a two week diary which induces problems with infrequency.

[12]We use the UK data since it gives a very long time series of cross-sections with consistent coding throughout the period.

have means for, say, high educated households aged 37 in 1981 and those aged 38 in 1982. This allows us to follow quasi-individuals through time. We consider cohort/year means of log nondurable consumption and equivalent household size. To construct the latter we first assign each member a consumption weight according to their age; we take values of 0.1, 0.15, 0.25, 0.35 and 0.65 for children aged $0-2$, $3-4$, $5-10$, $11-16$ and $17-18$ respectively. Each adult is given a weight of unity. We then sum these weights for each household and raise this to the power 0.7 to capture scale effects.[13] In Figures 1.37 and 1.38 we show the smoothed paths of cohort means of log nondurable consumption and equivalent household size against the wife's age. As can be seen, the patterns of consumption and family size coincide very closely. The variation over the life-cycle is substantial and much larger than variation induced by fluctuations in income or employment.

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1.7 Tables and Figures

Year	*Never married*		*Married*		*Divorced*		*Widowed*	
	Women	*Men*	*Women*	*Men*	*Women*	*Men*	*Women*	*Men*
				Canada				
1951	17.6	24.0	71.4	71.5	0.4	0.3	10.6	4.3
1981	15.3	21.5	69.6	73.5	3.5	2.5	11.3	2.5
2003	19.1	26.0	63.5	66.1	7.1	5.4	10.3	2.5
				Denmark				
1950	18.2	21.8	67.5	70.5	3.9	2.7	10.7	5.1
1980	16.8	24.5	62.1	65.5	7.2	5.8	14.0	4.2
2002	24.8	33.5	51.8	54.2	12.8	3.7	10.5	8.6
				France				
1954	16.7	21.6	62.6	71.7	2.2	1.5	18.6	5.3
1980	16.5	23.0	63.4	70.4	4.3	3.1	15.8	3.5
2001	24.8	31.9	53.2	59.0	7.8	6.2	14.2	3.0
				Italy				
1950	24.1	28.5	61.3	66.5	0.3	0.3	14.3	4.8
1980	16.7	22.2	66.3	73.1	1.5	1.3	15.6	3.5
2001	22.0	29.8	60.7	65.8	1.7	1.3	15.7	3.1
				United Kingdom				
1950	19.8	20.7	65.5	73.7	0.7	0.5	13.9	5.1
1980	14.0	19.8	66.2	72.9	4.2	3.3	15.6	4.0
2002	22.9	30.1	54.0	58.1	9.8	7.9	13.2	3.9
				USA				
1950	11.8	17.0	72.3	76.1	2.7	2.2	13.3	4.7
1980	14.1	19.7	64.1	71.4	7.9	6.0	13.9	2.9
2002	18.5	25.0	56.3	61.5	14.3	10.8	10.9	2.7
Source: Census of different countries								

TABLE 1.1. Marital Status of Men and Women, over 20 Years Old, in different Countries and Years

Age group	*USA*			*Denmark*		
	1980	*1990*	*2000*	*1980*	*1990*	*2000*
20 − 24	11.5	25.6	36.3	59.2	76.1	80.8
25 − 29	7.0	12.4	20.2	25.4	41.7	53.1
30 − 34	3.8	7.2	11.7	10.5	20.8	20.4
35 − 39	2.0	5.1	7.0	6.2	9.7	11.5
40 − 44	1.6	3.4	5.3	4.4	6.6	9.0
45 − 49	1.3	2.4	5.1	3.9	5.9	9.0
50 − 54	1.2	2.2	4.1	3.9	5.6	7.2
55 − 59	1.2	1.6	5.0	4.1	5.4	8.2
60 − 64	1.4	1.8	3.1	4.4	5.3	6.1
Source: US Census and Statistics Denmark						

TABLE 1.2. Cohabitation in the US and Denmark by Age of the Household Head

	Single head		*Married couples*		*Cohabiting couples*	
Age	*% of all HH*	*% of single HH with child.*	*% of all HH*	*% of married with child.*	*% of all HH*	*% of cohab. with child.*
20 − 24	69.1	2.1	5.3	51.7	25.6	14.2
25 − 29	42.0	8.3	22.4	72.8	35.7	33.4
30 − 34	28.9	23.4	46.2	88.0	25.0	61.5
35 − 39	25.7	37.4	56.1	91.7	18.3	73.6
40 − 44	25.4	37.4	61.1	82.7	13.4	66.3
45 − 49	25.4	20.0	64.5	51.4	10.2	43.0
50 − 54	23.6	6.9	69.0	18.7	7.5	20.0
55 − 59	24.3	1.8	69.8	5.5	6.0	7.9
60 − 64	26.4	0.3	68.7	1.5	4.9	2.5
Source: Statistics Denmark.						

TABLE 1.3. Household Arrangements, Denmark 2000

	Single head		*Married couples*		*Cohabiting couples*	
Age	*% of all HH*	*% of single HH with child.*	*% of all HH*	*% of married with child.*	*% of all HH*	*% of cohab with child.*
20 − 24	57.4	15.4	27.9	58.3	14.7	36.5
25 − 29	42.1	21.5	47.7	65.4	10.2	39.1
30 − 34	34.4	29.5	59.1	77.9	6.5	47.1
35 − 39	33.6	36.2	61.7	83.9	4.7	47.1
40 − 44	33.9	33.3	62.3	77.2	3.8	37.6
45 − 49	34.8	22.6	62.4	56.3	2.8	20.8
50 − 54	35.5	9.5	62.0	28.6	2.5	12.8
55 − 59	36.1	3.4	62.0	11.0	1.9	5.2
60 − 64	38.3	1.4	60.2	4.4	1.5	1.6
Source: Current Population Surveys						

TABLE 1.4. Household Arrangements, USA, 2000-2005

Sex,Age	*NumberofMarriages*					*Divorced*		*Widowed*	
	0	1	2	3+	1+	Now	Ever	*Now*	*Ever*
Women									
30 − 34	21.7	67.3	10.0	1.0	78.3	9.3	18.6	0.4	0.6
35 − 39	15.6	66.8	15.7	1.8	84.4	13.7	28.1	0.6	1.1
40 − 49	10.5	65.1	19.8	3.3	89.5	16.8	35.4	2.4	3.5
50 − 59	6.4	65.2	22.1	4.1	93.6	17.9	38.9	7.1	9.5
60 − 69	4.1	72.9	17.4	3.1	95.9	12.6	28.4	19.7	23.3
Men									
30 − 34	29.5	60.8	8.7	1.1	70.5	7.0	15.4	–	0.3
35 − 39	21.5	66.2	10.9	1.4	78.5	12.5	22.9	0.2	0.5
40 − 49	14.2	65.1	17.1	3.6	85.8	12.5	29.5	12.5	1.3
50 − 59	6.3	62.6	23.2	8.0	93.7	16.9	40.8	1.8	2.9
60 − 69	4.3	67.5	21.3	6.8	95.7	9.7	30.9	4.5	7.6
Source: U.S. Census Bureau, Survey of Income and Program Participation (SIPP), 2001 Panel, Wave 2 Topical Module									

TABLE 1.5. Marital History by Age and Sex, US, 2001 (percents)

Birth cohort 1931-36								
		1960	*1965*	*1970*	*1975*	*1980*	*1985*	*1990*
Married, first time	*men*	60.84	74.25	74.56	70.25	65.06	61.39	57.45
	women	69.45	69.00	65.28	58.92	52.51	45.36	38.75
Married, second time or more	*men*	2.08	5.57	8.26	12.16	15.15	18.88	20.21
	women	4.95	8.79	11.28	12.95	13.87	13.84	11.5
Divorced, first time	*men*	3.05	3.73	5.02	8.67	8.31	7.76	8.97
	women	4.45	6.37	7.86	13.25	13.50	15.64	17.24
Never married	*men*	26.90	14.64	10.74	9.01	8.30	7.90	7.52
	women	20.10	13.30	10.64	9.29	6.78	8.41	8.03
Birth cohort 1937-41								
Married, first time	*men*	19.59	60.57	71.33	67.38	62.49	58.52	55.09
	women	47.13	66.95	66.05	58.52	50.61	44.05	36.72
Married, second time or more	*men*	0.47	2.60	7.47	12.27	16.92	19.24	21.49
	Women	2.01	6.31	10.17	12.89	14.96	16.33	16.82
Divorced, first time	*men*	1.04	4.20	5.40	6.90	9.86	10.52	9.78
	women	2.65	5.59	8.88	11.03	16.68	18.48	21.40
Never married	*men*	63.90	26.43	13.57	9.88	8.32	7.66	7.24
	women	47.70	20.22	12.54	10.24	9.23	8.81	8.59
Source: Health and Retirement Survey, 1992								

TABLE 1.6. Marital History, US, of the 1931-36 and 1937-41 Birth Cohorts

Marital and educational Status	*Males N=2085*	*Females N=2202*
Not married no child at age 35	0.15	0.08
Married before age 36	0.81	0.89
Had child before age 36	0.67	0.78
Divorced before age 36	0.26	0.35
Finished school before age 36	0.90	0.90
Had child before first marriage	0.10	0.16
Married before finishing school	0.16	0.21
Source: National Longitudinal Survey, Youth, 1979		

TABLE 1.7. Marital History of the NLS Panel

	%
Belgium	9
Denmark	21.9
Germany	17
Greece	9
Spain	5
France	13
Ireland	8
Italy	10
Netherlands	14
Austria	14
Portugal	5
Finland (2000)	23
UK	13
USA	13.7
Source: Census of different countries	

TABLE 1.8. Individuals Living Alone, various countries

Age group	*Percentage living alone*
18-30	19.8
31-40	13.2
41-50	13.4
51-60	17.7
61-70	25.9
71+	51.9
Source: Statistics Denmark	

TABLE 1.9. Individuals Living Alone, Denmark

	USA		Canada		UK		Norway	
Year of survey	1975	2003	1971	1998	1975	2000	1971	2000
Paid work								
Single men	5.55	5.39	5.31	5.30	6.48	4.80	6.01	5.18
Single women	4.39	4.71	4.84	4.27	4.11	3.49	4.17	3.59
Married men, no children	6.13	6.32	6.37	6.39	6.64	5.91	6.20	5.89
Married women, no child	3.42	4.51	3.38	4.39	3.38	3.99	3.12	4.60
Married men, child 5-17	7.17	6.40	6.16	6.80	6.70	6.17	6.06	5.78
Married women, child 5-17	2.71	3.68	1.97	4.08	2.46	3.54	1.86	4.28
Married men, child < 5	6.98	6.39	6.13	6.21	6.59	6.16	6.58	5.73
Married women, child < 5	1.55	2.81	1.11	2.64	0.82	2.45	0.91	2.58
Leisure								
Single men	6.94	6.82	7.20	7.29	6.49	7.22	5.91	7.04
Single women	6.23	6.04	5.86	6.43	6.05	6.44	5.13	6.86
Married men, no children	6.14	6.09	6.25	5.96	5.83	6.13	5.33	6.21
Married women, no child	6.29	5.99	5.93	5.99	5.86	5.87	5.17	6.11
Married men, child 5-17	5.38	5.49	5.92	5.41	5.67	5.66	5.10	6.06
Married women, child 5-17	6.14	5.61	5.57	5.51	5.76	5.38	4.75	5.98
Married men, child < 5	5.43	4.93	5.39	4.93	5.78	5.10	4.93	5.43
Married women, child < 5	5.98	5.01	5.17	4.87	6.25	5.09	4.98	5.70
Source: Multinational Time Use Study								

TABLE 1.10. Hours of Work and Leisure per Day

	USA		Canada		UK		Norway	
Year of survey	1975	2003	1971	1998	1975	2000	1971	2000
Home production								
Single men	1.05	1.27	1.19	1.14	0.61	1.28	1.12	1.19
Single women	2.06	1.72	1.84	2.03	2.47	2.34	2.74	1.77
Married men, no children	1.25	1.52	9.79	1.57	1.07	1.65	1.53	1.64
Married women, no child	2.88	2.51	3.80	2.77	3.38	3.02	4.20	2.47
Married men, child 5-17	1.18	1.52	1.56	1.63	0.97	1.70	1.61	1.93
Married women, child 5-17	3.63	2.83	4.55	3.29	4.01	3.37	5.48	2.75
Married men, child < 5	1.10	1.38	1.83	1.66	0.90	1.42	1.37	1.64
Married women, child < 5	3.67	2.64	4.79	3.03	4.13	3.03	5.03	2.65
Child care								
Single men	0.03	0.08	0.03	0.04	0.02	0.03	0.02	0.04
Single women	0.36	0.48	0.15	0.43	0.23	0.47	0.19	0.33
Married men, no children	**	**	**	**	**	**	**	**
Married women, no child	**	**	**	**	**	**	**	**
Married men, child 5-17	0.20	0.57	0.14	0.41	0.06	0.26	0.23	0.32
Married women, child 5-17	0.65	1.13	0.64	0.77	0.30	0.58	0.65	0.59
Married men, child < 5	0.40	1.24	1.21	1.47	0.28	1.04	0.54	1.23
Married women, child < 5	1.63	2.67	2.16	2.97	1.28	2.57	2.08	2.61
Shopping								
Single men	0.24	0.35	0.31	0.41	0.24	0.33	0.21	0.28
Single women	0.49	0.49	0.23	0.57	0.53	0.55	0.29	0.46
Married men, no children	0.32	0.37	0.82	0.42	0.24	0.33	0.19	0.31
Married women, no child	0.53	0.54	0.37	0.53	0.56	0.54	0.28	0.37
Married men, child 5-17	0.24	0.34	0.33	0.35	0.25	0.33	0.22	0.35
Married women, child 5-17	0.59	0.61	0.54	0.59	0.63	0.57	0.34	0.39
Married men, child < 5	0.28	0.39	0.23	0.37	0.23	0.34	0.22	0.26
Married women, child < 5	0.50	0.60	0.55	0.58	0.66	0.58	0.36	0.42
Source: Multinational Time Use Study								

TABLE 1.11. Hours per Day of Home Production, Childcare and Shopping

Country	Male Participation Rates 1965	1975	1985	1995	2005	Female Participation Rates 1965	1975	1985	1995	2005
US	80.7	77.9	76.3	75.0	73.3	39.3	46.3	54.5	58.9	59.3
Canada	79.9	78.4	77.4	72.7	72.7	33.8	44.4	54.9	57.3	61.4
Australia	85.1	82.2	76.7	74.6	73.0	34.8	44.5	47.1	74.7	58.1
Japan	81.1	81.2	77.9	77.5	73.1	48.8	44.8	47.6	49.3	47.7
France	79.2	74.4	68.4	63.4	63.3*	38.2	41.7	46.4	48.2	51.1*
Germany	80.9	73.4	70.1	68.1	63.9*	40.0	39.3	41.1	47.1	49.6*
Italy	77.5	70.6	65.3	61.6	61.1*	27.8	26.8	30.7	34.4	38.2*
Nether.	NA	80.0	73.8	69.8	72.7	NA	29.5	37.9	48.1	57.8
Sweden	82.2	77.0	72.5	68.9	67.8*	46.6	55.2	61.5	59.5	59.7*
UK	85.4	81.2	76.1	72.0	70.5	41.7	46.6	50.7	53.5	56.2

Source: Comparative Civilian Labor Force Statistics, 10 Countries, 1960-2005,
US Department of Labor, 2006.
Note: * Observation from 2004.

TABLE 1.12. Labor Force Participation of Women and Men in Ten Countries

Marital Status	*School years*							
	Women				*Men*			
	< 12	12	$13-15$	16+	< 12	12	$13-15$	16+
Unmarried	9.7	9.2	10.6	14.3	19.7	18.8	18.9	18.6
Married	28.3	50.8	50.2	61.9	34.9	50.8	52.7	64.0
Separated	31.4	19.1	22.0	11.4	26.1	15.2	16.9	7.8
Divorced	30.6	20.9	17.2	12.7	19.4	17.2	11.6	9.6

Source: National Longitudinal Study, Youth, 1979.

TABLE 1.13. Marital Status at Age 35, by Gender and Education at Age 35

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Marital Status at age 35						
	Women			*Men*		
		Yearsofeducation			*Years of education*	
	%	*at age35*	*at marr.*	*%*	*at age 35*	*at marr.*
Never married	8.5	13.9	–	15.7	13.6	–
Married,never divorced	54.0	13.9	13.5	56.0	13.8	13.5
Married with prior separation	37.6	12.7	11.9	28.3	12.3	11.9
Source: National Longitudinal Study, Youth, 1979.						

TABLE 1.14. Years of Schooling at Marriage and at Age 35, by Gender and Marital Status at Age 35

	Number of Children				
	0	1	2	3	4+
Born 1932-1936	10.2	9.6	21.7	22.7	35.8
Born 1956-1960	19.0	16.4	35.0	19.1	10.5
Source: US Census					

TABLE 1.15. Completed Fertility for Two US Cohorts

Year	*Children with two parents*	*Children with mother*	*Children with father*
1950	93	5.9	1.3
1960	91	7.4	1.2
1970	87	10.7	1.5
1980	81	16.8	2.7
1990	76	19.5	4.4
2000	72	21.9	6.3
2005	68	23.4	4.7
Source: US Census (percentage)			

TABLE 1.16. Living Arrangements of U.S. Children, Aged less than 18, by Year

Mother's Age	*20-30*		*31-40*		*41-60*	
1979-92	*Div.*	*Mar.*	*Div.*	*Mar.*	*Div.*	*Mar.*
Prop. with CS>0	0.452	0.062	0.509	0.068	0.428	0.037
CS, if CS>0	1905	1320	2947	1797	3660	1859
Mother's Inc. if CS>0	10728	7190	15230	11444	17138	12858
Mother's Inc.	8834	5218	12952	7618	13398	7725
Family Inc. if CS>0	11210	25868	16085	36945	20499	44399
Family Inc.	9918	23867	14045	33247	17894	38095
Observations	8071	74900	14410	107108	7536	57936
1993-2004	*Div.*	*Mar.*	*Div.*	*Mar.*	*Div.*	*Mar.*
Prop. with CS>0	0.463	0.049	0.502	0.054	0.454	0.034
CS, if CS>0	1920	1664	2959	2368	4023	2775
Mother's Inc. if CS>0	11351	9195	16873	13854	21958	17564
Mother's Inc.	9699	7086	14544	10989	17779	13550
Family Inc. if CS>0	11731	27313	17644	39240	23929	49227
Family Inc.	10825	26298	15720	39599	21675	49148
Observations	4171	40686	12312	88472	10427	63332
Source: Current Population Surveys.						

TABLE 1.17. Child Support and Alimony Received by Mothers with Children 0-18 (in 1982-84 dollars) by Mother's Age and Time Period

	No children	*Children, at*	*Children*	*No children*
	wife ≤ 40	*least one* ≤ 6	*all aged* > 6	*wife* > 40
Sample size	1,255	2,367	1,965	1,217
Net income	50,060	48,425	52,889	50,045
Nondur expend	23,484	25,768	27,947	21,560
Ex/inc ratio	0.50	0.56	0.56	0.47
Selected budget shares (%)				
Food at home	16.1	22.1	23.1	21.7
Restaurants	10.9	6.6	7.0	8.1
Clothing	13.7	11.6	13.6	10.9
Alc and tob	9.2	6.1	6.0	7.7
Recreation	13.6	11.3	12.9	11.1
Source: Canadian Family Expenditure Surveys.				
All monetary values in 1992 Canadian dollars				

TABLE 1.18. Consumption Through Life Stages

Variable	*Constant*	*Log income*	*Stage 2*	*Stage 3*	*Stage 4*
Coefficient	3.38	0.62	0.12	0.14	−0.08
t-value		78	13	16	−8
$R^2 = 0.51$					
Source: Canadian Family Expenditure Surveys.					

TABLE 1.19. Descriptive Regression for Log Nondurable Consumption

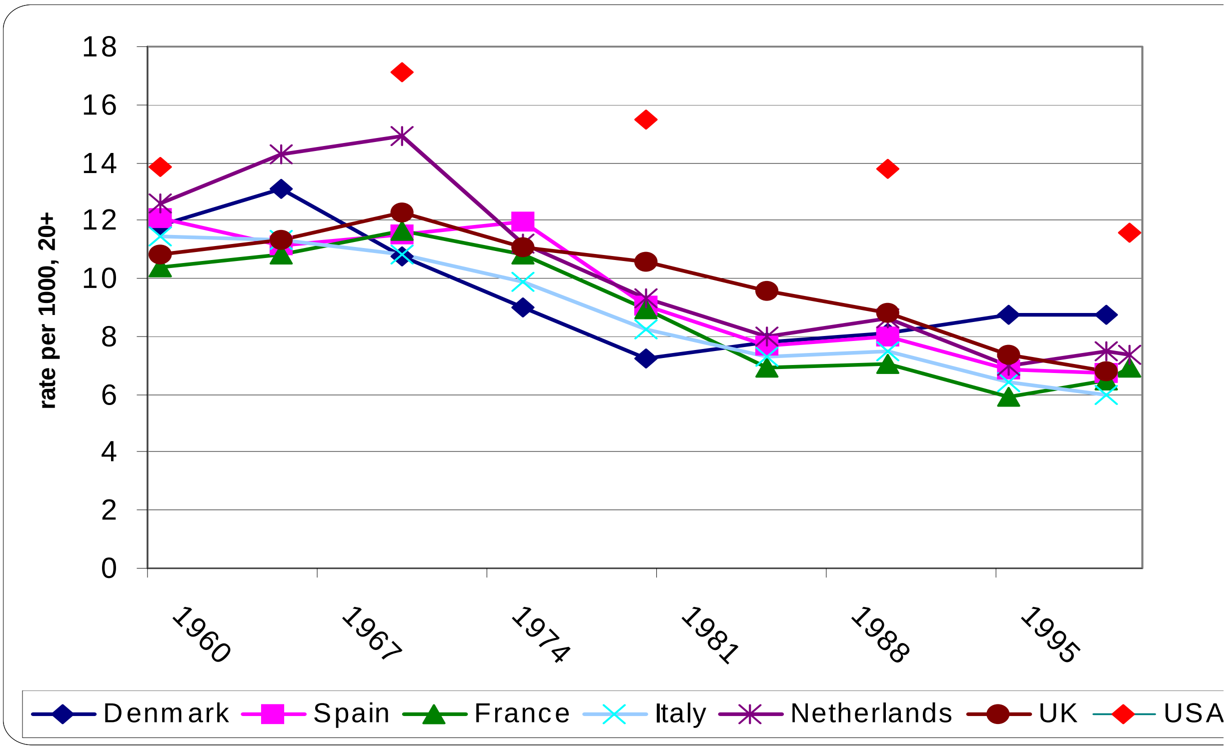

FIGURE 1.1. Marriage Rates for Selected Countries. Source: Eurostat.

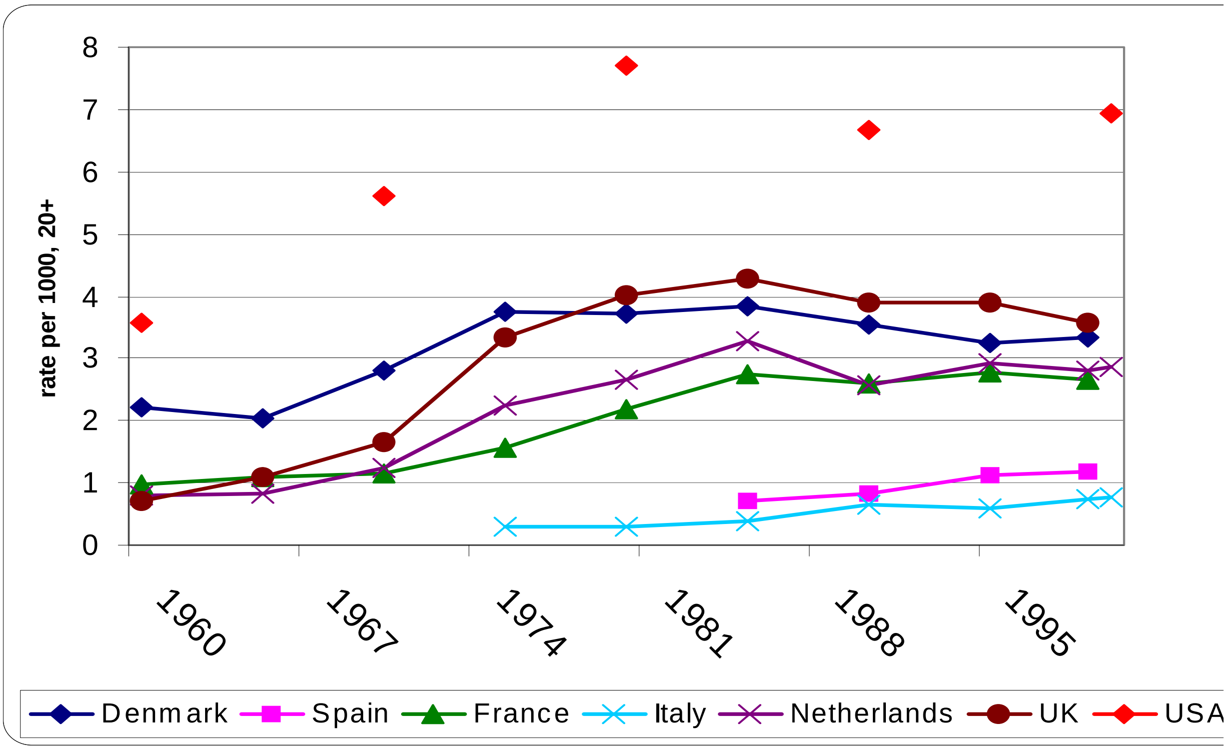

FIGURE 1.2. Divorce Rates for Selected Countries. Source: Eurostat.

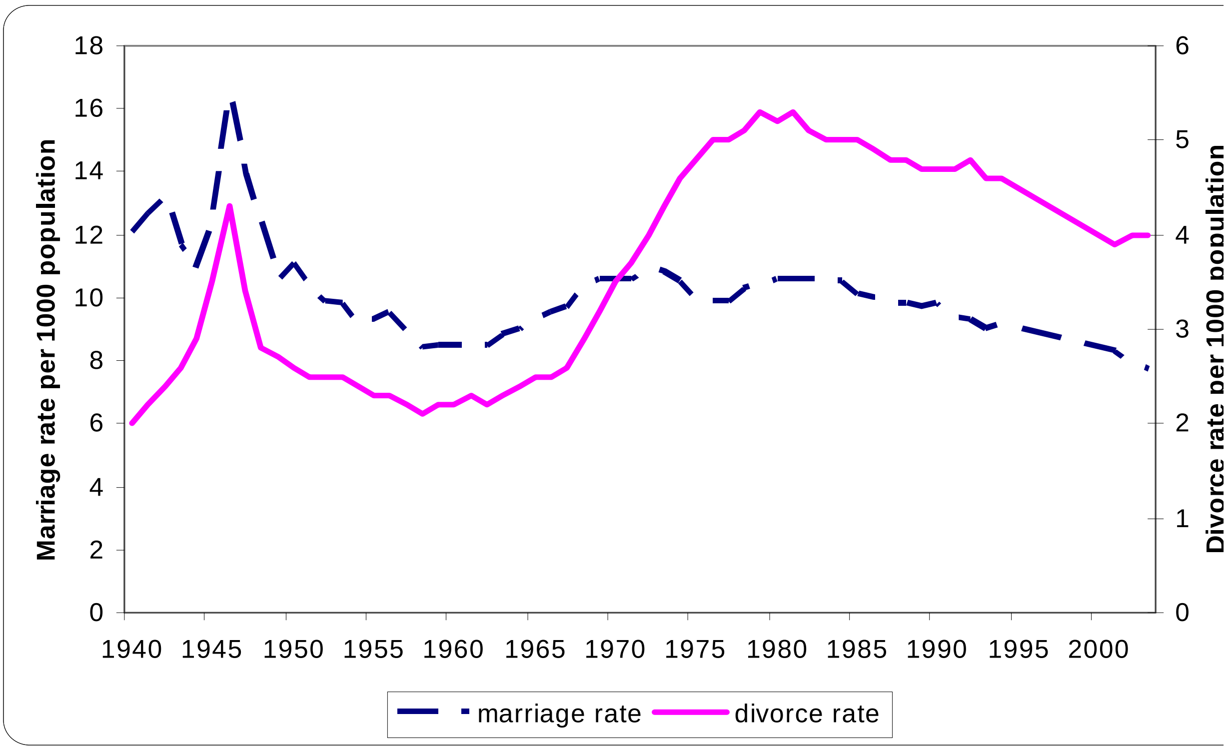

FIGURE 1.3. Marriage and Divorce Rates, US 1940-2002. Source: National Center of Health Statistics.

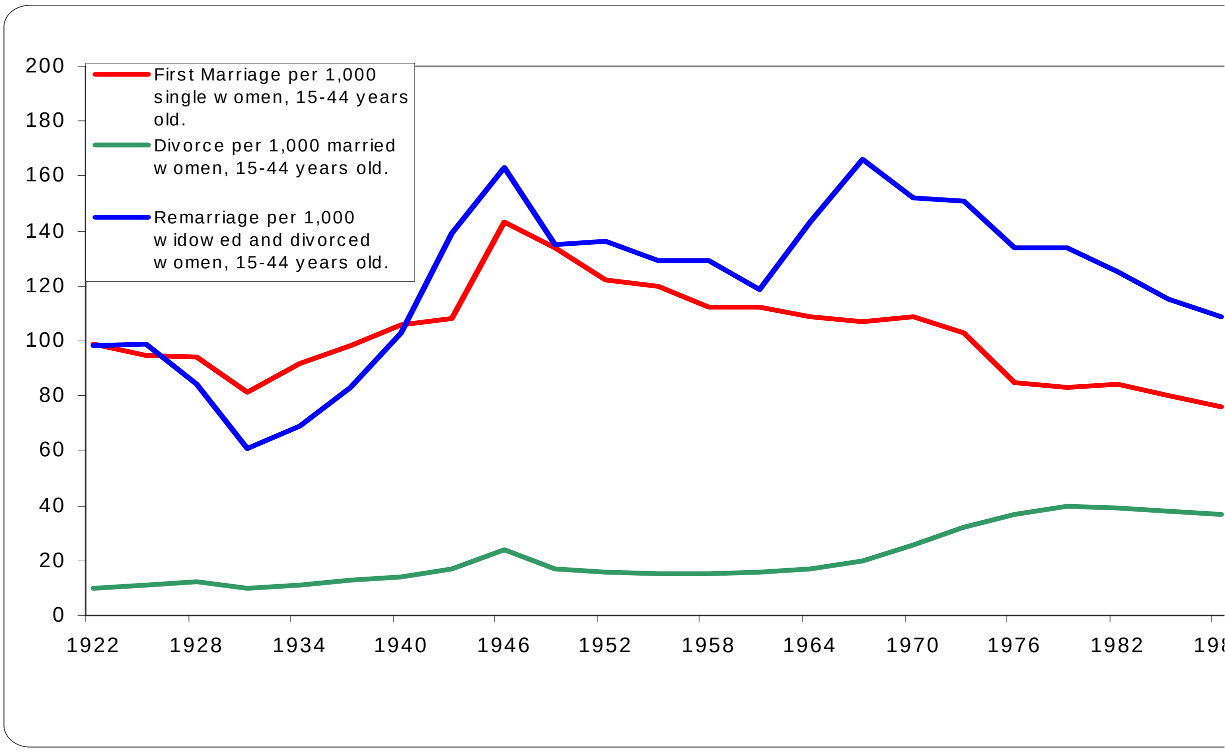

FIGURE 1.4. Marriage, Divorce and Remarriage Rates, US 1921-1989. Source: National Center of Health Statistics.

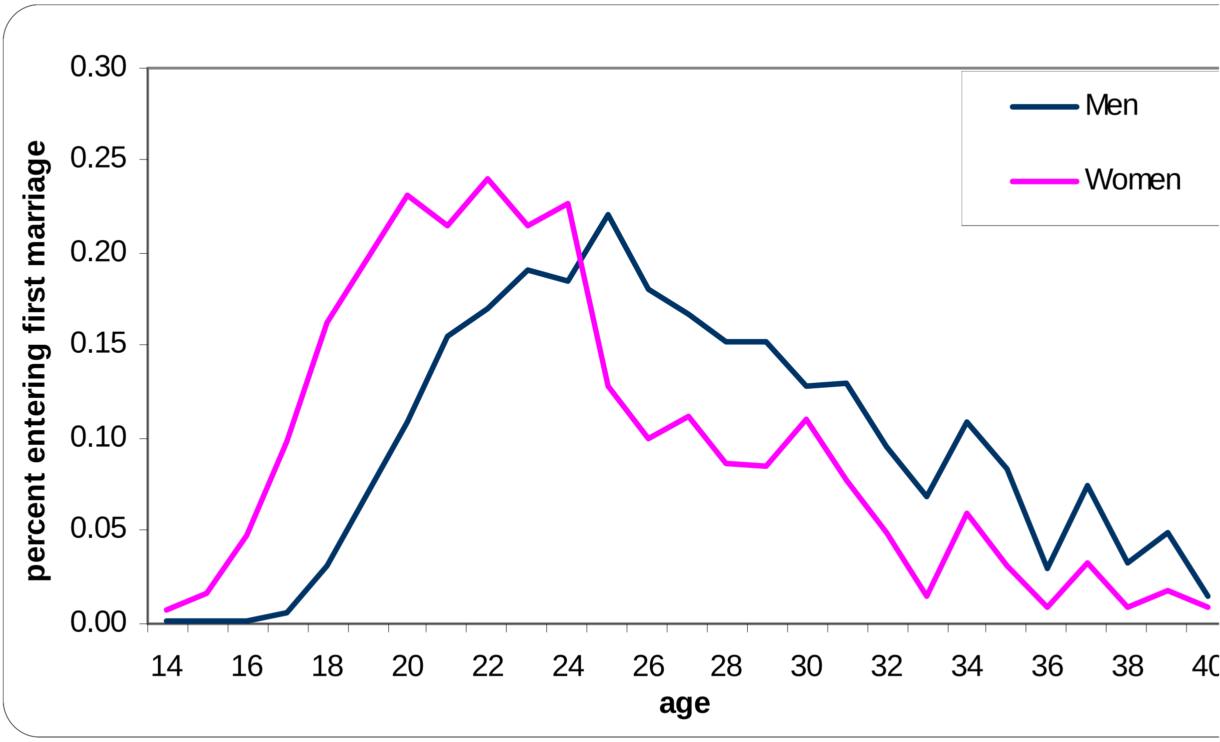

FIGURE 1.5. Entry into First Marriage, US, HRS Cohort. Source: Health and Retirement Survey, 1992 Wave.

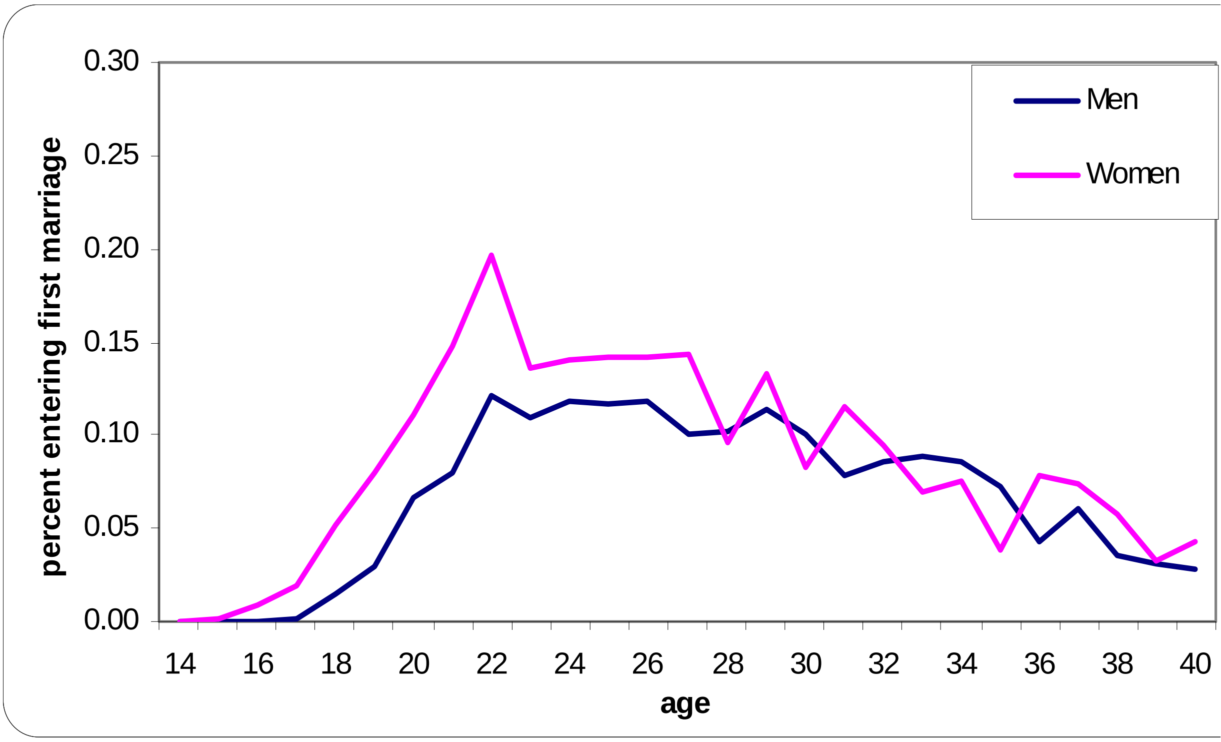

FIGURE 1.6. Entry into First Marriage, US, NLS Panel. Source: National Longitudinal Survey, Youth,1979.

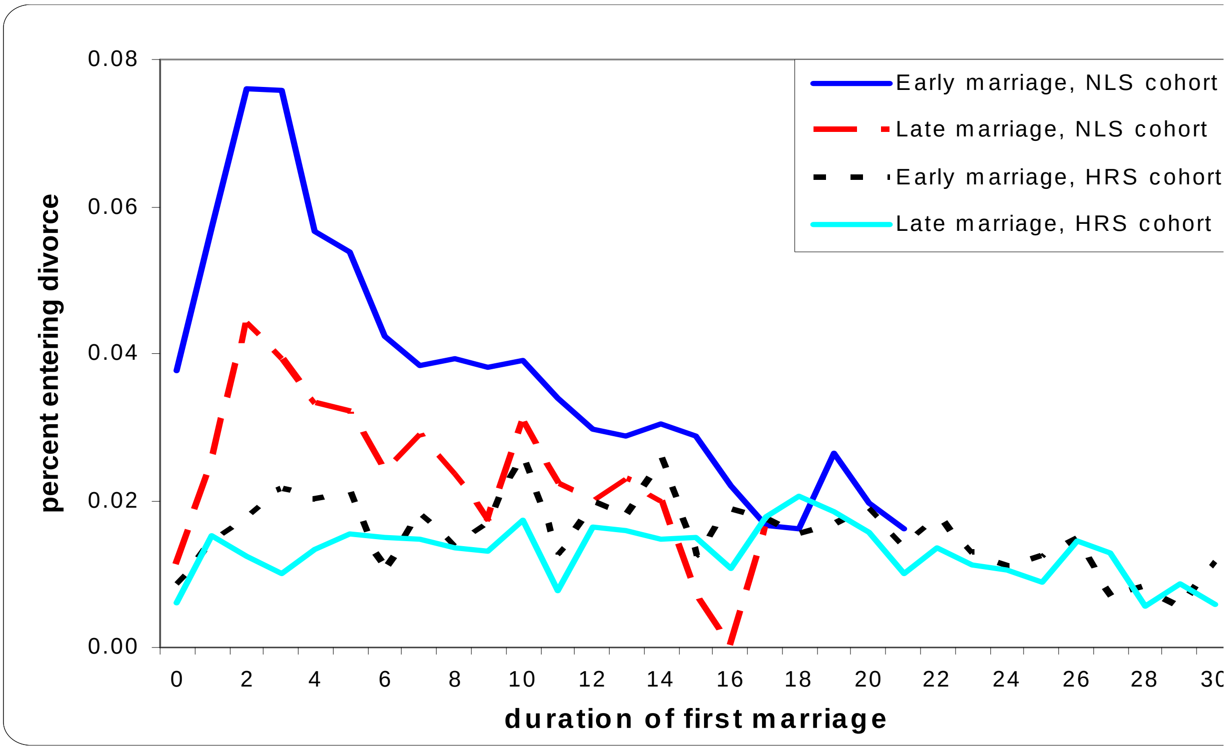

FIGURE 1.7. Hazard of Divorce for two Birth Cohorts, US. Source: National Longitudinal Survey, Health and Retirement Survey.

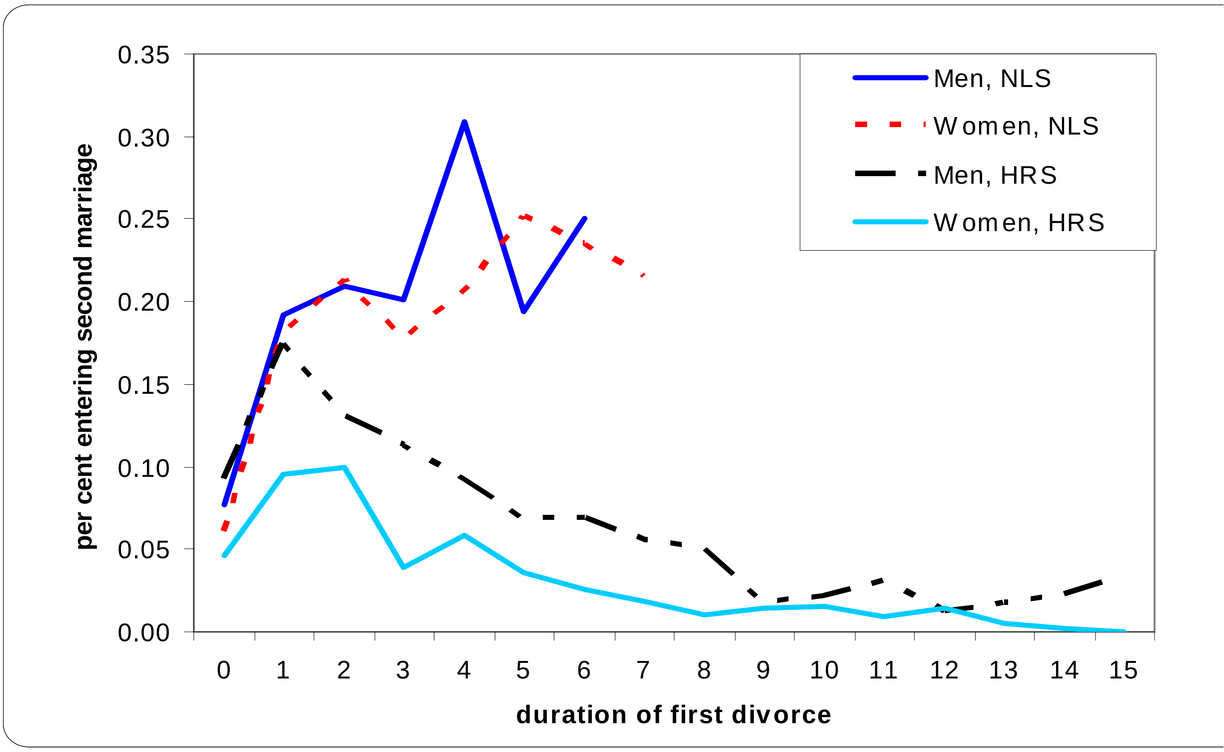

FIGURE 1.8. Hazard of Reamarriage, US. Source: National Longitudinal Survey, Health and Retirement Survey.

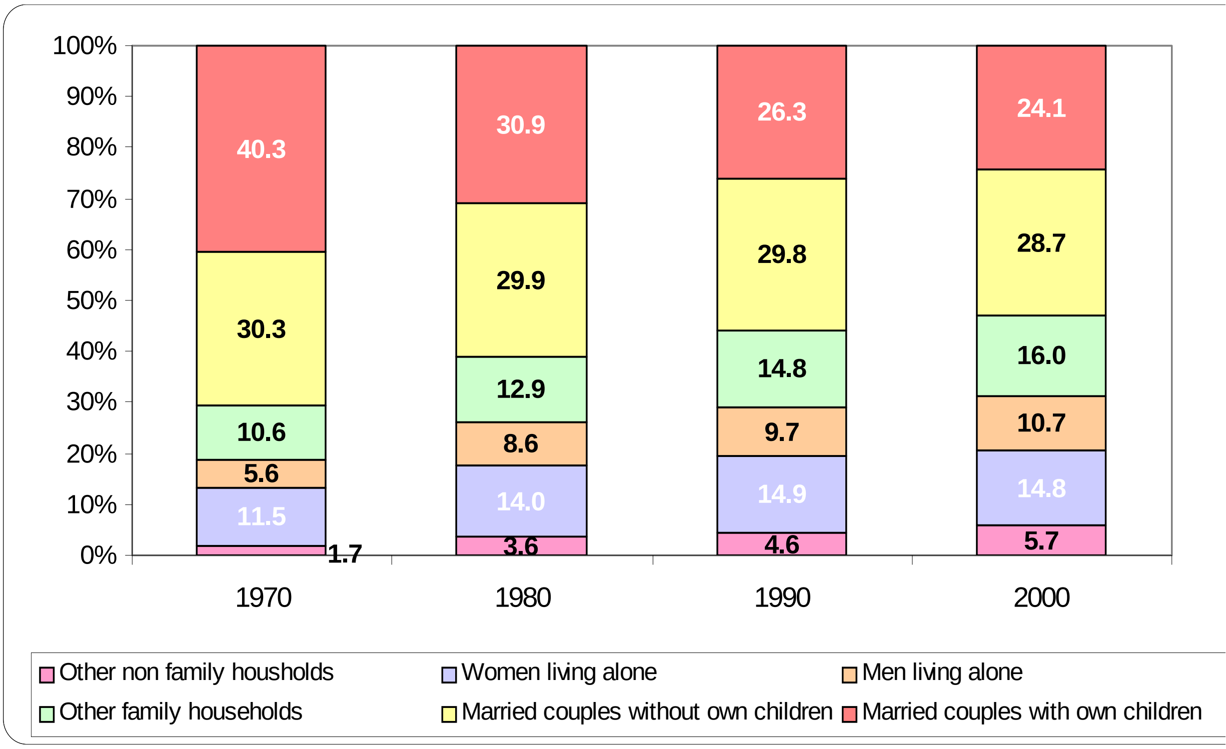

FIGURE 1.9. Households by Type: Selected Years, US. Source: US Census.

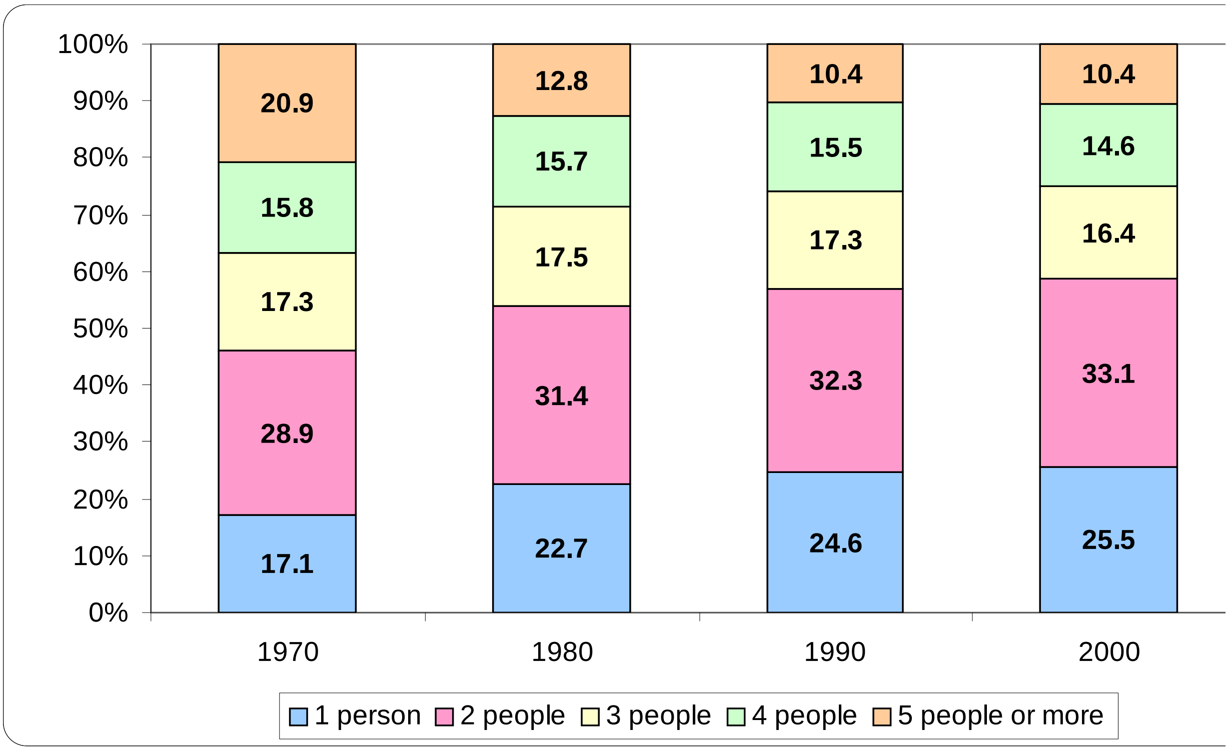

FIGURE 1.10. Households by Size, Selected Years, US. Source: US Census.

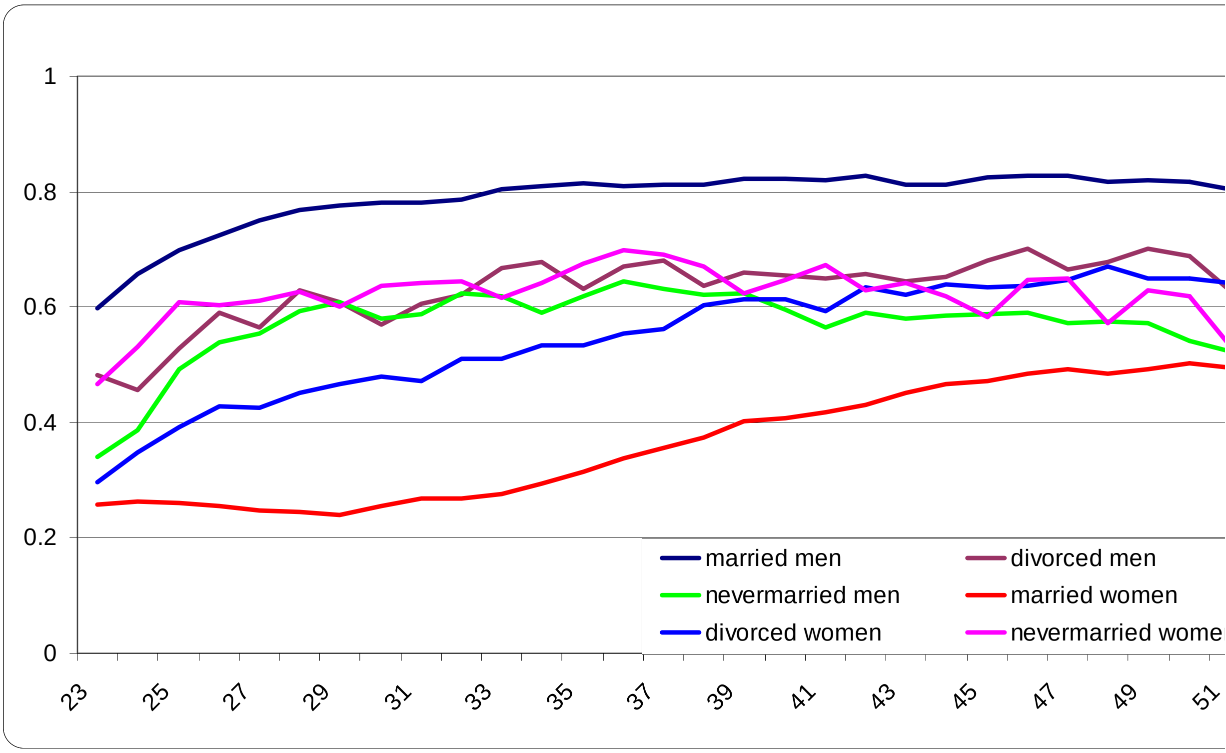

FIGURE 1.11. Full-Time Workers, by Marital Status and Sex, US Birth Cohort 1945-54. Source: Current Population Surveys.

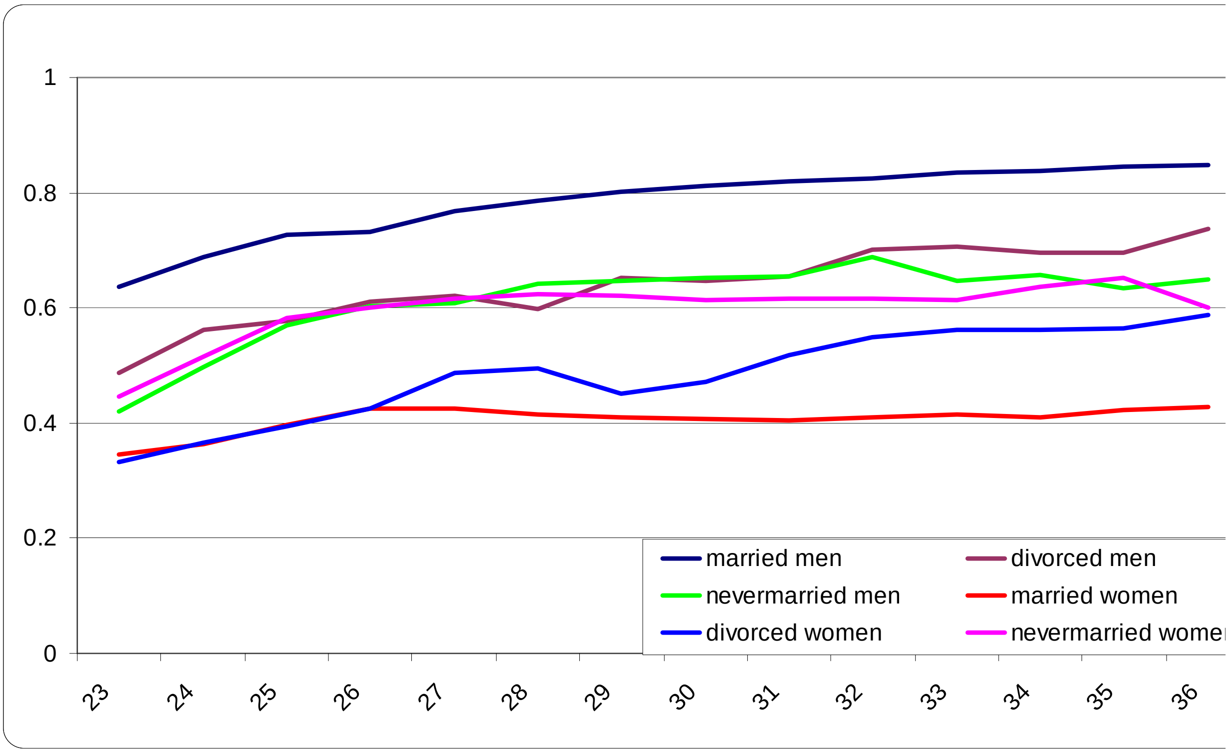

FIGURE 1.12. Full-Time Workers, by Marital Status and Sex, US Birth Cohort 1960–69. Source: Current Population Surveys.

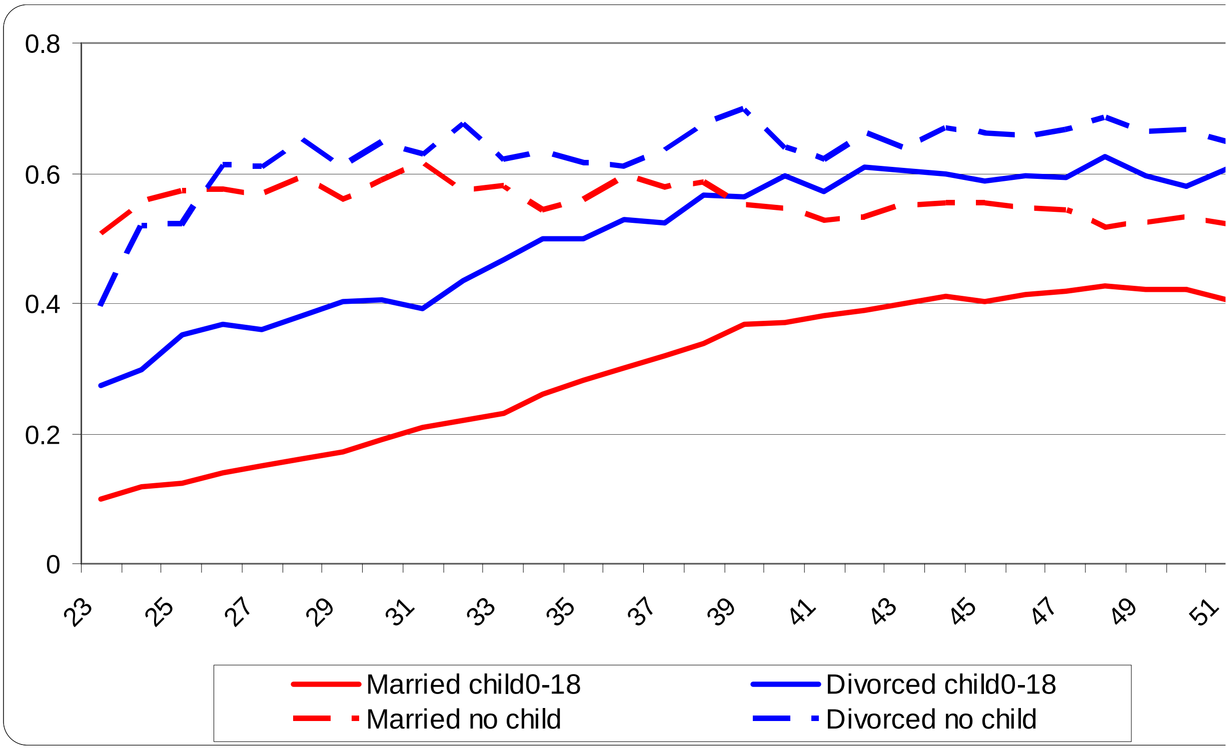

FIGURE 1.13. Full-Time Female Workers, by Marital Status and Child, US Birth Cohort 1945-54. Source: Current Population Surveys.

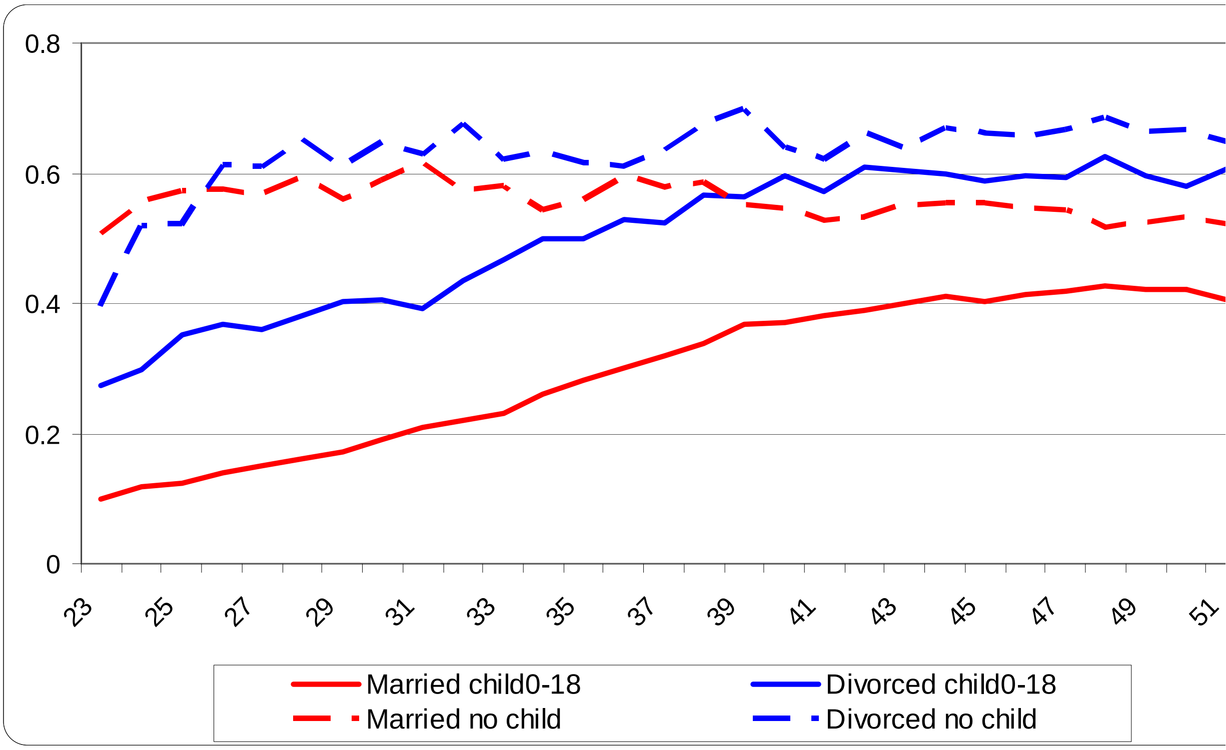

FIGURE 1.14. Full-Time Female Workers, by Marital Status and Child, US Birth Cohort 1960-69. Source: Current Population Surveys.

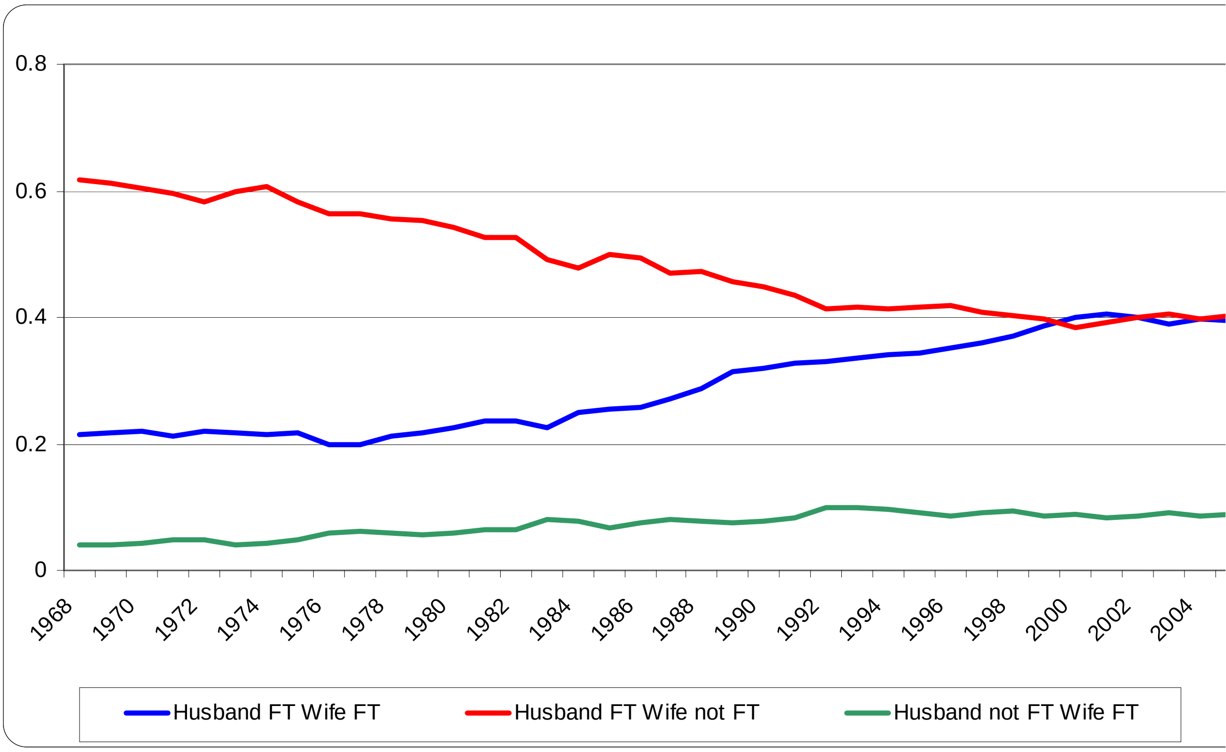

FIGURE 1.15. Work Patterns of Husbands and Wives (Aged 40-60). Source: Current Population Surveys.

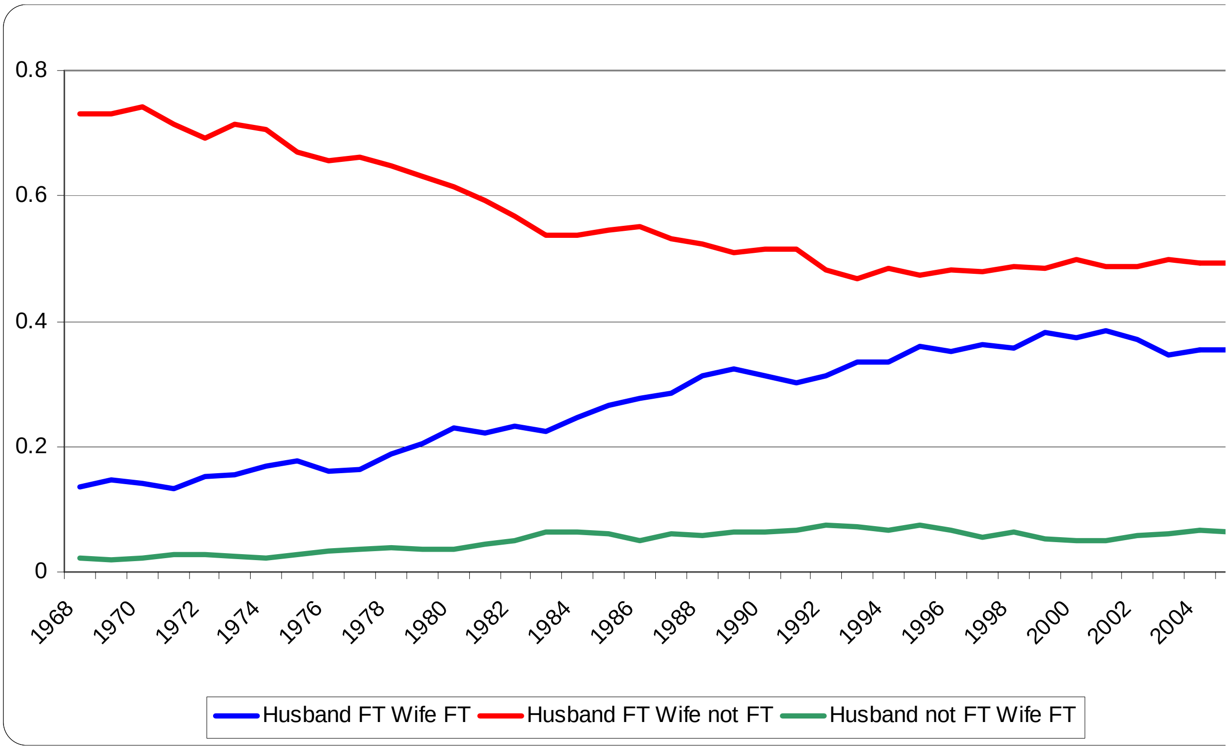

FIGURE 1.16. Work Patterns of Husbands and Wives (Aged 30-40). Source: Current Population Surveys.

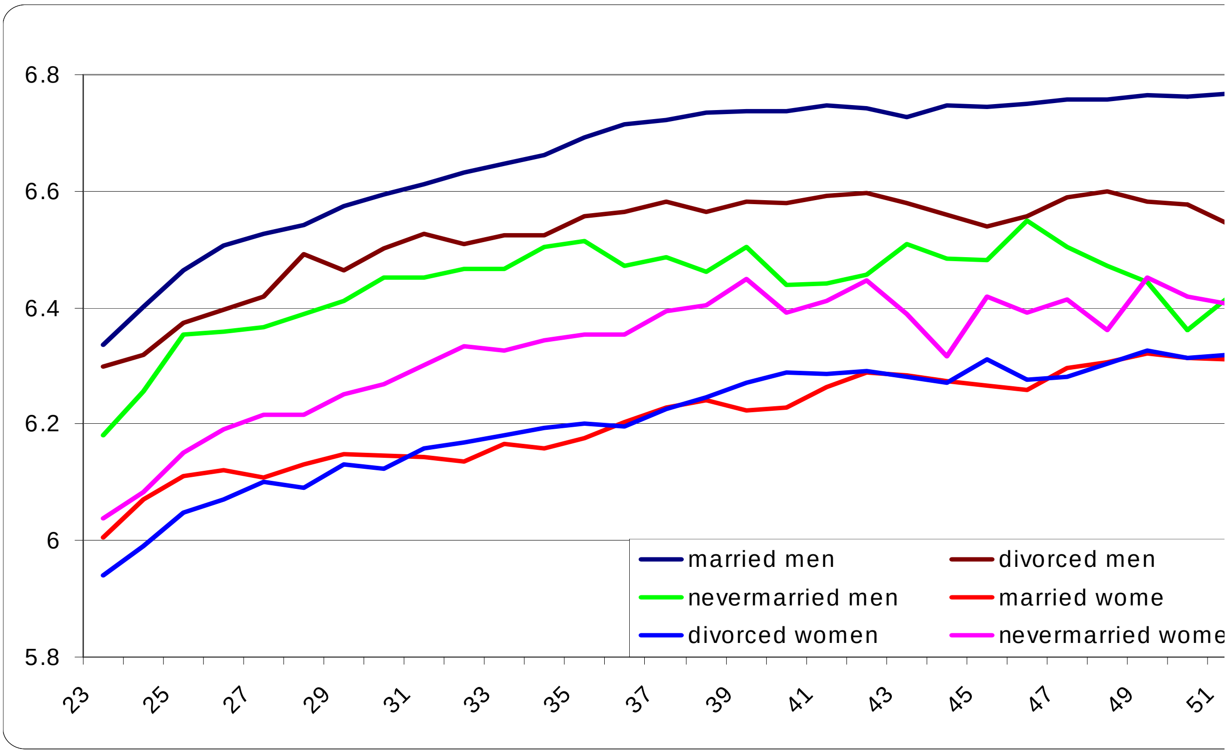

FIGURE 1.17. Weekly Wages of FT Workers, by Marital Status and Sex, US Birth Cohort 1945-54. Source: Current Population Surveys.

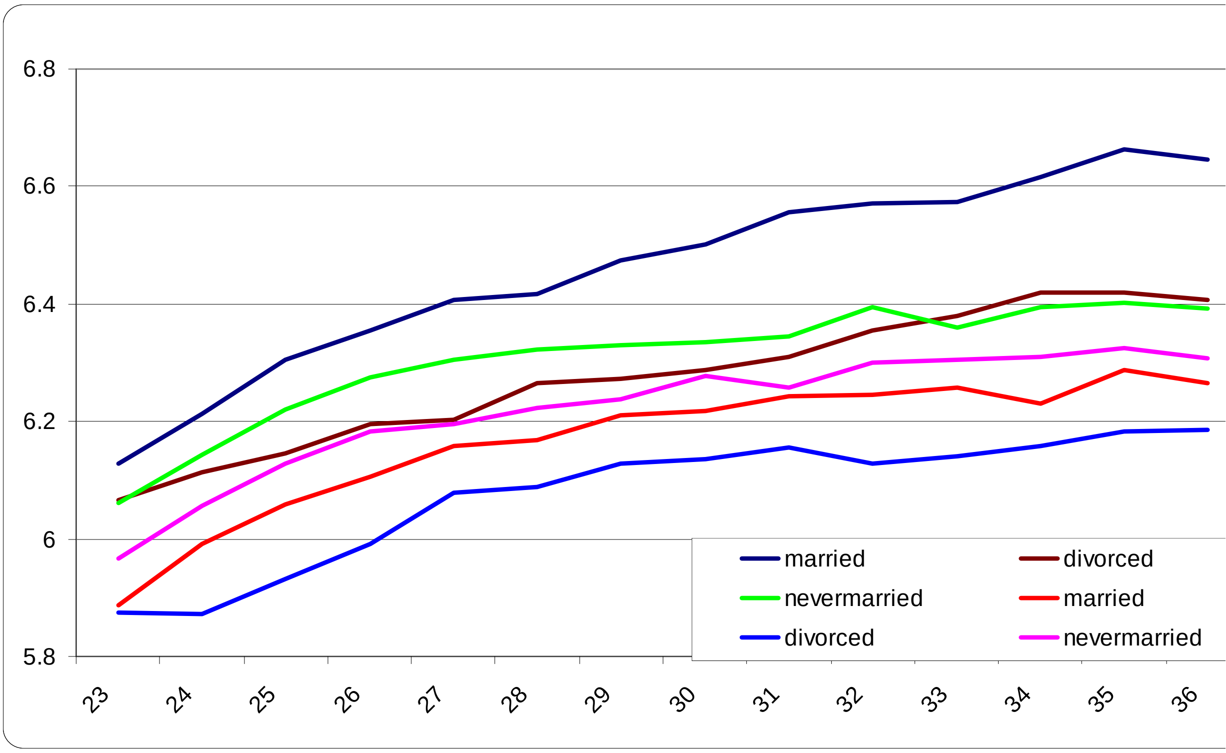

FIGURE 1.18. Weekly Wages of FT Workers, by Marital Status and Sex, US Birth Cohort 1960-69. Source: Current Population Surveys.

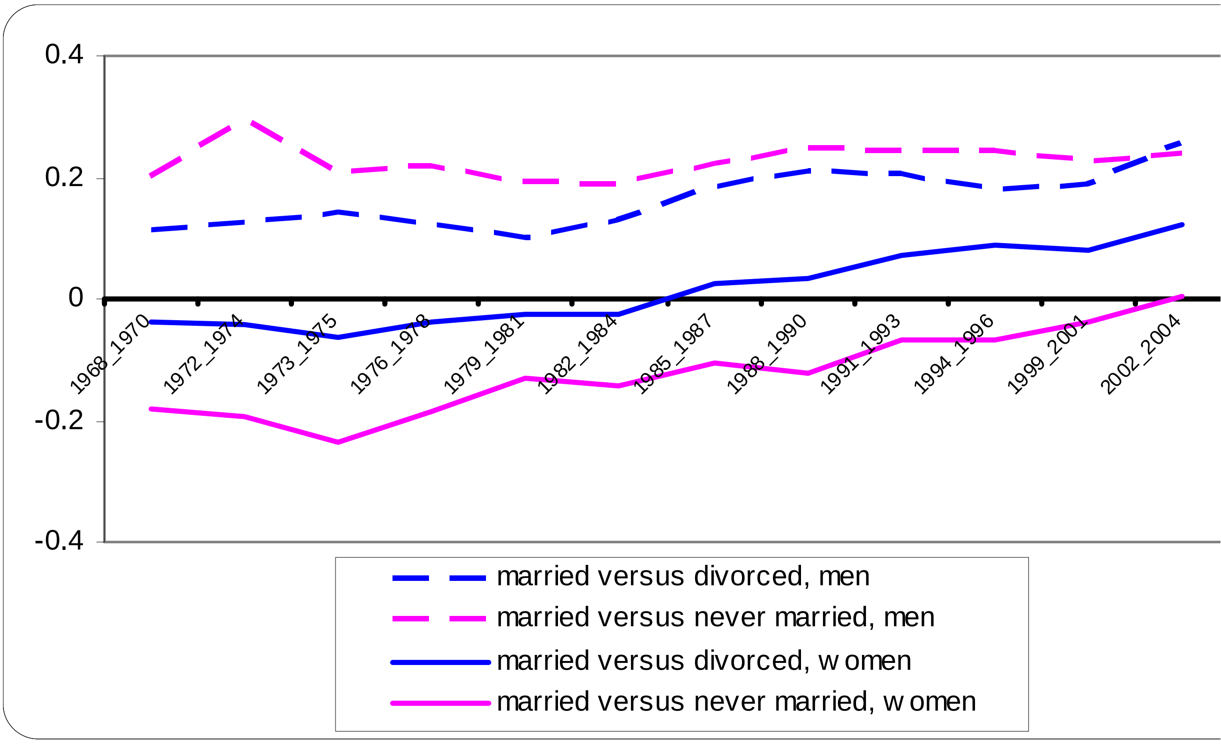

FIGURE 1.19. Log Wages Differnces between Married and Singles, by Sex, US 1968–2005. Source: Current Population Surveys.

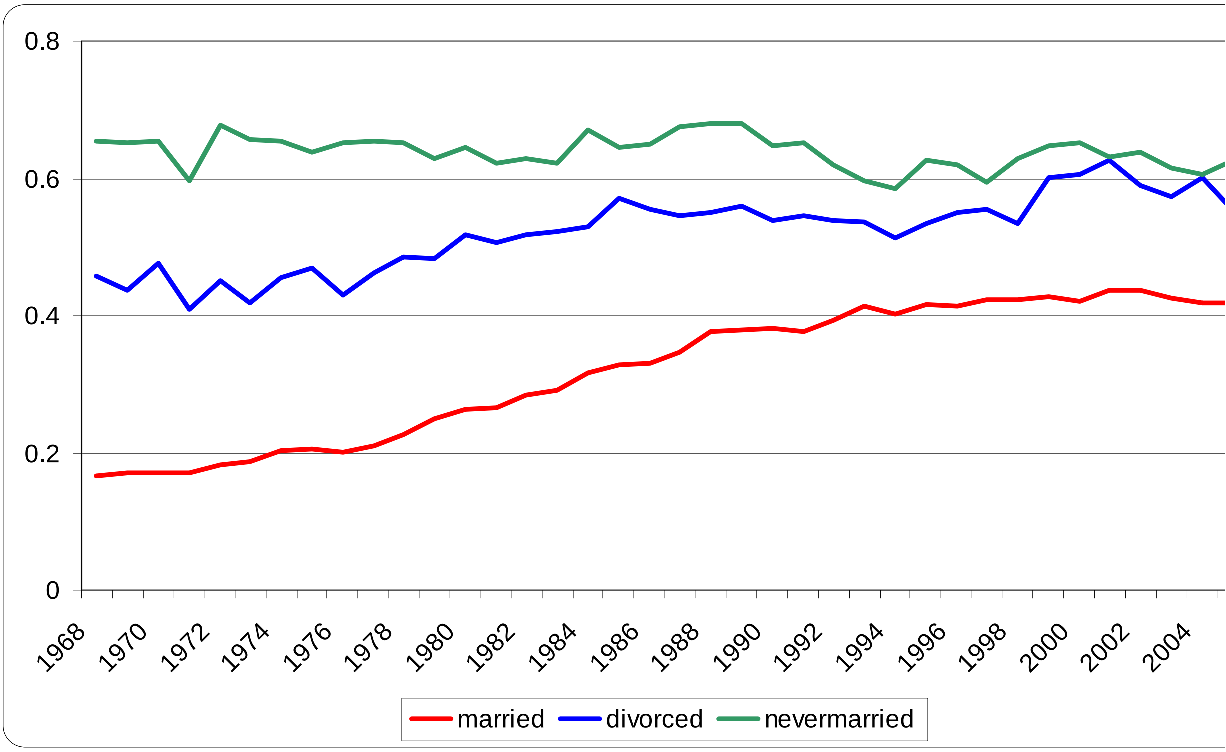

FIGURE 1.20. Women Working Full Time, by Marital Status, US 1968–2005.

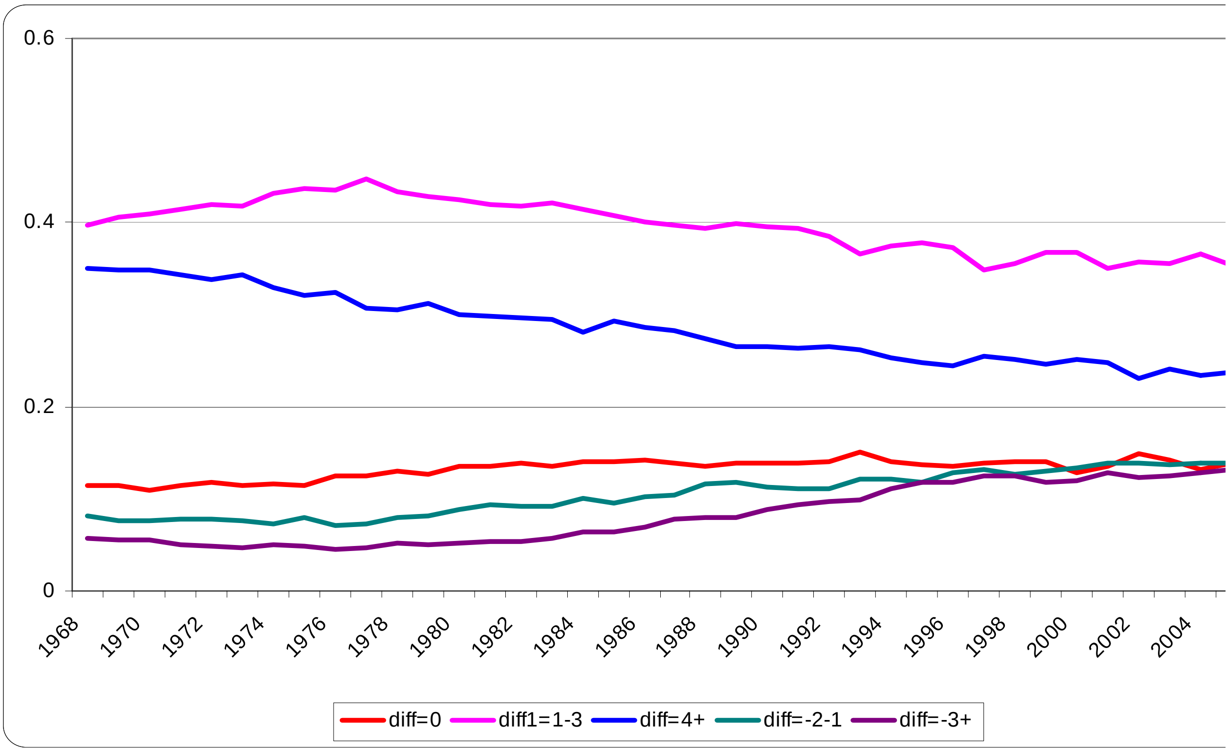

FIGURE 1.21. Age Differences between Husbands and Wives, US 1968-2005. Source: Current Population Surveys.

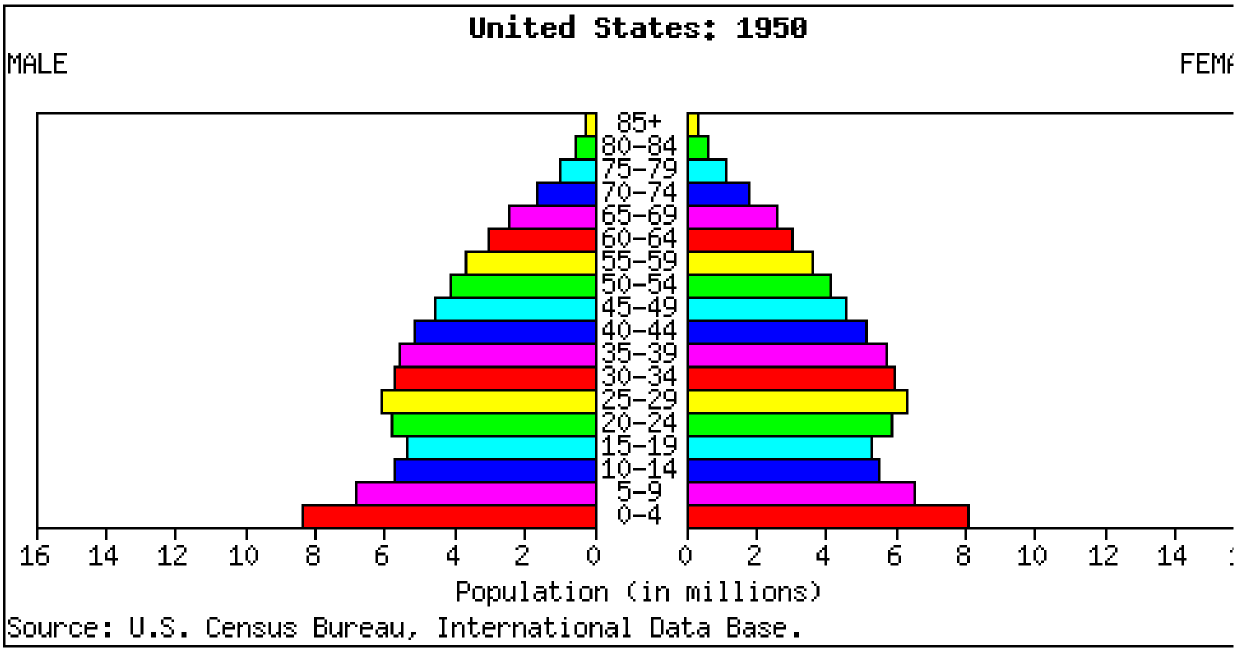

FIGURE 1.22. Age Pyramid , US 1950. Source: US Census.

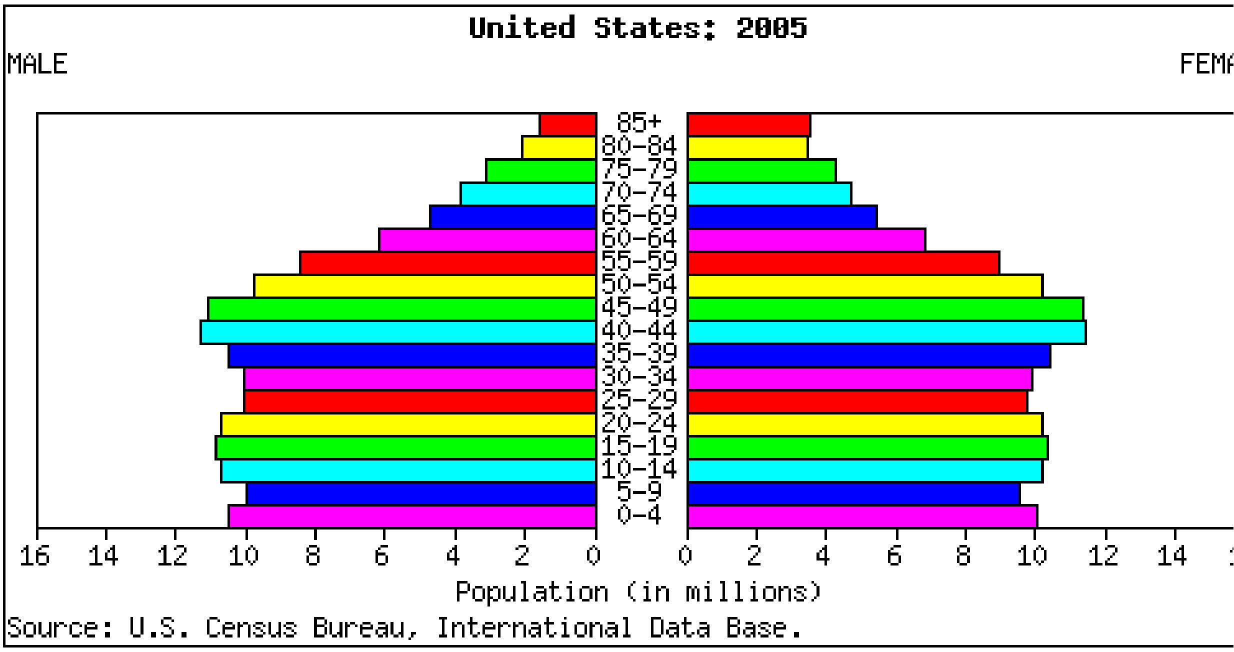

FIGURE 1.23. Age Pyramid , US 2005. Source: US Census.

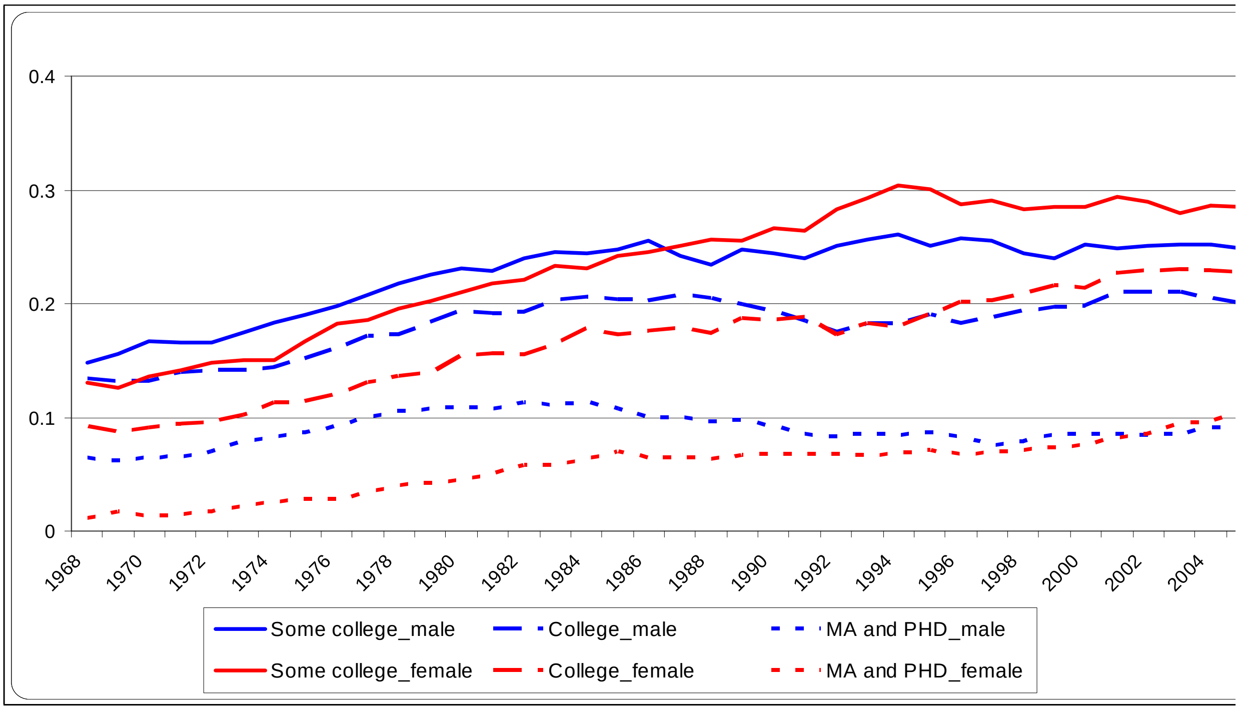

FIGURE 1.24. Completed Education by Sex, Age 30-40, US 1968-2005. Source: Current Population Surveys.

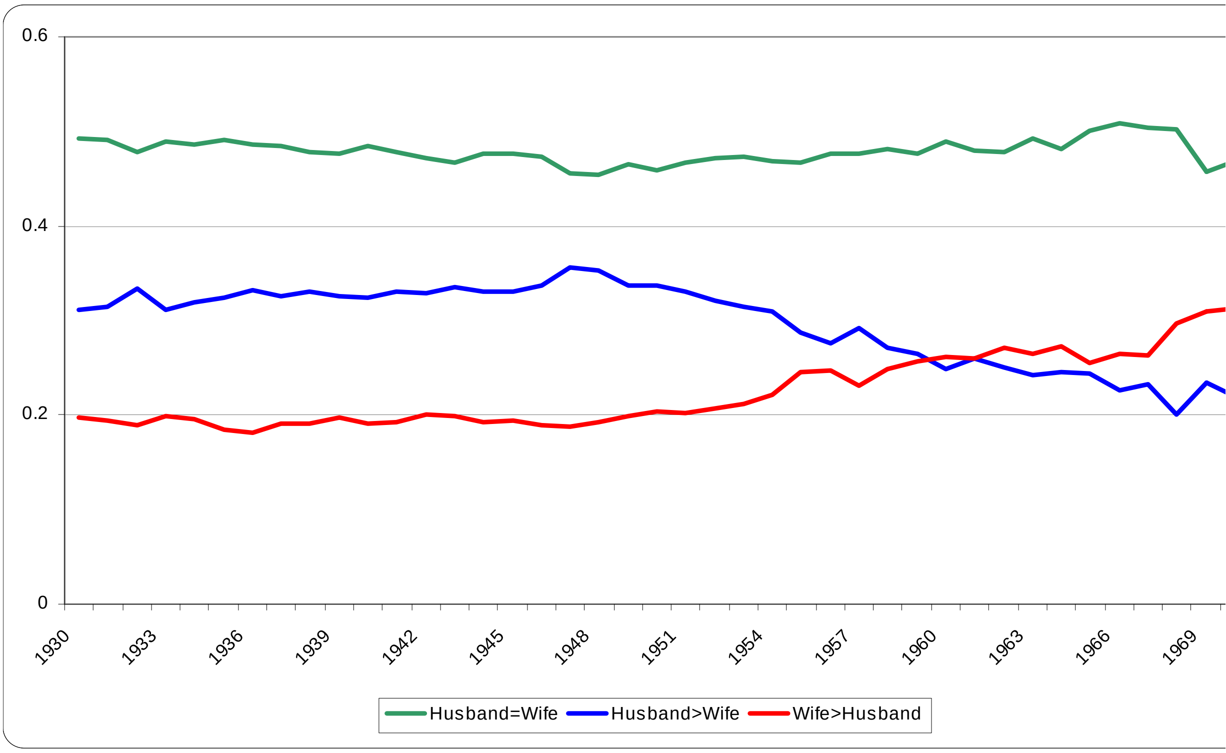

FIGURE 1.25. Education of Spouses, by Husband's Year of Birth, US. Source: Current Population Surveys.

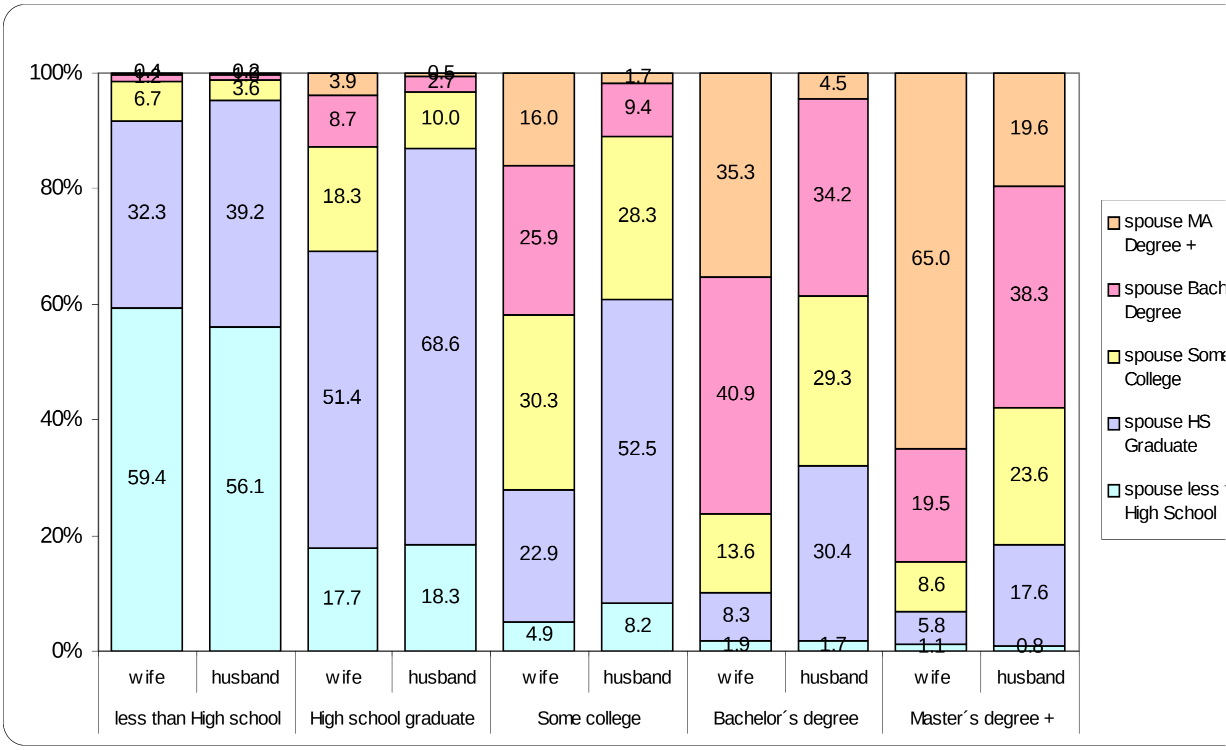

FIGURE 1.26. Spouse Education by own Education, Ages 30-40,US 1970-79. Source: Current Population Surveys.

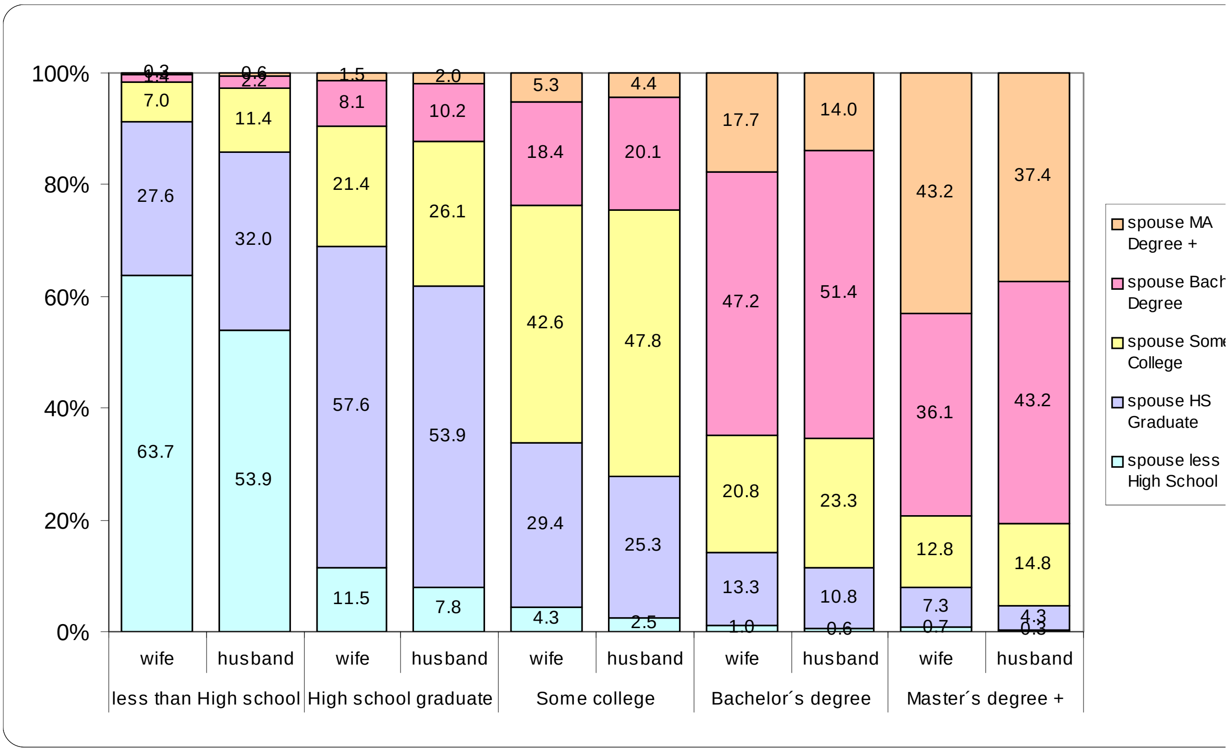

FIGURE 1.27. Spouse Education by own Education, Ages 30-40, US 1996-2005. Source: Current Population Surveys.

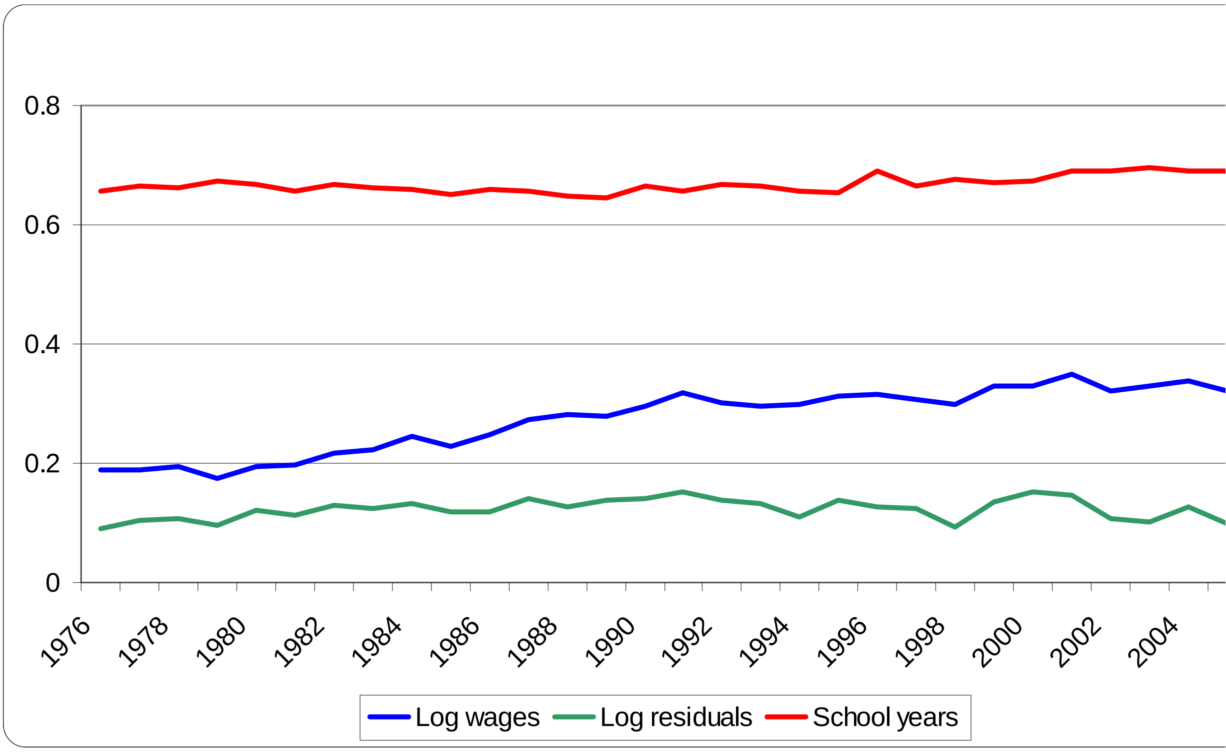

FIGURE 1.28. Husband-Wife Correlations of Schooling and Wages, Ages 25-40, US 1976-2005. Source: Current Population Surveys.

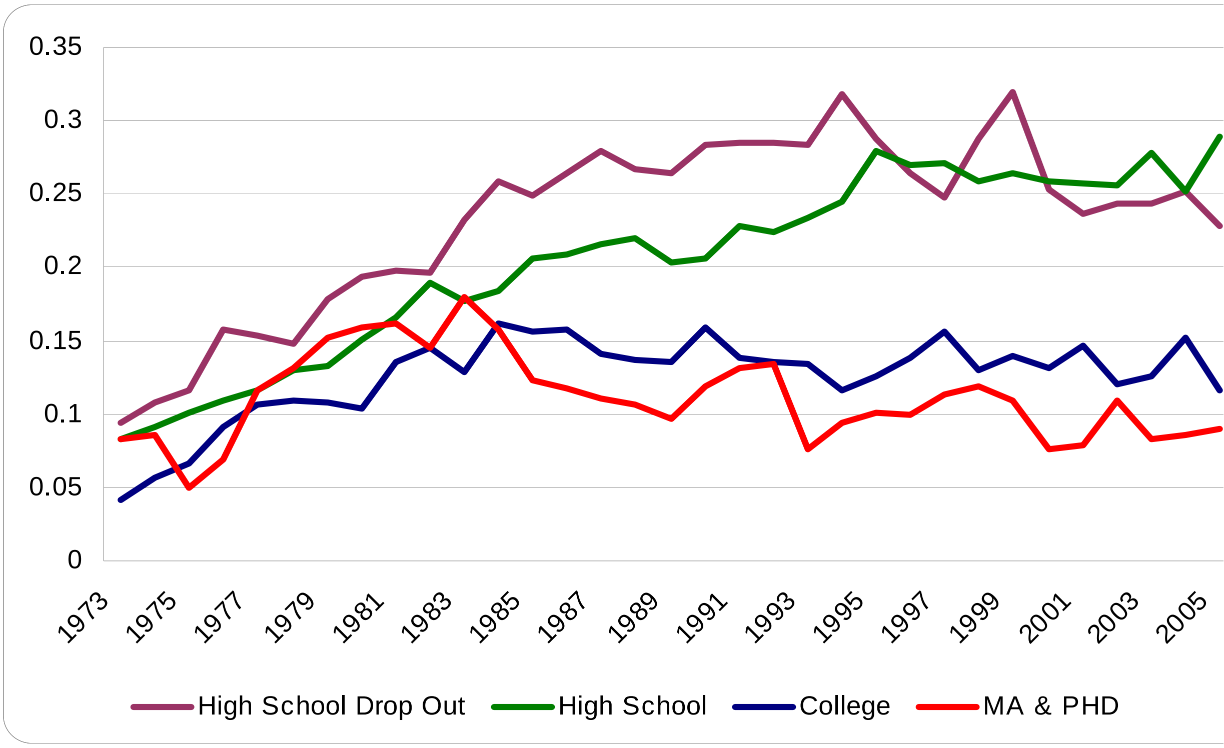

FIGURE 1.29. Divorce Rates of Men+Women by Schooling, US 1973-2005 (% divorced age $30-35$ year t) /(% marrried age $25-30$ year $t-5$). Source: Current Population Surveys.

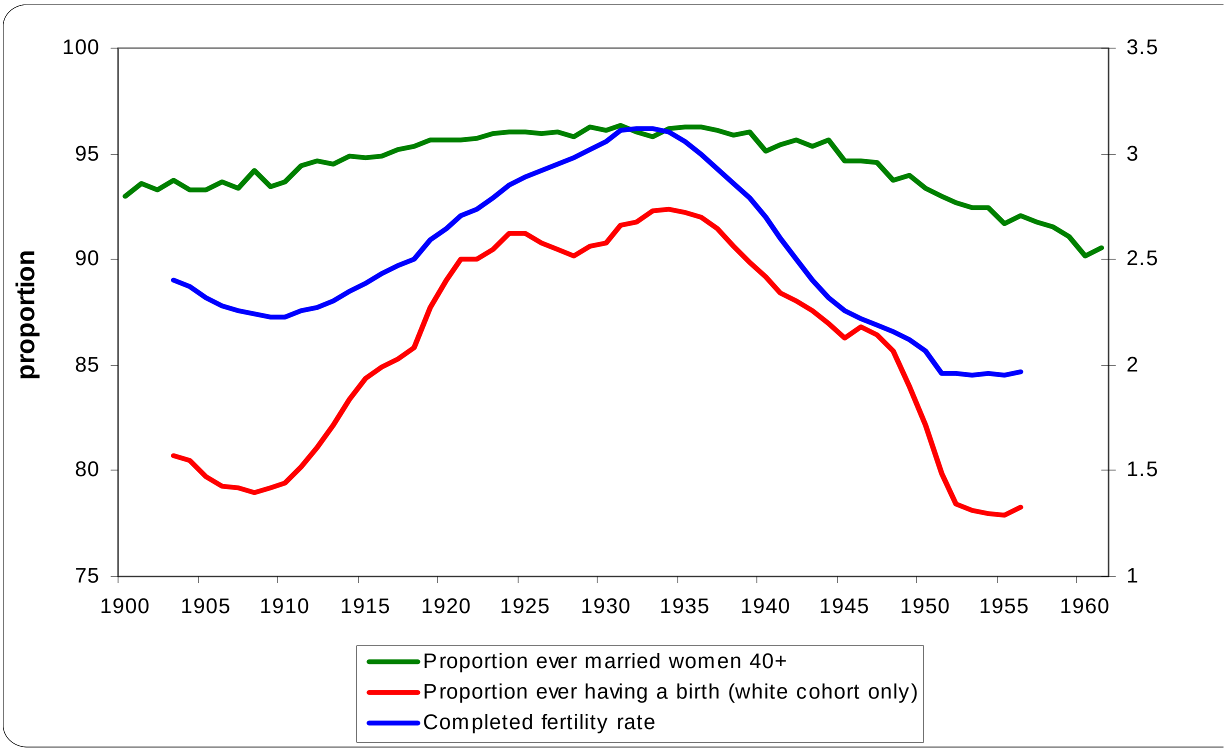

FIGURE 1.30. Completed Fertility, US by Birth Cohort. Source: National Center of Health Statistics.

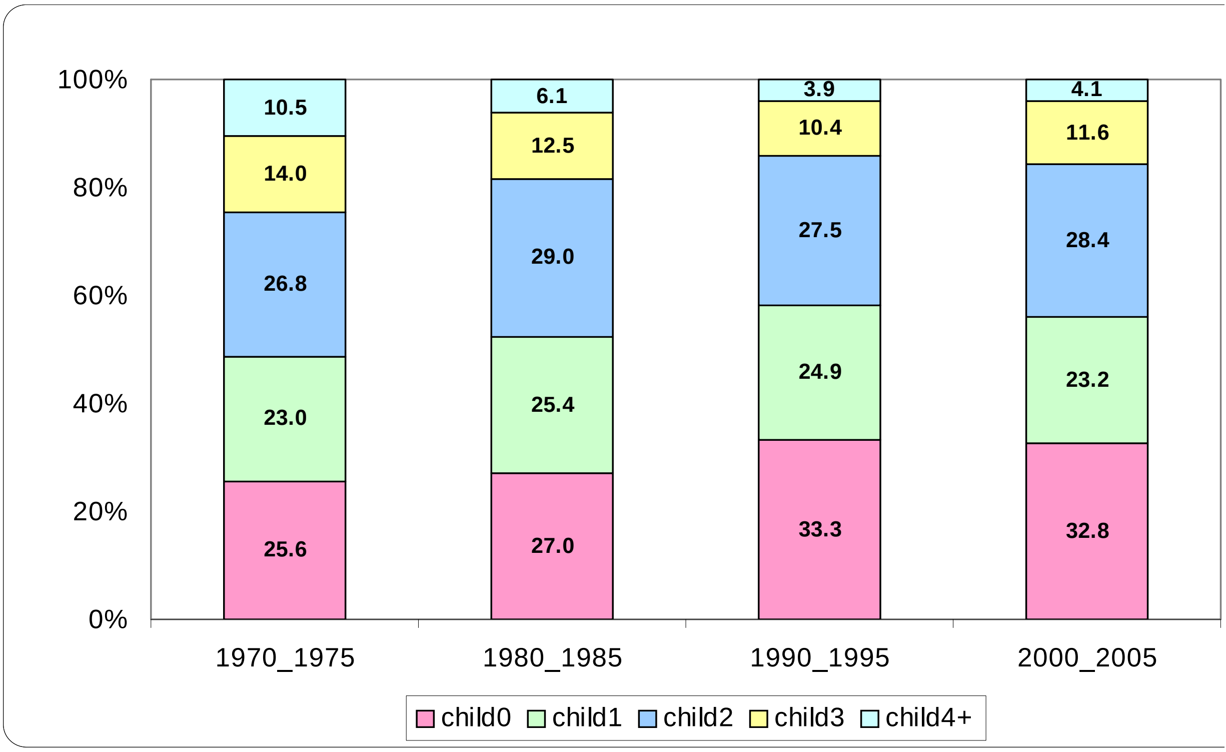

FIGURE 1.31. Number of Children 0-18 of US Women, Aged 35-45 at Selected Years. Source: Current Population Surveys.

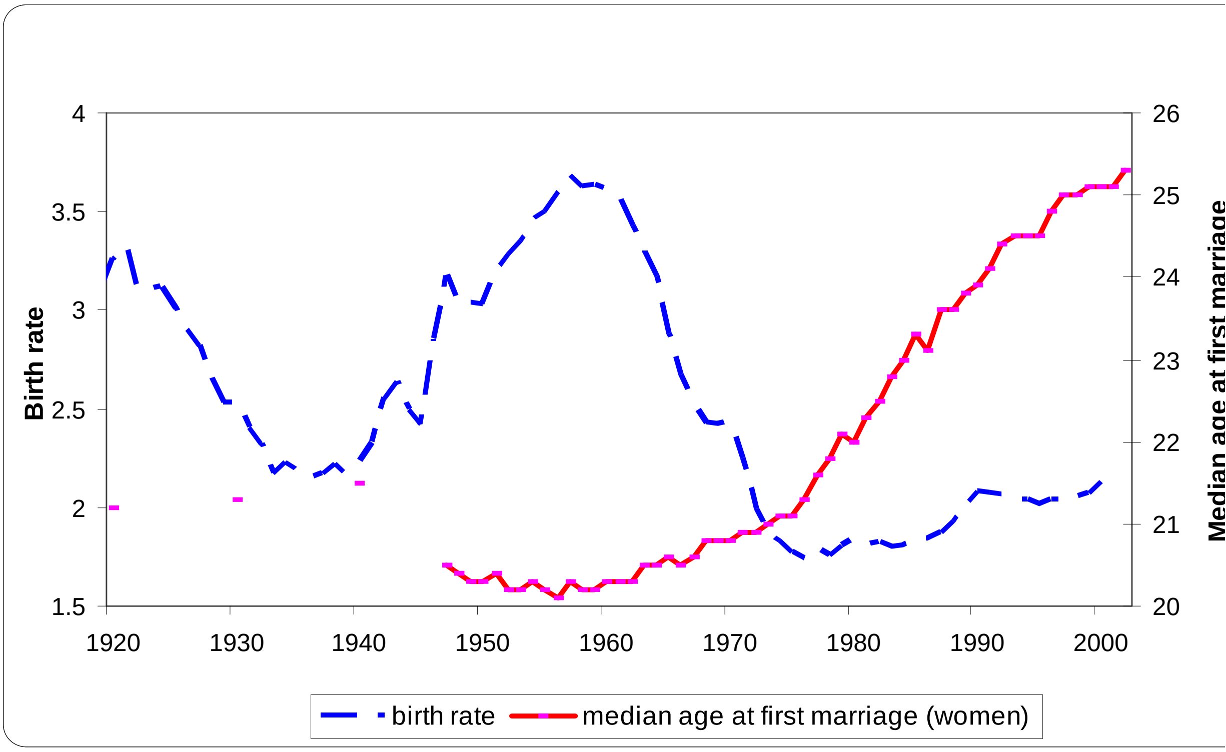

FIGURE 1.32. Birth Rates and Median Age at First Marriage, US 1900-2000.
Source: National Center of Health Statistics.

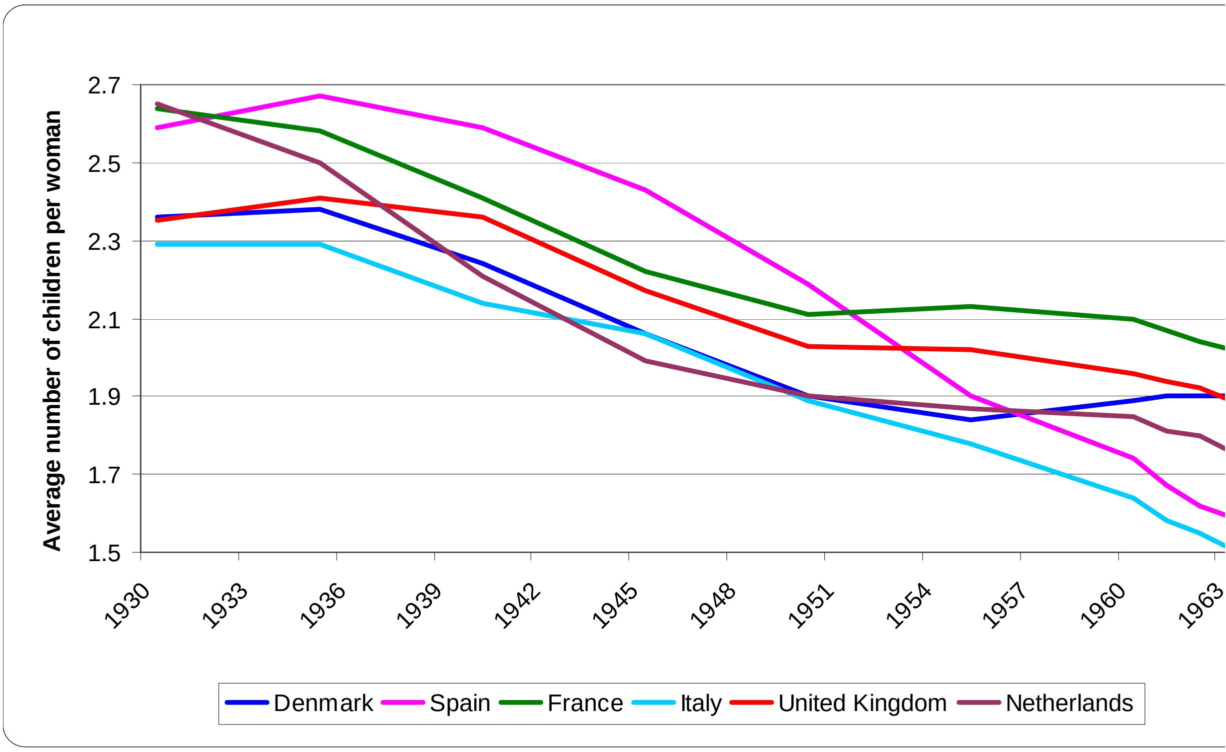

FIGURE 1.33. Completed Fertility by Generation. Source: Eurostat.

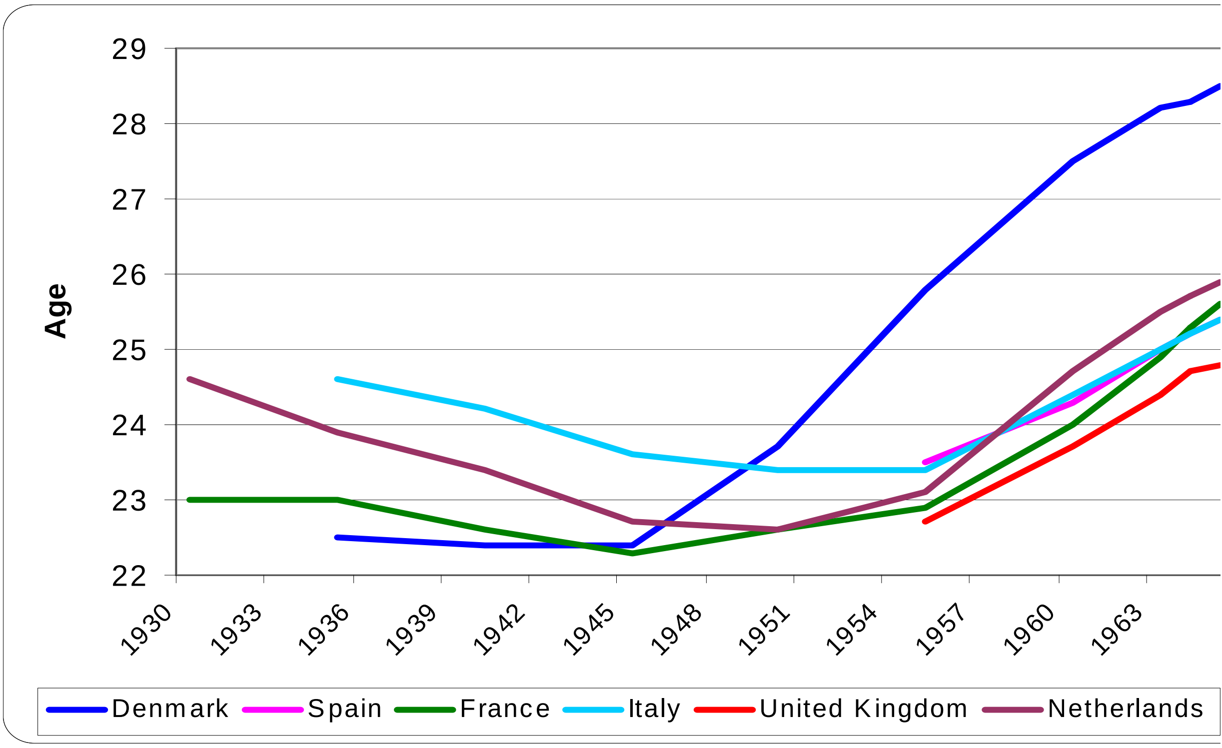

FIGURE 1.34. Mean Age at First Marriage by Generation, Mean Age at First Marriage by Generation. Source: Eurostat.

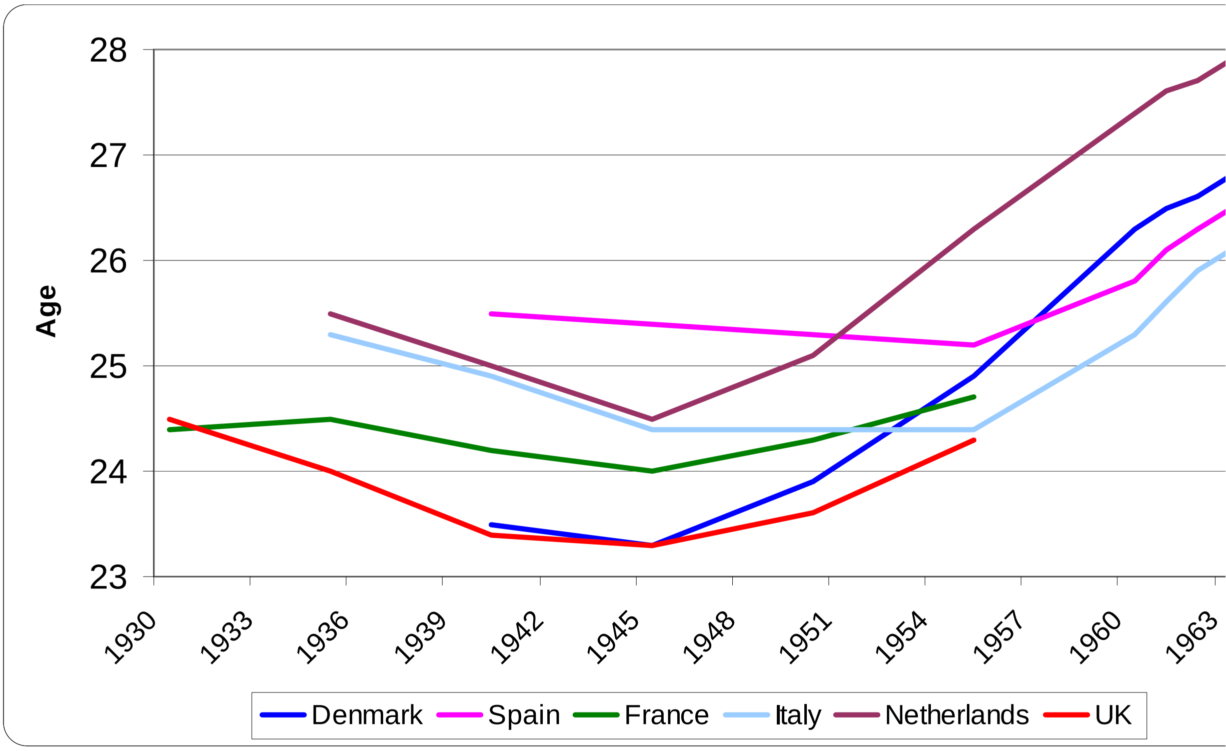

FIGURE 1.35. Age of Women at First Birth by Generation. Source: Eurostat.

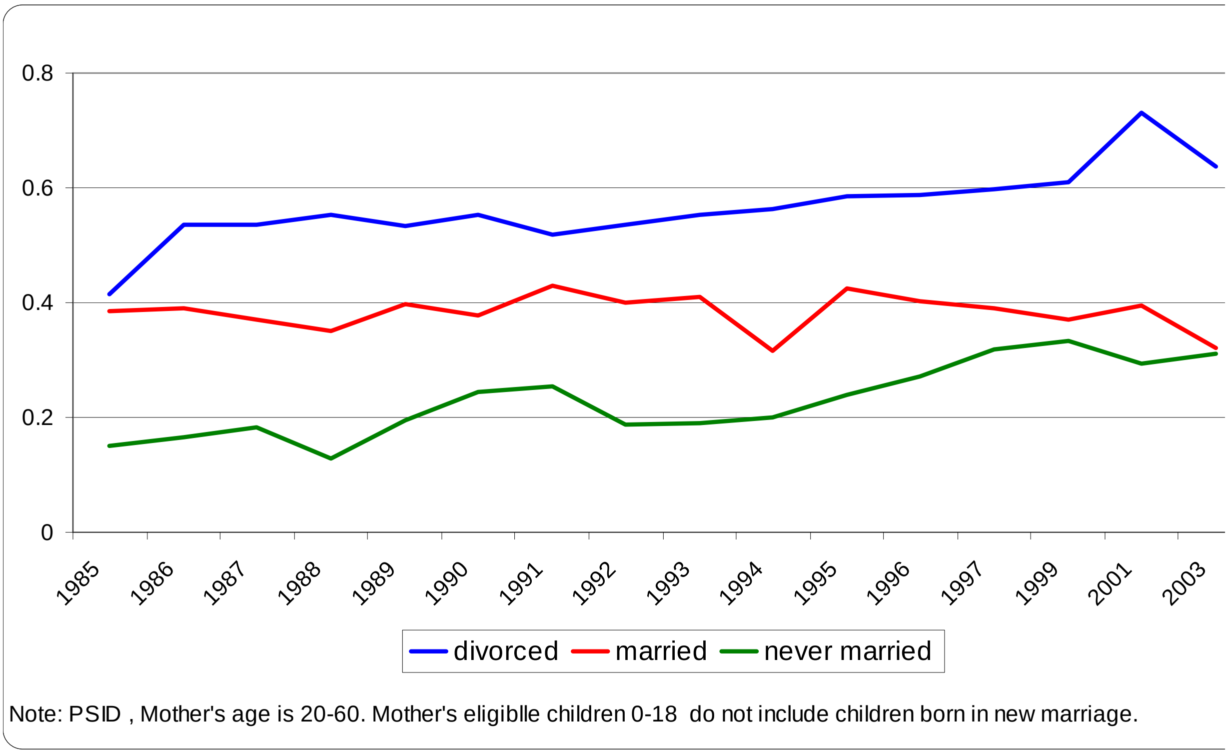

FIGURE 1.36. Percent of US Mothers Receiving Child Support, by Marital Status. Source: Panel Study of Income Dynamics.

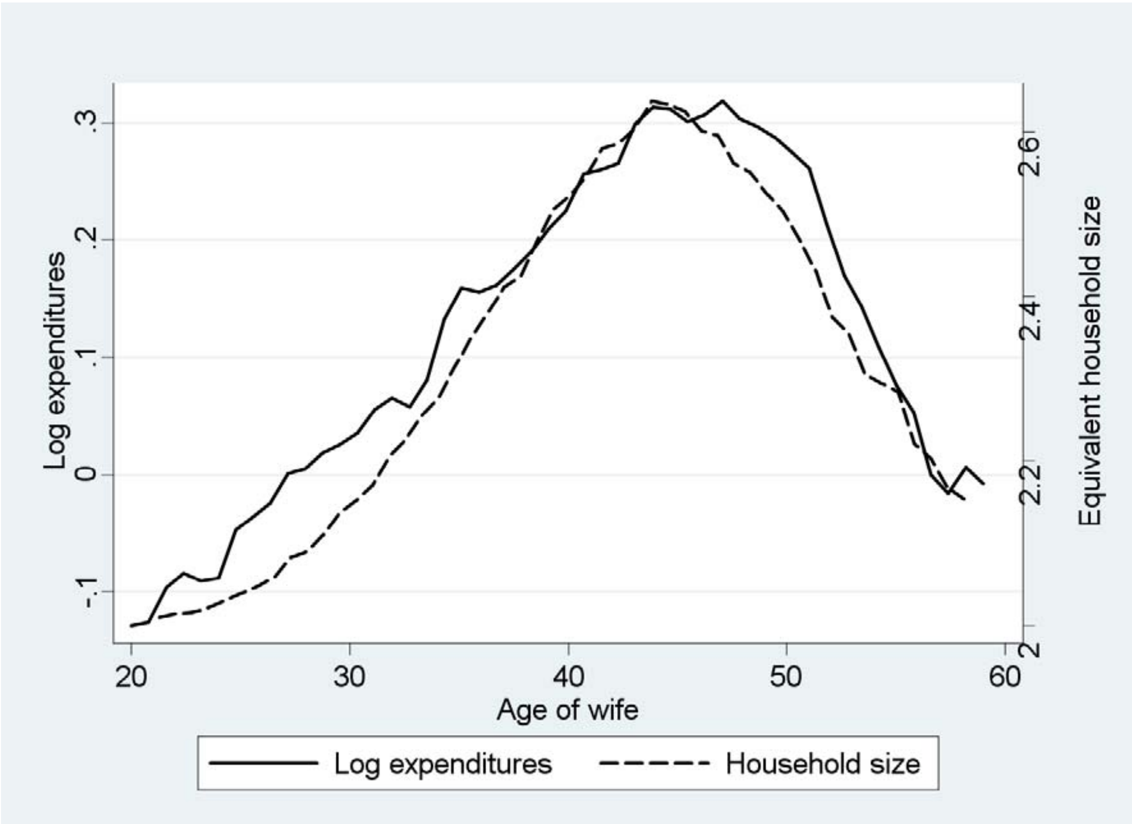

FIGURE 1.37. Consumption and household size - more educated wives. Source: UK Family Expenditure Surveys.

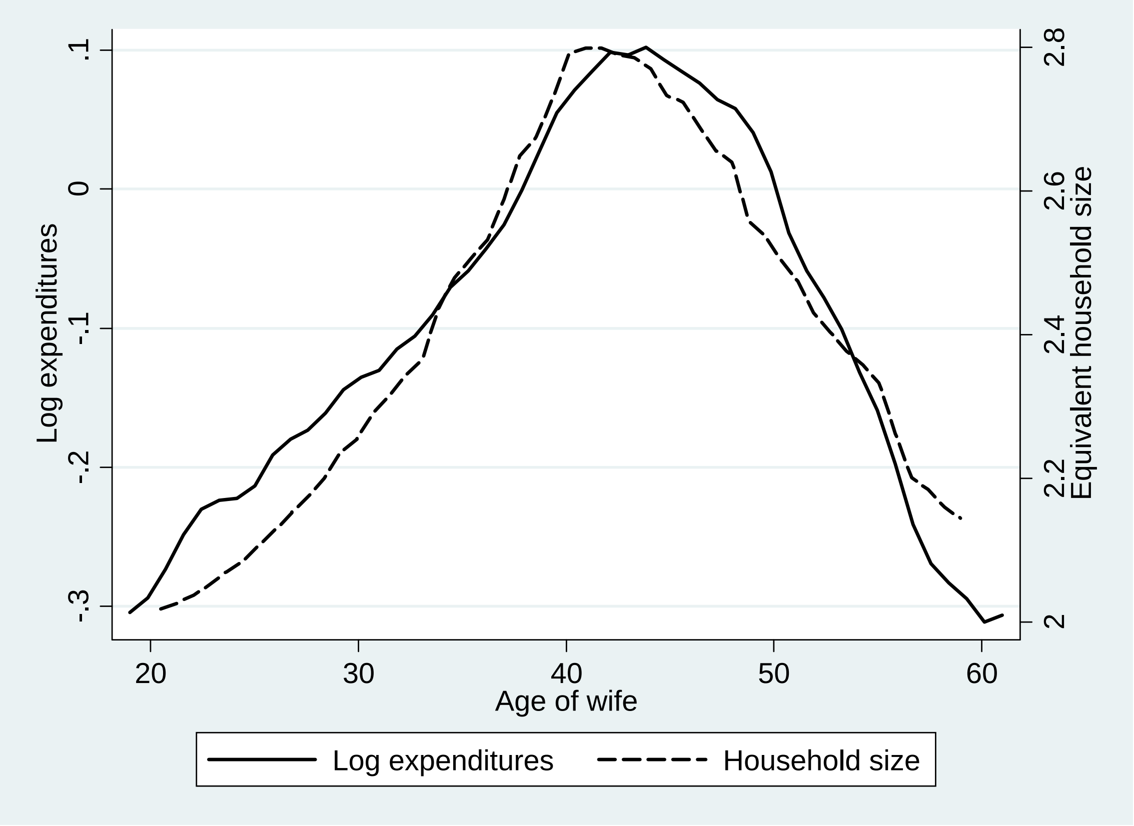

FIGURE 1.38. Consumption and household size - less educated wives. Source: UK Family Expenditure Surveys.

2

The gains from marriage

From an economic point of view, marriage is a partnership for the purpose of joint production and joint consumption. However, consumption and production are broadly defined to include goods and services such as companionship and children. Indeed, the production and rearing of children is the most commonly recognized role of the family. But there are other important gains from marriage, both economic and emotional.[1] Although the economic gains may not be the most important motivation for living together with someone ('marrying'), we focus on them here and examine five broad sources of potential material gain from marriage, that is, why "two are better than one":[2]

1. The sharing of public (non rival) goods. For instance, both partners can equally enjoy their children, share the same information and use the same home.
2. The division of labor to exploit comparative advantage and increasing returns to scale. For instance, one partner works at home and the other works in the market.
3. Extending credit and coordination of investment activities. For example, one partner works when the other is in school.
4. Risk pooling. For example, one partner works when the other is sick or unemployed.
5. Coordinating child care, which is a public good for the parents.

We emphasize that the gains discussed here are only potential - if they are realized to their full extent and who benefits from them is the subject matter of much of the rest of this book. We shall cast our discussion in terms of two agents who choose to live together but many of the points apply generally to a many person household. We also note that the gains

[1] In this book we shall often make a distinction between the material gains and the non-material gains and assume that the latter do not impinge upon valuations of the latter. This is done mainly for tractability. Generally, the two sets of factors need not be additive and the economic gains could interact with the "quality of match".

[2] According to of Ecclesiastes (4: 9-10) ; "Two are better than one, because they have a good reward for their toil. For if they fall, one will lift up the other; but woe to one who is alone and falls and does not have another to help. Again, if two lie together, they keep warm; but how can one keep warm alone?"

for one person may be different depending on the potential partner. In later sections of the book we shall expand and elaborate on many of the issues presented in this chapter.

2.1 Public goods

We begin with the most obvious potential gain, the publicness of some consumption. Some of the consumption goods of a family are public (non-rival) and both partners can consume them equally; expenditures on housing, children and heating are clear examples.[3] The sharing of housing clearly requires that the partners live in the same household. However, parents may enjoy their children (not necessarily equally) even if the parents live in different households. In this respect, children continue to be a public good for the parents even if the marriage dissolves. In practice, most goods display some publicness and some privateness. For example, housing has a strong public element in that both partners share the location and many of the amenities of the house. Nonetheless there is some private element if, for example, one or both of the partners requires a room of their own or if there is some congestion.[4]

To illustrate some of the issues, we begin with a simple situation in which we have two agents, a and b, and two goods.[5] One of the goods is a purely public good, Q, and the other is a single purely private good, q. We denote the incomes of these persons y^a and y^b, respectively, and normalize the prices of the two goods to unity. To focus on the issues associated with sharing, we shall also assume that the two agents do not care for each other and each has a private utility function that is used to order their own levels of private and public goods; in the next chapter we return to this issue. Let q^s denote the consumption of the private good by person s and let the felicity (private utility) functions be given by $u^s(Q, q^s)$ for $s = a, b$.

If the two agents live apart then each individual s solves

$$\begin{aligned} &\max_{Q,q^s} u^s(Q, q^s) \\ &\text{subject to } Q + q^s = y^s \end{aligned} \tag{2.1}$$

Let the optimal choices be $\left(\hat{Q}^s, \hat{q}^s\right)$ respectively. If the agents live together, they can pool their income and their joint budget constraint is

$$Q + q^a + q^b = y^a + y^b. \tag{2.2}$$

[3] 'Public' refers to the point of view of the two partners only. Such goods are sometimes known as collective goods or local public goods.

[4] As famously noted by Virginia Wolfe in "A room of one's own".

[5] In all that follows we assume that a is female and b is male.

If the preferences of both partners are increasing in the level of the public good then the two will always be potentially better off by living together in the sense that we can find feasible allocations that Pareto dominate the separate living case. Suppose, for example, that $\hat{Q}_a \geq \hat{Q}_b$; then the couple can set:

$$Q = \hat{Q}^a,\ q^b = \hat{q}^b \text{ and } q^a = \hat{q}^a + \hat{Q}^b \tag{2.3}$$

Such an allocation is feasible given the joint income and it maintains or improves the welfare of both b and a. This demonstration can be generalized to any number of private and public goods. A couple can always replicate the private consumption of the two partners as singles, purchase the *maximal* amount of each public good that the partners bought as singles and still have some income left over.

This result relies on the assumption that both partners have positive marginal utility from Q. Although a standard assumption, one can think of realistic situations in which preferences are not monotone in the public good; for example, for heating, too much may be as bad as too little and the partners may differ in what is the optimal level of heating. Then, there may be no gains from marriage at all, despite the reduced costs resulting from sharing. An obvious example is one in which the public good is beneficial for one partner and a nuisance to the other. Then publicness can be a curse rather than a blessing, because it may be impossible to avoid the jointness in consumption. Clearly, potential partners with such opposing preferences would not marry. In general, some concordance of preferences is required to generate gains from marriage (Lich-Tyler, 2003). Positive gains from marriage require that the preferred sets for each partner, relative to the situation when single, have a non-empty intersection on the budget line if they live together. This is illustrated in Figure 2.1 for two people who have the same income. In the left panel the two partners have preferences such that, *if there are no other gains*, they will not choose to live together. In the right panel they can find feasible allocations if they live together which give both more than if they live apart.

In the example of the last paragraph, we do not have any private goods; if we do have a private good then there may be possibilities for compensation to achieve positive gains from marriage. To see the nature of the requirements, suppose we have two public goods (Q_1, Q_2) and one private good. The program is:

$$\begin{gathered} \max u^a\left(Q_1, Q_2, q^a\right) \\ \text{subject to } Q_1 + Q_2 + q^a + q^b \leq y^a + y^b \\ \text{and } u^b\left(Q_1, Q_2, q^b\right) \geq u^b\left(\hat{Q}_1^b, \hat{Q}_2^b, \hat{q}^b\right). \end{gathered} \tag{2.4}$$

We need to show the solution of this program exceeds the utility of a as single, $u^a\left(\hat{Q}_1^a, \hat{Q}_2^a, \hat{q}^a\right)$. Because the minimum cost required to obtain the

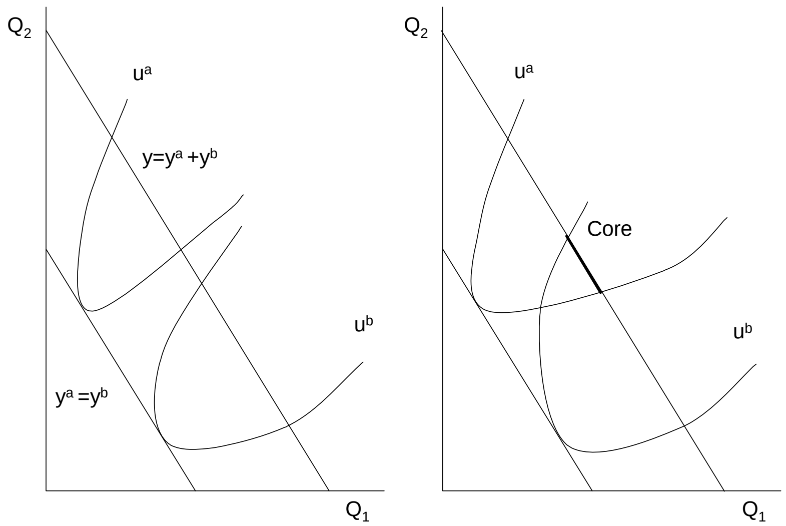

FIGURE 2.1. Preferences over two public goods

level of welfare that b had as single is y^b, it is possible to give a private consumption level of at most y^a without hurting b. Thus, a sufficient condition for positive gains from marriage is

$$u^a\left(\hat{Q}_1^b, \hat{Q}_2^b, y^a\right) > u^a\left(\hat{Q}_1^a, \hat{Q}_2^a, \hat{q}^a\right). \tag{2.5}$$

That is, it is possible to 'bribe' a to conform to b's preferences for public goods by giving her additional private consumption. By a similar logic

$$u^b\left(\hat{Q}_1^a, \hat{Q}_2^a, y^b\right) > u^b\left(\hat{Q}_1^b, \hat{Q}_2^b, \hat{q}^b\right) \tag{2.6}$$

is also a sufficient condition. Which of these two conditions is relevant depends on the initial wealth of the parties. If b is wealthier and public goods are normal goods then he would consume more public goods when single, and it would be easier to satisfy condition (2.5) and attract a into the marriage.

We return now to the simple case with one public good and one private good and monotone preferences and illustrate some further issues associated with sharing. Specifically, suppose that $u^a(Q, q^a) = q^a Q$, $u^b\left(Q, q^b\right) = q^b Q$. If the two live separately then we have $\hat{Q}^s = \hat{q}^s = \frac{y^s}{2}$ and $u^s = (\frac{y^s}{2})^2$ for $s = a, b$. If they live together, they have household income of $y^a + y^b$. The efficient program is to set $\hat{Q} = \frac{y^a+y^b}{2}$ and then divide the remaining household income so that $q^a + q^b = \frac{y^a+y^b}{2}$. This gives a utility possibility frontier of:

$$u^a = (\frac{y^a + y^b}{2})^2 - \bar{u}_b \text{ where } \bar{u}_b \in \left[0, (\frac{y^a + y^b}{2})^2\right]. \tag{2.7}$$

Figure 2.2 illustrates the case when $y^a = 1$ and $y^b = 3$. The Pareto frontier in this case is given by $u^a + u^b = 4$. Not all points on this frontier will be realized, because each partner has some reservation utility to enter the marriage (if the gains from sharing public goods are the only gain). Alone, partner a obtains $u^a = \frac{1}{4}$ and partner b obtains $u^b = \frac{9}{4}$. Clearly, these individual utility levels are well within the frontier and any choice of $\bar{u}_b$ between $\frac{9}{4}$ and $\frac{15}{4}$ will give *both* partners more than they would receive if they lived separately.

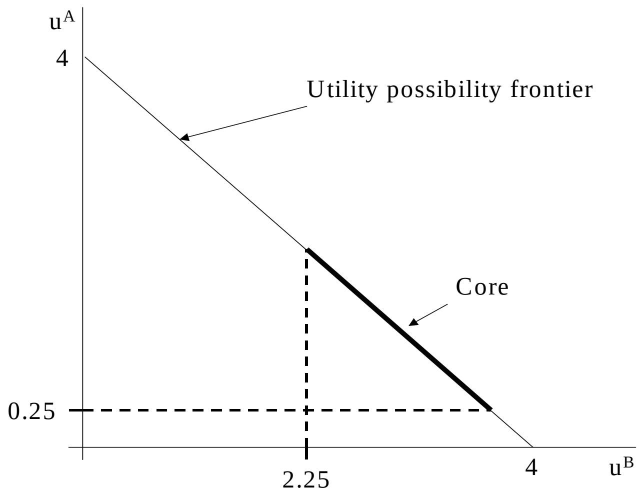

FIGURE 2.2. Gains from public goods.

This example has two related special features that are due to the assumed preferences. First, the level of the public good is independent of the distribution of the private good but this will not generally be the case. Second, the utility possibility frontier is linear (with a slope of -1) but generally it will be nonlinear (see Bergstrom, Blume and Varian, 1986).[6] Despite

[6] It is possible for the public good to be independent of the division of income also when the Pareto frontier is concave. This is the case, for instance, when $u^i = \ln Q + \beta \ln c^i$. Then $Q = \frac{y^a+y^b}{1+\beta}$ and, for $0 < c^a < \frac{\beta(y^a+y^b)}{1+\beta}$, the slope of the utility frontier is

this simplicity, this example brings out a number of important ideas. First, there are potentially large gains from the publicness of goods, which arises from the *complementarity* between the incomes that the partners bring into marriage. Second, although the distribution of the gains may not be uniquely determined, there may exist a unique efficient level of the public good, which depends only on the joint income of the partners. Thus the partners may agree on the level of the public good and restrict any disagreement to the allocation of private goods. Third, if there are cultural or legal constraints that limit inequality within the family then the high income person may not want to marry. For example, equal sharing in this example gives b a utility level of 2, which is lower than his utility level if single. Thus the gains from publicness are outweighed by the requirement to share with the partner. Finally, even if the final allocation is *not* Pareto efficient it may still pay to live together (if the allocation gives utility levels inside the UPF but above the singles levels).

That there are potential gains from the publicness of some consumption is uncontroversial. We would like to quantify how large these gains are. To do this we use the concept of 'equivalent income' which is the amount of income needed by two singles to achieve the same outcome as when they live together. There are two outcomes of interest: buying the same bundle and achieving the same utility levels (see Browning, Chiappori and Lewbel (2003)). For the former, we compute the cost of buying the bundle that the couple buys and the cost of the same bundle for each of partners if single. The ratio of what the two partners would spend if single to what the couple pays is the '*relative cost of an equivalent bundle*'. For our example this bundle is $Q = 2$ and q^a and q^b are such that $q^a + q^a = 2$. Whatever the distribution of the private good, the same bundle of goods would cost 6 units since each has to be given a level of public good equal to 2. The relative cost of an equivalent bundle is thus 1.5 so that the couple, if single, would need 50% more income to buy the bundle they consume as a couple.

Although the calculation of the relative cost of an equivalent bundle gives the two agents the same bundle and hence the same utility as when living together, the cost of achieving the same utility level may be lower since agents may choose to substitute away from the bundle they had when married. In our example, the utilities when together are $u^a = 2q^a$ and $u^b = 2(2 - q^a)$. If a is single then she spends half her money on the public good and half on the private good. Hence she needs an income y^a that solves:

$$2q^a = u^a = \left(\frac{y^a}{2}\right)\left(\frac{y^a}{2}\right) \Rightarrow y^a = \sqrt{8q^a} \tag{2.8}$$

Similarly, b needs an income of $y^b = \sqrt{8(2 - q^a)}$ so that the relative cost

$\frac{du^a}{du^b} = 1 - \frac{\beta(y^a+y^b)}{(1+\beta)c^a}$.

of equivalent utilities is:

$$\frac{y^a + y^b}{3+1} = \frac{\sqrt{8q^a} + \sqrt{8\left(2-q^a\right)}}{4} \tag{2.9}$$

For example, if $q^a = 0.5$ then $y^a = 2$ and $y^b = \sqrt{12} \simeq 3.46$ so that the cost of achieving the same utilities when single as when together is 5.46 and the *relative cost of equivalent utilities* is 1.375.

To use the 'relative cost of an equivalent bundle' with household expenditure data, we need to identify which goods are public and which are private and also to estimate budget shares for these goods for couples. To compute the 'relative cost of equivalent utilities' we need more information. Specifically, we need to know both the distribution of the private good in the couple household and preferences when single. This is a significantly higher informational level.

Rather than distinguishing goods into being entirely private or public, one can use a parameter η_j that indicates how 'public' is each particular good. Thus, if the quantity of good j bought in the market is q_j, then together the two partners can obtain $q_j^a + q_j^b = \eta_j q_j$ units of consumption where η_j is between 1 and 2. We refer to η_j as the *degree of jointness* of good j. If $\eta_j = 2$ then good j is purely public and $2q_j$ is available for consumption which is necessarily the same for the two agents: $q_j^a = q_j^b = q_j$. If $\eta_j = 1$ then good j is purely private and any allocation $q_j^a + q_j^b = q_j$ is feasible. Generally, the share that each one receives of this total must satisfy the restrictions that $q_j \geq q_j^a \geq \left(\eta_j - 1\right) q_j$ and $q_j \geq q_j^b \geq \left(\eta_j - 1\right) q_j$ to allow for the non-exclusion of each person from the public element of the good. As η_j rises and the good becomes more public, the utility frontier shifts up and, at the same time, the set of possible divisions narrows. In the demand literature this is known as Barten scaling (formally the Barten scale for good j equals the inverse of the degree of jointness $\left(\eta_j\right)^{-1}$); see, for example, Deaton and Muellbauer (1980), chapter 8. In the next chapter we shall discuss household production in more detail; for now it suffices to note that Barten scaling defines a simple household production technology in which n market goods are transformed into n household commodities in a linear and non-joint way. The cost of giving each partner the consumption they have when together is $\sum_{j=1}^{n} \eta_j q_j$ and an index of the degree of publicness is

$$\eta = \frac{\sum_{j=1}^{n} \eta_j q_j}{x} = \sum_{j=1}^{n} \eta_j \omega_j \tag{2.10}$$

where q_j is the couple's demand for good j, ω_j is the budget share for good j in the married household (recall that all prices are normalized to unity) and x is total expenditure. This index will vary from household to household even if all households have the same technology (the same η_j's) since different couples spend in different ways. It gives an upper bound on the cost of providing the same level of utility when the partners are single as when they were together because, as discussed above, the actual cost may be lower since the singles may optimize and choose different bundles than when together. In the example given above we have $\eta_1 = 2$, $\eta_2 = 1$ and $\omega_1 = \omega_2 = 0.5$ so that the relative cost of an equivalent bundle is 1.5, as derived above.

Although we can conceptually formulate precise measures of the gains from the jointness of goods, in practice we have very little idea of how important these gains are. As an informal application of the Barten approach, we consider the expenditure patterns of US households in taken from the Consumer Expenditure Survey (2003) and assign a degree of jointness to each of the composite commodities such as food, housing, clothing, etc.. Table 2.1 gives details for a nine commodity grouping.[7] For each commodity we assign a minimum and maximum for the jointness of the good (η_j) and then we compute the minimum and maximum values of the jointness of total expenditure ('consumption'). We do this for three different income groups (gross household incomes of \$10,000-\$20,000, \$30,000-\$40,000 and \$50,000-\$70,000, respectively) to allow that demand patterns differ between rich and poor. Of course, the bounds for jointness are somewhat arbitrary but they capture the idea that food, for example, is mostly private and housing is largely public. The implied scales for rich and poor do not vary much; this reflects the fact that public goods are a mix of necessities (housing) and luxuries (durables, transport and cars). The relative costs are bounded between singles needing one third and two thirds as much as couples to buy the equivalent bundles.

The bounds in Table 2.1 are rather wide. To pin down the values more precisely we need to make additional (and strong) assumptions and use the data more carefully. Lazear and Michael (1980) use a single cross-section family expenditure survey and estimate that two single individuals can almost double their purchasing power by forming a union. However, their identification rests on very strong identifying assumptions. Browning, Chiappori and Lewbel (2003) use Canadian nondurable expenditure data on cross-sections of single people and two person households and employ a Barten scheme of the variety outlined above. This exploits the variation in relative prices that arises from changes over time and variations across provinces. The estimates are only for nondurables and services and exclude housing and durables. They estimate that a couple who share private expenditures equally when married require 41 percent more total expenditure to replicate the bundles when single; that is, the relative cost of an equivalent bundle, η, is 1.41. This is at the low end of the bounds given in Table 2.1, perhaps because housing and durables are not included.

[7] Housing includes the costs of housing plus utilities and house operations. Durables are white goods, furniture and small durables. Electronic goods are included under entertainment. Transportation includes all transportation costs except for the purchase of cars. We exclude health and education expenditures.

			Low income	Medium income	High income
Net hhold income	-	-	$\$12,761$	$\$33,381$	$\$56,360$
	Degree of jointness				
	Min	Max	Budget shares ($\times 100$)		
Food	1	1.2	19.7	17.9	16.3
Alcohol and tobacco	1	1.2	3.1	2.5	2.8
Housing	1.5	1.9	38.6	34.2	33.7
Durables	1.5	1.9	3.8	4.8	4.8
Clothing	1	1.2	5.6	4.9	4.6
Transportation	1.3	1.7	11.9	13.8	14.2
Car purchases	1.5	1.9	10.3	12.4	14.1
Entertainment	1.3	1.8	5.2	7.8	7.5
Personal care	1	1.5	1.8	1.8	1.7
Relative cost of an equivalent bundle					
Minimum			131.5	132.2	133.0
Maximum			166.5	168.1	169.2

TABLE 2.1. Bounds for the relative cost of equivalent bundles

2.2 Specialization and Increasing Returns to Scale

The idea that agents can gain by specializing in different tasks is one of the most venerable and useful in economics. Becker, in particular, has emphasized this when considering the gains from marriage (see Becker, 1991). To illustrate its application within the family we consider a very simple household production model. Suppose that we have two people a and b who can spend their time in market work or home production of a single non-market good denoted by z. For a single person the household production function is:

$$z = xt \tag{2.11}$$

where t denotes time spent on production and x denotes purchased goods. This production function displays increasing returns to scale in the sense that doubling the inputs of home production time and market purchases raises output by a factor of more than two (see Crossley and Lu (2005) for evidence on the returns to scale for food preparation). Expenditure on the market good is given by $x = w(1-t)$, where w_s is the market wage for person s.

We assume that agents only derive utility from the amount of z consumed. This assumption implies that any agent is indifferent between time spent on household production and time spent in market work. We assume that other uses of time (leisure and personal care) are held fixed and normalize the total amount of work time to unity. Given this, an agent living alone will choose to maximize the output of the home produced good subject to $0 \leq t \leq 1$ and person s, when single, sets:

$$t_s = \frac{1}{2}, z_s = \frac{w_s}{4} \tag{2.12}$$

If the couple lives together, we assume that the household production function is given by:

$$z = x \left(t^a + t^b \right) \tag{2.13}$$

so that a and b are perfect substitutes in home production. Observe that total output is determined by the aggregate time spent at home by both partners and the total amount of goods purchased by the family in the market. The household budget constraint is

$$x = w^a(1 - t^a) + w^b(1 - t^b) \tag{2.14}$$

Thus the agents living together can produce aggregate output:

$$z = \left(t^a + t^b \right) \left(w^a \left(1 - t^a \right) + w^b \left(1 - t^b \right) \right) \tag{2.15}$$

We assume that z is a private good which can be divided between the two partners and that the partners agree to maximize the total output available to both of them. If they set the time allocation to the optimal levels for singles their total output will be $\frac{w^a+w^b}{2}$, which is larger than the aggregate output if they live separately, $\frac{w^a+w^b}{4}$. This outcome, which is due to increasing returns, is similar to the gains from jointness discussed in the previous section. However, the couple acting together can improve even on this higher output if their wages differ. To see this, suppose that $w^a > w^b$ and set $t^a = 0$ and $t^b = 1$; thus the higher wage person specializes in market work and the lower wage person specializes in home production. This gives a total output of the home produced good of w^a which is, of course, greater than the output with no specialization $\frac{w^a+w^b}{2}$. It can be shown that this choice maximizes aggregate output. Comparing the results for a single person household and a couple, we see that there is always a positive gain from marriage of $\max \left(w^a, w^b \right) - \frac{w^a+w^b}{4}$. The gain due to *specialization* according to *comparative advantage* is given by $\max \left(w^a, w^b \right) - \frac{w^a+w^b}{2}$ which is zero if and only if the wages are the same.

This example illustrates the potential gains from specialization but the specific implications depend on a number of special features of this model. First, the two partners are assumed equally productive at home production. This can be trivially extended to allow for different fixed productivities in which case specialization will depend on the ratios of productivity in the market (that is, the wage) to productivity at home of the two partners. Second, the technology is linear in the time inputs. If, instead, we allowed for some concavity and complementarity between partners time use, specialization need not occur and interior solutions would arise. Yet we would still expect the high wage spouse to work more in the market when wages differ.

As emphasized by Becker (1991, chapter 2), comparative advantage can be developed via differential investments or learning by doing. Within marriage or in the market each party can use their own human capital to a larger extent, yielding convexity and dynamic increasing return. In particular, if one partner may specialize in home production while the other specializes in market work then both of them acquire skills relevant to their specific activity. Thus, a small innate difference can be magnified, and strengthen the incentives to specialize (see Chicilinsky 2005, Pollak 2007).

There is ample evidence for a division of labor within the household (see Chapter 1). Married men work longer hours in the market and have substantially higher wages than unmarried men. Married women have lower wages and work more at home than unmarried women; see Gronau 1986, Korenman and Neumark, 1992 and Daniel, 1992.

2.3 Imperfect Credit Markets

Consider two potential partners denoted by a and b. Each person lives for two periods which we denote by 1 and 2. Utility in period t is derived from consumption and the per period utility is

$$u\left(c_t\right) = \ln c_t. \tag{2.16}$$

For simplicity, we assume that the discount factor is unity and the real rate is zero. Each person has an initial wage of 1 that he/she can augment by spending the first period in school, obtaining a second period wage of w.

If there is a perfect capital market, a person can smooth his consumption through borrowing and will set $c_1 = c_2 = c$. Thus, with investment in schooling, one can obtain $c = \frac{w}{2}$ each period, while without investment consumption each period will be 1. Investment is profitable if the increase in wage is sufficient to compensate for the earnings forgone in the first period, that is if the second period wage w exceeds 2. However, if borrowing is impossible there is no investment in schooling since consumption in the first period would be zero.

Now assume that a and b marry each other. Under a perfect capital market, marriage will not influence their investment choices. However, if there is an imperfect capital market, marriage allows a couple to partially overcome the no borrowing constraint. This is accomplished by extending credit *within* the family, whereby one partner (b, say) works in the market while the other goes to school. To evaluate the potential gains from marriage, consider an efficient program that maximizes the utility of partner a given that partner b receives the lifetime utility he would have in the single state, without schooling. With our choice of units, life time utility in the absence of investment is 0. We thus solve

$$
\begin{aligned}
& \max \{\ln c_1^a + \ln c_2^a\} \qquad (2.17)\\
\ln c_1^b + \ln c_2^b &\geq 0 \\
c_1^a + c_1^b &= 1 \\
c_2^a + c_2^b &= 1 + w
\end{aligned}
$$

A necessary condition for efficiency is that consumption in each period is distributed between the partners so as to equalize the ratios of their marginal utilities from consumption in the two periods

$$\frac{u'(c_1^a)}{u'(c_2^a)} = \frac{u'(c_1^b)}{u'(c_2^b)}. \qquad (2.18)$$

With a logarithmic utility function, this implies that the consumption of both partners must grow at the *same* rate, $1+w$. Using the requirement that the lifetime utility of partner b remains zero, we obtain that $c_1^b = (1+w)^{-\frac{1}{2}}$ and $c_2^b = (1+w)^{\frac{1}{2}}$. Because the consumption of a grows at the same rate, her lifetime utility will be positive if and only if the first period consumption, $c_1^a = 1 - (1+w)^{-\frac{1}{2}}$, exceeds that of b. A brief calculation will confirm that this is true whenever $w > 3$.

We conclude that the potential for coordination of investment activities through credit can motivate marriage when credit markets are not operative. Notice that marriage does not completely eliminate the borrowing constraint, because only one person will invest in schooling and he/she will do so only at higher rates of return from schooling than in the case of perfect capital market. An important aspect of this example is that individuals who are ex-ante identical may voluntarily agree to pursue different careers, allowing both partners to share in the gains from this efficient program. Obviously, specialization in investment activities can also be motivated by differences in innate abilities. Typically, the family will choose to invest in the person with the higher return from human capital investment. In either case, *commitments* are crucial for the implementation of such a program, see Dufwenberg (2002). A woman will be hesitant to support her husband through medical school if she expect him to break the marriage (and marry a young nurse) when he finishes.

Evidence of implicit credit arrangements within marriage is sometimes revealed at the time of divorce, when the wife claims a share of her ex-husband's earnings on the grounds that she supported him in school; see Borenstein and Courant (1989). However, recent empirical work casts doubt on the importance of liquidity constraints for schooling choices see Carneiro and Heckman (2003). This important issue is still a matter of controversy; see Acemoglu and Pischke (2001).

2.4 Risk sharing

Individuals who face idiosyncratic income risk have an obvious incentive to provide mutual insurance. This can be done within the family. Here we present a simple example. Consider two risk averse partners with random incomes, y^s, $s = a, b$. Acting alone, if there are no possibilities for saving or borrowing, each partner will have an expected utility given by $E(u^s(y^s))$ respectively. Acting together, they can trade consumption in different states of nature. To see the potential gains from trade, consider the maximization:

$$\max E(u^a(c^a))$$
$$\text{subject to } E(u^b(y^a + y^b - c^a)) \geq E(u^b(y^b)).$$

Clearly, setting in each state $c^a = y^a$ and $c^b = y^b$, is a feasible solution which will replicate the allocations in the single state. However, the optimal risk sharing rule is

$$u'(c^a) = \lambda u'(c^b) \tag{2.19}$$

where λ is a positive constant. That is, the slope of the utility frontier, given by $-\frac{u\prime(c^a)}{u\prime(c^b)}$ is equalized across all states, where a state is defined by the realized sum of the individual incomes, $y^a + y^b$, that is, total family income. Otherwise, both partners can be made better off by transferring resources to a person in a state where his marginal utility of consumption is relatively high, taking resources away from him in another state where his marginal utility is relatively low. Following this optimal rule, both partners can be made strictly better off, provided that their incomes are not perfectly correlated (or that risk aversions differ).

A strong testable implication of efficient risk sharing is that the consumption of each family member varies only with *family income.* That is, holding family income constant, the idiosyncratic shocks to individual incomes will induce transfers between the partners, but consumption levels will remain the same.

Depending upon the particular risk, the potential gains from mutual insurance can be quite large. For instance, Kotlikoff and Spivak (1981) who consider the risk of uncertain life, in the absence of an annuity market, estimate that the gains that a single person can expect upon marriage are equivalent to 10 to 20 percent of his wealth. In a different application, Rosenzweig and Stark (1989) show that marriages in rural India are arranged between partners who are sufficiently distant to significantly reduce the correlation in rainfall, thereby generating gains from insurance. Hess (2004) finds that couples with a higher correlation in incomes are more likely to divorce, suggesting that effects of mutual insurance on the gains from marriage are higher when the partners' incomes are less correlated. Shore (2007) finds that the correlation in spouses' earnings respond to the business cycle; it is higher for couples whose marriage spans longer periods of high economic activity.

2.5 Children

2.5.1 Technology and preferences

One of the principal gains from marriage is the production and rearing of children. Although the biological and emotional gains may dominate here, we can also consider the economic aspects. In particular, we wish to discuss the gains to the child that arise from living with their natural parents in an intact family. Consider two partners, a and b, who choose to have a child (or some other fixed number of children) denoted by k. We allow that the two partners have alternative uses for their time; in this case they can spend time in child care, t^a and t^b, respectively or in market work at the wages w^a and w^b. In this example we shall assume that there is a single private good with market purchases of q of this good being allocated between the three family members in amounts c^a, c^b, c^k. The utility of children depends additively on their consumption of goods and the time spent with each of the parents:

$$u^k = c^k + \alpha t^a + \beta t^b, \tag{2.20}$$

where the parameters α and β represent the efficiency of parents a and b, respectively, in childcare. This is, of course, a very special assumption and implies that consumption can fully compensate the child for the absence of parents and that the two parents' childcare time are perfect substitutes. Usually we assume that α and β are positive (perhaps an arguable assumption for teenagers). The utility of each parent is assumed to be multiplicative in their own consumption and the child's utility level:

$$u^s = c^s u^k \text{ for } s = a, b. \tag{2.21}$$

Thus, children are assumed to be a public good to their natural parents and *both* care about their welfare.

We consider here situations in which parents differ in their earning capacity and efficiency in child care. The linearity of the parents' utility functions in their own consumption implies that the parents would agree on an efficient program that maximizes the joint "pie" that is available for distribution between them.[8] That is, the parents would agree to:

$$\max_{t^a, t^b, c^k} \left\{ w^a \left((1 - t^a) + w^b \left(1 - t^b\right) - c^k \right) \left(c^k + \alpha t^a + \beta t^b\right) \right\} \tag{2.22}$$
$$\text{subject to } 0 \leq t_s \leq 1, \text{ for } s = a, b$$

[8] Thus, the amount of time spent on the child is determined by efficiency considerations, independently of the distribution of the consumption good. The two stage decision process, whereby production and distribution are separable, is an important consequence of transferable utility that will be discussed later in the book.

2.5.2 Intact families

We have three regimes, depending on the parameter values. We always assume:

$$w^b > w^a, \alpha > \beta$$

implying that the high wage spouse, b, has a *comparative* advantage in market work and the low wage person a has comparative advantage in home production:

$$\frac{w^b}{\beta} > \frac{w^a}{\alpha} \tag{2.23}$$

If both wages are high relative to efficiency at home production, (if $w^a > \alpha$ and, consequently, $w^b > \beta$) then both parents will work full-time in the market and use only market goods for caring for the child. Conversely, if both wages are low relative to efficiency at home production (if $w^b < \beta$ and $w^a < \alpha$) then parents will use only time to care for the child. An intermediate case is the one in which the high wage partner, b, has *absolute* advantage in market work and the low wage person a has *absolute* advantage working at home ,

$$\alpha > w^a, \beta < w^b$$

For this intermediate case b will spend all his time in market work and a will spend all her time looking after the child. This intermediate case has two distinct sub-cases that differ in the expenditures on the child. For case 1 we have:

$$w^b > \alpha. \tag{2.24}$$

In this case, the intact family spends part of its income on child goods, $c^k > 0$. Specifically, $t^a = 1$ and $t^b = 0$ and $c^k = \frac{w^b - \alpha}{2}$. The utility of the child is then $u^k = \frac{w^b + \alpha}{2}$ and the utility possibility frontier facing the parents is given by

$$u^a + u^b = \frac{(w^b + \alpha)^2}{4}. \tag{2.25}$$

In case 2 we have the converse:

$$w^b < \alpha, \tag{2.26}$$

which gives $c^k = 0$. In this case, the utility of the child is $u^k = \alpha$ and the UPF facing the parents is then given by

$$u^a + u^b = w^b \alpha. \tag{2.27}$$

2.5.3 Divorce with no transfers

What happens if the partners split and one of the partners receives custody, *without any transfers*? It is quite likely that if the marriage breaks up and the parents live in separate households, the utility of the non custodial parent from the child is reduced. Nevertheless, it is only natural that the non-custodial parent continues to care about the child and for simplicity we shall continue to assume that the utility of both parents is given by (2.21). We shall further assume that only the custodial parent can spend time with the child. If custody is assigned to parent b he will work fulltime in the market ($t^b = 0$ since $w^b > \beta$) and will set $c^k = 0.5w^b = u^k$. If custody is assigned to parent a she will work part time to finance her own consumption, setting $t = 0.5$, but will spend no money on child goods (since $\alpha > w^a$). In this case, the child's utility is $u^k = 0.5\alpha$. If we now choose the custodial parent to maximize the welfare of the child, we obtain a very simple rule for the assignment of custody. In the absence of post divorce transfers, the high wage parent b should obtain custody if and only if his\her wage, w^b, exceeds the efficiency of the low wage spouse a at home, α.

Table 2.2 compares the utility of the child when the parents are married and separated, when custody is assigned optimally for the two cases discussed above. We also show the utilities of each parent when they are separated and the *sum* of their utilities when they are married. Examining the entries in the table, it is seen that the child is always worse off when the parents split, because the custodial parent spends less time with the child or less goods on the child. We also have that at least one of the parents is worse off materially when the parents live apart, because their post divorce payoffs are below the utility possibility frontier in an intact family. That is:

$$(\frac{w^a}{2})^2 + (\frac{w^b}{2})^2 < \frac{(w^b + \alpha)^2}{4}. \tag{2.28}$$

Such results are quite typical and can be traced to the inefficient allocation of time following divorce. For example, for case 2 the custodial parent is pushed into the labor market, despite her comparative advantage in child care. The custodial parent who chooses how much time to spend with the child does not (or cannot) take into account the interests of the other parent, which is the source of the inefficiency. Following separation, the non- custodial parent can be better off than the custodial parent, because they can free ride on the custodial parent who takes care of the child. This is the case if the low wage parent a is the custodial parent and also holds if the high wage parent b is the custodian and $2w^a > w^b$. Thus, although the child is better off under the custody of the parent who is more efficient in caring for it, this parent may be better off if the *other* parent had the custody. The most natural way to deal with this "hot potato" problem, as well as with the low welfare of the child, is to force the non custodian parent

Case 1: $w^b > \alpha$, b is the custodian				
	Married		Separated	
Family member	Work at home	Utility	Work at home	Utility
a	1	$\frac{(w^b+\alpha)^2}{4}$	0	$\frac{w^a w^b}{2}$
b	0		0	$(\frac{w^b}{2})^2$
k	–	$\frac{w^b+\alpha}{2}$	–	$\frac{w^b}{2}$
Case 2, $w^b < \alpha$, a is the custodian				
Family member	Work at home	Utility	Work at home	Utility
a	1	$w^b\alpha$	$\frac{1}{2}$	$\frac{\alpha w^a}{4}$
b	0		0	$\frac{\alpha w^b}{2}$
k	–	α		$\frac{\alpha}{2}$
Note: when married the utility of a and b is shared.				

TABLE 2.2. Work patterns and material welfare of family members

to pay child support. Post divorce transfers will be discussed in detail in a subsequent chapter, but it should be noted at the outset that, in practice, custodial mothers often receive no transfer from the ex-husband and when they do the transfer is often quite small.

There is ample evidence that children with single or step parents are worse off than children in intact families (see Argys *et al*, 1998, Hetherington and Stanley-Hagan, 1999), suggesting that the break up of marriage can be quite costly. However, Piketty (2004) and Bjorklund and Sundstrom (2006) show that much of the differences in child attainments precede the divorce, so that the reduction in the child's welfare is caused by a bad quality of the match (for example, fights between the parents) rather than the divorce itself. In either case, the risk of separation may reduce the incentives to produce children and to specialize in home production.

2.6 Concluding remarks

None of the gains that we have discussed in this chapter actually require the traditional family institution. If all goods and work activities are marketable, there is no need to form marriages to enjoy increasing returns or to pool risks. In fact, the role of the family varies depending on market conditions and vice versa. For instance, with good medical or unemployment insurance one does not need to rely on his spouse. Similarly, sex and even children can be obtained commercially. Nevertheless, household production persists because it economizes on search, transaction costs and monitoring. However, to fully exploit these advantages requires a durable relationship.

This shifts attention to the question which types of partnerships are likely to last.

Gains from human partnerships need not be confined to a couple of the opposite sex. One also observes "extended families" of varying structures which coordinate the activities of their members and provide self insurance. The prevalence of male-female partnerships has to do with sexual attraction which triggers some initial amount of blind trust. (The Bible is quite right in puzzling over why "shall a man leave his father and mother and cleave unto his wife"[9].) Equally important is a strong preference for own (self produced) children. These emotional and biological considerations are sufficient to bring into the family domain some activities that could be purchased in the market. Then, the accumulation of specific "marital capital" in the form of children, shared experience and personal information increases the costs of separation and creates incentives for a lasting relationship. In this sense, there is an accumulative effect where economic considerations and investments reinforce the natural attachment. Other glues, derived from cultural and social norms also support lasting relationships. But in each case customs interact with economic considerations. The weaker is the market, the more useful is the extended family and social norms (commands) are added to the natural glue.

Keeping these considerations in mind, we can now address the question which activities will be carried out within the family. One argument is that the family simply fills in gaps in the market system, arising from thin markets, or other market failures, see Locay (1990). Another line of argument (see Pollak (1985)) is that the family has some intrinsic advantages in monitoring (due to proximity) and in enforcement (due to access to non-monetary punishments and rewards). A related but somewhat different argument is that family members have already paid the (sunk) costs required to acquire information about each other, see Ben-Porath (1980). Thus, credit for human capital investments may be supplied internally either because of a lack of lending institutions or because a spouse recognizes the capacity of her partner to learn and is able to monitor the utilization of his human capital better than outsiders. Similarly, annuity insurance is provided internally, either because of lack of annuity markets or because married partners have a more precise information on their spouse's state of health than the market at large. It is clear that these three considerations interact with each other and cannot be easily separated. The main insight is that the gains from marriage depend on the state of the market and must be determined in a general equilibrium context.

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[9] Genesis 2: 24.

2.7 References

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3

Preferences and decision making

3.1 Preferences

In the last chapter we informally reviewed the gains from marriage in some generality. The existence of potential gains from marriage is not sufficient to motivate marriage and to sustain it. Prospective mates need to form some notion as to whether families realize the potential gains and how they are divided. In this chapter we consider these issues in a very specific context. The context is a two person (woman a and man b) household[1] in which the only (static) decision is how much to spend on various market goods that are available at fixed prices, given fixed total household expenditure on all goods. Although very special this context allows us to discuss formally many of the issues that will be used in other contexts in later chapters.

Some commodities are private and some public. Private goods are consumed non-jointly by each partner and public goods, such as heating, are consumed jointly and non-exclusively by the two partners. In other words, private goods are characterized by an *exclusion restriction* property: the fact that I consume a particular apple *de facto* excludes anyone else from consuming the same apple. With public goods, on the contrary, no such restriction exists: that I enjoy seeing a beautiful painting on my wall does not preclude my spouse from enjoying it just as much (or even disliking it).

Several remarks can be made at that point. First, several commodities are sometimes used publicly and sometimes privately; for instance, I can drive my car alone to go to work, or the whole family may take a ride together. Second, the privateness or publicness of a good is quite independent of the type of control existing on that good and who exerts it; typically, parents have control over the (private) consumption of their young children. Finally, and more crucially, there exist subtle interactions between the ('technical') nature of a good and how it enters the member's utilities. The private consumptions of member a certainly enter a's utility; but it also may enter b's - we then call it an *externality.* Conversely, some commodities, although public by nature, may in fact be exclusively consumed by one member; for instance, although both spouses may in principle watch television together without exclusion, one of them may simply dislike TV and never use it. Throughout most of the book, we assume, to keep things simple, that any

[1] Children will be introduced at a later point.

particular commodity is either purely public or purely private, although many of our results would extend to more general settings.

We introduce some notation that will be used throughout the chapter. There are N public goods, and the market purchase of public good j is denoted Q_j; the N-vector of public goods is given by $\mathbf{Q}$. Similarly, private goods are denoted q_i with the n-vector $\mathbf{q}$. Each private good bought is divided between the two partners so that a receives q_i^a of good i and b receives $q_i^b = q_i - q_i^a$. Hence the vectors a and b receive are $\mathbf{q}^a$ and $\mathbf{q}^b$ respectively, with $\mathbf{q}^a + \mathbf{q}^b = \mathbf{q}$. An *allocation* is a $N+2n$-vector $\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$. The associated market prices are given by the N-vector $\mathbf{P}$ and the n-vector $\mathbf{p}$ for public and private goods respectively.

We assume that each married person has her or his own preferences over the allocation of family resources. Denote a's utility function by $U^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$ and b's by $U^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$. This general formulation allows that a is concerned directly with b's consumption and also that b's consumption of private goods impacts on a's preferences between her own private goods and the public goods. Any kind of externality is allowed. The presence of the partner's private consumption does not mean necessarily that members are altruistic to each other; for instance, it could simply represent the partner's smoking that bothers the other member by reducing their utility. Throughout the book, we assume, unless stated otherwise, preference orderings are continuous and convex and can be represented by utility functions $U^s(.)$, $s = a, b$, that are continuously differentiable and strictly concave.

In the subsequent chapters in the first half of this book we shall be discussing the resolution of conflicts that arise between partners if $U^a(.)$ and $U^b(.)$ represent different preferences. It is important to acknowledge, however, that if marriage is sometimes a battleground, it is can also be a playground. In the context of the family, love or affection might be operating and conflicts are thereby considerably attenuated. We return to this below.

Although quite reasonable, the form just described is too general to be used in most contexts - if only because it is difficult to incorporate such preferences into a model in which agents live alone for some part of their life-cycle. Consequently the literature generally takes a special case which is known as *caring*.[2] For this we first assume agents a and b have felicity functions $u^a\left(\mathbf{Q}, \mathbf{q}^a\right)$ and $u^b\left(\mathbf{Q}, \mathbf{q}^b\right)$ respectively. The most general form has

$$\begin{aligned} U^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) &= W^a\left(u^a\left(\mathbf{Q}, \mathbf{q}^a\right), u^b\left(\mathbf{Q}, \mathbf{q}^b\right)\right) \\ U^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) &= W^b\left(u^a\left(\mathbf{Q}, \mathbf{q}^a\right), u^b\left(\mathbf{Q}, \mathbf{q}^b\right)\right), \end{aligned} \tag{3.1}$$

where both $W^a(.,.)$ and $W^b(.,.)$ are monotone increasing functions. The

[2] Sometimes the term *altruistic* is used for preferences taking this form. Pollak (2006) has suggested the term *deferential* since each person defers to the judgment of the other regarding their consumption.

weak separability of these 'social' preferences represents an important moral principle; a is indifferent between bundles $\mathbf{q}^b, Q$ that b consumes whenever b is indifferent (and similarly for b). In this sense caring is distinguished from paternalism. Caring rules out direct externalities because a's evaluation of her private consumption $\mathbf{q}^a$ does not depend directly on the private goods that b consumes (and *vice versa*). A more commonly used form is the restricted version:

$$\begin{aligned} U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= u^a\left(\mathbf{Q},\mathbf{q}^a\right)+\delta^a u^b\left(\mathbf{Q},\mathbf{q}^b\right), \\ U^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= u^b\left(\mathbf{Q},\mathbf{q}^b\right)+\delta^b u^a\left(\mathbf{Q},\mathbf{q}^a\right). \end{aligned} \tag{3.2}$$

Generally we take the weights δ^a and δ^b to be non-negative parameters such that each person cares for the other but not as much as they care for themselves. For this formulation, $\delta^a = \delta^b = 0$ corresponds to *egotistic* preferences and $\delta^s > 0$ represents altruism. If $\delta^a \delta^b = 1$ then the two partners have the same ordinal preferences.

Some authors use a slightly different representation of altruism, namely

$$\begin{aligned} U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= u^a\left(\mathbf{Q},\mathbf{q}^b\right)+\tilde{\delta}^a U^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) \\ U^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= u^b\left(\mathbf{Q},\mathbf{q}^b\right)+\tilde{\delta}^b U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) \end{aligned} \tag{3.3}$$

The logic here is that a should cares about b's ultimate utility U^b, which includes also b's altruistic feelings towards a. We can then think of (3.2) as a reduced form obtained by the substitution:

$$U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) = u^a\left(\mathbf{Q},\mathbf{q}^b\right)+\tilde{\delta}^a[u^b\left(\mathbf{Q},\mathbf{q}^b\right)+\tilde{\delta}^b U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right)] \tag{3.4}$$

If $\tilde{\delta}^a\tilde{\delta}^b \neq 1$ we have:

$$U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) = \frac{1}{1-\tilde{\delta}^a\tilde{\delta}^b}u^a\left(\mathbf{Q},\mathbf{q}^a\right)+\frac{\tilde{\delta}^a}{1-\tilde{\delta}^a\tilde{\delta}^b}u^b\left(\mathbf{Q},\mathbf{q}^b\right) \tag{3.5}$$

Such a reduction yields logical results only if $\tilde{\delta}^a\tilde{\delta}^b < 1$. Clearly, too much caring ($\tilde{\delta}^a\tilde{\delta}^b > 1$) can lead to paradoxical results in which a puts negative weights on both felicity functions. See Bergstrom (1989) and Bernheim and Stark (1988) for further discussion and examples of how excessive altruism can lead to unpalatable outcomes.

In some contexts we wish to impose stronger restrictions on preferences. For example, we shall often consider only one private good. This can be justified if prices are fixed by an appeal to the composite commodity theorem. In that case we can consider the unique private good to be 'money'. A second, particular case that we shall consider in many contexts relies on the assumption of *transferable utility* (TU). This holds if we have egotistic preferences and each felicity function can be (possibly after an increasing transform and a renaming of the private goods) put into a form that is similar to the Gorman polar form:

$$\begin{aligned} u^a(\mathbf{Q},\mathbf{q}^a) &= f^a(q_2^a,...,q_n^a,\mathbf{Q}) + G(\mathbf{Q})\, q_1^a \\ u^b(\mathbf{Q},\mathbf{q}^b) &= f^b(q_2^b,...,q_n^b,\mathbf{Q}) + G(\mathbf{Q})\, q_1^b \end{aligned} \tag{3.6}$$

where $G(\mathbf{Q}) > 0$ for all $\mathbf{Q}$. Note that the $G(.)$ function is identical for both members, whereas the $f(.)$ functions can be individual-specific. In words, the transferable utility assumption implies that, for some well chosen cardinalization of individual preferences, the marginal utility of an additional dollar spent on private consumption of commodity 1 is always the same for both members. Hence utility can be 'transferred' between them (using commodity 1 transfers) at a fixed rate of exchange. Repeatedly in this book, we shall develop examples in which the transferability assumption drastically simplifies the problem to hand.

We shall often need to compare the utility of a given individual in two different marital situations, for instance when married versus when single (or divorced). Various assumptions can be made here. One extreme hypothesis states that marriage may change preferences in an arbitrary way. Then there is simply no relation between pre-marital and post-marital utility functions - not a very useful property for our purpose. Conversely, we may assume that preferences *over commodities* are not changed by marriage. This by no means implies that the satisfaction derived from any consumption is the same inside and outside marriage, but simply that the ranking of the various consumption bundles is not affected by the individual's marital status. With egotistic preferences, this will hold if the utility in marriage, u^s, is related to the pre-marital preferences represented by the utility function, $\bar{u}^s(\mathbf{Q},\mathbf{q}^s)$ by:

$$u^s(\mathbf{Q},\mathbf{q}^s) = F^s(\bar{u}^s(\mathbf{Q},\mathbf{q}^s)) \tag{3.7}$$

where the mapping $F^s(.)$ is strictly increasing. A particularly convenient special case that we shall employ when we consider explicitly the full gains from marriage is:

$$u^s(\mathbf{Q},\mathbf{q}^s) = F(\bar{u}^s(\mathbf{Q},\mathbf{q}^s) + \theta^s) \tag{3.8}$$

Here, θ^s represents non-monetary, marriage-specific aspects of s's idiosyncratic desire to be married. With caring preferences, the same obtains if we normalize the contribution of the spouse's utility to be uniformly zero when the agent is single. This assumption has important consequences on the empirical estimation of the models. If condition (3.8) is satisfied, then the preferences of married individuals amongst private and public goods are the same when married or single. These preferences can then be recovered from data on singles' behavior.

Finally, an intermediate assumption states that single and married individuals have the same basic preferences, but marriage involves a change in the consumption technology, a concept we define in the next subsection.

3.2 Household production

3.2.1 The general framework.

Household activities are not limited to private or public consumptions. They are also the source of important production activities that should not be disregarded. In low income countries, a large fraction of GDP consists of agricultural commodities produced at the household (or the village) level. Even in high income economies, a significant fraction of individual available time is spent on household production. This entails immediate tasks (cleaning, cooking, etc.) but also long term investments in health, education and others. In a sense, even such 'commodities' as love, affection or mutual care are 'produced' (and consumed) at the household level. In Becker's (1965) model, the only commodities that are ultimately consumed by individuals are those produced at the household level. Becker views goods that are purchased in the market as inputs in a production system that transforms these purchased goods into final commodities that are actually consumed (and enter individual utilities). These home produced goods can be either public or private for the two partners, denoted by C_j and c_j respectively. The production of commodities also requires time inputs that are provided by the household members in addition to market purchased goods. The technology is described by a production possibility set $\Omega\left(\mathbf{q}, t^a, t^b\right)$ that gives the possible vector of outputs $(\mathbf{c}, \mathbf{C})$ that can be produced given a vector of market purchases $\mathbf{q}$ and the total time spent in household production by each of the two partners, t^a and t^b. This allows that time spent on any activity may produce many goods.

Household production function

A special case is when the feasible set can be described by *household production functions* that specify the amount of each commodity that can be produced given the amount of market goods and time assigned to that commodity. We denote the vector of market goods assigned to commodity j by $\mathbf{q}^j$ and the time inputs of a and b devoted to commodity j by t_j^a and t_j^b, respectively. Thus:

$$c_j = f^j\left(\mathbf{q}^j, t_j^a, t_j^b\right) \tag{3.9}$$

The associated constraints are:

$$\begin{aligned} \sum_j \mathbf{q}^j &= \mathbf{q} \\ \sum_j t_j^s &= t^s, \quad s = a, b \end{aligned} \tag{3.10}$$

Each person has preferences defined over household produced goods and the vectors of time use, $U^s\left(\mathbf{C}, \mathbf{c}^a, \mathbf{c}^b, \mathbf{t}^a, \mathbf{t}^b\right)$ for $s = a, b$, where $\mathbf{t}^s$ is the vector of time inputs for j. This framework allows time activities to have two distinct roles. For example, a father who spends time with his child contributes to the development of the child (through $f^j(.)$) and may also enjoy spending time with the child (captured by the presence of t_j^b in $U^b(.)$). Of course, either of these effects could be negative (although not both).

A standard problem with this approach is that the production function, despite its conceptual interest, cannot be estimated independently of the utility function unless the home produced commodities are independently observable; see Pollak and Wachter (1975) and Gronau (2006). Observability of outputs may be acceptable for agricultural production, or even for children's health or education; it is less likely for, say, cleaning, and almost impossible for personal caring.

If only inputs are observed and not outputs we may be able to recover information about the technology if we make auxiliary assumptions such as constant returns to scale and assumptions on preferences. To illustrate this, consider two partners who consume one single public good C and one private good c such that a consumes c^a and b consumes c^b with preferences given by $u^s(C, c^s)$, $s = a, b$. Assume that the private good is purchased in the market and that the public commodity is produced using only the time inputs of the two partners. That is,

$$C = f\left(t^a, t^b\right). \tag{3.11}$$

Assuming that both partners participate in the labor market at wages w^a and w^b respectively, it can then be shown that for any efficient allocation the partners will minimize the cost of producing the public commodity in terms of the forgone private commodity, yielding

$$\frac{f_1\left(t^a, t^b\right)}{f_2\left(t^a, t^b\right)} = \frac{w^a}{w^b} \tag{3.12}$$

in any interior solution. If we assume constant returns to scale, we can write:

$$C = f\left(t^a, t^b\right) = t^b \phi(r) \tag{3.13}$$

for some function $\phi(r)$ where $r = \frac{t^a}{t^b}$. The condition (3.12) then reduces to:

$$\frac{\phi'(r)}{\phi(r) - r\phi'(r)} = \frac{w^a}{w^b} \tag{3.14}$$

The testable implication of this equality is that r only depends on the wage ratio ω; this can be tested on a data set that reports wages and time spent on household production. Defining,

$$h(r) = \frac{\phi'(r)}{\phi(r) - r\phi'(r)} \tag{3.15}$$

this equation can be re-written as:

$$\frac{\phi'(r)}{\phi(r)} = \frac{1}{r + \frac{1}{h(r)}}$$

Integrating, we have:

$$\phi(r) = K \exp\left(\int_0^r \frac{ds}{s + \frac{1}{h(s)}}\right)$$

where K is an unknown constant. In other words, when outputs are not observable, knowledge of the input supply (as a function of relative wages) allows us to determine the household production function up to a multiplicative scale factor.

It is important to note that the assumptions of constant returns to scale and no joint production (in the sense that t^a and t^b do not appear directly in the utility function) are critical for this particular identification result; see Pollak and Wachter (1975) and Gronau (2006) for further discussion of the role of these assumptions. A further issue that was not challenged in this literature is whether or not the partners are cooperating. The example above shows that in some cases it is sufficient to assume efficiency; other assumptions may also guarantee identification.

Marital technology and indifference scales

Let us briefly come back to the previous discussion on the changes in preferences that may result from marriage. The two extreme assumptions described were either that there are no such changes (in the sense that an individual's preference relationship over consumption bundles was independent of the person's marital status) or that they were arbitrary (that is, there is no relationship between pre- and post- marital utilities). The first assumption is often too restrictive, whereas the second is too general to be useful.

An intermediate approach, proposed by Browning, Chiappori and Lewbel (2003), relies on the notion of production technology. The idea is that marriage leaves ordinal preferences over commodities unchanged, but allows a different (and more productive) technology to be used. Formally, they apply the simple Barten household production technology in which n market goods are transformed into n household commodities in a linear and non-joint way; see the discussion in chapter 2, section 2.1. This setting allows us to separate the identification of preferences (which can be done on a subsample of singles) and that of the production function (for which household level data are needed). Not surprisingly, being able to observe identical individuals operating under different technologies (that is, as single or married) considerably facilitates identification. Browning *et al* show that the model can be estimated from the observation of demand functions for individuals and couples.

A crucial outcome of this approach is the computation of each member's *indifference scale,* defined as the minimum fraction of the household's income that this member would need to buy (at market prices) a bundle of privately consumed goods that put her *on the same indifference curve* *over goods* that she attained as a member of the household. Note that this amount is different (and lower) than what would be needed to purchase, as a single, the same *bundle* the member was consuming when married. Indeed, an obvious effect of the household technology is that the prices implicitly used within the household may differ from market prices; see chapter 2, section 2. It follows that even for a given level of expenditures, the consumption profile of a couple typically differs from that of single individuals.

3.2.2 Children

Modeling children is a complex issue, and one in which even basic methodological choices may be far from innocuous in terms of normative implications. A general approach relies on two basic ideas. One is that, in general, parents care about their children. This could take the form of parent s caring directly about the goods that the child consumes:

$$U^s = U^s\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b, \mathbf{q}^k, t^a, t^b\right) \tag{3.16}$$

where t^s are the time inputs of the parents and $\mathbf{q}^k$ denotes the vector of private consumption by the child. A widely used special case posits the existence of a child utility function:

$$u^k = u^k\left(t^a, t^b, \mathbf{Q}, \mathbf{q}^k\right) \tag{3.17}$$

Then the preferences of adult s can be defined recursively by:

$$U^s\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b, t^a, t^b\right) + \kappa^s u^k\left(\mathbf{Q}, \mathbf{q}^k, t^a, t^b\right) \tag{3.18}$$

where κ^s is the weight that parent s gives to the children.[3]

Of course, this approach can be used with any number of children. Depending on the problem to hand, one may either introduce one sub-utility per child or only distinguish between broader 'classes' (for example, boys versus girls, younger children versus older ones, etc.). Timing introduces additional issues, since parents care not only about their children but also their grandchildren. Barro and Becker (1988) have introduced the concept of dynastic utilities, whereby parents actually consider the sum of utility levels of *all* of their descendents, weighted according to some 'distance' factor $\kappa^s < 1$. Then adult s's utility takes the form $u^s + \sum_{t=1}^{\infty} (\kappa^s)^t u^{k(t)}$, with the convention that $u^{k(1)}$ denotes the utility of s's children, $u^{k(2)}$ of his grandchildren, and so on. This model, which relies on restricting (3.16) to (3.18), may have strong policy implications. For instance, government subsidies given to children can be completely offset by lower contribution

[3] A more general formulation would have utilities of the form $u^s\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b, u^k\right)$.

of their parents without any effect on the final outcome. This type of neutrality is often termed Ricardian Equivalence.

It is important to note that in this context, children matter for the household's choices, but only through the utility their parents derive from their well-being. This is a strong assumption, that can be relaxed in two directions. First, one may, alternatively, consider the child as another decision maker within the household. In this case a couple with one child would be considered as a three person household. Whether a child should be considered as a decision maker or not is a very difficult question, which may depend on a host of factors (age, education, occupation, income, etc.); moreover, its empirical translation introduces subtle differences that are discussed below.

Secondly, throughout this book we stick to a standard practice in economics, and we assume that preferences are given, that is, exogenous and stable. This assumption may be acceptable for adults, but maybe less so for children; after all, many parents invest time and resources into influencing (or 'shaping') their children's preferences regarding work, risk, or 'values' in some general sense. Indeed, a growing literature analyzes the formation of preferences from an economic viewpoint, as a particular 'production' process. These contributions are outside the scope of this book; the interested reader is referred to Becker (1998).

3.3 The unitary model

We now consider how the partners in the household make decisions on how to spend their time and money. To bring out the main ideas we consider a context in which there are no time allocation decisions, income is given and there is no household production. We take the incomes of a and b to be given at levels Y^a and Y^b respectively and we assume that there is no other income into the household. We further assume that household total expenditure, x, is set equal to household income $Y = Y^a + Y^b$, so that there is no borrowing or lending. The household budget constraint for allocations is given by:

$$\mathbf{P}'\mathbf{Q} + \mathbf{p}'\left(\mathbf{q}^a + \mathbf{q}^b\right) = x \tag{3.19}$$

In general the agents will differ on how to spend household income. There are three broad classes of decision processes: the unitary assumption, non-cooperative processes and cooperative processes.

The most widely used assumption is that choices are made according to a 'unitary' household utility function $\tilde{U}\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$. In subsection 3.5.6 we shall investigate when such an assumption is justified, but for now we simply consider the consequences. One natural assumption, due to Samuelson (1956), is to impose on the household utility function that it respects the individual preferences in the sense that:

$$\tilde{U}\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) = W\left(U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right), U^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right)\right) \tag{3.20}$$

where W is a *utility weighting function* which is strictly increasing in the individual utilities. The important feature of this weighting function is that it is fixed and does not vary with prices or income. Given a unitary utility function we define a *household utility function* over market goods by:

$$U\left(\mathbf{Q},\mathbf{q}\right) = \max_{\mathbf{q}^a} \tilde{U}\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}-\mathbf{q}^a\right) \tag{3.21}$$

Given this household utility function we can derive market demands in the usual way; namely, it solves the program

$$\max_{(\mathbf{Q},\mathbf{q})} U\left(\mathbf{Q},\mathbf{q}\right) \text{ subject to } \mathbf{P}'\mathbf{Q} + \mathbf{p}'\mathbf{q} \leq x$$

We assume enough on preferences (continuous differentiability, strict concavity), so that this leads to demands for market goods:

$$\mathbf{Q} = \mathbf{\Xi}\left(\mathbf{P},\mathbf{p},x\right) \tag{3.22}$$

$$\mathbf{q} = \boldsymbol{\xi}\left(\mathbf{P},\mathbf{p},x\right) \tag{3.23}$$

The unitary assumption has two important sets of implications. First, market demand functions satisfy the usual *Slutsky conditions;* adding-up, homogeneity, symmetry and negativity of the Slutsky matrix; see, for example, Mas-Colell, Whinston and Green (1995), chapter 3. Second, the demands only depend on prices and total household income, and are independent of the distribution of income; that is, they display *income pooling.* As we shall see below the latter has been the focus of much testing in the empirical literature.

3.4 Non-cooperative models

3.4.1 No public goods.

If we are not willing to assume a unitary utility function then we must specify a decision process. As always there are very many possibilities here but we shall only explore a small subset of these. We begin with non-cooperative procedures.[4] If household behavior is modeled non-cooperatively, then no binding agreements between members are assumed and the optimal decisions need not be Pareto efficient. However, in some cases efficiency obtains automatically as an outcome of independent decision making. Take

[4]Several authors take the Nash position that any cooperative game should be preceded by a non-cooperative game. Some of the authors cited in this section only develop a noncooperative interaction for this purpose.

the simple situation in which preferences are egotistic, and all commodities are privately consumed. The noncooperative solution boils down to the following two programs:

$$\max_{\mathbf{q}^a} \left\{ u^a\left(\mathbf{q}^a\right) \text{ subject to } \mathbf{p}'\mathbf{q}^a = Y^a \right\}$$
$$\max_{\mathbf{q}^b} \left\{ u^b\left(\mathbf{q}^b\right) \text{ subject to } \mathbf{p}'\mathbf{q}^b = Y^b \right\} \tag{3.24}$$

In words, the noncooperative solution simply implies in that case that each agent chooses his/her level of consumption independently of the other; that is, they live side by side, but without any economic interaction.[5] Then the consumption of individual s is simply this individual's demands at prices $\mathbf{p}$ and income Y^s. Denote the demand functions for s by $\boldsymbol{\xi}^s\left(\mathbf{p}, Y^s\right)$. Note that the allocation $\left(\boldsymbol{\xi}^a\left(\mathbf{p}, Y^a\right), \boldsymbol{\xi}^b\left(\mathbf{p}, Y^b\right)\right)$ is Pareto efficient: clearly, the utility of, say, a can only be increased by an income transfer from b, which would reduce b's welfare. Generally the associated household demands

$$\boldsymbol{\xi}\left(\mathbf{p}, Y^a, Y^b\right) = \boldsymbol{\xi}^a\left(\mathbf{p}, Y^a\right) + \boldsymbol{\xi}^b\left(\mathbf{p}, Y^b\right) \tag{3.25}$$

will not satisfy income pooling or the Slutsky conditions. The special case in which income pooling and the Slutsky conditions will hold is if the classic aggregation conditions hold. That is, if the two agents have linear Engel curves with each partner having the same slope for any good:

$$\begin{aligned} \xi_i^a &= \phi_i^a\left(\mathbf{p}\right) + \varphi_i\left(\mathbf{p}\right) Y^a \\ \xi_i^b &= \phi_i^b\left(\mathbf{p}\right) + \varphi_i\left(\mathbf{p}\right) Y^b \end{aligned} \tag{3.26}$$

so that the household demand for good i is given by:

$$\begin{aligned} \xi_i\left(\mathbf{p}, Y^a, Y^b\right) &= \phi_i^a\left(\mathbf{p}\right) + \phi_i^b\left(\mathbf{p}\right) + \varphi_i\left(\mathbf{p}\right)\left(Y^a + Y^b\right) \\ &= \phi_i^a\left(\mathbf{p}\right) + \phi_i^b\left(\mathbf{p}\right) + \varphi_i\left(\mathbf{p}\right) Y \end{aligned} \tag{3.27}$$

In this very special case income pooling holds in the sense that the household demands do not depend on the distribution of income. The distribution of the goods within the household will, however, depend on the distribution of income.

3.4.2 One public good.

Whenever a direct interaction between members is introduced - either because of public consumption, or because one member's consumption has an external effect on the other member's well being - inefficiencies are very

[5] Of course, this does not preclude the existence of non-economic interactions - love, sex, conversation or others.

likely to appear. To bring out the essential features of the analysis, let us assume that there is only one public good and one private good and that each person has egotistic preferences; see Chen and Wooley (2001) and Browning, Chiappori and Lechene (2010). Given that we have a public good and individual incomes a natural, noncooperative process to consider is a voluntary contributions game in which each person contributes to the purchase of the public good and then uses any money remaining to buy the private good for themselves. That is, the two agents have the problems:

$$\begin{aligned}
&\max_{Q^a,q^a} \left\{u^a\left(Q^a+Q^b,q^a\right) \text{ subject to } PQ^a+pq^a=Y^a\right\} \\
&\max_{Q^b,q^b} \left\{u^b\left(Q^a+Q^b,q^b\right) \text{ subject to } PQ^b+pq^b=Y^b\right\} \qquad (3.28)
\end{aligned}$$

where Q^s denotes agent s's contribution to the public good. Assuming that both goods are normal, this interaction has exactly one Nash equilibrium, which can take one of two forms. In the first form, both agents contribute to the public good. Since this is an interior solution for both we have:

$$\begin{aligned}
\frac{u^a_Q}{u^a_q}\left(\hat{Q},\hat{q}^a\right) &= \frac{P}{p} \\
\frac{u^b_Q}{u^b_q}\left(\hat{Q},\hat{q}^b\right) &= \frac{P}{p} \qquad (3.29)
\end{aligned}$$

If we sum the budget constraints we have:

$$P\hat{Q}+p\left(\hat{q}^a+\hat{q}^b\right)=Y^a+Y^b \qquad (3.30)$$

Thus we have three equations in three unknowns $\left(\hat{Q},\hat{q}^a,\hat{q}^b\right)$ with a solution:

$$\begin{aligned}
\hat{q}^a &= \xi^a\left(P,p,Y^a+Y^b\right) \\
\hat{q}^b &= \xi^b\left(P,p,Y^a+Y^b\right) \\
\hat{Q} &= \Xi\left(P,p,Y^a+Y^b\right) \qquad (3.31)
\end{aligned}$$

We conclude that the household's market demand for both the public good, $\hat{Q}$, and the private good, $\hat{q}=\hat{q}^a+\hat{q}^b$ depends only on total household income Y^a+Y^b and not on how it is distributed. In other words, we have *income pooling* even though we have a non-unitary model. This is an example of the remarkable neutrality result due to Warr (1983) (see also Bergstrom, Blume and Varian (1986) and Bernheim (1986)). This shows that while income pooling is a necessary condition for the unitary model, it is not sufficient.

It is important to note that income pooling here is a local property and holds only as long as both partners contribute to the public good. The other case we have to consider is the one in which only one person contributes. If this is person a, the first order condition in (3.29) holds for her. Person b spends all of his income on the private good, so that:

$$\frac{u_Q^b}{u_q^b}\left(\hat{Q}, \frac{Y^b}{p}\right) \leq \frac{P}{p} \tag{3.32}$$

with a strict inequality if the agent is not on the margin of contributing to the public good. In this case a redistribution of income from a to b will generally change the market demand since b will increase his demand for the private good and a generally will not change her demands to exactly offset these revisions. Thus we have market demands:

$$\begin{aligned} \hat{q} &= \hat{q}^a + \frac{Y^b}{p} = \xi^a\left(P, p, Y^a\right) + \frac{Y^b}{p} \\ \hat{Q} &= \hat{Q}^a = \Xi\left(P, p, Y^a, Y^b\right) \end{aligned} \tag{3.33}$$

In both cases, the non-cooperative procedure leads to an inefficient outcome (except for the cases in which one or the other has all of the income); this is the standard under-provision for the voluntary contributions public goods game. To see that for the case of an interior solution, add the two first order conditions (3.29), yielding

$$\frac{u_Q^a}{u_q^a}\left(\hat{Q}, \hat{q}^a\right) + \frac{u_Q^b}{u_q^b}\left(\hat{Q}, \hat{q}^b\right) = 2\frac{P}{p},$$

while Samuelson's (1954) condition for an efficient allocation of public goods requires that

$$\frac{u_Q^a}{u_q^a}\left(\hat{Q}, \hat{q}^a\right) + \frac{u_Q^b}{u_q^b}\left(\hat{Q}, \hat{q}^b\right) = \frac{P}{p}. \tag{3.34}$$

That is, the sum of the willingness to pay for the public good of the two partners, should equal to the opportunity cost of the public good in terms of the private good. In this regard, there is an under provision of the public good. [6]

We now present an example to illustrate some of the points made here. Normalize prices to unity, $P = p = 1$, and take preferences represented by $u^a = q^a Q^\alpha$ and $u^b = q^b Q$. The parameter α governs how much a likes the public good; if $\alpha > 1$ then she values it more than b if they have the same private consumption. We set $Y^a = \rho$ and $Y^b = (1 - \rho)$ so that household

[6] Results on dynamic contributions games suggest that inefficiencies cam be eliminated if players contribute sequentially and cannot reduce previous contributions; see, for example, Matthews (2006).

income is unity and ρ is a's share of household income. It is straight forward to show that the decisions of the agents are given by:

$$\begin{aligned}\hat{Q}^a &= \min\left(\max\left(0, \rho - \frac{1}{1+2\alpha}\right), \frac{\alpha\rho}{1+\alpha}\right)\\ \hat{Q}^b &= \min\left(\max\left(0, (1-\rho) - \frac{\alpha}{1+2\alpha}\right), \frac{(1-\rho)}{2}\right)\end{aligned}$$

(with the demands for private goods being given by the budget constraints). It is easiest to see the implications of this if we graph $\hat{Q} = \hat{Q}^a + \hat{Q}^b$ against a's share of income, ρ. In figure 3.1 we do this for two values of α, 0.8 and 1.2. There are a number of notable features to figure 3.1. First, if b has all the income ($\rho = 0$) then the level of public goods provision corresponds to his preferred level; here a value of one half. If we now redistribute some income from b to a we see that the level of the public good falls whether or not a has a higher valuation for the public good ($\alpha \lesseqgtr 1$). This is because a uses all of her income for her own private good and b reduces spending on both the public good and his private good. As we continue shifting income from b to a the level of the public good falls until at some point a starts to contribute. The level at which a starts to contribute is lower the higher is the value of her liking for the public good (compare the curves for $\alpha = 0.8$ and $\alpha = 1.2$). Once both partners are contributing to the public good, further small transfers from b to a leave all allocations unchanged as a increases her contributions and b reduces his in an exactly offsetting way (this is the local income pooling result). At some point b's income falls to the point at which he stops contributing. This level of income is lower the higher is the level of the provision of the public good. After this, transfers of income from b to a cause a to increase her contribution to the public good until she has all of the income ($\rho = 1$).

To illustrate that the level of provision of the public god is inefficiently low for any value of $\rho \in (0, 1)$, consider the case $\alpha = 1$ and $\rho = 0.5$. The equilibrium choices are $\hat{Q} = 1/3$ and $q^a = q^b = 1/3$. This gives each a utility level of $1/9$. If we instead impose that each contributes 0.25 to the public good and spends 0.25 on their own private good then each has a utility level of $1/8$, which is a Pareto improvement on the equilibrium outcome.

3.4.3 Altruism and the Rotten Kid Theorem.

An important extension to this analysis is to move beyond the egoistic assumption and to allow for altruism. In this case, if one person has most (or all) of the income and cares for the other then they may make a transfer of private goods to the poorer partner as well as being the sole contributor to the public good. This adds flat segments at the extremes of figure 3.1, as shown in figure 3.2. In this figure the demand for the public good if a's income share is between ρ_1 and ρ_4 is of the same form as for the egoistic case of the last subsection. If, however, we have an extreme distribution of income then the figure changes. For example, if b has most of the income ($\rho < \rho_1$) and cares for a then he will transfer some private good to her and will be the only contributor to the public good (since ρ_2 is the distribution at which a starts to contribute). In this region we have three important implications. First, there is local income pooling and small re-distributions of income within the household would not change the allocations $\left(\hat{Q}, \hat{q}^a, \hat{q}^b\right)$. Second, the outcome is efficient since b is an effective dictator; any other feasible allocation will make b worse off. Third, the household demands for private goods $(\hat{q}^a(P,p,Y) + \hat{q}^b(P,p,Y))$ and public goods $(\hat{Q}(P,p,Y))$ will satisfy the Slutsky conditions. Note, however, that the range of this unitary-like behavior and efficiency will depend on the degree of altruism; as drawn, b cares more for a than a cares for b (the interval $[0, \rho_1]$ is wider than the interval $[\rho_4, 1]$).

In chapter 8 of his revised Treatise of 1991 Becker refers to the unitary style implications (efficiency, income pooling and the Slutsky conditions) as the Rotten Kid Theorem (RKT); see also Becker (1974). If one person has enough income relative to the other and cares for them then they internalize all decisions and the household behaves as though it is one. A corollary is that each household member will be motivated to maximize total household income. For example, if we have $\rho < \rho_1$ and a can take some action that lowers her income but increases b's income by more, she will choose to do it, safe in the knowledge that b will increase the transfer to her sufficiently to make her better off. This presentation makes it clear that the scope of the RKT (in this version) is limited; it only applies locally and requires an extreme distribution of household income and altruism. In subsection 3.5.10 below we present a more general version of the RKT that is closer in spirit to Becker's original formulation in Becker (1974). This version widens the scope at the cost of imposing restrictions on preferences.

3.4.4 Many public goods.

When we turn to the more realistic case with more than one public good, the important features we saw above persist but some new ones emerge. The main points can be seen in a model with no altruism, N public goods, a single private good and prices normalized to unity. The voluntary contributions model has:

$$\begin{aligned}
&\max_{\mathbf{Q}^a, q^a} \left\{ u^a\left(\mathbf{Q}^a + \mathbf{Q}^b, q^a\right) \text{ subject to } \mathbf{e}'\mathbf{Q}^a + q^a = Y^a \right\} \\
&\max_{\mathbf{Q}^b, q^b} \left\{ u^b\left(\mathbf{Q}^a + \mathbf{Q}^b, q^b\right) \text{ subject to } \mathbf{e}'\mathbf{Q}^b + q^b = Y^b \right\} \qquad (3.35)
\end{aligned}$$

where $\mathbf{e}$ is an N-vector of ones. Let $\left(\hat{Q}_1^s, ...\hat{Q}_N^s\right)$ for $s = a, b$ be a Nash equilibrium.[7] We say that person s *contributes* to good j if $\hat{Q}_j^s > 0$. Let m^a (respectively, m^b) be the number of goods to which a (respectively, b) contributes. Browning *at al* (2010) show that if all public goods are bought ($\hat{Q}_j^s > 0$ for at least one s) then either $m^a + m^b = N$ or $m^a + m^b = N + 1$ (generally). This striking result shows that there is at most one public good to which both partners contribute.[8]

To see the result informally, suppose that both partners simultaneously contribute to two public goods, i and j. Then both set the marginal rates of substitution between the two goods to unity (the relative prices) and hence equalize the mrs's:

$$\frac{u_i^a}{u_j^a} = \frac{u_i^b}{u_j^b} \tag{3.36}$$

Without restrictions on preferences and incomes, this is unlikely to hold. Moreover, if it does hold, if we make an infinitesimal change in Y^a or Y^b the property (3.36) will generally not hold.

If there is some overlap in contributions ($m^a + m^b = N + 1$) then we have the local income pooling result, just as in the one public good case when both contribute. The result that each partner has a set of public goods which are his or her 'domain' suggests a gender division of allocation within the household. Note, however, that the goods that each takes as their domain is determined endogenously by preferences *and* the division of income within the household. As we move from b having all the income to a having all the income (holding total income constant) the number of goods that she contributes to will generally rise and the number of goods to which he contributes will generally fall.

We illustrate with an example with egoistic preferences from Browning *et al* (2010) for the case of two public goods, G and H. Let the two partners have preferences represented by the pair of Cobb-Douglas utility functions

$$\begin{aligned} u^a(q^a, G, H) &= \ln q^a + \frac{5}{3}\ln G + \frac{8}{9}H \\ u^b(q^b, G, H) &= \ln q^b + \frac{15}{32}\ln G + \frac{1}{2}\ln H \end{aligned}$$

The relative weights on the two public goods are $\frac{45}{24}$ and $\frac{15}{16}$ for a and b respectively; that is, a likes good G relative to good H, more than b. Figure 3.3 shows the purchases of public goods against a's share of income. When a has a low share of income (region I on the x-axis) she does not

[7] We assume enough to ensure the existence of at least one Nash equilibrium. We do not impose uniqueness.

[8] This result is generic in the sense that it is possible to find 'knife-edge' configurations of preferences and incomes for which the two partners contribute to more than one common public good.

contribute to either public good ($m^a = 0$ and $m^b = 2$). In this region, increases in a's income share lead her to spend more on the private good and lead b to spend less on both public goods. If her income is increased to region II then she starts contributing to one of the public goods (good G in this case) and he continues contributing to both ($m^a = 1$ and $m^b = 2$); this is a region of income pooling. As we continue to take income from b and give it to a we move to region III where she contributes to one public good and he contributes to the other ($m^a = 1$ and $m^b = 1$). This is again a region in which the distribution of income matters (locally). Regions IV and V are analogous to regions II and I, with a and b exchanged. One feature to note about this model is the point at which flat segments begin is the same for the two goods (and household expenditures on the private good).

Lundberg and Pollak (1993) introduce a model that is similar to the many public goods version above, except that they restrict contributions to exogenously given sets of public goods for each partner, which they term *separate spheres*.[9] They have two public goods and assume that each partner has a public good to which they alone can contribute; this is their 'sphere' of responsibility or expertise. These spheres are determined by social norms; this is the principal difference from the model developed in this subsection in which the 'sphere' of influence depends on preferences and the distribution of income within the household and is hence idiosyncratic to each household.

3.5 Cooperative models: the collective approach

The main problem with non-cooperative procedures is that they typically lead to inefficient outcomes. In a household context this is a somewhat unpalatable conclusion. If each partner knows the preferences of the other and can observe their consumption behavior (the assumption of symmetric information) and the two interact on a regular basis then we would expect that they would find ways to exploit any possibilities for Pareto improvements. This does not preclude the existence of power issues; as we shall see, the notion of 'power' plays a crucial role (and has a very natural interpretation) in cooperative models. The cooperative approach does recognize that the allocation of resources within the household may (and generally) will depend on the members' respective 'weights'; it simply posits that however resources are allocated, none are left on the table.

There are various ways of modeling cooperative behavior. In what follows, we mainly explore the implications of the only restriction common to all cooperative models, namely that household decisions are Pareto efficient,

[9] In section 5 below we give a fuller account of the separate spheres model.

in the usual sense that no other feasible choice would have been preferred by *all* household members. This approach was originally suggested by Chiappori (1988, 1992) and Apps and Rees (1988). Following Chiappori, we refer to such models as *collective* and refer to households that always have Pareto efficient outcomes as *collective households*. More specific representations, based on bargaining theory, are briefly discussed at the end at this section. In the remainder of this chapter we briefly introduce the collective model. Chapters 4 and 5 expand on this discussion.

The collective approach relies on two fundamental assumptions. First, there exists a decision process in the household and it is stable. Second, this process leads to Pareto efficient outcomes. We discuss these aspects successively.

3.5.1 Decision processes

A fundamental assumption in unitary demand theory is that individual preferences are *stable*, in the sense of not changing capriciously from moment to moment. This is not a logical requirement; in principle, the world could be such that people are intrinsically inconsistent, and a person's preferences today are unconnected with those of yesterday. Clearly, in such a world, very little could be say about individual behavior: a minimum level of stability is necessary if we wish to make predictions based on our models.

The same requirement applies to any model aimed at describing the behavior of a group. The notion of stability, in that case, must be given a broader interpretation: it relates not only to preferences, but also to the decision process. Again, the world could be such that a given household, faced with the same environment in different time periods, adopts each time a different decision process leading to different outcomes. And again, in such a world not much could be predicted about household behavior. We rule out these situations by assuming the existence of a stable decision process. Formally, we define the fundamentals of the model as the preferences of the members and the domestic technologies they can use. A *decision process* is a mapping that associates, to given fundamentals and given vectors of prices, incomes and factors that affect preferences and the decision process, a probability distribution over the set of consumption bundles. Our first basic assumption is thus the following:

Axiom 3.1 (Stability) *Each household is characterized by a unique decision process.*

In words: there is a stable relationship between the fundamentals of the model, the economic environment and the chosen outcomes. Note that, in full generality, this relationship needs not be deterministic. It may be the case, for instance, that in some circumstances the process will lead to explicit randomization.[10] What the stability axiom requires in that case is that the randomization be stable, in the sense that, keeping the fundamentals fixed, a household faced with the same economic environment will always randomize in exactly the same way (that is, using the same probability). Nevertheless, in what follows we essentially consider deterministic decision processes.

An important remark, however, is that while *household members* can observe all the factors influencing the decision process, the *econometrician* analyzing their behavior may not have such luck. If some determinants of the process are not observed, we are in a classical situation of unobserved heterogeneity. Then the model may (and typically will), for empirical purposes, involve probability distributions (of unobserved heterogeneity), even when the decision process in each household is purely deterministic. The corresponding randomness should not be considered as intrinsic; it simply reflects the econometrician's inability to observe all the relevant variables.

Clearly, the stability axiom is not specific to the collective approach; *any* model of group behavior must assume some kind of stability to be able to make predictions. Most of the time, the stability is implicit in the formulation of the model. For instance, in the unitary framework, a unique utility is maximized under a budget constraint; the outcome is the solution to the maximization problem. Similarly, in noncooperative models based on Nash equilibria, the decision process selects, for given fundamentals and a given environment, the fixed point(s) of the best response mapping. In the collective approach, one way to justify the stability axiom could be to assume that the household uses a well specified bargaining protocol, which can be cooperative (Nash, Kalai-Smorodinsky) or noncooperative (for example, Rubinstein's 'shrinking cake' model). In all cases, the concept under consideration exactly pins down the particular outcome of the negotiations as a function of prices, incomes and factors which influence the bargaining game (for example, the status quo points). But, of course, assuming bargaining is by no means necessary for stability.

3.5.2 Assuming efficiency

The second key assumption of the collective approach is that the outcomes of the decision process are always efficient, in the (usual) sense that no alternative decision would have been preferred by all members. The efficiency assumption is standard in many economic contexts, and has often been applied to household behavior. For instance, the axiomatic concepts of bargaining used in cooperative game theory typically assume efficiency,

[10] For instance, a basic conclusion of second best theory is that in the presence of non convexities, randomization may be needed to achieve Pareto efficiency. See Chiappori (2009) for an application to collective labor supply.

and noncooperative models of bargaining under symmetric information typically generate Pareto efficient outcomes. Among the alternative approaches that have been proposed in the literature, many, from simple dictatorship (possibly by a 'benevolent patriarch', as in Becker, 1974) to the existence of some household welfare function (as in Samuelson, 1956) assume or imply Pareto efficiency. In the same line, the 'equilibrium' approaches of Becker (1991) and Grossbard-Schechtman (1993), in which household members trade at existing market prices, typically generate efficient outcomes.

Natural as it seems for economists, the efficiency assumption nevertheless needs careful justification. Within a static context, this assumption amounts to the requirement that married partners will find a way to take advantage of opportunities that make both of them better off. Because of proximity and durability of the relation, both partners are aware of the preferences and actions of each other. They can act cooperatively by reaching some binding agreement. Enforcement of such agreements can be achieved through mutual care and trust, by social norms and by formal legal contracts. Alternatively, the agreement can be supported by repeated interactions, including the possibility of punishment. A large literature in game theory, based on several 'folk theorems', suggests that in such situations, efficiency should prevail.[11]

There are, however, two situations (at least) in which the efficiency assumption may fail to apply. One is when existing social norms impose patterns of behavior that may conflict with efficiency. One example for apparent inefficiency is when, due to the traditional norms, the wife is expected to stay at home and the husband to work in the market. Although such a division of labor may have been efficient in the past, it certainly conflicts with efficiency in modern societies in which women are often more educated than their husband. Another example is the finding by Udry (1996) that households in Burkina-Faso fail to allocate inputs efficiently among various crops because of the socially imposed division of labor between genders, which implies that some crops can only be grown by a particular gender.[12]

Secondly, some decisions are taken only once (or a few times), which implies that the repeated game argument does not apply; see Lundberg and Pollak (2003). Deciding whether to engage in a long training program or to move to another city, are not frequent decisions. The kind of 'equilibrium punishments' that are needed to implement efficient outcomes in repeated games may then be unavailable. The main theoretical insight here is that for

[11] Note, however, that folks theorems essentially apply to infinitely repeated interactions.

[12] A program of research in economics tries to explain existing institutions (including social norms) as an efficient response to a particular context; the argument being that competition will tend to promote the 'best performing' institutions. However, when technology changes deviations from efficiency can arise, because social norms may change slowly.

medium or long-term decisions, efficiency requires commitment; conversely, any limitation of the members' ability to commit may result in inefficient outcomes. As we know, commitment within a household is only partial; for instance, it is impossible to commit not to divorce, although marriage contracts can be used to make divorce costly for one or both of the members. For that reason, we will treat the issue of efficiency in a different manner depending upon whether we deal with a dynamic or static context. In most of the first half of this book the setting is static and efficiency is assumed. However, in chapters 6, 7, 8 and 12, which discuss saving and investment, we allow departures from efficiency.

In the remainder of this Chapter, we investigate the properties of models based on the collective approach. Before entering the technical analysis, however, one point should be stressed. The great advantage of the collective model is that we do not have to specify the (stable) mechanism that households use, but only assume that such a mechanism exists. In other words, the collective strategy does not make assumptions about decision processes (beyond efficiency); it rather lets the data tell what these processes are. This naturally leads to two crucial questions. One is whether the efficiency assumption is testable; that is, whether empirically falsifiable predictions can be derived from it. The second question relates to identification: when (and under which assumptions) is it possible to recover the decision process from observable behavior? Answering these questions is a strong motivation for what follows.

3.5.3 Distribution factors

The generality of the collective approach comes at a cost. While the efficiency assumption restricts the set of possible allocations, we are still left with 'many' of them - a continuum, in fact. If we want to make more specific predictions on household behavior, additional information - more precisely, additional sources of variations - may be useful. Among the various factors that can influence household behavior, many have a 'direct' impact on either preferences or the budget constraint. A change in prices, wages and non labor income are likely to affect demands and labor supplies, simply because they modify the set of options available. A more subtle influence goes, indirectly, through the decision process. A change in the economic environment may not affect either the preferences or the budget opportunities but still have an impact on the decision process. This idea is incorporated into the collective model by introducing distribution factors. Any variable that has an impact on the decision process but affects neither preferences nor budget constraints is termed a *distribution factor*. In theory, a large number of variables fit this description. Factors influencing divorce, either directly (for example, the legislation governing divorce settlements and alimony payments) or indirectly (for example, the probability of remarriage, which itself depends on the number of available potential mates - what

Becker calls the 'marriage market') should matter, at least insofar as the threat (or the risk) of divorce may play a role in the decision process. Individuals' income or wealth can also be used as distribution factors. Suppose, for example, that earned and unearned income is given for any individual and let Y^s denote the total income of person s. Then total household income, given by $Y = Y^a + Y^b$, is all that matters for the budget constraint. For any *given* level of Y, the individual contribution of a to total income, measured for instance by the ratio Y^a/Y, can only influence the outcome through its impact on the decision process; it is thus a distribution factor.

In the collective framework, changes in distribution factors typically lead to variations in outcomes while the set of efficient allocations remains unchanged; as such, it provides very useful information on the decision process actually at stake in the household. For that reason, it is in general crucial to explicitly take then into account in the formal model. In what follows, therefore, $\mathbf{z}$ denotes a vector of distribution factors.

3.5.4 Modeling efficiency

The basic framework

The characterization of efficient allocations follows the standard approach. The basic definition is that an allocation is Pareto efficient if making one person better off makes the other worse off:

Definition 3.1 *An allocation* $\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$ *is* Pareto efficient *if any other allocation* $(\bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b, \bar{\mathbf{Q}})$ *that is feasible:*

$$\mathbf{P}'\bar{\mathbf{Q}} + \mathbf{p}' \left(\bar{\mathbf{q}}^a + \bar{\mathbf{q}}^b\right) \leq \mathbf{P}'\mathbf{Q} + \mathbf{p}' \left(\mathbf{q}^a + \mathbf{q}^b\right)$$

and is such that $u^a\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right) > u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$*, it must be such that* $u^b\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right) < u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$ *(and conversely).*

In practice the basic definition is not very tractable and we often use one of two alternative characterizations. A first characterization is:

Definition 3.2 *For any given vector* $(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$ *of prices, total expenditure and distribution factors, an allocation* $\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$ *is* Pareto efficient *if there exists a feasible* $\bar{u}^a$*, which may depend on* $(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$*, such that* $\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$ *solves the problem:*

$$\max_{\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b} u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) \tag{3.37}$$

$$\text{subject to } \mathbf{P}'\mathbf{Q} + \mathbf{p}'\left(\mathbf{q}^a + \mathbf{q}^b\right) \leq x \tag{3.38}$$

$$\text{and } u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) \geq \bar{u}^a\left(\mathbf{P}, \mathbf{p}, x, \mathbf{z}\right) \tag{3.39}$$

Thus the Pareto efficient allocation can be derived from maximizing the utility of one partner holding the utility of the other at a given level: among all allocations providing a with exactly $\bar{u}^a$, the efficient one(s) generate the maximum possible utility for b. It goes without saying that this approach - just like most microeconomics - should not be taken literally. No one believes that agents actually write and solve a program such as (3.37). Our claim is simply that when a decision process, whatever its exact nature, always lead to efficient outcomes, then for any choice of prices, income and distribution factors there exists a $\bar{u}^a$ such that the household behaves *as if* it was solving program (3.37).

The solution to (3.37), when it exists (that is, if $\bar{u}^a$ is feasible), depends on prices, total expenditure and on the arbitrary level $\bar{u}^a$; it can be denoted as $u^b = \Upsilon(\mathbf{P}, \mathbf{p}, x, \bar{u}^a)$. The set of all pairs $(\bar{u}^a, \Upsilon(\mathbf{P}, \mathbf{p}, x, \bar{u}^a))$ when $\bar{u}^a$ varies over a domain of feasible allocations for a is the set of all efficient allocations; it is known as the *Pareto frontier* or *utility possibility frontier*, UPF. Under the assumption that the utility functions are strictly concave it is straight forward to show that the function $\Upsilon(.)$ is strictly concave in $\bar{u}^a$ (see below). This allows us to write Program (3.37) in a different but equivalent way. Let μ denote the Lagrange multiplier for constraint (3.39); note that μ is always nonnegative (and is a function of $(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$). Then the program is equivalent to:

$$\max_{\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b} \mu u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) + u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) \tag{3.40}$$

under the constraint (3.38). The coefficient μ is known as the *Pareto weight* for a. That is, a Pareto efficient outcome always maximizes a weighted sum of the two individual utilities. A slightly modified form that keeps the formal symmetry of the problem is sometimes used:

$$\max_{\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b} \tilde{\mu} u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) + (1 - \tilde{\mu})\, u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right), \tag{3.41}$$

where $\tilde{\mu} \in [0, 1]$. The Pareto weight plays a critical role in all that follows.

Finally, an equivalent formulation directly generalizes Samuelson's household welfare. Specifically, take any smooth function $W\left(u^a, u^b, \mathbf{P}, \mathbf{p}, x, \mathbf{z}\right)$ that is strictly increasing in its first two arguments, and consider the program:

$$\max_{\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b} W\left(u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right), u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right), \mathbf{P}, \mathbf{p}, x, \mathbf{z}\right) \tag{3.42}$$

under the constraint (3.38). Clearly, a solution to (3.42) is Pareto efficient; for otherwise some alternative allocation would increase both u^a and u^b, one of them (at least) strictly, but that would strictly increase W, a contradiction. Conversely, any allocation that is Pareto efficient maximizes a weighted sum of the form (3.40), which is a particular (linear) case of a W function. This establishes equivalence: an allocation is Pareto efficient if and only if there exists some W such that it maximizes (3.42) under budget constraint.

This setting generalizes Samuelson because the welfare index W depends on individual utilities, but also directly on prices, total expenditure and distribution factors. In other words, the household maximizes an index that is *price (and income) dependent*, which distinguishes this setting from a unitary representation. The surprising property is that under tsrict concavity one can assume without loss of generality that the index W is indeed linear (as in (3.40)). We shall come back to this issue below.

Ordinal versus cardinal representation of preferences

It is important to understand what, in the previous discussion, requires a *cardinal* representation of preferences, and what can be defined using only a standard, ordinal representation. The *set* of Pareto efficient allocations is an ordinal concept; it is not modified when u^s is replaced with $F(u^s)$ for a strictly increasing mapping $F(.)$. Under smoothness conditions, the set is one-dimensional, and therefore can be described by one parameter. However, the parametrization entails cardinality issues. For instance, a natural parametrization is through the weight μ. But μ depends on the particular cardinal representation that has been adopted for u^a and u^b: if u^s is replaced with $F(u^s)$, then the parameter μ corresponding to a particular efficient allocation has to be modified accordingly. Moreover, the convexity properties of the Pareto set are also of a cardinal nature. Assuming smooth preferences, for any given price-income vector, one can find cardinal representations of preferences such that the Pareto frontier is convex, linear or concave. In most of what follows, we adopt the convention of always using a strictly concave representation of utilities. In this case, the Pareto set is strictly convex. Indeed, for a given price-income vector, take two points $(\bar{u}^a, \bar{u}^b)$ and (u'^a, u'^b) on the Pareto frontier, and let $(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b)$ and $(\mathbf{Q}', \mathbf{q}'^a, \mathbf{q}'^b)$ be the corresponding consumption vectors. The vector

$$(\mathbf{Q}'', \mathbf{q}''^a, \mathbf{q}''^b) = \frac{1}{2}(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b) + \frac{1}{2}(\mathbf{Q}', \mathbf{q}'^a, \mathbf{q}'^b)$$

satisfies the budget constraint, and by strict concavity,

$$u^s(\mathbf{Q}'', \mathbf{q}''^a, \mathbf{q}''^b) > \frac{1}{2}u^s(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b) + \frac{1}{2}u^s(\mathbf{Q}', \mathbf{q}'^a, \mathbf{q}'^b) = \frac{1}{2}\bar{u}^s + \frac{1}{2}u'^s$$

for $s = a, b$. We conclude that the point $\frac{1}{2}(\bar{u}^a, \bar{u}^b) + \frac{1}{2}(u'^a, u'^b)$ belongs to the interior of the Pareto set.

Graphically, on Figure 3.4, the Pareto set is indeed strictly convex. We see that any point on the UPF can be defined either by its coordinate on the horizontal axis, here u^a, as in program (3.37), or by the negative of the slope of the Pareto frontier at that point, here μ as in program (3.40). Given that the UPF is strictly concave there is an increasing correspondence between $\bar{u}^a$ and μ: a larger $\bar{u}^a$ (or μ) corresponds to an allocation that is more favorable to a (hence less favorable for a). Note that the correspondence between $\bar{u}^a$ and μ is one-to-one; that is, for any $\bar{u}^a$, there exists exactly one μ that picks up the efficient point providing a with exactly $\bar{u}^a$, and conversely for any μ there is only one allocation that maximizes (3.40) under budget constraint, therefore only one corresponding utility level $\bar{u}^a$.

We can also understand from Figure 3.5 why the maximization of generalized Samuelson index $W\left(u^a, u^b, \mathbf{P}, \mathbf{p}, x, \mathbf{z}\right)$ is equivalent to that of a *linear* combination $\mu u^a + u^b$. The maximization of a non linear index W will select a point where the Pareto frontier is tangent to some indifference curve of W. If $-\mu$ denotes the slope of the corresponding tangent, maximizing $\mu u^a + u^b$ leads to exactly the same point. Replacing W with its linear equivalent can be done at any point, provided that μ varies adequately; technically, this simply requires that:

$$\mu = \frac{\partial W / \partial u^a}{\partial W / \partial u^b}$$

The main drawback of the generalized index version is that a continuum of different welfare indices lead to the same choices. Indeed, for any function F strictly increasing in its first argument, the indices W and $\bar{W} = F\left(W, \mathbf{P}, \mathbf{p}, x, \mathbf{z}\right)$ are empirically indistinguishable. The linear version, from this perspective, has an obvious advantage in terms of parsimony; in addition, it has a natural interpretation in terms of distribution of powers (see below).

Finally, we may briefly discuss two particular cases. One obtains when the cardinal representations of utilities are concave but not strictly concave. In that case, the UPF may include 'flat' (that is, linear) segments (Figure 3.6). Then Program (3.37) is still equivalent to Program (3.40), but the relationship between $\bar{u}^a$ and μ is no longer one-to-one. It is still the case that for any $\bar{u}^a$, exactly one μ picks up the efficient point providing a with $\bar{u}^a$. But the converse property does not hold; that is, to some values of μ are associated a continuum of utility levels $\bar{u}^a$; graphically, this occurs when $-\mu$ is exactly the slope of a flat portion of the UPF.[13] This case is particularly relevant for two types of situations, namely transferable utility on the one hand (then the cardinalization is usually chosen so that the entire UPF is a straight line) and explicit randomization on the other hand.

The second particular case relates to local non differentiability of utility functions (Figure 3.7). Then the UPF may exhibit a kink, and the one-to-one relationship breaks down for the opposite reason - namely, many values of μ are associated with the same $\bar{u}^a$.

[13] However, a strictly quasi concave generalized welfare index would still pick up exactly one point.

Stability and the Pareto weight

In what follows, we concentrate on deterministic decision processes. Then the stability axiom has a very simple implication - namely that in program (3.37), the coefficient $\bar{u}^a$ is a well-defined *function* of prices, income and possibly distribution factors, denoted $\bar{u}^a(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$. It follows that, for given fundamentals and price-income bundle, the outcome of the decision process is such that the utility of a is $\bar{u}^a(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$, and that of b is $\Upsilon(\mathbf{P}, \mathbf{p}, x, \bar{u}^a(\mathbf{P}, \mathbf{p}, x, \mathbf{z}))$. Note that under strict quasi-concavity, these utility levels are reached for only one consumption bundle.

If, in addition, we adopt a strictly concave cardinalization of individual utilities, then the Pareto weight is also a well-defined function of prices, income and possibly distribution factors, denoted $\mu(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$. For analytic tractability, we often add some structure to the problem by assuming that the function μ has convenient properties such as continuous differentiability. Such assumptions will be stated wherever they are needed.

In summary: under our two assumptions of stability and efficiency, using a strictly concave cardinalization of preferences, the behavior of the household can be modelled in a simple way; that is, there exists a function $\mu(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$ such that the household solves:

$$\max \mu(\mathbf{P}, \mathbf{p}, x, \mathbf{z})\, u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) + u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) \tag{3.43}$$

under the budget constraint (3.38).

3.5.5 Pareto weights and 'power'

A major advantage of the formulation in (3.40) or (3.43) is that the Pareto weight has a natural interpretation in terms of respective decision *powers*. The notion of 'power' may be difficult to define formally, even in a simplified framework like ours. Still, it seems natural to expect that when two people bargain, a person's gain increases with the person's power. This somewhat hazy notion is captured very effectively by the Pareto weights. Clearly, if μ in (3.40) is zero then it is as though b is a dictator, while if μ is large then a effectively gets her way. A key property of (3.40) is precisely that increasing μ will result in a move along the Pareto set, in the direction of higher utility for a (and lower for b). If we restrict ourselves to economic considerations, we may thus consider that the Pareto weight μ 'reflects' a's power, in the sense that a larger μ corresponds to more power (and better outcomes) being enjoyed by a.

The empirical implications of this remark are quite interesting. For instance, when a reform is known or predicted to improve the relative situation of a particular member (say, paying some family benefits to the wife instead of the husband), we should find that the reform increases the member's Pareto weight. More generally, the intuitive idea that a specific distribution factor z is favorable to member a can readily be translated by the fact that μ is increasing in z. Conversely, we shall see later on that it is sometimes possible to recover the Pareto weights from a careful analysis of the behavior of the households at stake. Then one can find out which factors increase or decrease the power of each member - quite a fascinating outcome indeed.

Another important insight of the analysis is that, broadly speaking, *co-operation does not preclude conflict*. In other words, the Pareto efficiency assumption by no means implies that the members always agree on what to do. On the contrary, each agent will plausibly try to obtain a favorable Pareto efficient outcome. In other words, who gets what is a crucial but difficult and potentially conflictual issue, that the efficiency assumption leaves totally open. It can be resolved in a number of different ways - bargaining, legally binding contracts, tradition, social norms or less formal ways that reflect the feelings of the two partners towards each other. Pareto efficiency does not preclude any of these aspects; it just imposes that whichever solution is found, no money is ultimately left on the bargaining table. In a sense, the collective approach provides the tools needed to concentrate on the interesting issue of who gets what - or, technically, what do the Pareto weights look like as functions of prices, income and distribution factors.

3.5.6 Household utility

If the Pareto weight is not a function of prices and income, then we have a unitary model and we can define a household utility function as a function of household public and private goods. It turns out that for the collective model we can also define a household utility function over household purchases of public and private goods but this function has one extra argument as compared to the unitary model. We define the *household utility function* by:

$$\begin{aligned} & u^h\left(\mathbf{Q},\mathbf{q},\mu\left(\mathbf{P},\mathbf{p},x,\mathbf{z}\right)\right) = \\ & \max_{\mathbf{q}^a,\mathbf{q}^b}\left\{\mu\left(\mathbf{P},\mathbf{p},x,\mathbf{z}\right)u^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right)+u^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right)\right\} \\ & \text{subject to } \mathbf{q}^a+\mathbf{q}^b=\mathbf{q} \end{aligned} \tag{3.44}$$

With this definition of the household utility u^h, program (3.40) is equivalent to the maximization of u^h under the budget constraint. This looks a lot like standard utility maximization in a unitary model. However, the critical feature of this household utility function is that it depends on the Pareto weight $\mu\left(\mathbf{P},\mathbf{p},x,\mathbf{z}\right)$. This remark is important for two reasons. First, it explains why an efficient household needs not (and will not in general) behave like an individual: since the utility u^h is price-dependent, the demand derived from its maximization under budget constraint needs not (and will not in general) satisfy the standard conditions of consumer theory. Secondly, while the idea of introducing prices into the utility function is an old one, the important feature in our case is how it is done. Following standard demand theory we do not allow prices to enter the individual utility functions; *prices and income can only affect the respective weights* given to individual utilities. As we shall see below, this gives very specific predictions for household demands. Additionally, it makes analysis using a collective model almost as easy as using a unitary model which is an important consideration when considering non-unitary alternatives.

This approach allows us to decompose changes in the utility levels of the two partners following a change in the environment into changes that would follow in a unitary model and the additional effect due to the collective framework. This is illustrated in figure 3.8, where we ignore distribution factors. Here we consider a change in prices and incomes that moves the UPF from $UPF(\mathbf{P}, \mathbf{p}, x)$ to $UPF(\mathbf{P}', \mathbf{p}', x')$. Initially the point I is chosen on $UPF(\mathbf{P}, \mathbf{p}, x)$. If we hold μ constant when prices and income change (the unitary assumption) then the utility levels move to point II at which point the tangent to $UPF(\mathbf{P}', \mathbf{p}', x')$ is parallel to the tangent at point I on $UPF(\mathbf{P}, \mathbf{p}, x)$. However, a change in the economic environment may also lead to a change in the Pareto weight. This is the 'collective' effect, illustrated by the move around $UPF(\mathbf{P}', \mathbf{p}', x')$ from II to III.

Finally, the collective formalization provides a natural way of introducing distribution factors within the framework of household decision process. If some distribution factors $\mathbf{z}$ influence the process by shifting the individual weights, then μ will depend on these variables. The fact that distribution factors matter only through their impact on μ plays a key role in the results of Chapter 4. As we shall show, efficiency can be tested using cross equation restrictions that arise from the fact that the same function $\mu(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$ appears in the demand for all goods. Moreover, there is an important difference between prices and total income, on the one hand, and distribution factors on the other hand. A change in prices or total income will affect not only the weight μ, but also the shape of the Pareto set; hence it final impact on individual welfare may be difficult to assess. On the contrary, a change in a distribution factor can by definition *only influence the weight* μ. In general, its effect on welfare is not ambiguous. In terms of figure 3.8 a distribution factor shifts the tangent point but not the frontier itself.

As an illustration of this point, we may briefly come back to the example discussed in subsection 3.4.2 of the impact of individual incomes Y^a and Y^b on household behavior. From a collective perspective, this impact should be decomposed into two components. One is the resulting change in total income $Y = Y^a + Y^b$ (hence of total expenditures x in our static framework); this affects the shape of the Pareto frontier as well as the weight μ, and its effect is *a priori* ambiguous. The second component is the change in *relative* incomes, say $z = Y^a/Y^b$, keeping the sum constant. The latter should be analyzed as a variation of a distribution factor, and its consequences are much easier to assess. For instance, if we assume, as is natural, that increasing the relative income of a increases a's weight, then it must increase a's welfare. However, how this improvement in a's situation will be translated into observable household behavior (for example, which consumptions will increase) is a difficult issue, for which a more precise formalization is needed.

3.5.7 Caring

The way in which partners care about each other may also affect the Pareto utility frontier. To take a simple example, consider the caring preferences introduced in section 3.1:

$$\begin{aligned} U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= u^a\left(\mathbf{Q},\mathbf{q}^a\right)+\delta^a u^b\left(\mathbf{Q},\mathbf{q}^b\right) \\ U^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= u^b\left(\mathbf{Q},\mathbf{q}^b\right)+\delta^b u^a\left(\mathbf{Q},\mathbf{q}^a\right) \end{aligned} \tag{3.45}$$

The maximand is now

$$\begin{aligned} & \mu\left[u^a\left(\mathbf{Q},\mathbf{q}^a\right)+\delta^a u^b\left(\mathbf{Q},\mathbf{q}^b\right)\right]+u^b\left(\mathbf{Q},\mathbf{q}^b\right)+\delta^b u^a\left(\mathbf{Q},\mathbf{q}^a\right) \\ = & \left(\mu+\delta^b\right)u^a\left(\mathbf{Q},\mathbf{q}^a\right)+\left(1+\mu\delta^a\right)u^b\left(\mathbf{Q},\mathbf{q}^b\right) \end{aligned}$$

Since $(1+\mu\delta^a)>0$, we can then represent household preferences by:

$$\tilde{\mu}u^a\left(\mathbf{Q},\mathbf{q}^a\right)+u^b\left(\mathbf{Q},\mathbf{q}^b\right) \tag{3.46}$$

where

$$\tilde{\mu}=\frac{\mu+\delta^b}{1+\mu\delta^a}$$

Formally, (3.46) is identical to the egotistic case ($\delta^a=\delta^b=0$), indicating that *any allocation that is Pareto efficient for the caring preferences is also Pareto efficient for the egotistic ones.* The argument underlying this conclusion is quite general, and goes as follows: if an allocation fails to be efficient for egotistic preferences, there exist another allocation that entails higher values of both u^a and u^b. But then it also entails higher values of both U^a and U^b, showing that the initial allocation was not efficient for caring preferences as well. In other words, *the Pareto set for caring preferences is a subset of the Pareto set for egotistic preferences.* Note, however, that the two sets do not coincide: some allocations may be efficient for egotistic preferences, but not for caring ones. Indeed, an allocation that gives all resources to one member may be efficient for egotistic agents, but not for caring persons - a redistribution in favor of the 'poor' member would then typically be Pareto improving. Technically, when μ varies from 0 to infinity, μ' only varies from δ^b to $1/\delta^a$, and the new Pareto set is a strict interior subset of the initial one.

One important feature of (3.46) is that if b's caring for a increases (giving an increase in δ^b) then it is as though a's Pareto weight increases (and conversely). This is entirely reasonable: increased caring gives the other partner a greater weight with respect to egoistic preferences.

A variant of this is if the two partners care for each other in the following way:

$$\begin{aligned} U^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= \min\left\{u^a\left(\mathbf{Q},\mathbf{q}^a\right),u^b\left(\mathbf{Q},\mathbf{q}^b\right)\right\} \\ U^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) &= \min\left\{u^a\left(\mathbf{Q},\mathbf{q}^a\right),u^b\left(\mathbf{Q},\mathbf{q}^b\right)\right\} \end{aligned} \tag{3.47}$$

This formalizes the maxim that 'no man can be happier than his wife'. In this very special case the utility possibility frontier shrinks to a single point at which $u^a = u^b$.

3.5.8 *Children*

Finally, let us briefly come back to the distinction sketched above between children being modeled as public goods or genuine decision makers. In the first case, using parental utilities of the form $u^s + \kappa^s u^k$ described above, the maximand in (3.40) becomes

$$\mu\left(u^a + \kappa^a u^k\right) + \left(u^b + \kappa^b u^k\right)$$

which is equivalent to

$$\frac{1}{1+\mu+\mu\kappa^a+\kappa^b}\left[\mu u^a + u^b + \left(\mu\kappa^a+\kappa^b\right)u^k\right] \tag{3.48}$$

the initial fraction in (3.48) gives a normalization that the weights sum to unity.

Alternatively, we may model the child as a decision maker. Then (s)he is characterized by an additional Pareto weight, say μ^k, and the household maximizes the weighted sum:

$$\mu u^a\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) + u^b\left(\mathbf{Q},\mathbf{q}^a,\mathbf{q}^b\right) + \mu^k u^k \tag{3.49}$$

Although the two forms (3.48) and (3.49) look similar, they are, in fact, quite different. Recall that the key insight of collective models is that Pareto weights may depend on prices, wages, incomes and distribution factors, and that this fact explains why collective households do not generally behave as unitary ones. In (3.48) all Pareto weights are defined by the knowledge of the function μ ; in (3.49), however, μ and μ^k can be defined independently, and the location of the final outcome on the Pareto frontier now depends on *two* parameters. Broadly speaking, the deviation from the unitary model is one dimensional in the first case (it is summarized by a unique function μ) whereas it is two-dimensional in the second case. As it turns out, this distinction has testable implication; that is, we shall see later on that a household with three decision makers does *not* generally behave as a couple, pretty much in the same way as couples do not generally behave as singles. Another fascinating implication is that, in principle, *one can assess the number of actual decision makers in a household from the sole examination of the household's behavior*, even in a fairly general context!

3.5.9 The unitary model as a special case

It is clear, from the discussion above, that the unitary model is a special case of the collective framework. An obvious example obtains when the household utility (or equivalently the Pareto weight μ) does not depend directly on prices, incomes and distribution factors. As a matter of fact, the unitary assumption is far and away the most common assumption in the modelling of household decisions. This, however, is certainly due to its very great convenience, rather than any intuitive plausibility. If one is to take seriously the idea of a decision process actually taking place between the members, it hard to believe that neither prices (including respective wages), nor respective incomes, nor any exterior factor will influence the 'weights' of individual agents in the decision process.

Nonetheless there are circumstances under which the household will act as though it has a single utility function. One obvious example is if custom or strong social traditions give all the power to one person (usually the husband) in the household.

An alternative is given in Samuelson (1956). Samuelson considers the family to be the fundamental unit on the demand side of the economy. However, because such a unit consists of several members, we cannot expect a consensus (that is, consistent family indifference curves). He recognizes that preferences within a family are interrelated and that external consumption effects (*a la* Veblen and Dusenberry) are the "essence of family life". Nevertheless, if such external effects are put aside, and a restricted form of altruism is assumed, families may behave *as if* they maximize a single social utility. In particular Samuelson considers a common social welfare function for the family that is restricted to depend on the individual consumptions of family members only through the preferences of these members. This restriction, together with the assumption of no external consumption effects and no public goods, implies that all family decisions can be decentralized via a distribution of income.[14] The important point is the distribution of income depends on prices and income and will not be constant. Thus schemes such as 'a receives 60% of income and b receives 40%' are generally not consistent with the maximization of a family SWF.[15]

[14] If we have public goods and externalities then we also need Lindahl prices and Pigouvian taxes to decentralize.

[15] Although it may in special cases (e.g., Cobb Douglas preferences).

The main result that Samuelson provides is that if income is redistributed so as to maximize a given social welfare function, then the family aggregate consumptions will satisfy the Slutsky conditions. That is a family will act in the same manner as a single person.

Becker (1991) criticizes Samuelson for not explaining how a social welfare function arises. In the context of moral judgements, each person can have a private utility that is defined on outcomes affecting them directly, and a social utility function that reflects preferences on the outcomes for all family members. So it is unclear how partners agree on a single common social welfare function. One mechanism suggested by Becker is that one person, the 'head', has most of the family resources and is sufficiently altruistic that they will transfer resources to the other member. If the dependents' consumption is a normal good for the head, all family members will align their actions with the head's preferences, as any improvement in the income under the command of the head raises their utilities. It is then the case that the family as a group acts as if a single objective is being maximized. This is the Rotten Kid Theorem mechanism outlined in subsection 3.4.3. In that noncooperative voluntary contributions model, one of the partners may effectively be a dictator if they control most (but not necessarily all) of household resources. In that case a unitary model obtains locally if one partner is wealthier and they are the sole contributor to the public good.

Another important case is when the preferences display *transferable utility* (TU); see subsection 3.1. Indeed, under TU, program (3.40) becomes

$$\max \mu \left(f^a\left(\mathbf{q}_{-1}^a, \mathbf{Q}\right) + G(\mathbf{Q})\, q_1^a\right) + \left(f^b\left(\mathbf{q}_{-1}^b, \mathbf{Q}\right) + G(\mathbf{Q})\, q_1^b\right) \tag{3.50}$$

(where $\mathbf{q}_{-1}^s$ denotes the quantity of s's private goods, except the first one) under the budget constraint. The first surprising feature of the TU assumption is that if the optimum has q_1^a and q_1^b both positive, then μ is necessarily equal to unity. To see this, set the price of the first good to unity and substitute for q_1^b using the budget constraint:

$$\begin{aligned} &\max \mu \left(f^a\left(\mathbf{q}_{-1}^a, \mathbf{Q}\right) + G(\mathbf{Q})\, q_1^a\right) \\ &+ f^b\left(\mathbf{q}_{-1}^b, \mathbf{Q}\right) + G(\mathbf{Q})\left(x - \mathbf{P}'\mathbf{Q} - \mathbf{p}'_{-1}\left(\mathbf{q}_{-1}^a + \mathbf{q}_{-1}^b\right) - q_1^a\right) \end{aligned} \tag{3.51}$$

Taking the derivative with respect to q_1^a we see that:

$$\mu G(\mathbf{Q}) - G(\mathbf{Q}) = 0 \tag{3.52}$$

which implies $\mu = 1$ so that the UPF is a line with a constant slope of -1. Thus the Pareto weight cannot depend on prices, income or any distribution factors. Therefore the partners will always agree to act in a manner which shifts the frontier out as far as possible by the choice of $\left(\mathbf{Q}, \mathbf{q}_{-1}^a, \mathbf{q}_{-1}^b\right)$. In fact they will agree to maximize the sum of their individual utilities given by:

$$f^a\left(\mathbf{q}_{-1}^a, \mathbf{Q}\right) + f^b\left(\mathbf{q}_{-1}^b, \mathbf{Q}\right) + G(\mathbf{Q})\left(x - \mathbf{P}'\mathbf{Q} + \mathbf{p}'_{-1}\left(\mathbf{q}_{-1}^a + \mathbf{q}_{-1}^b\right)\right) \tag{3.53}$$

Thus, under transferable utility and assuming efficiency, married partners will agree on almost all consumption choices. The only conflict will be in how to divide the private good q_1 which is often referred to as 'money' but in the family context it may be interpreted more broadly as a medium of exchange.

3.5.10 The Rotten Kid Theorem revisited

As we have just seen, under transferable utility and efficiency, a couple acts as a single decision unit in the sense that both partners would agree on the set of actions which maximizes the *joint* marital output, defined as the sum of the partners's individual utilities. In contrast to the case of dictatorship, where the issue of implementation does not arise, for the case of transferable utility we also need to ask how the actions that maximize the joint output are actually enforced. One possibility is that bargaining takes place at the outset of marriage, and some sort of binding agreement is signed and then carried out. However, if the partners are altruistic towards each other, these emotional ties generate commitments that can replace legally binding contracts. In particular, commitments that arise from altruistic preferences can be exploited in the design of a mechanism that implements the maximization of the total output (sum of utilities) and is self enforcing. One such scheme (see Becker, 1974) is to select a principal (a family head) who is given control over family resources and can make transfers as she or he sees fit. The only requirement is that the principal should care about all family members in the sense that their utilities enter his or her own preferences as normal goods. Once this scheme is put in place, each person is allowed to choose their own actions selfishly. It had been observed by Becker that such a mechanism is efficient and each participant voluntarily acts in the interest of the group. The reason is that any productive action which increases total output is rewarded by an increased transfer from the principal. Conversely, any destructive action is punished by reduced transfers. In this way the interests of the group are internalized by every member. Although the allocation of income depends on who is the head, family decisions will be invariant to his or her preferences. The crucial aspect is that every partner should trust the principal to truly care about all family members and that the principal should be able to fully control the distribution of income (in the sense that her/his resources are such that she/he gives everyone a positive transfer that can be reduced or increased).[16]

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[16] Becker has two slightly different versions of the Rotten Kid Theorem. The early one stated in Becker (1974, page 1080) is "If a head exists, other family members are also motivated to maximize family income and consumption, even if their utility depend on their consumption alone " The later version in Becker (1991, p288) is set in context of mutual altruism where each person is a potential contributor to the other and states that

To illustrate the working of the 'family head mechanism', let each spouse have two private actions: consumption and work. Time not spent at work is used to produce a household good which is a public good (for example, child quality). Let us assume transferable utility and write the person specific utility as

$$U^s(Q, q^s, l^s) = Qq^s + v^s(l^s), \;\; s = a, b \tag{3.54}$$

where where, q^s denotes private consumption, l^s is leisure time and Q is a public good produced at home. The household production function is

$$Q = \phi(t^a, t^b) \tag{3.55}$$

where t^s denotes time spent by s on the production of the public good. The family budget constraint is

$$q^a + q^b = \; (1 - t^a - l^a)w^a + (1 - t^b - l^b)w^b, \tag{3.56}$$

where w^s is the wage of person s. Applying the results on transferable utility, it is easy to verify that any Pareto efficient allocation must maximize the sum of private utilities given by:

$$\pi = [(1 - t^a - l^a)w^a + (1 - t^b - l^b)w^b]\phi(t^a, t^b) + v^a(l^a) + v^b(l^b) \tag{3.57}$$

To show that this is an equilibrium outcome of the 'family head mechanism', we consider a two stage game, such that in the first stage each agent s chooses independently the amount of work at home, t^s and in the market $1 - t^s - l^s$. In the second stage, the head, say partner a, chooses the level of the private good, q^s that each partner receives based on a's *social* preferences, $W^a(U^a(Q, q^a, l^a), U^b(Q, q^b, l^b))$. We can solve this problem backwards. In the last stage, the levels of work at home, t^s and l^s are given to a and she can only transfer private goods. This means that the head faces a *linear* Pareto frontier (see Figure) and will select the best point for her on this frontier. Now assume that the two private utilities appear as 'normal goods' in a's social welfare function so that whenever the Pareto frontier shifts up the head reallocates private goods to raise the private utilities of *both* agents. Anticipating that, each agent who chooses actions selfishly in the first stage will realize that their private utility is a monotone increasing function of the total resources available for the head for redistribution (π in equation (3.57)). Therefore, each agent will choose the actions under their control to maximize the pie and the outcome is the same efficient outcome that would arise if the head could directly control all family decisions.

"Each beneficiary, no-matter how selfish, maximizes the family income of his benefactor and thereby internalizes all effects of his actions on other beneficiaries." In both versions, there is only one good that is distributed. Following Bergstrom (1989) we consider here a problem with two goods and show that under transferable utility similar results are obtained.

The family head mechanism was first proposed by Becker and is discussed in detail in Becker (1991, chapter 8). One application of the analysis is parent child relationship and the main result is that selfish children can act in a manner that internalizes the consequences of their actions, yielding an efficient outcome. This result was coined by Becker as 'the rotten kid theorem'. His analysis, however, was much more general, dealing with various forms of altruism and preference dependence. The subsequent literature addressed the generality of the efficiency head mechanism. Bergstrom (1991) shows that this result generally fails in the absence of transferable utility, because agents can then affect not only the location of the Pareto frontier but also its slope, destroying the monotonicity result required for the theorem to hold. Another issue is the precise sequence of events. Suppose that the children can consume in both periods 1 and 2. Then, efficiency requires that, for each child, the ratio of the marginal utilities of consumption in the two periods is equated to the cost of transferring goods over time that is facing the household, $1 + r$. However, in choosing consumption, the child will take into account that his first period consumption also influences the transfer from the head. Being poor in the second period causes the parent to transfer more, causing the child to under-save. This pattern of behavior, where giving leads to under provision, is referred to as the Samaritan Dilemma (Bruce and Waldman, 1990). This example shows that altruism can also be a constraint on mechanism design. The parent could in principle impose the efficient outcome by conditioning the payment on past performance. However, an altruistic parent may not be able to commit to punish a deviating child - a restriction that is captured in modeling the stages of game and seeking a subgame perfect equilibrium.

3.5.11 Bargaining models

Throughout the chapter, we have stressed that the collective model, in its fully general version, is agnostic about the specific decision process provided that the latter generates Pareto efficient outcomes. Because of this generality, it is thus compatible with a host of more specific models that further specify the way a particular point on the Pareto frontier is selected. For instance, we shall show in detail in chapter 8 that under some conditions, this choice can be fully determined by the competition in the marriage market, where considerations such as what are the individual characteristics that generate marital surplus, what is the matching process and does a person have a close substitute play a crucial role. However, much of the literature pursues a more partial view and concentrates on the relative strengths of two individuals who are already matched and use tools from cooperative game theory to derive the bargaining outcome.[17]

Any bargaining model requires a specific setting: in addition to the framework described above (two agents, with specific utility functions), one has to define a *threat point* T^s for each individual s. Intuitively, a person's threat point describes the utility level this person could reach in the absence of an agreement with the partner. Then resources are allocated between public and private consumption, resulting in two utility levels $\bar{u}^a$ and $\bar{u}^b$. Typically, bargaining models assume that the outcome of the decision process is Pareto efficient. Bargaining theory is used to determine how the threat points influence the location of the chosen point on the Pareto frontier. Clearly, if the point $T = \left(T^a, T^b\right)$ is outside of the Pareto set, then no agreement can be reached, since at least one member would lose by agreeing. However, if T belongs to the interior of the Pareto set so that both agents can gain from the relationship, the model picks a particular point on the Pareto utility frontier.

Before describing in more detail some of the standard solutions to the bargaining problem, however, it is important to note that the crucial role played by threat points - a common feature of all bargaining models - has a very natural interpretation in terms of distribution factors. Indeed, *any variable that is relevant for threat points only is a potential distribution factor.* For instance, the nature of divorce settlements, the generosity of single parent benefits or the probability of re-marriage do not directly change a household's budget constraint (as long as it does not dissolve), but may affect the respective threat points of individuals within it. Then bargaining theory implies that they will influence the intrahousehold distribution of power and, ultimately, household behavior. This intuition is perfectly captured in the collective framework by the idea that the Pareto weight depends on distribution factors. Moreover, it provides a clear idea of the direction of these effects. That is, a change in a variable that increases the wife's threat point should always positively affect her Pareto weight. These notions potentially provide a number of powerful tests, which are moreover independent of the particular bargaining concept at stake.

Nash bargaining

The most commonly used bargaining solution was proposed by John Nash in the early 1950's. Nash derived this solution as the unique outcome of a set of axioms that any 'reasonable' solution must satisfy. Some of the axioms are uncontroversial. One is individual rationality: an agent will never accept an agreement that is less favorable than her threat point. Another is Pareto efficiency, as discussed above. A third mild requirement is invariance with

[17] Bargaining approaches to household decision making were first introduced by Manser and Brown (1980) and McElroy and Horney (1981). For a more complete discussion of two person bargaining, see Myerson (1991, ch.8).

respect to affine transformations[18]: if both the utility and the threat point of an agent are transformed by the same increasing, affine mapping the prediction about the equilibrium outcome of cooperation does not change. Note, however, that a non linear transform will change the outcome; that is, Nash bargaining requires a *cardinal* representation of preferences.

The last two axioms are more specific. One is symmetry; it states that if utilities and threat points are permuted between members (u^a and T^a are replaced with u^b and T^b, and conversely) then the outcomes are simply switched ($\bar{u}^a$ is replaced with $\bar{u}^b$ and conversely). Natural as it may sound, this assumption may still sometimes be too strong. In many socioeconomic contexts, for instance, male and female roles are by no means symmetric. Fortunately, Nash bargaining can easily be extended to avoid the symmetry assumption.

The last and crucial axiom is independence. It can be stated as follows. Assume that the set of available opportunities (the Pareto set) shrinks, so that the new Pareto set is within the old one, but the initial equilibrium outcome is still feasible; then the new equilibrium outcome will be the same as before. In other words, the fact that one member misses some opportunities that he had before does not affect his bargaining position towards the other member. This requirement alone implies that the Nash solution maximizes some function of the utilities of the two partners.

If one accepts these axioms, then only one outcome is possible. It given by the following rule: find the pair $\left(\bar{u}^a, \bar{u}^b\right)$ on the Pareto frontier that maximizes the product $(u^a - T^a)\left(u^b - T^b\right)$.[19] That is, the Nash bargaining allocation $\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$ solves

$$\max_{\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b} \left(u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) - T^a\right)\left(u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) - T^b\right) \tag{3.58}$$

under the budget constraint 6.18. Thus the product $(u^a - T^a)\left(u^b - T^b\right)$ can be considered as a household utility function, that is maximized on the Pareto set. Note that $(u^s - T^s)$ is the surplus derived from the relationship by agent s. The main implication of Nash bargaining is that the product of surpluses should be maximized.[20]

Clearly, if the threat points do not depend on prices, incomes and distribution factors, the maximand can be seen as a standard, unitary utility and the Nash bargaining solution boils down to a unitary model; the outcome satisfies in particular the properties of a regular consumer demand. This case, however, is of little interest. Typically, threat point depends on a number of parameters, and the previous formalization allows us to study how these effects translate into behavioral patterns. An important result

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[18] An affine mapping is of the form $f(x) = ax + b$.

[19] Or equivalently the sum $\log(u^a - T^a) + \log\left(u^b - T^b\right)$.

[20] Note that simply maximizing the sum of surpluses, $\left(u^A - T^A\right) + \left(u^B - T^B\right)$, would violate the invariance axiom.

is that at the Nash bargaining equilibrium $(\bar{u}^a, \bar{u}^b)$, $\bar{u}^a$ is increasing in T_a and decreasing in T_b (while, obviously, $\bar{u}^b$ is decreasing in T_a and increasing in T_b). Hence, any change that increases a member's threat point without changing the Pareto frontier (the typical impact of a distribution factor) will ameliorate this member's situation.

Finally, the symmetry axiom can be relaxed. Then the general form is a straightforward generalization of the previous one: instead of maximizing the sum of log surpluses, one maximizes a weighted sum of the form $\gamma_a \log(u^a - T^a) + \gamma_b \log(u^b - T^b)$. In this form, the weights γ_s introduce an asymmetry between the members' situations.

Kalai-Smorodinsky

An alternative concept has been proposed by Kalai and Smorodinsky (1975). It relies on the following, *monotonicity* property. Consider two bargaining problems such that (i) the range of individually rational payoffs that player a can get is the same in the two problems, and (ii) for any given, individually rational utility level for player a, the maximum utility that player b can achieve (given the Pareto frontier) is never smaller in the second problem than in the first. Then player b should do at least as well in the second problem than in the first. In other words, if one enlarges the Pareto set by inflating b's opportunities while keeping a's constant, this change cannot harm b.

Kalai and Smorodinsky prove that there exists a unique bargaining solution that satisfies all the previous axioms except for independence, which is replaced with monotonicity. Moreover, the solution has an interesting interpretation. Define the *aspiration level* A^s of player s as the maximum utility he/she can get that is compatible with feasibility and the partner's individual rationality constraint; this corresponds to the point on the Pareto frontier that leaves the partner, say s', at their threat point utility $T^{s'}$. Define, now, the *ideal point* as the point (A^a, A^b); obviously, this point lies outside of the Pareto frontier. The solution, now, is to chose a point $U = (u^a, u^b)$ on the Pareto frontier so that

$$\frac{u^a - T^a}{u^b - T^b} = \frac{A^a - T^a}{A^b - T^b}$$

In words, the bargaining is here influenced, in addition to the threat points, by the players' aspirations about what they might receive within marriage. The surplus share received by player s, $u^s - T^s$, is directly proportional to the maximum gain s could aspire to, $A^s - T^s$.

Non cooperative foundations

Finally, an on going research agenda, initially proposed by Nash, is to provide noncooperative foundations to the bargaining solutions derived from axioms. The most influential framework is the model of Rubinstein (1982), in which players make alternating offers until one is accepted. When time matters through a constant discount factor, there exists a unique, subgame-perfect equilibrium of this noncooperative game, which is characterized by the requirement that each player should be indifferent between accepting the current offer and waiting to an additional round and making an offer that the opponent would accept. Binmore, Rubinstein and Wolinsky (1987) have analyzed the link between these non cooperative formulations and the axiomatic approaches. Specifically, they study a model in which the bargaining process may, with some probability, be *exogeneouly* interrupted at each period. This model has a unique, subgame perfect equilibirum; moreover, if one allows the time interval between successive offers in both models to decrease to zero, then the equilibirum converges to the Nash bargaining solution.

Empirical content of bargaining models

Since the bargaining models just described all assume (or imply) Pareto efficiency, their solutions will satisfy the general properties generated by the collective model; these will be detailed in the next Chapter. But do these models allow us to go one step further? That is, which additional insights (if any) can we obtain from the use of bargaining concepts?

The answer to that question depends on what is known on the threat points. Indeed, a first result (Chiappori, Donni and Komunjer 2010) is that *any* Pareto efficient allocation can be derived as the Nash bargaining solution for an *ad hoc* definition of the threat points. Hence the additional information provided by the bargaining concepts (with respect to the sole efficiency assumption) must come from specific hypotheses on the threat points - that is, on what is meant by the sentence: 'no agreement is reached'.

Several ideas have been used in the literature. One is to refer to divorce as the 'no agreement' situation. Then the threat point is defined as the maximum utility a person could reach after divorce. Such an idea seems well adapted when one is interested in the effects of laws governing divorce on intrahousehold allocation. Another interesting illustration would be public policies such as single parent benefits, or the guaranteed employment programs that exist in some Indian states; Kanbur and Haddad convincingly argue that the main impact of the program was to change the opportunities available to the wife outside marriage (or cohabitation). It is probably less natural when minor decisions are at stake: choosing who will walk the dog is unlikely to involve threats of divorce.[21]

A second idea relies on the presence of public goods, and the fact that

[21] An additional difficulty is empirical. The estimation of utility in case of divorce is delicate, since most data sets allow us to estimate (at best) an ordinal representation of preferences, whereas Nash bargaining requires a cardinal representation. See Chiappori (1991)

non-cooperative behavior typically leads to inefficient outcomes. The idea, then, is to take the non-cooperative outcome as the threat point: in the absence of an agreement, both members provide the public good(s) egotistically, not taking into account the impact of their decision on the other member's welfare. This version captures the idea that the person who would suffer more from this lack of cooperation (the person who has the higher valuation for the public good) is likely to be more willing to compromise in order to reach an agreement. Interestingly, in this context some of the results derived in the non-cooperative case extend to the cooperative context as well. For instance, the income pooling result for interior solutions, derived in subsection 3.4.4, applies here as well: total income being kept constant, a change in respective incomes that does not affect the noncooperative consumption pattern leaves the threat point unchanged and hence has no impact on the bargaining outcome. Thus local income pooling is inherited by the bargaining solution.

Finally, it must be remarked that assumptions on threat points tend to be strong, somewhat *ad hoc*, and often not independently testable.[22] Given this cost, models based on bargaining should be used parsimoniously, and preferably when there is good evidence that the actual structure of the decision process is close to what is implicitly assumed by the concept referred to. An alternative approach is to concentrate on more general assumptions, the implications of which hold for a large class of models. Efficiency is one natural example. Another is that some distribution factors, whatever the distribution process, can only be favorable to one partner (hence unfavorable to the other) - an intuition that can often be documented using sociological or ethnographic studies. This point should be kept in mind for the next chapters.

3.5.12 Other approaches

Finally, we may briefly review three approaches that have been proposed for analyzing household and family behavior. Two of them (the equilibrium models of Grossbard-Shechtman and Haller, and the 'separate spheres' model of Lundberg and Pollak) lead to efficient outcomes, therefore are consistent with the cooperative/collective model; the third (Basu's 'inefficient bargaining') is not, although it relies on a bargaining framework.

Equilibrium models

Following the seminal contributions of Becker[23], several papers by Grossbard-Schechtman[24] analyze marriage in a general equilibrium framework, in

[22] See Chiappori, Donni and Komunjer (2010) for a formal analysis.
[23] See Becker (1991) for a general overview.
[24] See Grossbard-Schechtman (1993) for a unified presentation.

which intrahousehold allocations are directly driven by the competitive constraints that exist on the marriage market. In some of these models, the women's role is essentially to produce domestic commodities. Men employ women to produce for them, and compensate them with transfers (which, in developing societies, may take the form of provision of basic needs) and/or non-pecuniary benefits. From this perspective, marriage can essentially be analyzed as an employment relationship, which allows to apply the standard concepts of labor economics. The framework is then generalized to situations where both men and women engage in household production work. In all these models, the emphasis is put on a general equilibrium analysis, and specifically on the impact of the economy on intrahousehold decisions. One may remark, at this stage, that the outcome of the decision process thus described is efficient; therefore these models belong to the cooperative/collective family.[25]

In a related line, Gerbach and Haller (1999) and Haller (2000) study the general equilibrium implications of competitive exchange among multimember households, in a context in which consumptions are exclusively private but consumption externalities may exist within the household. They compare two benchmark cases: one in which decision making within households is always efficient (therefore households can be described using the collective representation), and one ('individual decentralization') in which each household member 'goes shopping on his or her own, following his or her own interests, after receiving a share of household income' (Haller, (2000), page 835). They first analyze whether competitive exchange among efficient households leads to a Pareto-optimal allocation at the global level. The answer is positive as long as each household's demand exhausts its budget.[26] They then ask whether such an optimal allocation can be 'individually decentralized' in the sense just defined. They show that, generically on preferences, the answer is now negative; they conclude that some specific household decision processes are needed to internalize the externalities.

Separate spheres

The 'separate sphere' approach of Lundberg and Pollack (1993) considers a model with two public goods and assumes that each partner is assigned a public good to which they alone can contribute; this is their 'sphere' of responsibility or expertise. These spheres are determined by social norms. The question Lundberg and Pollak address is how the contributions to the individual spheres are determined. If the partners cooperate, they pool

[25] The relationship between intrahousehold decision processes in a collective framework and equilibrium on the market for marriage will be the main topic of the second part of the present book.

[26] That the household should spend its entire budget may seem an obvious implication of efficiency, et least in the static context under consideration here. However, the authors show that the property may be violated in the presence of negative externalities.

their incomes and set the levels of all goods at the Nash bargaining solution, which is efficient. The Nash solution is enforced by a binding agreement. The resulting allocation then depends on the respective threat points of the husband and wife. They consider the threats of continued marriage in which the partners act non-cooperatively and each chooses independently the level of public good under their domain. In this case, the outcome is inefficient. Specifically, if the partners' individual utilities are additively separable in the two public goods (implying no strategic interactions) each partner will choose the level desired by him\her given their respective incomes. If the wife is poor and the child is under her sphere, the outcome will be under provision of child services. This solution can be modified, however, by transfers that the husband voluntarily commits to pay his wife (before incomes are known) or by a direct purchase of child services in the market.

Inefficient bargaining

Basu (2006) considers a model in which agents bargain in a cooperative way, but the respective threat points depend in part on *endogenous* decisions. For instance, when deciding on labor supply and consumption, a spouse's bargaining position may depend not only on her wage and non labor income, but also on the labor income she generates. Basu analyzes the corresponding model, and shows in particular that multiple equilibria may coexist; moreover, decisions may not be monotonic in a member's power (for instance, child labor can first decline, then rise as the wife's power increases). It is important to note, here, that although it uses a bargaining framework, Basu's model leads to Pareto inefficient decisions, because of the noncooperative ingredient implicit in the framework. Typically, linking a person's weight to that person's labor income leads to oversupply of labor: once an efficient allocation has been reached, it is individually rational for each spouse to marginally boost their Pareto weight through additional labor supply. Both members could benefit from a *simultaneous* reduction of their labor supply that would leave Pareto weights unchanged, but strategic incentives prevent this Pareto improvement from taking place.

A similar intuition had actually been proposed earlier by Brossolet (1993) and Konrad and Lommerud (1995). In the two-period model of Konrad and Lommerud, individuals first invest in education, then marry; when married, their decisions are derived from a Nash bargaining framework. Since investments in human capital are made noncooperatively and current investments will serve to improve future bargaining power, there is again inefficient overinvestment in human capital. Unlike Basu, the second period outcome is efficient in the static sense (that is, labor supply choices, conditional on education, are *ex post* Pareto efficient); the inefficiency, here, is dynamic, and can be seen in the initial overinvestment.

In both cases, efficiency could be restored through adequate commitment devices. In practice, such devices are likely to exist in Basu's setting (since the Pareto improvement could be reached during marriage) but not in Konrad and Lommerud's framework (because investments are made before the spouses meet). All in all, these models emphasize the key role of commitment, a point that has been evoked earlier and that will be extensively discussed in Chapter 6. They also indicate that the interaction between ex ante investements and ex post matching on the marriage markets are both important and complex; we shall analyze them in full detail in the second part of the book.

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FIGURE 3.1. Demand for public good

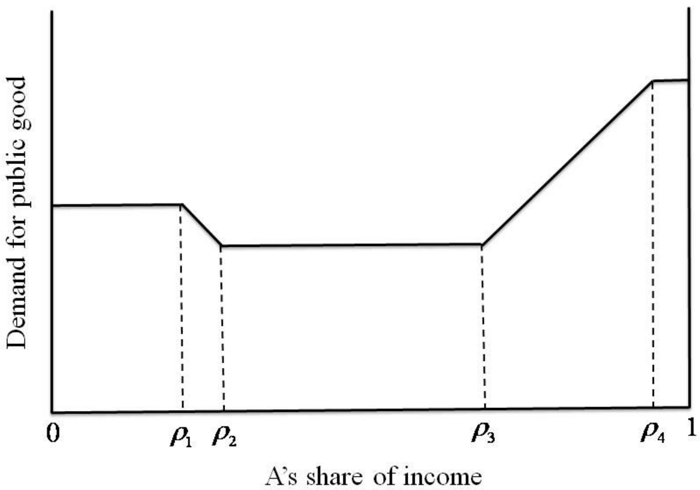

FIGURE 3.2. The demand for public goods with altruism.

FIGURE 3.3. Household demands for public goods.

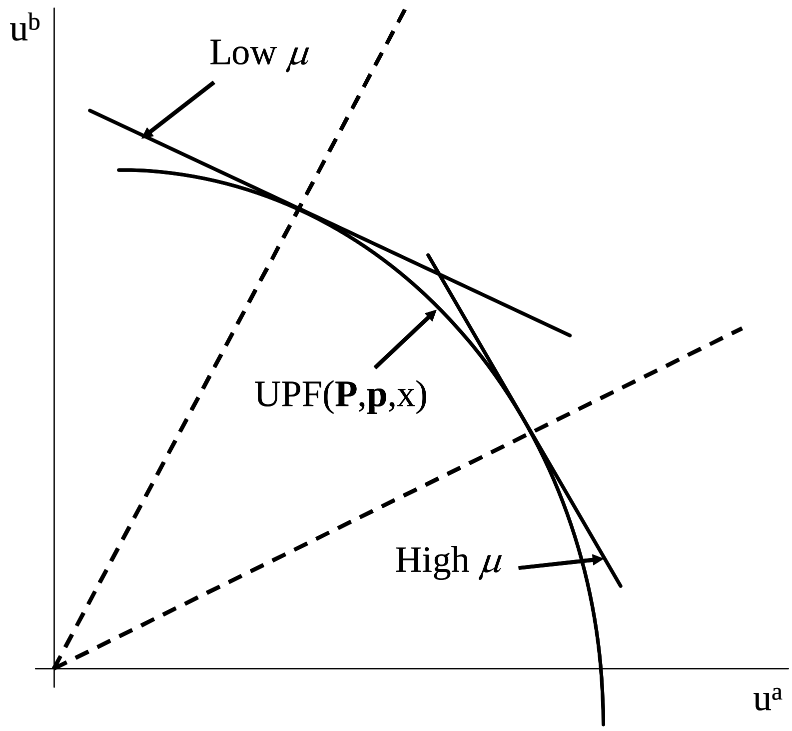

FIGURE 3.4. The utility possibility frontier.

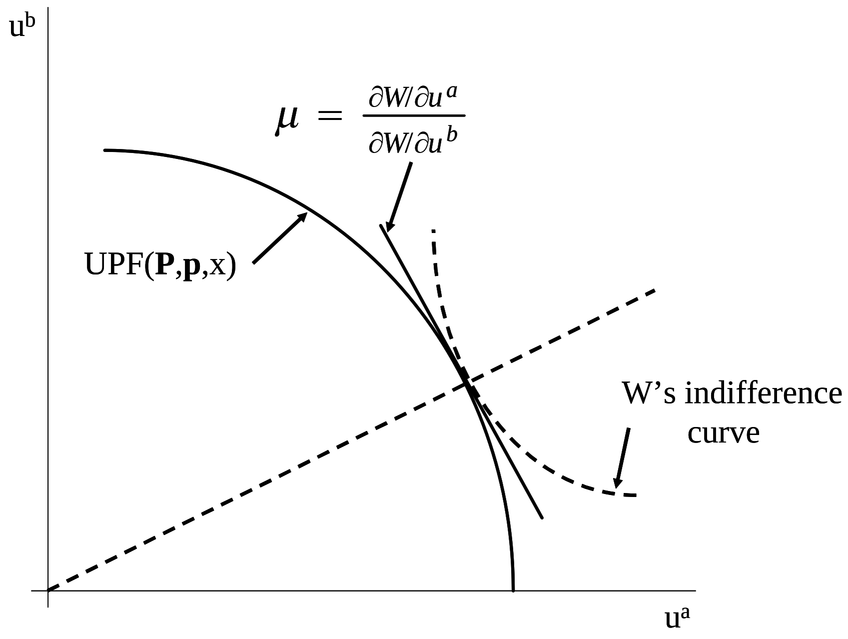

FIGURE 3.5. Linear and non linear generalized Samuelson index

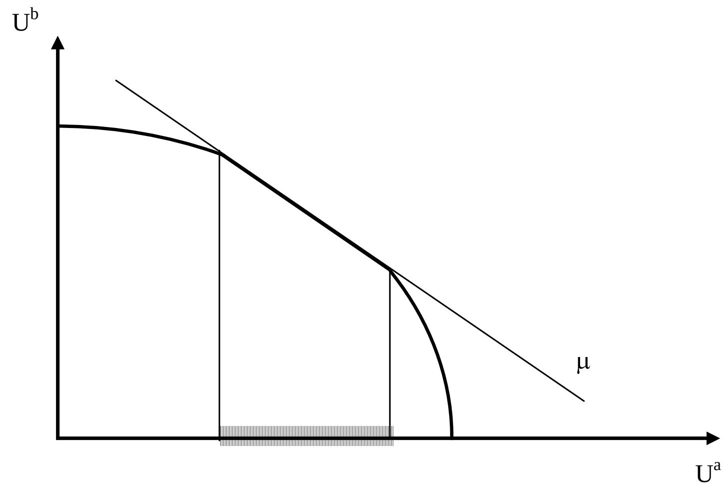

FIGURE 3.6. All utilities in the shaded area correspong to the same μ

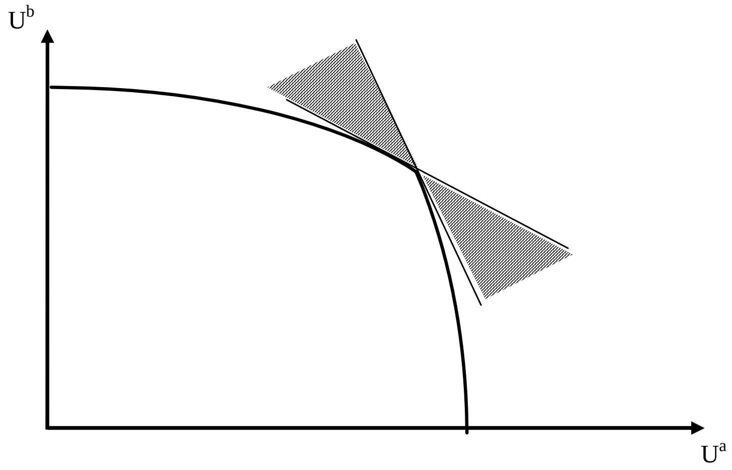

FIGURE 3.7. All μ in the shaded cone correspond to the same u^a

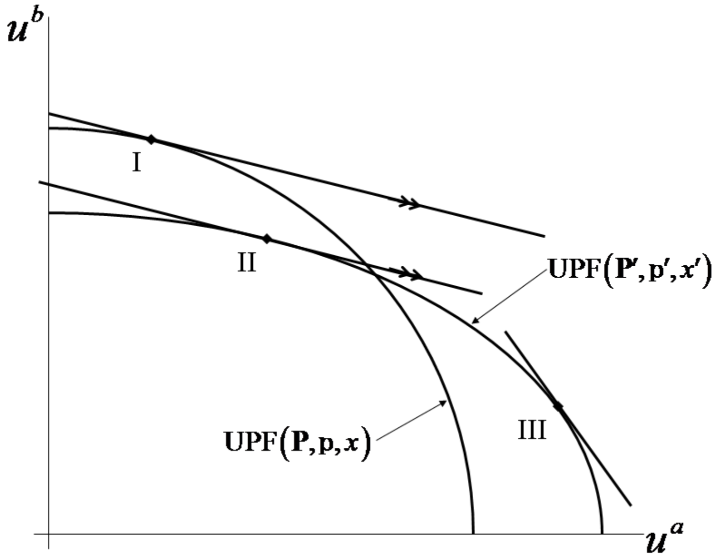

FIGURE 3.8. The effects of changes in prices

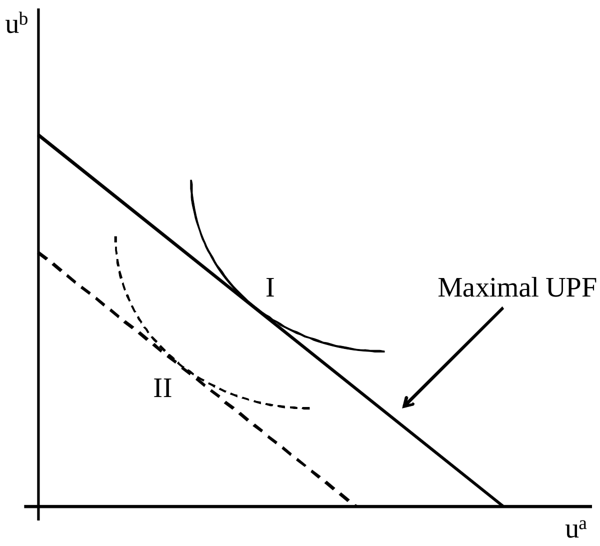

FIGURE 3.9. The RKT utility possibility frontier.

4

The collective model: a formal analysis

4.1 Collective demand functions: a general characterization

4.1.1 The collective household utility function.

The basic aspects of the collective model have been described in the previous chapter. As stated earlier, the particular form adopted has testable implications for demand functions. We now describe these implications in detail. We start with the most general version of the model with individual preferences of the form $u^s\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right)$. This allows for any type of consumption externalities between agents. We define the *collective household utility function* by

$$u^f\left(\mathbf{q}, \mathbf{Q}, \mu\right) = \max_{\mathbf{q}^a} \left\{\mu u^a\left(\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q} - \mathbf{q}^a\right)\right) + u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q} - \mathbf{q}^a\right)\right\} \tag{4.1}$$

where μ may be a function of $(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$ where $\mathbf{z}$ is a vector of distribution factors. We shall always assume that $\mu\left(.\right)$ is zero homogeneous in $(\mathbf{P}, \mathbf{p}, x)$ and any elements of $\mathbf{z}$ that are denominated in monetary terms.

At this level of generality, the distinction between public and private goods is somewhat blurred, and we can leave it aside for the moment. We thus adopt a general notation with $\mathbf{g} = (\mathbf{q}, \mathbf{Q})$ denoting the quantities consumed by the household and $\mathbf{r} = (\mathbf{p}, \mathbf{P})$ denoting the corresponding price vector. Then the household's behavior is described by the maximization of $u^f\left(\mathbf{g}, \mu\right)$ under the household budget constraint $\mathbf{r}'\mathbf{g} = x$.

4.1.2 Structural and observable demand.

The household's program is:

$$\max_{\mathbf{g}} u^f\left(\mathbf{g}, \mu\right) \text{ subject to } \mathbf{r}'\mathbf{g} = x \tag{4.2}$$

which generates *collective demand functions*, $\tilde{\mathbf{g}}\left(\mathbf{r}, x, \mu\right)$. It is important to emphasize that this program is *not* equivalent to standard utility maximization (the unitary model) because u^f varies with μ, which in turn depends on prices, income and distribution factors. Yet, for any *fixed* μ, $\tilde{\mathbf{g}}\left(., \mu\right)$ is a standard demand function. From standard consumer theory, we therefore know that it satisfies Slutsky symmetry and negativeness. This property is crucial in what follows; it can be exploited in a more formal way. Define the generic Slutsky matrix element of $\tilde{\mathbf{g}}\,(.,\mu)$, always holding μ constant, as:

$$\sigma_{ij} = \frac{\partial \tilde{g}_i}{\partial r_j} + \tilde{g}_j \frac{\partial \tilde{g}_i}{\partial x} \tag{4.3}$$

and denote its Slutsky matrix by $\Sigma = [\sigma_{ij}]_{i,j}$. We then have that Σ is symmetric and negative[1]. Rearranging (4.3), we get the standard interpretation of a Slutsky matrix; namely, the *Marshallian response* of the demand for good i to changes in the price of good j ($\frac{\partial \tilde{g}_i}{\partial r_j}$) can be decomposed into the difference between a substitution effect (σ_{ij}) and an income effect ($\tilde{g}_j \frac{\partial \tilde{g}_i}{\partial x}$). The intuition is that a marginal increase in the price of any good i affects, among other things, the real income (the purchasing power) of all agents. The *substitution* term σ_{ij} represents the effect of the infinitesimal variation if it was fully *compensated* in income (that is, accompanied by a variation in income sufficient to exactly offset the loss in purchasing power); for that reason, we often talk of compensated demand. The *income* effect, on the other hand, reflects the fact that the price increase decreases the agent's purchasing power in proportion to the quantity purchased, which in turn influences the demand.

Although the analysis of $\tilde{\mathbf{g}}\,(\mathbf{r}, x, \mu)$, holding μ constant is conceptually useful, it is crucial to realize that $\tilde{\mathbf{g}}$ *cannot be observed directly*; indeed, such an observation would require changing prices and income *without modifying* μ. Since, in general, μ does depend on $(\mathbf{r}, x)$ this can be, at best, a thought experiment. What we *do* observe is the household demand, in which price and income variations affect *both* $\tilde{\mathbf{g}}$ and μ. Thus the empirically relevant concept is the demand function defined by:

$$\hat{\mathbf{g}}\,(\mathbf{r}, x) = \tilde{\mathbf{g}}\,(\mathbf{r}, x, \mu\,(\mathbf{r}, x)) \tag{4.4}$$

where we have, for notational economy, dropped the distribution factors.[2] Thus, we make a distinction between the 'structural' demand function, $\tilde{\mathbf{g}}\,(\mathbf{r}, x, \mu)$, and the observable demand function, $\hat{\mathbf{g}}\,(\mathbf{r}, x)$. Again, the difference between these collective demand functions and the unitary model (Marshallian) demand functions is the presence of the Pareto weight in the demands.

[1] Throughout the book we use 'negative' as shorthand for negative semidefinite; it does *not* imply that all the elements of the matrix are negative.

[2] We shall maintain the ˆ notation for an observable function and ˜ for structural throughout the book. Think of the ˆ as denoting a function that could be estimated.

For the observable demand function we have:

$$\begin{aligned}\frac{\partial \hat{g}_i}{\partial r_j} &= \frac{\partial \tilde{g}_i}{\partial r_j} + \frac{\partial \tilde{g}_i}{\partial \mu}\frac{\partial \mu}{\partial r_j} \\ \frac{\partial \hat{g}_i}{\partial x} &= \frac{\partial \tilde{g}_i}{\partial x} + \frac{\partial \tilde{g}_i}{\partial \mu}\frac{\partial \mu}{\partial x}\end{aligned} \tag{4.5}$$

Thus we can decompose the price effect into a Marshallian response (the first term on the right hand side) and a collective effect (the second term), which operates through variations of the Pareto weight μ. Figure 4.1 illustrates for two goods. We start with prices and income $(\mathbf{r}, x)$ and the demand at point I. We then change prices so that good 1 is cheaper; denote the new environment $(\mathbf{r}', x)$. The substitution effect is given by the move from I to II and the income effect is II to III. The collective effect associated with the change in μ is represented by the final term in (4.5) which is shown as the move from III to IV.

4.1.3 The Slutsky matrix for collective demands.

Using the *observable* functions $\hat{\mathbf{g}}(.)$, we can define the *observable* or quasi-Slutsky matrix $S = [s_{ij}]_{i,j}$ by its general term:

$$s_{ij} = \frac{\partial \hat{g}_i}{\partial r_j} + \hat{g}_j \frac{\partial \hat{g}_i}{\partial x} \tag{4.6}$$

From (4.5) this can be written as:

$$s_{ij} = \left[\frac{\partial \tilde{g}_i}{\partial r_j} + \tilde{g}_j \frac{\partial \tilde{g}_i}{\partial x}\right] + \frac{\partial \tilde{g}_i}{\partial \mu}\left[\frac{\partial \mu}{\partial r_j} + \tilde{g}_j \frac{\partial \mu}{\partial x}\right] \tag{4.7}$$

From (4.3), the first term between brackets is the substitution term σ_{ij} with associated matrix Σ. We adopt the following notation:

$$\begin{aligned}D_\mu \tilde{\mathbf{g}} &= \left[\frac{\partial \tilde{g}_i}{\partial \mu}\right]_i \\ \mathbf{v} &= \left[\frac{\partial \mu}{\partial r_j} + \tilde{g}_j \frac{\partial \mu}{\partial x}\right]_j\end{aligned} \tag{4.8}$$

This gives:

$$S = \Sigma + (D_\mu \tilde{\mathbf{g}}) . \mathbf{v}' = \Sigma + R \tag{4.9}$$

so that the Slutsky matrix of the observable collective demand $\hat{\mathbf{g}}(\mathbf{r}, x)$ is the sum of a conventional Slutsky matrix Σ, which is symmetric and negative, and an additional matrix R. The latter is the product of a column vector $(D_\mu \tilde{g})$ and a row vector $(\mathbf{v}')$. Note that such an outer product has rank of at most one; indeed, for any vector $\mathbf{w}$ such that $\mathbf{v}'.\mathbf{w} = 0$ we have that

$R.\mathbf{w} = 0$. Finally, this analysis and the homogeneity assumption on $\mu(.)$ yields that the necessary conditions for the collective model demands are the *generalized Slutsky conditions*:

$$\hat{\mathbf{g}}(\mathbf{r}, x) \text{ is zero homogeneous} \tag{4.10}$$

$$\mathbf{r}'\hat{\mathbf{g}}(\mathbf{r}, x) \equiv x \tag{4.11}$$

$$S \text{ is the sum of a symmetric, negative matrix and a rank 1 matrix} \tag{4.12}$$

(Browning and Chiappori (1998)). We denote the third property *SNR1*.

One can readily see that these conditions generalize the conventional Slutsky conditions in the unitary setting. In the particular case of $R = 0$, indeed, we are back to the predictions of the unitary model. This is the case, in particular, when either μ is constant (so that $\mathbf{v} = 0$) or when $\tilde{\mathbf{g}}$ does not depend on μ (so that $D_\mu \tilde{g} = 0$). The latter case corresponds to the two partners having the same cardinal preferences; $u^b(\mathbf{g}) = k_0 + k_1 u^a(\mathbf{g})$ with $k_1 > 0$. In general, however, R is not zero, and the predictions of the model deviate from those of the unitary model; in a sense, matrix R summarizes this deviation. The main result is that *this deviation is only one-dimensional* - which formally translates into the rank of R being at most one. This is a strong result because the *size* of matrix R can be quite large - as many as goods the household buys.[3]

The result has a simple, geometric intuition, given by Figure 3.8 in chapter 3. The move from I to II represents the variation that would obtain if μ was kept constant; as such, it does not violate Slutsky symmetry. The violation comes from the second component,that is, the move from II to III which reflects the impact of changes in μ. This change takes place *along the Pareto frontier*. But this frontier is one dimensional, independently of the number of commodities in the economy. Consequently the matrix R has at most rank 1.

Finally, it can be shown that these conditions are also (locally) sufficient for the existence of a collective model. Chiappori and Ekeland (2006) show that any 'smooth'[4] demand function satisfying the three properties above (homogeneity, adding-up and SNR1) can *locally* be constructed as the collective demand of a well chosen household. This is a very difficult result, that requires complex mathematical tools; it constitutes a generalization of the classical 'integrability' result in standard consumer theory.

[3] In general, R has $(n+N)$ eigenvalues (possibly complex); the rank condition means that all of them, but maybe one, are equal to zero. Equivalently, one can find a basis in which all of the $(n+N)$ columns of R but one are identically zero.

[4] Technically, the result has been proved for twice continuously differentiable demand functions.

4.1.4 Distribution factors

We may now reintroduce distribution factors. An interesting feature is that such factors do not change the Pareto frontier, but only the Pareto weight. In geometrical terms, thus, they can only generate moves along the Pareto frontier (from II to III in Figure 3.8). This suggests that analyzing the impact of distribution factors may help understanding the nature and the form of such moves. This intuition can be given a formal translation. Equation (4.4) above can now be rewritten as:

$$\hat{\mathbf{g}}(\mathbf{r}, x, \mathbf{z}) = \tilde{\mathbf{g}}(\mathbf{r}, x, \mu(\mathbf{r}, x, \mathbf{z})) \tag{4.13}$$

Because the same $\mu(.)$ function appears in all goods the collective model yields *cross-equation restrictions*. To see this, consider the consequences of a marginal change in distribution factor z_k on the collective demand for commodity i:

$$\frac{\partial \hat{g}_i}{\partial z_k} = \frac{\partial \tilde{g}_i}{\partial \mu} \frac{\partial \mu}{\partial z_k} \tag{4.14}$$

Comparing the effect of different distribution factors, say z_k and z_l, we find that (assuming $\partial g_i / \partial z_l \neq 0$):

$$\frac{\partial \hat{g}_i / \partial z_k}{\partial \hat{g}_i / \partial z_l} = \frac{\partial \mu / \partial z_k}{\partial \mu / \partial z_l} \tag{4.15}$$

The right hand side term is independent of the good we are considering. Hence we have the *proportionality property* that the ratio of derivatives with respect to two sharing factors is the same for all goods. The result that the impact of z_k and z_l must be proportional across commodities is very important empirically, and can be given various equivalent forms; for instance, we can write that[5]

$$\frac{\partial \hat{g}_i}{\partial z_k} = \frac{\partial \mu / \partial z_k}{\partial \mu / \partial z_l} . \frac{\partial \hat{g}_i}{\partial z_l} \tag{4.16}$$

If the impact of a change in z_k on household demand for good i is, say, twice as large as that of z_l, then the same must be true for *all* commodities; and we can actually conclude that the impact of z_k on the Pareto weight μ is twice as large as that of z_l. Intuitively, whatever the number of distribution factors, they only operate through their impact on μ; hence their impact is one-dimensional. In a sense, it is as if there was one distribution factor only. This prediction is empirically testable (subject to having at least two distribution factors); possible tests will be discussed in the next chapter.

Another interesting feature of (4.14) is that it provides additional information about the structure of price and income effects in the collective

[5] Equivalently, the matrix $D_{\mathbf{z}}\mathbf{g}$ with general terms $\frac{\partial g_i}{\partial z_k}$ is of rank (at most) one.

demand. From (4.14), we have that:

$$\begin{aligned} \frac{\partial \tilde{g}_i}{\partial \mu} &= \frac{1}{\partial \mu / \partial z_k} \frac{\partial \hat{g}_i}{\partial z_k} \text{ for all } i, k \\ &= \lambda_k \frac{\partial \hat{g}_i}{\partial z_k} \text{ for all } i, k \end{aligned} \tag{4.17}$$

so that (4.9) becomes

$$S = \Sigma + R = \Sigma + \lambda_k . \left(D_{z_k} \hat{\mathbf{g}} \right) . \mathbf{v}' \text{ for any } k \tag{4.18}$$

Thus regarding price and income effects, not only is the deviation from the unitary model (the 'collective effect') one-dimensional, but it is closely related to the impact of distribution factors on demand. This is a surprising property, since it establishes links between the impact of purely economic factors - prices and incomes - and that of variables of a different type (say, divorce laws or sex ratios). Again, empirical tests of this property will be discussed in the next chapter.

4.1.5 Larger households

The analysis developed above can be extended to larger households. Suppose there are T agents in the household. We continue to assume efficiency so that the collective household utility function is defined as:

$$\begin{aligned} u^f \left(\mathbf{q}, \mathbf{Q}, \boldsymbol{\mu} \right) = \max_{\mathbf{g}} & \left\{ \sum_{s=1}^{T} \mu_s u^s \left(\mathbf{Q}, \mathbf{q}^1, ..., \mathbf{q}^T \right) \right\} \\ & \text{subject to } \sum_{s=1}^{T} \mathbf{q}^s = \mathbf{q} \end{aligned} \tag{4.19}$$

where the vector $\boldsymbol{\mu} = (\mu_1, ..., \mu_T)$ of Pareto weights is normalized by $\mu_T = 1$. Again, the μ_t are functions of prices, income and distribution factors. The household maximizes this utility under budget constraint. With the same notations as above, we can define a 'structural' demand function, $\mathbf{g}\left(\mathbf{r}, x, \mu_1, ..., \mu_{T-1}\right)$ as the solution to (4.19); note that it now depends on $T-1$ Pareto weights. As before, the empirically relevant concept is the observable demand function, defined by:

$$\hat{\mathbf{g}} \left(\mathbf{r}, x, \mathbf{z} \right) = \tilde{\mathbf{g}} \left(\mathbf{r}, x, \mu_1 \left(\mathbf{r}, x, \mathbf{z} \right), ..., \mu_{T-1} \left(\mathbf{r}, x, \mathbf{z} \right) \right) \tag{4.20}$$

Similar computations to the two person case yield:

$$s_{ij} = \left[\frac{\partial \tilde{g}_i}{\partial r_j} + \tilde{g}_j \frac{\partial \tilde{g}_i}{\partial x} \right] + \sum_{t=1}^{T-1} \frac{\partial \tilde{g}_i}{\partial \mu_t} \left[\frac{\partial \mu_t}{\partial r_j} + \tilde{g}_j \frac{\partial \mu_t}{\partial x} \right] \tag{4.21}$$

Again, the collective Slutsky matrix is the sum of a symmetric, negative matrix Σ and of a 'deviation' R. However, R is now the sum of $T-1$ terms of the form $\left(D_{\mu_s}\tilde{\mathbf{g}}\right).\mathbf{v}'_t$, in which the vector $\mathbf{v}_t$ is of general term $\left[\frac{\partial \mu_t}{\partial r_j} + \hat{\xi}_j \frac{\partial \mu_t}{\partial x}\right]$; indeed, the deviations now come from the $T-1$ functions μ_t. In particular, its rank is at most $T-1$.

The generalized Slutsky conditions for a T person household are given by:

$$\begin{gathered}\mathbf{g}\left(\mathbf{r}, x, \mathbf{z}\right) \text{ is zero homogeneous} \\ \mathbf{r}'\hat{\mathbf{g}}\left(\mathbf{r}, x, \mathbf{z}\right) \equiv x \\ T \text{ is the sum of a symmetric, negative matrix and a rank } T-1 \text{ matrix}\end{gathered} \tag{4.22}$$

These conditions are sometimes called the SNR$(T-1)$ conditions. They have a nicely nested structure, in the sense that SNR(k) is a special case of SNR$(k+1)$. They are more restrictive, the larger the number of goods and the smaller the size of the household. Note, in particular, that when the number of persons in the household is equal to (or larger than) the number of commodities, the SNR$(T-1)$ conditions are not restrictive at all: any $(n+N)\times(n+N)$ matrix satisfies them (just take $\Sigma = 0$). This is by no means a problem in real life, since the number of commodities available is very large. However, it may be an issue in econometric estimation, which typically use a small number of aggregate 'commodities'.

4.1.6 Children

Finally, we may briefly come back to the issue of children. We described in the previous chapters two different ways of modelling children: either as a 'public good' that enters parents' utility or as a genuine decision maker. The previous analysis sheds light on the respective implications of these options. In the first case the household has two decision makers, whereas it has three in the second. According to the generalized Slutsky conditions, the demand function should satisfy SNR1 in the first case, but not in the second (it only satisfies SNR2). In words: one can devise a test allowing to find out how many decision makers there are in the household (the precise implementation of the test will be described in the next chapter).

Clearly, one has to keep in mind the limits of this exercise. What the theory predicts is that the rank of the R matrix is *at most* $T-1$. Still, it can be less. For instance, if μ_s and $\mu_{s'}$ have a similar impact on household demand (in the sense that $D_{\mu_s}\tilde{\mathbf{g}}$ and $D_{\mu_{s'}}\tilde{\mathbf{g}}$ are colinear) then matrix R will be of rank $T-2$. In other words, if a household demand is found to satisfy SNRk, the conclusion is that there are *at least k* decision makers; there may be more, but there cannot be less. Or, in the case of children: a demand satisfying SNR1 is consistent with children being decision makers; however, if it satisfies SNR2 and not SNR1, then the hypothesis that children are not decision makers is rejected.

4.2 Duality in the collective model

4.2.1 The collective expenditure function.

The standard tools of duality theory which have been developed in consumer theory can readily be extended to collective models. They provide useful ways of analyzing welfare issues in the collective setting. We introduce these notions for a two-person household; the extension to larger units is straightforward. The first concept is that of *collective expenditure function*, denoted E, which is defined by:

$$\begin{aligned} E\left(\mathbf{r}, u^a, u^b\right) &= \min_{\mathbf{g}, \mathbf{q}^a \mathbf{q}^b, \mathbf{Q}} \mathbf{r}'\mathbf{g} \\ \text{subject to } u^s\left(\mathbf{q}^a, \mathbf{q}^b, \mathbf{Q}\right) &\geq \bar{u}^s, \;\; s = a, b. \\ \text{and } \mathbf{g} &= \left(\mathbf{q}^a + \mathbf{q}^b, \mathbf{Q}\right) \end{aligned} \tag{4.23}$$

The collective expenditure function depends on prices and on two utility levels $\left(\bar{u}^a, \bar{u}^b\right)$; it represents the minimum level of expenditures needed at these prices to achieve these utilities. One can then define the *compensated collective demand function*, $\breve{\mathbf{g}}\left(\mathbf{r}, \bar{u}^a, \bar{u}^b\right)$, as a solution to program (4.23). A key remark is that the definition of household collective expenditure and demand functions depends only on individual preferences and not on the household's decision process.

The properties of the functions just defined are analogous to those of their standard counterpart. The basic one is the following. Consider the 'primal' model stated in Chapter 3:

$$\max_{\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b} u^b\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) \tag{4.24}$$

$$\text{subject to } \mathbf{r}'\mathbf{g} \leq x$$

$$u^a\left(\mathbf{Q}, \mathbf{q}^a, \mathbf{q}^b\right) \geq \bar{u}^a$$

$$\text{and } \mathbf{g} = \left(\mathbf{q}^a + \mathbf{q}^b, \mathbf{Q}\right) \tag{4.25}$$

The two programs (4.23) and (4.24) are closely related. Indeed, let $\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$ denote the solution to (4.24) and let $\bar{u}^b = u^b\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$. Then:

$$E\left(\mathbf{r}, \bar{u}^a, \bar{u}^b\right) = x$$

and $\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$ solves (4.23). Conversely, if $\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$ denotes the solution to (4.23) for some $\bar{u}^a, \bar{u}^b$, then for $x = E\left(\mathbf{r}, \bar{u}^a, \bar{u}^b\right)$ we have that

$$u^b\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right) = \bar{u}^b$$

and $\left(\bar{\mathbf{Q}}, \bar{\mathbf{q}}^a, \bar{\mathbf{q}}^b\right)$ solves (4.24). The intuition is simply that if a particular bundle maximizes b's utility subject to constraints on a's utility and total expenditures - this is program (4.24) - then one cannot reach the same utilities at a lower total cost than this bundle (if that was possible, the difference in costs could be used to buy extra public commodities and increase both members' utilities, a contradiction). Conversely, if a bundle minimizes total cost for two given utility levels - and therefore solves Program (4.23) - then one cannot increase b's utility without either reducing a's utility or spending more.

The notion of collective expenditure function - and the duality property just described - is a direct generalization of the standard expenditure function of consumer theory; the only difference is that, now, there are two utility levels that should be reached. Many results follow that generalize standard theorems of consumer theory; in particular:

Proposition 4.1 *We have:*

$$\breve{\mathbf{g}}\left(\mathbf{r}, u^a, u^b\right) = \nabla_r E\left(\mathbf{r}, u^a, u^b\right) \tag{4.26}$$

where $\nabla_r E$ denotes the gradient of E with respect to $\mathbf{r}$ (that is, the vector of partial derivatives $\partial E / \partial r_j$).

The result is a consequence of the envelope theorem applied to program (4.23).

In the case of egotistic preferences of the form $u^s\left(\mathbf{q}^s, \mathbf{Q}\right)$, we have further results. Define the compensated demand for public goods by $\breve{\mathbf{Q}}\left(\mathbf{p}, \mathbf{P}, u^a, u^b\right)$. Then:

Proposition 4.2 *If u^s only depends on $\left(\mathbf{q}^s, \mathbf{Q}\right)$, $s = a, b$, then:*

$$\begin{aligned}
E\left(\mathbf{p}, \mathbf{P}, u^a, u^b\right) &\leq e^a\left(\mathbf{p}, \mathbf{P}, u^a\right) + e^b\left(\mathbf{p}, \mathbf{P}, u^b\right) \\
E\left(\mathbf{p}, \mathbf{P}, u^a, u^b\right) &\geq e^a\left(\mathbf{p}, \mathbf{P}, u^a\right) \\
&\quad + e^b\left(\mathbf{p}, \mathbf{P}, u^b\right) - \mathbf{P}'\breve{\mathbf{Q}}\left(\mathbf{p}, \mathbf{P}, u^a, u^b\right)
\end{aligned} \tag{4.27}$$

where $e^s\left(\mathbf{p}, \mathbf{P}, u^s\right)$ denotes the (individual) expenditure function of member s.

Proof. *The last inequality stems from the definition of individual expenditure functions, since*

$$e^s\left(\mathbf{p}, \mathbf{P}, u^s\right) \leq \mathbf{p}'\mathbf{q}^s\left(\mathbf{p}, \mathbf{P}, u^a, u^b\right) + \mathbf{P}'\mathbf{Q}\left(\mathbf{p}, \mathbf{P}, u^a, u^b\right) \tag{4.28}$$

For the first inequality, let $\left(\bar{\mathbf{q}}^s, \bar{\mathbf{Q}}^s\right)$ denote the individual *compensated demand of s (corresponding to prices $\mathbf{p}, \mathbf{P}$ and utility u^s). If $\bar{\mathbf{Q}}^a = \bar{\mathbf{Q}}^b$ the conclusion follows. If not, say $\bar{\mathbf{Q}}^a > \bar{\mathbf{Q}}^b$, then*

$$\begin{aligned}
u^a\left(\bar{\mathbf{q}}^a, \bar{\mathbf{Q}}^a\right) &= u^a \\
u^b\left(\bar{\mathbf{q}}^b, \bar{\mathbf{Q}}^a\right) &> u^b
\end{aligned} \tag{4.29}$$

therefore

$$\begin{aligned} E\left(\mathbf{p},\mathbf{P},u^a,u^b\right) &\leq \mathbf{p}'\left(\bar{\mathbf{q}}^a+\bar{\mathbf{q}}^b\right)+\mathbf{P}'\bar{\mathbf{Q}}^a \\ &\leq \mathbf{p}'\left(\bar{\mathbf{q}}^a+\bar{\mathbf{q}}^b\right)+\mathbf{P}'\bar{\mathbf{Q}}^a+\mathbf{P}'\bar{\mathbf{Q}}^b \\ &= e^a\left(\mathbf{p},\mathbf{P},u^a\right)+e^b\left(\mathbf{p},\mathbf{P},u^b\right) \end{aligned} \tag{4.30}$$

■

4.2.2 Indirect utilities

We can also define indirect utility functions. Consider first the program

$$\max_{\left(\mathbf{q}^a,\mathbf{q}^b,\mathbf{Q}\right)} \mu u^a\left(\mathbf{q}^a,\mathbf{Q}\right)+u^b\left(\mathbf{q}^b,\mathbf{Q}\right)$$

$$\text{subject to } \mathbf{r}'\left(\mathbf{q}^a+\mathbf{q}^b,\mathbf{Q}\right)=x \tag{4.31}$$

Let $\left(\mathbf{q}^{*a},\mathbf{q}^{*b},\mathbf{Q}^*\right)$ denote its solution. Then the function ω^s, defined for $s=a,b$ by:

$$\omega^s\left(\mathbf{r},x,\mu\right)=u^s\left(\mathbf{q}^{*s},\mathbf{Q}^*\right)$$

is the direct equivalent, in the collective setting, of the indirect utility concept in standard consumer theory. In particular, ω^s only depends on *preferences*, not on the *decision process*; technically, ω^s is a function of the Pareto weight μ, and a change in the decision process would result in the same function ω^s being applied to a different μ.

A second, and more important definition is obtained by plugging the particular Pareto weight adopted by the household into the previous definition. In this case, the *collective indirect utility of a member* is the level of utility ultimately reached by this member as a function of prices and income and distribution factors. Formally, if the decision process is characterized by a function $\mu\left(\mathbf{r},x,\mathbf{z}\right)$, the collective indirect utility of member s is defined for $s=a,b$ by:

$$V^s\left(\mathbf{r},x,\mathbf{z}\right)=\omega^s\left(\mathbf{r},x,\mu\left(\mathbf{r},x,\mathbf{z}\right)\right)$$

Now, the definition of s's collective indirect utility depends not only on s's preferences, but also on the whole decision process. In other words, collective indirect utilities are specific to a particular match between agents and a particular decision rule (summarized by the function μ). This is in sharp contrast with the unitary case, where there exists a one-to-one correspondence between direct and indirect utility at the individual level.

Also, a key remark, here, is that if one is interested in welfare analysis, then the collective indirect utility is the appropriate concept. Indeed, it preserves the basic interpretation of standard, indirect utilities in consumer theory - namely, it characterizes each agent's final welfare once all aspects of the decision process have been taken into account.

4.2.3 Welfare

An important application of consumer theory relates to welfare issues, such as the cost-benefit evaluation of economic reforms. A standard tool is the notion of *compensating variation.* Consider a reform that changes the price vector from $\mathbf{r}$ to $\mathbf{r}'$. For an agent with initial income x, the compensating variation (CV) is defined as the change in income that would be needed to exactly compensate the agent. That is, the income that would allow her to remain on the same indifference curve. For a single person this is defined by:

$$CV = e\left(\mathbf{r}', v\left(\mathbf{r}, x\right)\right) - x$$

where e and v respectively denote the agent's expenditure and indirect utility functions. This concept can directly be extended to a collective setting. This leads to the following definition:

Definition 4.1 *The* potentially compensating variation *is the function* $\Gamma_1\left(.\right)$ *such that:*

$$\Gamma_1\left(\mathbf{r}, \mathbf{r}', x, \mathbf{z}\right) = E\left(\mathbf{r}', V^a\left(\mathbf{r}, x, \mathbf{z}\right), V^b\left(\mathbf{r}, x, \mathbf{z}\right)\right) - x$$

In words, consider a household in which, before the reform, total income is x and member s's utility is $u^s = V^s\left(\mathbf{r}, x, \mathbf{z}\right)$. The potentially compensating variation measures the change in income that has to be given to the household for the previous utility levels to be affordable at the new prices $\mathbf{r}'$. Natural as this extension may seem, it nevertheless raises problems that are specific to a multi-person setting. The variation is *potentially* compensating, in the sense that the additional income thus measured could, *if allocated appropriately within the household*, enable both members to reach their pre-reform utility levels. That is, the income $x + \Gamma_1\left(\mathbf{r}, \mathbf{r}', x, \mathbf{z}\right)$ has the property that the utilities $\left(V^a\left(\mathbf{r}, x, \mathbf{z}\right), V^b\left(\mathbf{r}, x, \mathbf{z}\right)\right)$ belong to the Pareto frontier at prices $\mathbf{r}'$. What is *not* guaranteed, however, is that the point $\left(V^a\left(\mathbf{r}, x, \mathbf{z}\right), V^b\left(\mathbf{r}, x, \mathbf{z}\right)\right)$ will still be chosen on the new frontier. In other words, the compensation is such that the welfare level of each members could be maintained despite the reform. Whether the household will choose to do so is a different story.

The idea is illustrated in Figure 4.2. The potentially compensating variation is such that the new frontier (the dashed frontier) goes through $uu = \left(V^a\left(\mathbf{r}, x, \mathbf{z}\right), V^b\left(\mathbf{r}, x, \mathbf{z}\right)\right)$. However, the reform changes both the frontier and the Pareto weights. While the initial allocation uu is still affordable (it belongs to the new frontier), the household may instead choose the allocation uu'. It follows that although both members could have been exactly compensated, in practice one partner will strictly gain from the reform (a in Figure 4.2), whilst the other will strictly lose. This is despite the fact that, as drawn, the Pareto weight for a has actually gone down.

This suggests an alternative definition of the compensation, which is the following:

Definition 4.2 *The* actually compensating variation *is the function* Γ_2 *such that:*

$$\Gamma_2\left(\mathbf{r},\mathbf{r}',x,\mathbf{z}\right)=\min_{x'}\left\{(x'-x)\ \ \textit{subject to}\ V^s\left(\mathbf{r}',x',\mathbf{z}\right)\geq V^s\left(\mathbf{r},x,\mathbf{z}\right),\ s=a,b\right\} \tag{4.32}$$

Thus $\Gamma_2\left(\mathbf{r},\mathbf{r}',x,\mathbf{z}\right)$ is the minimum amount to be paid to the household for each agent to be actually compensated for the reform, taking into account the intrahousehold allocation of additional income. This is illustrated in figure 4.3. The actually compensating change moves the Pareto frontier out until b is no worse off. On the new frontier uu'' is the chosen allocation. Note, still, that while b is then exactly compensated for the reform, a gains strictly; the initial point uu lies strictly within the new frontier.

Clearly, both concepts raise specific difficulties. The concept of potential compensation disregards actual decision processes, and ignores intrahousehold inequality. In a fully compensated household, the reform may worsen the situation of one of the members. This may have a social cost, at least if we accept that the actual intrahousehold decision process need not always be optimal from a normative, social viewpoint. On the other hand, the notion of actual compensation may lead to costly compensations, resulting in a bias in favor of the *status quo*. Moreover, it *de facto* rewards (marginal) unfairness, since the amount paid to the household has to be larger when most of the additional transfers goes to the dominant member. These issues are still largely open. We may simply make two remarks. First, these issues are inherent to any context in which the social planner cannot fully control intragroup redistribution; they are by no means specific to the collective approach, or for that matter to cooperative models. The obvious conclusion is that welfare economics can hardly do without a precise analysis of intrafamily decision processes.

Secondly, the notion of distribution factors suggests an additional direction for public intervention. Some of these factors can indeed be controlled by the planner. For example, a benefit can be paid to the husband or to the wife, in cash or in kind. The benefit should then be designed taking into account the planner's ability to influence the decision process; technically, the maximization in (4.32) should be over x' *and* $\mathbf{z}$. For instance, several authors have suggested that a benefit aimed at improving the welfare of children should be paid to the mother, because such a shift may increase her weight in the decision process. Again, we may conclude that a theoretical and empirical analysis of intrahousehold allocation is a key step in any policy design.

4.3 The case of purely private consumptions

4.3.1 The sharing rule.

Although the Pareto weight captures very clearly our intuitive idea about power, it turns out that there is an equivalent concept which is easier to work with and to think about, if preferences are egotistic and we ignore public goods:

$$\begin{aligned} u^a\left(\mathbf{q}^a, \mathbf{q}^b\right) &= u^a\left(\mathbf{q}^a\right) \\ u^b\left(\mathbf{q}^a, \mathbf{q}^b\right) &= u^b\left(\mathbf{q}^b\right) \end{aligned} \tag{4.33}$$

It is a very familiar idea in convex economies with independent agents that if there are no externalities, then any efficient outcome can be decentralized by a choice of prices and the (re)distribution of income. This is the Second Fundamental Theorem of Welfare Economics. In collective models we can exploit a similar idea. The efficiency assumption has a very simple and natural translation. With preferences of this kind, the economic interactions within the household are minimal: neither externalities, nor public goods are involved - agents essentially live side by side and consume independently.[6] Efficiency simply means that *for each agent*, the consumption bundle is optimal, in the sense that no other bundle could provide more utility at the same cost. In other words, take any particular (re)distribution of total income between members, and assume each member chooses his/her preferred consumption bundle subject to the constraint that the corresponding expenditures cannot exceed his/her share of total income. Then the resulting consumption will be Pareto efficient. Conversely, when preferences are quasi-concave, any Pareto efficient allocation can be obtained in this way.

Suppose a household faces prices $\mathbf{p}$ and has decided on a level of total expenditure x. Let the resulting allocation be denoted $\left(\hat{\mathbf{q}}^a, \hat{\mathbf{q}}^b\right)$ so that $\mathbf{p}'\left(\hat{\mathbf{q}}^a + \hat{\mathbf{q}}^b\right) = x$. The decentralization procedure is simple: each person is given a share of total expenditure and allowed to spend it on their own private goods, using their own private sub-utility function $u^s\left(\mathbf{q}^s\right)$. In what follows, let x^s denote s's total expenditures; then $x^a = \mathbf{p}'\hat{\mathbf{q}}^a$, $x^b = \mathbf{p}'\hat{\mathbf{q}}^b$, and $x^a + x^b = x$. Traditionally, a's part of total expenditures x^a is denoted ρ (so that $x^b = x - \rho$), and called the *sharing rule*.[7] Hence the following statement:

Proposition 4.3 *Define $\rho = \mathbf{p}'\hat{\mathbf{q}}^a$ so that $x - \rho = \mathbf{p}'\hat{\mathbf{q}}^b$. We have:*

[6] This claim should be qualified. One could easily introduce additional, *non monetary* benefits of marriage (love, sex, companionship etc.).

[7] The terminology is not completely tied down with some authors referring to the fraction of expenditures going to A (that is, x^A/x) as the sharing rule.

- $\hat{\mathbf{q}}^a$ *solves*
$$\max u^a\left(\mathbf{q}^a\right) \text{ subject to } \mathbf{p}'\mathbf{q}^a = \rho \tag{4.34}$$

- $\hat{\mathbf{q}}^b$ *solves*
$$\max u^b\left(\mathbf{q}^b\right) \text{ subject to } \mathbf{p}'\mathbf{q}^b = x{-}\rho \tag{4.35}$$

Conversely, for any ρ, if $\hat{\mathbf{q}}^a$ and $\hat{\mathbf{q}}^b$ solve (4.34) and (4.35) then the allocation $\left(\hat{\mathbf{q}}^a, \hat{\mathbf{q}}^b\right)$ is Pareto efficient.

The demands functions $\tilde{\mathbf{q}}^a$ and $\tilde{\mathbf{q}}^b$, *as functions of* $(\mathbf{p},\rho)$ and $(\mathbf{p},x-\rho)$, are conventional demand functions and have all of the usual (Slutsky) properties.

In other words, when all commodities are privately consumed, the decision process can be decomposed into two phases: a *sharing* phase in which agents determine the sharing rule and a *consumption* phase, in which agents allocate their share between the various commodities available. In this context, efficiency only relates to the second phase: whatever the sharing rule, the resulting allocation will be efficient provided that agents maximize their utility during the consumption phase. On the other hand, the collective part of the process (whether it entails bargaining, formal rules or others) takes part in the first stage.

Also, note that a sharing rule can be defined for any decision process (one can always consider the outcome and compute the amount privately spent by member a). However, Proposition (4.3) is satisfied (that is, the outcome maximizes a's utility under a's budget constraint) if and only if the process is efficient. Clearly, there exists a close connection (actually, if u^a and u^b are strictly concave, a one-to-one, increasing mapping) between a's share ρ and a's Pareto weight; both reflect a's power in the bargaining phase of the relationship. This implies that the sharing rule depends not only on prices and total expenditures but also on distribution factors.[8]

An advantage of the sharing rule is that, unlike the Pareto weight, it is easy to interpret. In particular, it is independent of the cardinal representation of individual utilities. For this reason, it is often more convenient to use the sharing rule as an indicator of the agent a's 'weight' in the decision process: any change in, say, a distribution factor that increases ρ makes a better off. Of course, this quality comes at a price: the sharing rule interpretation, as presented above, *is valid only when all goods are privately consumed*. We will see in Section 5 to what extent it can be generalized to public goods

Finally, one should keep in mind that the functions $\tilde{\mathbf{q}}^a\left(\mathbf{p},\rho\right)$ and $\tilde{\mathbf{q}}^b\left(\mathbf{p},x-\rho\right)$, although 'structural' in the previous sense, cannot be observed, for two rea-

[8] The sharing rule depends on prices and income even if the Pareto weight is independent of the latter. Thus even in a unitary model with egotistic preferences we have a sharing rule and it depends on prices and total expenditure. However, the sharing rule cannot depend on distribution factors unless the Pareto weight does.

sons. One is that, in general, one cannot change prices without changing the sharing rule as well; what can be observed, *at best*, are the functions $\hat{\mathbf{q}}^a(\mathbf{p}, x, \mathbf{z})$ and $\hat{\mathbf{q}}^b(\mathbf{p}, x, \mathbf{z})$, which are related to the previous ones by the relationships:

$$\begin{aligned} \hat{\mathbf{q}}^a(\mathbf{p}, x, \mathbf{z}) &= \tilde{\mathbf{q}}^a(\mathbf{p}, \rho(\mathbf{p}, x, \mathbf{z})) \\ \hat{\mathbf{q}}^b(\mathbf{p}, x, \mathbf{z}) &= \tilde{\mathbf{q}}^b(\mathbf{p}, x - \rho(\mathbf{p}, x, \mathbf{z})) \end{aligned} \tag{4.36}$$

However, even these functions are in general unknown, because most of the time *the intrahousehold allocation of purchases is not observed*. Expenditure surveys invariably collect information about expenditures that are aggregated at the household level; but who consumes what remains largely unknown, except, maybe, for some specific commodities (for example, expenditure surveys typically distinguish between male and female clothing). In general what we observe is the household demand which is equal to the sum of the individual demands:

$$\begin{aligned} \hat{\mathbf{q}}(\mathbf{p}, x, \mathbf{z}) &= \hat{\mathbf{q}}^a(\mathbf{p}, x, \mathbf{z}) + \hat{\mathbf{q}}^b(\mathbf{p}, x, \mathbf{z}) \\ &= \tilde{\mathbf{q}}^a(\mathbf{p}, \rho(\mathbf{p}, x, \mathbf{z})) + \tilde{\mathbf{q}}^b(\mathbf{p}, x - \rho(\mathbf{p}, x, \mathbf{z})) \end{aligned} \tag{4.37}$$

As we shall see below, one can often use this relationship to derive the properties of collective demand functions.

4.3.2 Caring preferences

Let us now consider the case of preferences of the 'caring' type, namely

$$\begin{aligned} U^a\left(\mathbf{q}^a, \mathbf{q}^b\right) &= u^a(\mathbf{q}^a) + \delta^a u^b\left(\mathbf{q}^b\right) \\ U^b\left(\mathbf{q}^a, \mathbf{q}^b\right) &= u^b\left(\mathbf{q}^b\right) + \delta^b u^a(\mathbf{q}^a) \end{aligned} \tag{4.38}$$

Here, the Welfare Theorems do not directly apply, since caring involves an externality component. Two points should however be remembered. First, any allocation that is Pareto efficient for caring preferences is also Pareto efficient for the egotistic preferences u^a and u^b. This implies that the first part of Proposition 4.3 still applies: whenever an allocation is efficient, it can be decentralized through a sharing rule. The converse, however, no longer holds in general. We know that some allocations may be efficient for egotistic preferences, but not so for caring ones. It follows that *only a subset* of possible sharing rules generate efficient allocations for caring preferences. For instance, a sharing rule such as $\rho \simeq 0$ typically generates inefficient allocations since a redistribution of the resulting allocation in favor of a may increase both agents' welfare (if $\delta^b > 0$ and $\partial u^a / \partial q^a$ is sufficiently large when $\mathbf{q}^a$ is very small).

4.3.3 Indirect utilities

In the private good case, there exists a simple link between the collective indirect utilities defined above and the standard, individual indirect utilities. Denote the indirect utility corresponding to u^s (for $s = a, b$):

$$\begin{aligned} v^s(\mathbf{p}, x^s) &= \max u^s(\mathbf{q}) \\ \text{subject to } \mathbf{p}.\mathbf{q} &= x^s \end{aligned} \tag{4.39}$$

Thus $v^s(.)$ denotes the (maximum) utility level reached by s when facing prices $\mathbf{p}$ and consuming a total amount x^s. This is the standard, unitary concept, which makes no reference to the intrahousehold decision process. Now, in the case of private goods, the decision process is fully summarized by the sharing rule. It follows that:

$$V^a(\mathbf{p}, x, \mathbf{z}) = v^a(\mathbf{p}, \rho(\mathbf{p}, x, \mathbf{z})) \tag{4.40}$$

$$V^b(\mathbf{p}, x, \mathbf{z}) = v^b(\mathbf{p}, x - \rho(\mathbf{p}, x, \mathbf{z})) \tag{4.41}$$

where V^s is the *collective indirect utility* of member s, according to the definition of the previous section. In particular, the first phase of the decision process (deciding over the sharing rule) can readily be modeled using indirect utilities: whenever some ρ is chosen, a receives $v^a(\mathbf{p}, \rho)$ and b gets $v^b(\mathbf{p}, x - \rho)$. The program would therefore become:

$$\max_{\rho} \mu v^a(\mathbf{p}, \rho) + v^b(\mathbf{p}, x - \rho) \tag{4.42}$$

More specific processes can also be considered. For instance, Nash bargaining with respective threat points T^a and T^b would solve:

$$\max_{\rho} \left[v^a(\mathbf{p}, \rho) - T^a\right]\left[v^b(\mathbf{p}, x - \rho) - T^b\right] \tag{4.43}$$

It is important to note that, in general, many different structures (that is, individual preferences and a sharing rule) generate the same collective indirect utilities V^a, V^b. Indeed, for any given pair $\left(V^a, V^b\right)$, let $\left(v^a, v^b, \rho\right)$ be such that (4.40) and (4.41) are satisfied, and assume that v^a and v^b are strictly increasing and strictly quasi-concave. Pick up an arbitrary function $\phi(\mathbf{p})$, and define $\left(v^a_\varepsilon, v^b_\varepsilon, \rho_\varepsilon\right)$ by:

$$\begin{aligned} v^a_\varepsilon(\mathbf{p}, r) &= v^a(\mathbf{p}, r - \varepsilon\phi(\mathbf{p})) \\ v^b_\varepsilon(\mathbf{p}, r) &= v^b(\mathbf{p}, r + \varepsilon\phi(\mathbf{p})) \\ \rho_\varepsilon(\mathbf{p}, x, \mathbf{z}) &= \rho(\mathbf{p}, x, \mathbf{z}) + \varepsilon\phi(\mathbf{p}) \end{aligned}$$

then one can readily check that

$$V^a(\mathbf{p}, x, \mathbf{z}) = v^a_\varepsilon(\mathbf{p}, \rho_\varepsilon(\mathbf{p}, x, \mathbf{z})) \tag{4.44}$$

$$V^b(\mathbf{p}, x, \mathbf{z}) = v^b_\varepsilon(\mathbf{p}, x - \rho_\varepsilon(\mathbf{p}, x, \mathbf{z})) \tag{4.45}$$

In other words, the structures $\left(v^a, v^b, \rho\right)$ and $\left(v^a_\varepsilon, v^b_\varepsilon, \rho_\varepsilon\right)$, although different, generate the same collective indirect utilities. It follows that the welfare conclusions reached by the two structures are always identical. For instance, if a given reform is found to increase his welfare and decrease her welfare when the evaluation is made using the first structure, using the second instead will lead to the same conclusion. We say that different structures that generate the same collective indirect utilities are *welfare equivalent.*

The notion of welfare equivalence plays an important role, notably in the discussion of identification in Chapter 5. In many situations, welfare equivalent structures are hard to empirically distinguish; in some cases, only the collective indirect utilities can actually be recovered. The key remark is that as far are welfare judgment are concerned, identifying collective indirect utilities is sufficient.

4.4 Application: labor supply with private consumption

4.4.1 The general setting

An example that has been widely analyzed in the literature concerns labor supply. In the most stripped down model without household production, labor supply is modelled as a trade off between leisure and consumption: people derive utility from leisure, but also from the consumption purchased with labor income. In a couple, however, an additional issue is the division of labor and of labor income: who works how much, and how is the resulting income distributed between members? As we now see, the collective approach provides a simple but powerful way of analyzing these questions.

Let l^s denote member s's leisure (with $0 \leq l^s \leq 1$) and q^s the consumption by s of a private Hicksian composite good whose price is set to unity. We start from the most general version of the model, in which member s's welfare can depend on his or her spouse's consumption and labor supply in a very general way, including for instance altruism, public consumption of leisure, positive or negative externalities, etc. In this general framework, member s's preferences are represented by a utility function $U^s\left(l^a, q^a, l^b, q^b\right)$. Let w^a, w^b, y denote respectively real wage rates and household non-labor income. Finally, let z denote a K-vector of distribution factors. The efficiency assumption generates the program:

$$\max_{\{l^a, l^b, q^a, q^b\}} \mu U^a + U^b$$

$$\begin{aligned} \text{subject to } q^a + q^b + w^a l^a + w^b l^b &\leq w^a + w^b + y \\ 0 &\leq l^s \leq 1, \quad s = a, b \end{aligned} \tag{4.46}$$

where μ is a function of $\left(w^a, w^b, y, \mathbf{z}\right)$, assumed continuously differentiable in its arguments.

In practically all empirical applications we observe only $q = q^a + q^b$. Consequently our statement of implications will involve only derivatives of q, l^a and l^b. In this general setting and assuming interior solutions, the collective model generates one set of testable restrictions, given by the following result:

Proposition 4.4 *Let $\hat{l}^s(w^a, w^b, y, z)$, $s = a, b$ be solutions to program (4.46). Then*

$$\frac{\partial \hat{l}^a / \partial z_k}{\partial \hat{l}^a / \partial z_1} = \frac{\partial \hat{l}^b / \partial z_k}{\partial \hat{l}^b / \partial z_1}, \quad \forall k = 2, ..., K. \tag{4.47}$$

This result is by no means surprising, since it is just a restatement of the proportionality conditions (4.15). The conditions are not sufficient, even in this general case, because of the SNR1 condition (4.12). Namely, one can readily check that the Slutsky matrix (dropping the equation for q because of adding up) takes the following form:

$$S = \begin{pmatrix} \frac{\partial \hat{l}^a}{\partial w^a} - \left(1 - \hat{l}^a\right) \frac{\partial \hat{l}^a}{\partial y} & \frac{\partial \hat{l}^a}{\partial w^b} - \left(1 - \hat{l}^b\right) \frac{\partial \hat{l}^a}{\partial y} \\ \frac{\partial \hat{l}^b}{\partial w^a} - \left(1 - \hat{l}^a\right) \frac{\partial \hat{l}^b}{\partial y} & \frac{\partial \hat{l}^b}{\partial w^b} - \left(1 - \hat{l}^b\right) \frac{\partial \hat{l}^b}{\partial y} \end{pmatrix}$$

As above, S must be the sum of a symmetric negative matrix and a matrix of rank one. With three commodities, the symmetry requirement is not restrictive: any 2×2 matrix can be written as the sum of a symmetric matrix and a matrix of rank one. Negativeness, however, has a bite; in practice, it requires that there exists at least one vector $\mathbf{w}$ such that $\mathbf{w}'\mathbf{S}\mathbf{w} < 0$. With distribution factors, the necessary and sufficient condition is actually slightly stronger, For $K = 1$, there must exist a vector $\mathbf{w}$ such that $S - \begin{pmatrix} \frac{\partial \hat{l}^1}{\partial z} & \frac{\partial \hat{l}^2}{\partial z} \end{pmatrix}' \mathbf{w}'$ is symmetric and negative.

4.4.2 Egoistic preferences and private consumption

Much stronger predictions obtain if we add some structure. One way to do that is to assume private consumption and egotistic (or caring) preferences, that is utilities of the form $u^s\left(l^s, q^s\right)$. Then there exists a sharing rule ρ, and efficiency is equivalent to the two individual programs:[9]

$$\begin{aligned} & \max_{\{l^a, q^a\}} u^a\left(l^a, q^a\right) \\ \text{subject to } q^a + w^a l^a & \leq w^a + \rho \\ 0 & \leq l^a \leq 1 \end{aligned} \tag{4.48}$$

[9] In what follows, we shall assume for simplicity that only one distribution factor is available; if not, the argument is similar but additional, proportionality conditions must be introduced.

and

$$\max_{\{l^b,q^b\}} u^b(l^b, q^b)$$
$$\begin{array}{rcl}\text{subject to } q^b + w^b l^b & \leq & w^b + (y - \rho) \\ 0 & \leq & l^b \leq 1\end{array}$$

Note that now ρ may be negative or larger than y, since one member may receive all non-labor income plus part of the spouse's labor income. Two remarks can be made at this point. First, ρ is the part of total non-labor income *allocated to* member a as an outcome of the decision process. This should be carefully distinguished from a's *contribution* to household non-labor income (although the latter may be a distribution factor if it influences the allocation process). That is, if non-labor income comes either from a (denoted y^a, representing for instance return on a's capital) or from b (denoted y^b, representing, say, a benefit paid exclusively to b), so that $y = y^a + y^b$, then a's part of total expenditures, denoted ρ, may depend (among other things) on y^a or on the ratio y^a/y - just as it may depend on any relevant distribution factor. But it is *not* equal to y^a in general.

The second point is that ρ may be an arbitrary function of wages, non-labor income and distribution factors. However, our assumptions imply that ρ *cannot* depend on the agents' total labor income, $w^s(1 - l^s)$. Indeed, efficiency precludes a person's allocation to depend on an *endogenous* variable such as the labor supply of this person. The intuition is that such a link would act as a subsidy that would distort the price of leisure faced by the agents, as in Basu's (2006) model of inefficient bargaining described in the previous chapter.

4.4.3 Collective labor supply

In turn, these programs shed light on various aspects of household labor supply. First, we have that

$$\begin{array}{rcl} l^a & = & \tilde{l}^a(w^a, \rho) \qquad (4.49)\\ l^b & = & \tilde{l}^b\left(w^b, y - \rho\right) \qquad (4.50)\end{array}$$

where $\tilde{l}^s$ denotes the Marshallian demand for leisure corresponding to u^s. The function $\tilde{l}^s$ is structural (in the sense that it depends on preferences), but only l^s is observed. The first implication of this model is that the spouse's wage matters for an individual's demand for leisure, but only through its impact on the sharing rule; that is,, through an income effect.

The same is true of non-labor income and of distribution factors:

$$\begin{aligned}\frac{\partial \hat{l}^a}{\partial w^b} &= \frac{\partial \tilde{l}^a}{\partial \rho}\frac{\partial \rho}{\partial w^b}, \\ \frac{\partial \hat{l}^a}{\partial y} &= \frac{\partial \tilde{l}^a}{\partial \rho}\frac{\partial \rho}{\partial y} \\ \frac{\partial \hat{l}^a}{\partial z_k} &= \frac{\partial \tilde{l}^a}{\partial \rho}\frac{\partial \rho}{\partial z_k} \end{aligned} \tag{4.51}$$

The second equation can be rewritten in elasticity terms:

$$\frac{y}{\hat{l}^a}\frac{\partial \hat{l}^a}{\partial y} = \left(\frac{\rho}{\tilde{l}^a}\frac{\partial \tilde{l}^a}{\partial \rho}\right)\left(\frac{y}{\rho}\frac{\partial \rho}{\partial y}\right) \tag{4.52}$$

Thus the income elasticity of a's *observed* demand for leisure is the product of two terms. The first is the *structural* income elasticity which characterizes a's preferences - what would be observed if a's fraction of total non-labor income could be independently monitored. The second term is the income elasticity of ρ, reflecting the change (in percentage) of a's allocation resulting from a given percentage change in household non-labor income. Hence if a member's allocation is elastic, then the elasticity of this person's demands for leisure, *as computed as the household level*, will exceed (in absolute value) the 'true' value (as observed for instance on singles, assuming that preferences are not changed by marriage). Conversely, if the allocation is inelastic (< 1), then her income elasticity will be found to be smaller than the 'true' value.

The same argument applies to own wage elasticities. From (4.49), we have that:

$$\frac{w^a}{\hat{l}^a}\frac{\partial \hat{l}^a}{\partial w^a} = \frac{w^a}{l^a}\frac{\partial \tilde{l}^a}{\partial w^a} + \left(\frac{\rho}{\tilde{l}^a}\frac{\partial \tilde{l}^a}{\partial \rho}\right)\left(\frac{w^a}{\rho}\frac{\partial \rho}{\partial w^a}\right) \tag{4.53}$$

Thus the own wage elasticity observed at the household level is the sum of two terms. The first is the 'structural' elasticity, corresponding to the agent's preferences; the second is the product of the person's structural income elasticity by the wage elasticity of the sharing rule. To discuss the sign of the latter, consider the consequences for intrahousehold allocation of an increase in a's wage. If leisure is a normal good, then the observed own wage elasticity (the left hand side) is smaller than the structural value (the first expression on the right hand side) if and only if ρ is increasing in w^a. This will be case if the wage increase dramatically improves a's bargaining position, so that a is able to keep all the direct gains *and* to extract in addition a larger fraction of household non-labor income. Most of the time, we expect the opposite; that is, part of a's gain in labor income is transferred to b, so that ρ is decreasing in w^a. Then the observed own wage elasticity (the left hand side) will be larger than the structural value.

The impact of distribution factors is in principle much easier to assess, because they leave the budget set unchanged and can only shift the distribution of power. Assuming that leisure is normal we have that if a change in a distribution factor favors member a, then a's share of household resources will increase which will reduce labor supply through a standard income effect. This simple mechanism has been repeatedly tested, using distribution factors such as sex ratios and 'natural experiments' such as the legalization of divorce (in Ireland) or abortion (in the United States). Interestingly enough, all existing studies tend to confirm the theory. The effects are found to be significant and of the predicted sign; moreover, they are specific to married people and are typically not significant when singles are considered (see the discussion in the next Chapter).

4.5 Public goods

4.5.1 Lindahl prices

We now consider a more general version of the model with egotistic preferences in which we allow for public goods. Hence individual utilities are of the form $u^s(\mathbf{q}^s, \mathbf{Q})$, $s = a, b$. While the general form of the Pareto program remains unchanged, its decentralization is trickier, because the welfare theorems do not apply in an economy with public goods.[10] One solution, which generalizes the previous intuitions, is to use *individual* (or 'Lindahl') prices. It relies on an old idea in public economics, namely that decisions regarding public commodities can be decentralized using agent-specific prices; see, for example, Mas-Colell, Whinston and Green (1995). In a sense, this is part of the standard duality between private and public consumptions. When a good is private, all agents face the same price and choose different quantities; with public goods, they all consume the same quantity but would be willing to pay different marginal prices for it.

A precise statement is the following:

Proposition 4.5 *For any* $(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$*, assume that the consumption vector* $\left(\hat{\mathbf{Q}}, \hat{\mathbf{q}}^a, \hat{\mathbf{q}}^b\right)$ *is efficient. Then there exists a* ρ *and* $2N$ *personal prices* $\mathbf{P}^a = (P_1^a, ..., P_N^a)$ *and* $\mathbf{P}^b = \left(P_1^b, ..., P_N^b\right)$*, with* $P_j^a + P_j^b = P_j$*,* $j = 1, ..., N$ *, such that* $\left(\hat{\mathbf{q}}^a, \hat{\mathbf{Q}}\right)$ *solves:*

$$\max u^a(\mathbf{q}^a, \mathbf{Q})$$
$$\textit{subject to } \mathbf{p}'\mathbf{q}^a + (\mathbf{P}^a)'\mathbf{Q} = \rho \tag{4.54}$$

[10] Private contributions to the public goods are ruled out, since they generate inefficient outcomes (see Chapter 3).

and $\left(\hat{\mathbf{q}}^b, \hat{\mathbf{Q}}\right)$ *solves:*

$$\max u^b\left(\mathbf{q}^b, \mathbf{Q}\right)$$
$$\textit{subject to } \mathbf{p}'\mathbf{q}^b + \left(\mathbf{P}^b\right)'\mathbf{Q} = y - \rho \tag{4.55}$$

Note that both the function ρ and the personal prices $\mathbf{P}^a$ and $\mathbf{P}^b$ will in general depend on $(\mathbf{P}, \mathbf{p}, x, \mathbf{z})$.

These programs correspond to a decentralization of the efficient allocation in the sense that each agent is faced with their own budget constraint, and maximizes their utility accordingly. There is however a clear difference with the private good case, in which all relevant information was readily available to each agent as soon as the sharing rule has been decided upon. Here, individuals need to know not only the 'resources' devoted to them, as described by ρ, but also their personal prices. Computing the personal prices is a difficult task, that is basically equivalent to solving for the efficient allocation; hence the 'decentralization' only obtains in a specific sense.[11]

Still, the Lindahl approach generates interesting insights on the outcome of the model. Assuming an interior solution, the first order conditions of (4.54) give:

$$P_j^a = \frac{\partial u^a / \partial Q_j}{\partial u^a / \partial q_i} p_i \tag{4.56}$$

The right hand side of this equation is often called a's *marginal willingness to pay* (or MWP) for commodity j; indeed, it is the maximum amount a would be willing to pay to acquire an additional unit of public good j, if the amount was to be withdrawn from a's consumption of private good i. Note that this amount does *not* depend on the private good at stake since the marginal utility of any private good divided by its price is equalized across private goods. Intuitively P_j^a increases with a's preference for the public good; the intuition of Lindahl prices is precisely that agents with a higher private valuation of the public good should pay more for it. This is required for an efficient allocation of the family income between alternative uses.

Let us now compare the budget constraint the agent is facing in (4.54) with what the same agent would face if she was a single: $\mathbf{p}'\mathbf{q}^a + \mathbf{P}'\mathbf{Q} = y^a$, where y^a denotes a's income as single. An obvious difference is that the amount of resources has changed - from y^a to ρ; this is similar to the private goods case. However, another difference, which is specific to the public good case, is that *the relative prices of the public commodities have been changed*, from P_j/p_i to P_j^a/p_i. Since $P_j^a + P_j^b = P_j$ and $P_j^b > 0$, we have that $P_j^a < P_j$. Intuitively, the publicness of good j makes it less expensive

[11] The literature on planning has developed several procedures through which information exchanges may lead to the determination of Lindahl prices.

relatively to any private good, precisely because the other spouse will also contribute to the purchase of the public good.

4.5.2 The conditional sharing rule.

An alternative approach relies on the notion of the *conditional sharing rule*. Again, let $\left(\hat{\mathbf{Q}}, \hat{\mathbf{q}}^a, \hat{\mathbf{q}}^b\right)$ denote an efficient consumption vector. The total expenditure of a and b on private goods only are $x^a = \mathbf{p}'\hat{\mathbf{q}}^a$ and $x^b = \mathbf{p}'\hat{\mathbf{q}}^b$. This implies that $x^a + x^b = x - \mathbf{P}'\hat{\mathbf{Q}}$. Then:

Proposition 4.6 *For $s = a, b$, $\hat{\mathbf{q}}^s$ solves:*

$$\max_{\mathbf{q}} u^s \left(\mathbf{q}, \hat{\mathbf{Q}}\right) \text{ subject to } \mathbf{p}'\mathbf{q} = x^s \tag{4.57}$$

Note that, in the two programs above for $s = a, b$, individuals maximize over private consumptions taking public consumptions as given. The value x^a is called the *conditional sharing rule* precisely because its definition is conditional to the level of public expenditures. The proof is clear: if a could, through a different choice of her private consumption bundle, reach a higher utility level while spending the same amount, then the initial allocation had to be inefficient, a contradiction.

Again, the decision process can be interpreted as operating in two phases, although the precise definition of the phases differs from the private good case. Specifically, during the first phase agents determine *both* the level of public expenditures *and* the conditional sharing rule; then comes the *consumption* phase, when agents allocate their conditional share between the various private commodities available. It is important to note that in sharp contrast with the private good case, the existence of a conditional sharing rule is necessary for efficiency, but by no means sufficient. The reason for that is that, in general, efficiency introduces a strong relationship between the level of public expenditures and the conditional sharing rule. Broadly speaking, for any given level of public expenditures, most (actually, almost all) sharing rules would be incompatible with efficiency.

Before analyzing in more detail the first phase, it is useful to define a's indirect conditional utility $\tilde{v}^a$ as the value of program (4.57) above:

$$\begin{aligned} \tilde{v}^a \left(\mathbf{p}, x^a; \mathbf{Q}\right) &= \max_{\mathbf{q}^a} u^a \left(\mathbf{q}^a, \mathbf{Q}\right) \\ \text{subject to } \mathbf{p}'\mathbf{q}^a &= x^a \end{aligned} \tag{4.58}$$

That is, $\tilde{v}^a$ denotes the maximum utility a can ultimately reach given private prices and conditional on the outcomes $(x^a, \mathbf{Q})$ of the first phase decision. We may now consider the first phase, which determines the public consumption, Q, and the disposable income allocated to each spouse,

$\left(x^a, x^b\right)$. Efficiency leads to the following program:

$$\begin{aligned}&\max_{x^a;x^b;\mathbf{Q}} \left\{\mu \tilde{v}^a\left(\mathbf{p}, x^a; \mathbf{Q}\right) + \tilde{v}^b\left(\mathbf{p}, x^b; \mathbf{Q}\right)\right\}\\ &\text{subject to} \quad x^a + x^b + \mathbf{P}'\mathbf{Q} = x \end{aligned} \tag{4.59}$$

The first order conditions give:

$$\begin{aligned}\mu \frac{\partial \tilde{v}^a}{\partial x^a} &= \frac{\partial \tilde{v}^b}{\partial x^b}\\ \frac{\partial \tilde{v}^a/\partial Q_j}{\partial \tilde{v}^a/\partial x^a} + \frac{\partial \tilde{v}^b/\partial Q_j}{\partial \tilde{v}^b/\partial x^b} &= P_j,\ j = 1, ..., N \end{aligned} \tag{4.60}$$

The second set of conditions are often called the Bowen-Lindahl-Samuelson (BLS) conditions. The ratio $\frac{\partial \tilde{v}^a/\partial Q_j}{\partial \tilde{v}^a/\partial x^a}$ is exactly a's willingness to pay for public good j. To see this, note that the first order conditions of (4.57) imply that $\frac{\partial u^a}{\partial q_i^a} = \lambda^a p_i$, where λ^a is the Lagrange multiplier of a's budget constraint; and the envelope theorem applied to the definition of $\tilde{v}^a$ gives that $\frac{\partial \tilde{v}^a}{\partial x^a} = \lambda^a$, hence $\frac{\partial \tilde{v}^a}{\partial x^a} = \frac{1}{p_i}\frac{\partial u^a}{\partial q_i^a}$. Thus the conditions simply state that MWP's (or private prices) must add up to the market price of the public good, as argued above. The BLS conditions (the second set of (4.60)) are necessary and sufficient for efficiency. The choice of a particular allocation on the Pareto frontier is driven by the first condition in (4.60).

As an application, consider the model of collective labor supply proposed by Donni (2007), who assumes individual preferences of the form:

$$U_s(T - h_s, Q),$$

where Q is a Hicksian good which represents public consumption. Under this hypothesis, and taking into account the property of homogeneity, labor supplies can be written as:

$$h_s = h_s\left(\frac{w_s}{\pi_s(y, w_a, w_b)}, \frac{\rho_i(y, w_a, w_b)}{\pi_i(y, w_a, w_b)}\right),$$

where

$$\pi_i(y, w_a, w_b) = \frac{h_i w_i + \rho_i(y, w_a, w_b)}{y + h_a w_a + h_b w_b}$$

denotes member i's Lindahl price for the public good. In this context, Donni shows that the utility functions are identified, up to a positive transformation, from individual labor supplies.

4.5.3 Application: labor supply, female empowerment and expenditures on public good

While the previous concepts may seem somewhat esoteric, they have important practical applications. For instance, a widely discussed issue in development economics and welfare policy in general is the impact of intrahousehold redistribution on the structure of household consumption, and in particular on household demand for public goods. The notion of 'public goods' should be understood here in a very general sense - any expenditure that benefits both partners. A typical and normatively important example is expenditures on children, at least if we assume that both parents care about the well being of their children. The crucial question, then, is the following: if a given policy 'empowers' women, in the sense that it increases their weight in the household decision process, what will be the impact on household expenditures on children? For instance, by paying a given benefit to the wife instead of the husband, can we expect children health or education to be improved? A large and growing body of empirical evidence suggests that such redistributive effects do exist and can actually be quite large, at least in some countries. As an instance, Duflo (2003), studying elderly benefits in South Africa, concludes that the same transfer has drastically different impact of the health of female grandchildren depending on whether it is paid to the grandmother or the grandfather.

The collective framework provides a very adequate framework for studying these effects. The basic intuition is that while the amount received has a direct impact on the household's budget constraint, the gender of the recipient does not. It can only affect the respective Pareto weights; as such, it is a perfect example of a *distribution factor*. We therefore want to investigate the impact of distribution factors (or equivalently of exogenous changes in the Pareto weights) on household demand. Two questions are of particular interest. First, is it possible to predict, from the knowledge of preferences, which public consumptions will increase when the wife's weight raises? Second, is it always the case that female empowerment also results in more spending on the wife's *private* consumption - or could it be the case that she puts so much emphasis on public consumption that her private consumption actually declines when she has more power?

To investigate these issues, we start with a very simple example. Assume individual preferences are Cobb-Douglas:

$$U^s(x^s, Q) = \sum_k \alpha_k^s \log q_k^s + \sum_j \delta_j^s \log Q_j \tag{4.61}$$

where the coefficients are positive and normalized by $\sum_k \alpha_k^s + \sum_j \delta_j^s = 1$. As above, let μ denote a's Pareto weight. Prices are normalized to 1, so that the budget constraint is simply

$$\sum_k \left(q_k^a + q_k^b\right) + \sum_j Q_j = x$$

Straightforward computations give household demands:

$$\begin{aligned} q_k^a &= \frac{\mu\alpha_k^a}{1+\mu}x \\ q_k^b &= \frac{\alpha_k^b}{1+\mu}x \\ Q_j &= \frac{\mu\delta_j^a+\delta_j^b}{1+\mu}x \end{aligned}$$

and the following conclusions follow:

1. The private consumptions of a are all increasing in μ
2. The private consumptions of b are all decreasing in μ
3. Since
$$\frac{\partial Q_j}{\partial \mu} = \frac{\delta_j^a-\delta_j^b}{(\mu+1)^2}x$$
household consumption in public commodity j increases if and only if a 'cares more' about that commodity than b does, in the sense that $\delta_j^a > \delta_j^b$.

As above, it is natural to interpret these results in terms of marginal willingness to pay. These are given for any public good j by:

$$MWP_j^s = \delta_j^s \frac{x^s}{Q_j}, \ \ s = a, b$$

where $x^s = \sum_k q_k^s$ is the conditional sharing rule of member s. Interestingly enough, the condition $\delta_j^a > \delta_j^b$ is *not* equivalent to her MWP being larger than his; rather, it implies that

$$\frac{\partial MWP_j^a}{\partial x^b} > \frac{\partial MWP_j^b}{\partial x^a} \tag{4.62}$$

In words, the MWP of a must be more *income sensitive* than that of b. Still, it may be the case that $MWP^a < MWP^b$ (particularly if x^b is large with respect to x^a): the absolute magnitude of the respective MWP plays no role in the result.

The interpretation of these findings is quite intuitive. First, one may think of the wife's empowerment (as resulting from an increase in μ) in purely economic terms: she now receives a higher fraction of household resources. With Cobb-Douglas preferences, all commodities are normal, therefore more income always results in more consumption for her; conversely, his share has been reduced and he consumes less. Regarding public goods, however, things are more complex, because a transfer from the husband to the wife typically increases her MWP for each public good but reduces his. The question, here, is whether her increase is sufficient to compensate his reduction - which is exactly what is implied by equation (4.62). If the condition is satisfied, the impact of the change over total MWP for the public good is positive, and consumption grows; in the opposite situation, it is reduced.

The previous results, natural as they sound, are still dependent on the very specific functional form chosen for utilities. Whether they extend to non homothetic preferences, for instance, is not clear. In full generality, the comparative statics of the model just described are somewhat complex, if only because, unlike the Cobb-Douglas case, the MWP for a particular commodity depends in an *a priori* arbitrary way on the quantities of the other public goods. However, a clearer picture obtains when there is only one public good, a case considered by Blundell, Chiappori and Meghir (2005). They show that if preferences are such that both private expenditures and the public good are normal (in the usual sense that an increase in income would raise the corresponding, *individual* demands for these goods), then a marginal improvement in a member's Pareto weight increases the household's expenditures on the public good if and only if the marginal willingness to pay of this member is *more sensitive to changes in his/her share* than that of the other member. Again, it is not the magnitude of the MWP's that matters, but their income sensitivity. Moreover, the private consumptions of the beneficiary member are always increased.

Coming back to the initial motivation, consider the model discussed in Chapter 3 in which children's well being is modeled as a public good that enters the parents' utility. Assume that some policy measure may increase the relative weight of the wife within the household. It is often argued that children should benefit from such a change, the (somewhat hazy) intuition being that 'mothers care more about children than do fathers'. What is the exact meaning of such a statement, and what exactly does it assume about preferences? The answer is given by the previous result. She 'cares more' means, in this context, that her MWP for children is more income-sensitive: should she receive an additional dollar to be spent either on children or on her private consumption, she would spend a larger fraction of it on children than her husband would.

4.6 Household production in the collective model

Becker (1965) put forward a generalized approach of consumption and time use in which final consumption is produced within the household by intermediate goods purchased in the market and personal time withdrawn from market work. Although house production is important for singles, it is particular relevant for married (or cohabiting) couples. Household production generates several of the gains from marriage that we mentioned in chapter

3, including increasing returns, specialization and sharing (home produced) public goods. At a global level, household production represents, according to several estimates, up to 20% of total production in developed countries, although it is usually not explicitly taken into account in aggregate measures such as GDP, and much more in developing economies. At the household level, domestic production represents a significant fraction of resources (and especially of time) used and consumed. Finally, at an individual level, utility depends on leisure, which can be defined as time not spent working either at home or on the market (although such a definition raises delicate problems) and also on the consumption of internally produced commodity.

The analysis of household production raises several important issues. One is the choice of the commodities produced at home and their quantity. In many cases, a trade-off exists between home production and market trade. For instance, I can clean my apartment or hire a cleaning person; and in the opposite direction, the vegetables I grow in my garden can be consumed internally by my family or sold on the market.[12] The commodity is then said to be *marketable*. Alternatively, some commodities have to be at least partly internally 'produced'; for instance, a nanny cannot, in many cases, be a perfect substitute for parental care. Another issue is whether and how these decisions depend on the partners' respective 'powers'. Is it the case, for instance, that the allocation of work by each spouses to the domestic production process reflects the bargaining positions of the spouses - or is it exclusively determined by the production technology?

Finally, these issues must be analyzed in an equilibrium context, in which many key factors have drastically evolved over time. In particular, the division of labor within households has changed as married women have dramatically increased their labor force participation. Becker's framework allows one to conceptualize the distinct roles of technological advance in home production and in industrial production in explaining the observed changes in allocation of time. There is extensive research that applies the household production approach and tries to sort out the roles of technological advance and changes in norms that have made this revolution possible (Greenwood *et al.*, 2005, Fernandez, 2007). Mulligan and Rubinstein (2007) emphasize the role of higher rewards for ability (reflected in the general increase in wage inequality) in drawing married women of high ability into the labor market. See also Albanesi and Olivetti (2009), who emphasize the role of medical progress in child feeding that enabled women to stay out of home.[13]

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[12] This issue is particularly important in development economics, since a majority of the population of a developing economy typically work in agriculture, often producing marketable commodities at the household level.

[13] Another application is De Vries (1994, 2008) who applied this framework to identify an "industrious revolution", characterized by an increased production of marketable goods within households, which "preceded and prepared the way for the Industrial Rev-

Another crucial determinant of the time spent on household production is its opportunity cost, which is directly related to the wage the person could receive by working on the market. Over the last decades, a striking phenomenon is the global increase in female education, an evolution that has deeply modified the trade-off between domestic and market work by raising female market wages. Of course, education is not exogenous; it is the outcome of an investment decision based on future (expected) returns, therefore on (among other things) the fraction of time that individuals expect to spend working on the market. In other words, education and current wages affect current decisions regarding household production, but are themselves the outcomes of past expectations about future domestic work. The general equilibrium aspects will be left for the second part of the book; here, we concentrate on a providing a conceptual framework for analyzing the respective impacts of wages, technology and powers on domestic production.

4.6.1 The basic model

We have already discussed home production in section 3.2 in chapter 3; here we focus on the novel aspects that arise in a collective model. Let $\mathbf{c}^s$ denote the vector of private consumption of the home produced commodity by s and let $\mathbf{C}$ denote public home produced goods. For the time being, we ignore time inputs and let $\mathbf{q}$ denote the purchases of market goods that are used in home production. Assuming for the moment that household commodities are not marketable, the Pareto program thus becomes

$$\max \mu U^a\left(\mathbf{C},\mathbf{c}^a,\mathbf{c}^b\right)+U^b\left(\mathbf{C},\mathbf{c}^a,\mathbf{c}^b\right) \tag{4.63}$$

subject to

$$\begin{aligned}\mathbf{F}\left(\mathbf{C},\mathbf{c}^a+\mathbf{c}^b,\mathbf{q}\right) &= 0\\ \mathbf{p}'\mathbf{q} &= x\end{aligned} \tag{4.64}$$

where $\mathbf{F}$ is the production function. As above, what is observed is the the household's demand function $\mathbf{q}=\hat{\mathbf{q}}\left(\mathbf{p},x,\mathbf{z}\right)$. Note that the model implicitly assumes that *all* commodities are input for household production. This is without loss of generality: if commodity i is directly consumed, the corresponding row of the production equation simple reads $c_i^a+c_i^b=q_i$ for private consumption, or $C_i=q_i$ if the consumption if public.

When compared with the household production model in the unitary framework, (4.63) exhibits some original features. For instance, the outcome of the intrahousehold production process can be consumed either privately or publicly; the two situations will lead to different conclusions, in particular

olution".

in terms of identification. On the other hand, two main issues - whether the goods produced within the household are marketable or not, and whether the output is observable - remain largely similar between the collective and the unitary frameworks.

4.6.2 Domestic production and time use

Of particular interest are the various versions of the collective model with production involving labor supply. For simplicity, we present one version of the model, initially analyzed by Apps and Rees (1997) and Chiappori (1997), in which the two partners supply labor and consume two *private* consumption goods, one (denoted q and taken as numeraire) purchased on a market and the other (denoted c) produced domestically, according to some concave function $F\left(t^a,t^b\right)$, where t^s is member s's household work.[14] Market and domestic labor supplies for person s, h^s and t^s, are assumed observed as functions of wages w^a, w^b, non-labor income y and a distribution factor z. For simplicity, we ignore the tax system and assume that budget sets are linear;[15] similarly, we exclude joint production.[16] Finally, we assume that preferences are 'egoistic', so that s's are represented by $U^s(q^s,c^s,l^s)$, where l^s denotes leisure and total time is normalized to unity so that

$$l^s+t^s+h^s=1 \text{ for } s=a,b \tag{4.65}$$

When the domestic good is not marketable, the previous model therefore becomes:

$$\max \mu U^a\left(q^a,c^a,l^a\right)+U^b\left(q^b,c^b,l^b\right) \tag{4.66}$$

subject to

$$\begin{aligned} c^a+c^b &= F\left(t^a,t^b\right) && (4.67)\\ q^a+q^b &= y+w_ah^a+w_bh^b && (4.68)\end{aligned}$$

and the time constraint (4.65).[17] Conversely, if the commodity is marketable - that is, if good c can be bought and sold on a market, we let c^M denote the quantity sold (or bought if negative) on the market and p its market price, which the household takes as given. Then total production of the good is $c=c^a+c^b+c^M$; if $c^M>0$ then the household produces more than it consumes (c^a+c^b) and sell the difference, if $c^M<0$ the household

[14] The model can easily be generalized by adding other inputs to the production process; the main conclusions below do not change.

[15] For a comprehensive analysis of taxation with household production, the reader is referred to Apps and Rees (2009).

[16] See Pollak and Wachter (1975), and Apps and Rees (2009) for a general presentation.

[17] Note that utility depends only on consumption and leisure and that, by assumption, time spent at work either at home or in the market do not enter utility directly.

produces only a fraction of the amount it consumes and purchases the rest. The production equation is now:

$$c^a + c^b + c^M = F\left(t^a, t^b\right)$$

and the budget constraint at the household level becomes:

$$q^a + q^b = w^a h^a + w^b h^b + y + pc^M. \tag{4.69}$$

In our analysis of household production models, we shall first consider the benchmark situation in which both spouses are working outside the family, and their working time is flexible enough to allow for marginal variations. Then the opportunity cost of a person's time is determined by the person's wage, which is taken as given for the family decision process. We later consider 'corner' solutions, in which one spouse works exclusively at home.

Marketable production

Cost minimization

Let us first assume that good c is marketable. In this context, efficiency has an immediate implication, namely profit maximization. Specifically, t^a and t^b must solve:

$$\max_{(t^a,t^b)} pF\left(t^a, t^b\right) - w^a t^a - w^b t^b \tag{4.70}$$

implying the first order conditions:

$$\frac{\partial F}{\partial t^s}\left(t^a, t^b\right) = \frac{w_s}{p}, \quad s = a, b \tag{4.71}$$

The economic interpretation of these equations is clear. The opportunity cost of an additional unit of time spent on domestic production is the person's wage. If this is not equated to the marginal productivity of domestic labor, efficiency is violated. For instance, if this marginal productivity is smaller than the wage, then the person should spend less time working at home and more working for a wage, keeping total leisure constant. Intra-household production would decline, but household income would increase by more than the amount needed to purchase the missing production on the relevant market. To put it differently, the condition reflects cost minimization; if it not satisfied, then the household could achieve the same level of leisure and domestic consumption while saving money that could be used to purchase more of the consumption goods - clearly an inefficient outcome.

The same argument can be presented in a more formal way. Consider the household as a small economy, defined by preferences u^a and u^b and by *two* 'production' constraints - namely, the production of the household good (here $c = F\left(t^a, t^b\right)$) *and* the budget constraint. By the second welfare theorem, any Pareto efficient allocation can be decentralized as a market equilibrium. On the production side, the second constraint (the budget constraint) implies that the *intrahousehold* prices of the consumption goods q and c and the leisures l^a and l^b are proportional to $\left(1, p, w^a, w^b\right)$; we can normalize the proportionality factor to be one, and keep $\left(1, p, w^a, w^b\right)$ as intrahousehold prices as well. Then market equilibrium requires profit maximization, which does not depend on individual preferences. This is the well-known separation principle, according to which the production side is fully determined by profit maximization, irrespective of individual preferences.

Choosing domestic work

The first order conditions of the profit maximization program give

$$\frac{\partial F}{\partial t^s}\left(t^a, t^b\right) = \frac{w^s}{p}, \quad s = a, b \tag{4.72}$$

If F is strictly concave (that is, if the domestic technology exhibits decreasing returns to scale), these relations can be inverted to give:

$$t^s = f^s\left(\frac{w^a}{p}, \frac{w^b}{p}\right), \quad s = a, b \tag{4.73}$$

Knowing the $f^s(.)$ functions is strictly equivalent to knowing F. The relationships (4.73) can in principle be econometrically estimated, leading to a complete characterization of the production side. It is important to note that, in this logic, *the time spent by each spouse on domestic production is totally determined by 'technological' consideration*: it depends only on wages and on the household production function F, but neither on preferences nor on 'powers' (as measured by Pareto weights). The model predicts, for instance, that when a change in a distribution factor redistributes power in favor of the wife (say, a benefit that used to be paid to the husband is now paid to the wife), the result will be a different consumption pattern (as discussed above, the household now consumes more of the commodities preferred by the wife), but the times spent on domestic production by the husband and the wife remain unchanged. On the contrary, an exogenous increase in female wage reduces her domestic labor; the impact on his domestic work then depends on the domestic production technology (that is, are male and female housework complements or substitutes?).

It should be stressed that the marketability assumption is demanding. Strictly speaking, it requires that households can freely buy *or sell* the domestic good. Selling the domestic good is natural in some contexts (for example, agricultural production in developing countries), but less so in others (many people clean their own house but would not think of selling their cleaning services to a third party). If domestic goods can only be purchased but not sold, our analysis still applies whenever wages and technology are such that they always consume more than what they produce - that is, the household as a positive net demand of the domestic good. However, some households may reach a corner solution, in which the market purchase of domestic goods is nil, and the normalized marginal productivity of a person's domestic work exceeds the person's wage. In practice, this is equivalent to the domestic good not being marketable, a case we consider below.

Finally, the model above assumes that all forms of labor are equally costly - that is, that the subjective disutility of one hour of labor is the same, whether it is spent working in a factory or taking care of children. This assumption, however, can readily be relaxed. One may posit, for instance, that for some activities (say domestic work), one hour of work 'costs' to spouse s only a fraction α^s of one hour of leisure (intuitively, the remaining fraction $(1-\alpha^s)$ *is* leisure). Under this extension, the time constraint (4.65) should be replaced with:

$$l^s + \alpha^s t^s + h^s = 1 \text{ for } s = a, b \tag{4.74}$$

and the first order conditions become:

$$\frac{\partial F}{\partial t^s}\left(t^a, t^b\right) = \alpha^s \frac{w_s}{p}, \quad s = a, b \tag{4.75}$$

In words, the opportunity cost of domestic work should be adjusted for the associated amenities. Note, however, that the same logic applies; that is, the time spent by each spouse on domestic production is fully determined by wages, technology and the individual preferences captured here by amenity parameter α^s. However, they do *not* depend on the power of the spouses as measured by μ.

The demand side

The separability principle implies that the demand side is totally divorced from production. Indeed, the household's total 'potential' income is

$$Y = w^a\left(1 - t^a\right) + w^b\left(1 - t^b\right) + y + pc \tag{4.76}$$

This potential income has to be split between the members and spent on individual leisures and consumptions of the two goods. Since all commodities are private, efficiency is equivalent to the existence of a sharing rule. As above, thus, there exists two functions $\rho^a\left(w^a, w^b, y, p\right)$ and $\rho^b\left(w^a, w^b, y, p\right)$, with $\rho^a + \rho^b = Y$, such that each member s solves:

$$\max U^s\left(q^s, c^s, l^s\right)$$

under the member-specific budget constraint

$$q^s + pc^s + w^s l^s = \rho^s$$

At this stage, we are back to the standard collective model of labor supply.

Non-marketable production

The other polar case obtains when no market for the domestic good exists (then $c^M = 0$). Then we are back to maximizing (4.66) under the constraints (4.67), (4.68) and (4.65). One can still define a price p for the domestic good, equal to the marginal rate of substitution between the domestic and the market goods for each of the members (the MRS's are equalized across members as a consequence of the efficiency assumption). The difference, however, is that p is now *endogenous* to the model - that is, it is determined by the maximization program.

A particularly interesting case obtains when the domestic production function exhibits constant returns to scale (CRS). Then:

$$F\left(t^a, t^b\right) = t^b \Phi\left(\frac{t^a}{t^b}\right) \tag{4.77}$$

for some function $\Phi(.)$. First order conditions imply that:

$$\frac{\partial F/\partial t^a}{\partial F/\partial t^b} = \frac{w^a}{w^b},$$

which give in this case:

$$\Phi\left(\frac{t^a}{t^b}\right) - \frac{t^a}{t^b}\Phi'\left(\frac{t^a}{t^b}\right) = \frac{w^a}{w^b},$$

This relationship, which is a direct consequence of the efficiency assumption, pins down the ratio $\frac{t^a}{t^b}$ to be some function $\phi\left(\frac{w^a}{p}\right)$. In other words, it is now the case that *the ratio of male to female domestic work depends only on wages and household production technology* - a natural consequence of cost minimization. On the other hand, the *scale* of production - that is, the quantity eventually produced - is indeterminate from the production perspective; it depends on preferences and the decision process. We conclude that preferences and powers determine the total quantity of household goods produced; however, *conditional on that quantity*, the particular combination of male and female time is determined by respective wages and the production technology, and does not depend on preferences or powers.[18]

The (household-specific) price of the domestic good can readily be recovered. Indeed, an interior solution under constant returns require zero profits, therefore it must be the case that:

$$p = \frac{w^a\left(t^a/t^b\right) + w^b}{\Phi\left(t^a/t^b\right)}$$

[18] Pollak and Wachter (1975) discuss the roles of constant returns to scale and joint production. They show that with joint production (i.e., activities that generate more than one final good), it is generally impossible to separate household technology from preferences, even under a constant return to scale technology.

Again, this price depends only on wages and on the technology. It is household specific in the sense that two households with different wages will price the household good differently, even if they have access to the same domestic technology. However, for given wages and domestic technology, it depends neither on preferences nor on respective powers. Finally, the separation result still holds. That is, each member's decision can be modeled as if they were maximizing their own utility under the member specific budget constraint defined by a sharing rule; this mechanism determines all the components of consumption, including The only difference with the marketable case is that p is no longer a market price; instead, it is determined by the wages and the technological constraints.

Power and domestic work

While the previous conclusions are not really surprising, at least from a general equilibrium perspective - they basically illustrate standard results in welfare economics - their implications can be somewhat unexpected. Consider, for instance, a change in Pareto weights that benefits women - say, through the impact of a distribution factor - while wages and incomes are unaffected. As discussed in the previous subsection, a first consequence is that the structure of consumption will change; intuitively, the household will now consume more of the commodities that the wife 'likes more'. If, as it is often argued, women generally care more about the goods that are domestically produced (child care being a primary example), the total consumption of these commodities should increase. If the commodity is marketable and initially (partly) purchased on the market, the result will be higher market purchases of these goods, with no impact on domestic labor by the partners. In all other cases, domestic labor will increase, and the distribution of the additional effort between spouses is completely driven by the technology. For instance, under a standard, Cobb-Douglas production function, inputs are complements; at constant prices (here wages), more production requires increasing both inputs. We conclude that more power to the wife may actually imply more domestic work for *both* spouses. Note, however, that because a transfer of income to the wife does not affect her time input into home production, the income effect will induce her to reallocate the remaining time so that her market work should decline and her leisure increase. This conclusion should be contrasted with the impact of an increase in the wife's market wage, which *always* affect her domestic labor supply. When the commodity is marketable, her domestic work is always reduced. In the alternative situation, her domestic work decreases *with respect to her husband's*, but the absolute impact also depends on the structure of consumption - especially if her Pareto weight is boosted by her higher wage.

Extensions

Public goods

In the previous analysis, the internally produced commodity was privately consumed. What if, instead, the commodity is public within the household - as it is the case for childcare, for instance? Interestingly, not much is changed, because the separation principle still applies. If the commodity, although public within the household, is marketable, then its production is driven by profit maximization; the only change regards the demand side, where the decision process can no longer be decentralized using a sharing rule. Even in the non marketable case, the logic of cost minimization prevails. In particular, under constant returns to scale, it still the case that the level of production is determined by preferences and the decision process, while for any given level the time allocation of domestic work between spouses stems from technological considerations.[19]

Specialization

Another special (but empirically relevant) case obtains when one of the spouses - say b - does not enter the labor market, and specializes instead in home production. This happens when, for the chosen allocation of time and consumption, b's potential wage, $\bar{w}^b$, is smaller than both b's marginal productivity in household production and b's marginal rate of substitution between leisure and consumption. In words: the marginal hour can indifferently be spent in leisure or household production, and both uses dominate market work.[20]

The situation, here, is more complex, because the opportunity cost of labor for b is no longer exogenously given; instead, it is now endogenous to the program. Still, if we keep the assumption of constant return to scale domestic technology, some of the previous conclusions remain valid. Indeed, in the marketable case, efficiency in a's allocation of time still requires that:

$$\frac{\partial F}{\partial t^a}\left(t^a, t^b\right) = \frac{w^a}{p}$$

while the CRS condition (4.77) implies that

$$\frac{\partial F}{\partial t^a}\left(t^a, t^b\right) = \Phi'\left(\frac{t^a}{t^b}\right)$$

It follows that, again, the ratio t^a/t^b is pinned down by technological con-

[19] The reader is referred to Blundell, Chiappori and Meghir (2005) for a more detailed investigation.

[20] Technically, this result is true at the marginal level only in the absence of non convexities. In the presence of fixed costs of work or constraints on the number of hours worked, the same constraint must be redefined at a more global level.

straints - namely, it must be such that

$$\Phi'\left(\frac{t^a}{t^b}\right) = \frac{w^a}{p}$$

In words: the volume of domestic production is now determined by preferences, but the distribution (between spouses) of effort needed to produce that amount is fixed by the technology.

Finally, in the case of specialization into the production of a non marketable good, both the price of the domestic good and b's opportunity cost of labor are endogenous. Then all aspects of household production are potentially affected by the distribution of powers within the couple.

4.6.3 Empirical issues

To what extent can the previous analysis generate testable restrictions? Note first that, as discussed in section 2 of chapter 3, when the outcome is observable, efficiency can directly be tested empirically. Indeed, a straightforward implication of efficiency is cost minimization: whatever the value of the output, it cannot be the case that the same value of output could be produced with a cheaper input combination. Udry (1996) provides a test of this sort on data from Burkina-Faso. Also, it is in general possible to directly estimate the production function; then one can refer to the standard, collective setting, using the methods presented above. Usually, however, the output of the intrahousehold production process is not observable. Still, some of the techniques described for models without home production can be extended to the case of production. For instance, distribution factor proportionality should still hold in that case; the basic intuition (distribution factors matter only through the one-dimensional Pareto weight μ) remains perfectly valid in Program (4.63). The same is true for the various versions of the SNR conditions, with and without distribution factors, which rely on the same ideas.

Moreover, if time use data are available, then the previous models generate several, testable restrictions regarding the impact of wages, income and power on domestic production. If we consider the benchmark case of CRS technology, the basic prediction is that the *proportion* of total domestic time spent by each member only depends on wages and the technology. Therefore any variable that does not affect the production side of the household (but only, say, preferences or the decision process) should not be relevant for the determination of the ratio t^a/t^b. On the other hand, changes in wages do affect the ratio; as expected, a (proportionally) higher female wage reduces the ratio of her domestic work to his.

Regarding identification, note first that if the internally produced commodity is marketable (as will often be the case for, say, agricultural production in developing countries), then conditions (4.73) above can in principle be econometrically estimated, leading to a complete characterization of the production side. In the opposite case, however, the separability property no longer applies; the price p has to be estimated as well. As discussed by Chiappori (1997), identifiability does not obtain in general; however, it can still be achieved under additional assumptions.

Finally, a much stronger result obtains when the produced good is publicly consumed. Blundell, Chiappori and Meghir (2005) consider a model which is formally similar to the previous one, except that the second commodity is public and its production requires labor and some specific input, Q. Technically, individual utilities take the form $u^s(q^s, C, l^s)$, and the production constraint is $C = F\left(Q, t^a, t^b\right)$. A natural (but not exclusive) interpretation of C is in terms of children's welfare, which enters both utilities and is 'produced' from parental time and children expenditures Q. Blundell, Chiappori and Meghir show that strong testable restrictions are generated. Moreover, the structure (that is, utilities and the Pareto weights) are identifiable from labor supplies (both domestic and on the market) and children's expenditures, provided that one distribution factor (at least) is available.

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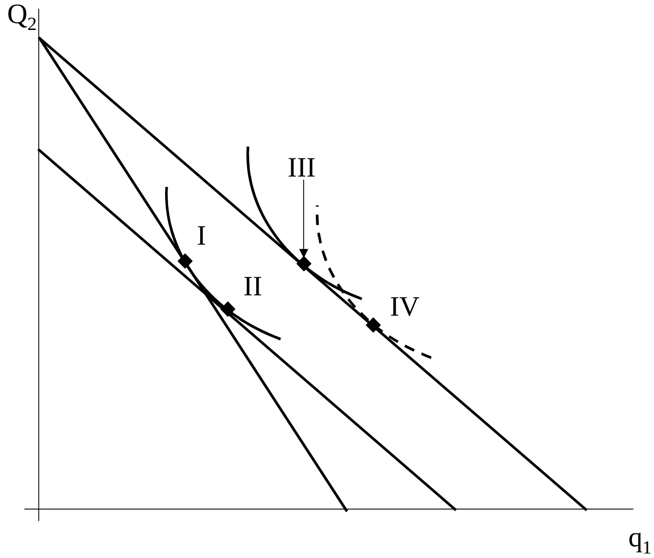

FIGURE 4.1. Collective price responses

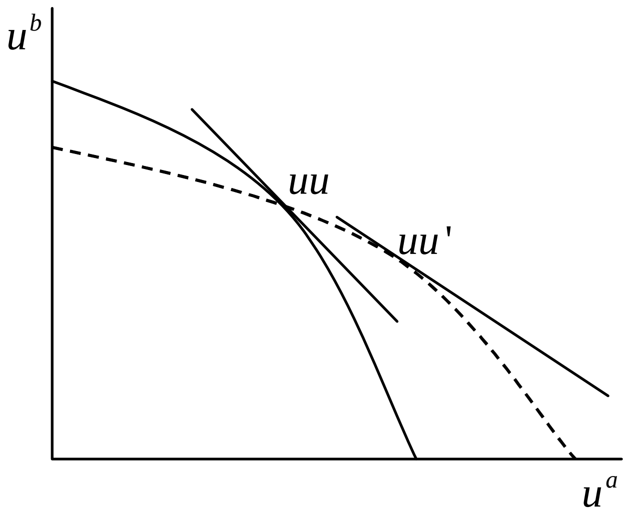

FIGURE 4.2. A potentially compensating variation.

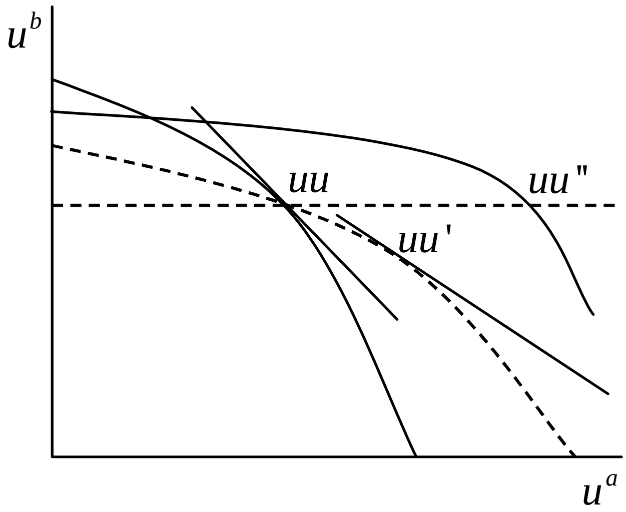

FIGURE 4.3. An actually compensating variation.

5

Empirical issues for the collective model

5.1 What are the objects of interest?

We have seen above that various approaches can be used to describe household behavior, from the unitary setting to noncooperative approaches and the collective model. Ultimately, the choice between these various frameworks will rely on particular considerations. First, general methodological principles may favor one approach over the others. For instance, one can argue that the unitary framework is not totally faithful to methodological individualism, a cornerstone of micro theory that postulates that individuals, not groups, are the ultimate decision makers. A second requirement is the model's ability to generate testable predictions for observable behavior, that can be taken to data using standard techniques. Standard consumer theory fares pretty well in this respect. Utility maximization under a linear budget constraint yields strong predictions (adding-up, homogeneity, Slutsky symmetry and negative semidefiniteness and income pooling) and adequate methodologies have been developed for testing these properties. Finally, a crucial criterion is the fruitfulness of the approach, particularly in terms of normative analysis and policy recommendations. A remarkable feature of standard consumer theory is that individual preferences can be uniquely recovered from demand functions (if these satisfy the Slutsky conditions); it is therefore possible to analyze welfare issues from the sole knowledge of observed behavior. This is a particular case of the general requirement that the model be *identifiable*, that is, that it should be possible to recover the underlying structure from observed behavior.

The first line of argument, concerning methodological individualism, has been evoked earlier. In this chapter, we concentrate on the remaining two aspects, namely testability and identifiability of preferences and processes from observed behavior. Most of the existing knowledge for non-unitary models concerns the cooperative framework, and especially the collective model. The testability requirement, *per se*, is not problematic. The idea that a model should generate predictions that can be taken to data belongs to the foundations of economics (or any other science!). Identifiability is more complex and it is useful to define more precisely what is meant by 'recovering the underlying structure'. The structure, in our case, is the (strictly convex) preferences of individuals in the group and the decision process. In the collective setting, because of the efficiency assumption, the decision process is fully summarized (for any particular cardinalization of individual utilities) by the Pareto weight corresponding to the outcome at stake. The structure thus consists of a set of individual preferences (with a particular cardinalization) and a Pareto weight - which, as we should remember, can be (and generally is) a function of prices, incomes and distribution factors.

The structure cannot be directly observed; instead we observe the outcomes of the interactions between preferences, constraints and the decision process. Often we observe only aggregate outcomes and not individual outcomes. In addition, the 'observation' of, say, a demand function is a complex process, that entails specific difficulties. For instance, one never observes a (continuous) function, but only a finite number of values on the function's graph. These values are measured with some errors, which raises problems of statistical inference. In some cases, the data are cross-sectional, in the sense that different groups are observed in different situations; specific assumptions have to be made on the nature and the form of (observed and unobserved) heterogeneity between the groups. Even when the same group is observed in different contexts (panel data), other assumptions are needed on the dynamics of the situation - for example, on the way past behavior influences present choices. All these issues lay at the core of what is usually called the *inference* problem.[1]

A second and different aspect relates to what has been called the *identifiability* problem, which can be defined as follows: when is it the case that the *(hypothetically) perfect knowledge* of a smooth demand function uniquely defines the underlying structure within a given class? This abstracts from the econometrician's inability to exactly recover the form of demand functions - say, because only noisy estimates of the parameters can be obtained, or even because the functional form itself (and the stochastic structure added to it) have been arbitrarily chosen. These econometric questions have, at least to some extent, econometric or statistical answers. For instance, confidence intervals can be computed for the parameters (and become negligible when the sample size grows); the relevance of the functional form can be checked using specification tests; etc. The non-identifiability problem has a different nature: even if a *perfect* fit to *ideal* data was feasible, it might still be impossible to recover the underlying structure from this ideal data.

In the case of *individual* behavior, as analyzed by standard consumer theory, identifiability is an old but crucial result. Indeed, it has been known for more than a century that an individual demand function uniquely identifies the underlying preferences. Familiar as this property may have become, it remains one of the strongest results in microeconomic theory. It implies, for

[1] In the original Koopmans discussion of identification, the step from sample information to inferences about population objects (such as demand functions) is referred to as identification. Here we follow modern terminology and refer to it as the inference step.

instance, that assessments about individual well-being can unambiguously be made based only on the observation of demand behavior with sufficiently rich (and 'exogenous') variation in prices and total expenditures; a fact that opens the way to all of applied welfare economics. It is thus natural to ask whether this classical identifiability property can be extended to more general approaches.[2]

Finally, it should be remembered that identifiability is only a necessary condition for identification. If different structures are observationally equivalent, there is no hope that observed behavior will help to distinguish between them; only ad hoc functional form restrictions can do that. Since observationally equivalent models may have very different welfare implications, non-identifiability severely limits our ability to formulate reliable normative judgments: any normative recommendation based on a particular structural model is unreliable, since it is ultimately based on the purely arbitrary choice of one underlying structural model among many. Still, whether an *identifiable* model is econometrically *identified* depends on the stochastic structure representing the various statistical issues (measurement errors, unobserved heterogeneity,...) discussed above. After all, the abundant empirical literature on consumer behavior, while dealing with a model that is always identifiable, has convinced us that identification crucially depends on the nature of available data.

The main properties of the collective model have been described in the previous chapter. However, which empirical test can actually be performed obviously depends on the nature of available data. Three different contexts can be distinguished. In the first context, individual demand can be estimated as a function of income and possibly distribution factors; this approach is relevant when no price variation is observed, for instance because data are cross-sectional and prices are constant over the sample. We then allow that we also observe price variation so that we can estimate a complete demand system. The analysis of labor supply raises specific issues that are considered in the third section. The final half of this chapter presents a review of empirical analysis using non-unitary models (including the results of applying the tests of the first half of the chapter). We conclude the chapter with an account of intra-household allocation based on two Danish data sets that were specifically designed to address research issues concerning intrahousehold allocation.

[2] Note, however, that only one utility function is identifiable in the standard case. In a 'unitary' framework in which agents are characterized by their own utility function (see chapter 3, subsection 3.5.9) but the household behaves as a single decision unit, it is typically *not* possible to identify the individual utility functions.

5.2 Data without price variation.

5.2.1 Necessary and sufficient conditions for a collective model.

In this section we consider testing and identification in the absence of price variation as is often the case with cross-sectional data. We begin with the case in which we observe only household (aggregate) demand of each good. Let x denote the household's total expenditures and let $\mathbf{z}$ be a K-vector of distribution factors. Recall that distribution factors, by definition, influence neither preferences nor the budget constraint. In a unitary setting, they have no impact on demand. In the collective framework, on the contrary, household behavior can be described by a program of the following form:

$$\begin{aligned} &\max \mu(x, \mathbf{z})\, u^a(\mathbf{g}) + u^b(\mathbf{g}) \\ &\text{subject to } \mathbf{e}'\mathbf{g} \leq x \end{aligned} \tag{5.1}$$

where $\mathbf{g}$ is the vector $\left(\mathbf{q}^a, \mathbf{q}^b, \mathbf{Q}\right)$ and quantities are normalized so that the price vector is a vector of ones, $\mathbf{e}$. The resulting vector of *collective demand functions* can be written $\mathbf{g} = \tilde{g}(x, \mu(x, \mathbf{z}))$ with a corresponding observable demand functions $\hat{\mathbf{g}}(x, \mathbf{z})$.

An alternative demand formulation which is useful for empirical work (see below) can be formulated if there is at least one good (good j, say) that is strictly monotone in one distribution factor (z_1, say); that is, $g_j(x, \mathbf{z})$ is strictly monotone in z_1. This demand function can be inverted on the first factor to give:

$$z_1 = \zeta(x, \mathbf{z}_{-1}, g_j)$$

where $\mathbf{z}_{-1}$ is the vector of distribution factors without the first element. Now substitute this into the demand for good i:

$$g_i = \hat{g}_i(x, z_1, \mathbf{z}_{-1}) = \hat{g}_i[x, \zeta(x, \mathbf{z}_{-1}, g_j), \mathbf{z}_{-1}] = \theta_i^j(x, \mathbf{z}_{-1}, g_j)\,.$$

Thus the demand for good i can be written as a function of total expenditure, all distribution factors but the first and the demand for good j. To distinguish this conditioning from the more conventional conditional demands used in the demand literature, we shall refer to them as *z-conditional* demands.[3]

We now address the issue of what restrictions a collective model imposes upon observable demands. Bourguignon, Browning and Chiappori (2009) provide a complete characterization of these conditions. Specifically they prove that the following equivalent conditions are necessary consequences of the collective model:

[3] In the unitary setting, distribution factors cannot influence demand, so that z-conditional demands are not defined in this case.

1. there exist real valued functions $\tilde{g}_1, ..., \tilde{g}_n$ and μ such that :

$$\hat{g}_i(x, \mathbf{z}) = \tilde{g}_i[x, \mu(x, \mathbf{z})] \qquad \forall i = 1, ..., n \tag{5.2}$$

2. household demand functions satisfy the *proportionality condition*:

$$\frac{\partial \hat{g}_i / \partial z_k}{\partial \hat{g}_j / \partial z_k} = \frac{\partial \hat{g}_i / \partial z_1}{\partial \hat{g}_j / \partial z_1} \qquad \forall i = 1, .., n;\ j = 1, .., n;\ k = 2, .., K \tag{5.3}$$

3. for any good j such that $\partial \hat{g}_j / \partial z_1 \neq 0$, the z-conditional demands satisfy:

$$\frac{\partial \hat{\theta}_i^j(x, \mathbf{z}_{-1}, g_j)}{\partial z_k} = 0 \qquad \forall i \neq j, k = 2, .., K \tag{5.4}$$

The intuition for this result relates to the discussion provided in earlier chapters. Again, the basic idea is that, by definition, distribution factors do not influence the Pareto set. They may affect consumption, but only through their effect upon the *location* of the final outcome on the Pareto frontier or, equivalently, upon the *respective weighting* of each member's utility that is implicit in this location. The key point is that this effect is one-dimensional (see chapter 4, subsection 1.3). This explains why restrictions appear only in the case where there is more than one distribution factor. Whatever the number of such factors, they can only influence consumption through a single, real-valued function μ. Conditions (5.2) and (5.3) are direct translations of this remark. By the same token, if we compute q_i as a z-conditional function of $(x, \mathbf{z}_{-1}, g_j)$, it should not depend on $\mathbf{z}_{-1}$. The reason is that, for any given value of x, whenever distribution factors $(z_1, \mathbf{z}_{-1})$ contain some information that is relevant for intra-household allocation (hence for household behavior), this information is one-dimensional and can be *fully* summarized by the value of g_j. Once we condition on g_j, $\mathbf{z}_{-1}$ becomes irrelevant. This is the meaning of condition (5.4).

The conditions (5.2)-(5.4) are also sufficient for the collective model: if they are satisfied for the observable demands $\hat{g}(x, \mathbf{z})$, then one can find utility functions and Pareto weights which rationalize the observed demands (see Bourguignon *et al* (2009)). An important implication of these conditions is that in the absence of price variation, proportionality is the *only* testable implication of the collective model. This means that if we have only one distribution factor, then we can never reject the hypothesis of collective rationality. Any extra restrictions for a collective model require that *additional assumptions* be made on the form of individual preferences. For instance, restrictions exist even for a single distribution factor when some goods are private and/or are consumed exclusively by one member of the household. It may surprise readers that in the absence of price variation, proportionality is the full empirical content of the collective model. Recall, however, that in the unitary model, *without price variation*, any demands as a function of total expenditure are compatible with utility maximization.

This result provides two distinct ways of testing for efficiency. Condition (5.3) leads to tests of cross-equation restrictions in a system of unconditional demand equations. An alternative method, implied by (5.4), tests for exclusion restrictions in a conditional demand framework. Empirically, the latter is likely to be more powerful for at least two reasons. First we can employ single equation methods (or even non-parametric methods). Second, single equation exclusion tests are more robust than tests of the equality of parameters across equations. Both tests generalize easily to a framework in which domestic goods are produced by the household. Adding a domestic production function that relates market inputs and domestic labor to goods actually consumed by household members does not modify the above tests on household demands for market goods.

As discussed in Chapter 3, the bargaining version of the collective model has attracted lot of attention. A bargaining framework should be expected to impose *additional* restrictions to those discussed above. Indeed, an easy test can be described as follows. Assume that some distribution factors, which are part of a K'-sub-vector $\mathbf{z}'$, are known to be positively correlated with member b's threat point, while others, constituting a K''-sub-vector $\mathbf{z}''$, are known to favor a. Then in program (5.1) μ should decrease with distribution factors in $\mathbf{z}''$ and increase with those in $\mathbf{z}'$. This property can readily be tested; it implies that,

$$\frac{\partial \hat{g}_i/\partial z'_k}{\partial \hat{g}_i/\partial z''_m} = \frac{\partial \hat{g}_j/\partial z'_k}{\partial \hat{g}_j/\partial z''_m} \leq 0 \text{ for } i,j = 1,..,n; k = 1,...,K'; m = 1,...,K''$$

Should one be willing to go further and assume, for instance, that only the ratio z'_1/z''_2 of distribution factors matters, then we have in addition:

$$\frac{\partial \hat{g}_i}{\partial \ln(z'_1)} + \frac{\partial \hat{g}_i}{\partial \ln(z''_2)} = 0 \qquad \forall i = 1,..,n$$

This is simple to test and easy to interpret.

5.2.2 *Identifiability.*

A more difficult issue arises when we consider identifiability. That is, when is it possible to recover the underlying structure from the sole observation of household behavior? Note, first, that the nature of the data strongly limits what can be recovered. For instance, one cannot hope to identify utility functions in the absence of price variations. 'Identifiability', in this context, essentially means recovering individual Engel curves (that is, demand as a function of income) and the decision process, as summarized by the Pareto weights or (in the private good case) by the sharing rule, again as functions of income and distribution factors only.

With these precautions in mind, we start with some mathematical results concerning integrability that are useful in the current context. Suppose we have a smooth unknown function $f(x,y)$ with non-zero partials f_x and f_y. Suppose first that we observe:

$$h^1(x,y) = f_x(x,y) \text{ and } h^2(x,y) = f_y(x,y) \tag{5.5}$$

If $f(.)$ is twice continuously differentiable, these two functions must satisfy the cross derivative restriction $h^1_y(x,y) = h^2_x(x,y)$. In general, these conditions can be translated into empirical tests of the hypothesis that $h^1(.)$ and $h^2(.)$ are indeed partials of the same function. Moreover, if this symmetry condition is satisfied, then $f(.)$ is identifiable up to an additive constant. That is, if $\bar{f}$ is a solution of (5.5), then any alternative solution must be of the form $\bar{f}(x,y) + k$ where k is an arbitrary constant.

Suppose now that rather than observing the partials themselves we only observe their ratio:

$$h(x,y) = \frac{f_y}{f_x} \tag{5.6}$$

Given $h(x,y)$, $f(x,y)$ is identifiable 'up to a strictly monotone transformation'. That is, we can recover some $\bar{f}(x,y)$ such that any solution is of the form $f(x,y) = G\left(\bar{f}(x,y)\right)$ where $G(.)$ is an arbitrary strictly monotone function.

In general, when $f(.)$ has more than two arguments, $f(x_1,...,x_n)$, assume that we observe $m < n-1$ ratios of partials, say those involving the $m+1$ first partials of f : $\frac{f_2}{f_1}, ..., \frac{f_{m+1}}{f_1}$. Then f is identifiable up to a function of the other variables. That is, we can identify some $\bar{f}(x_1,...,x_n)$ such that any solution is of the form

$$f(x_1,...,x_n) = G\left(\bar{f}(x_1,...,x_n), x_{m+2}...,x_n\right)$$

where $G(.)$ is an arbitrary function. In particular:

- if we observe only one ratio of partials, say $h(x_1,...,x_n) = f_1/f_2$, then $f(.)$ is identifiable up to a function of the other variables $(x_3,...,x_n)$.
- if we observe all ratios of partials, then $f(.)$ is identifiable up to an arbitrary, strictly monotone transformation. Note, as well, that whenever we observe more than one ratio of partials, testable restrictions are generated. These generalize the previous cross-derivative conditions.
- Finally, if in addition the $m+1$ first ratios $\frac{f_2}{f_1}, ..., \frac{f_{m+1}}{f_1}$ only depend on $(x_1,...,x_{m+1})$ then $\bar{f}$ can be chosen to only depend on $(x_1,...,x_{m+1})$ - this is an usual separability property.

We can now return to the identifiability problem for the collective model. Even in the most general case (no identifying restriction beyond efficiency), some (but by no means all) of the structure can be recovered from the observation of demand functions. To see why, note that by equation (5.3) we have:

$$\frac{\partial \hat{g}_i/\partial z_k}{\partial \hat{g}_i/\partial z_1} = \frac{\partial \mu/\partial z_k}{\partial \mu/\partial z_1} = \frac{\mu_k}{\mu_1} = \kappa_k \text{ for all } i \text{ and } k \tag{5.7}$$

The left hand side expression is potentially observable so that we can identify the ratio of partials of $\mu(x, \mathbf{z})$ with respect to distribution factors. Since the right hand side does not depend on the good, the ratio on the left hand side must be the same for all goods; this is the proportionality condition. Given the ratio of partials of the Pareto weight, we can recover $\mu(.)$ up to some function of x. That is, we can recover a particular Pareto weight $\bar{\mu}$ such the true Pareto weight μ must be of the form:

$$\mu(x, \mathbf{z}) = m(x, \bar{\mu}(x, \mathbf{z})) \tag{5.8}$$

for some unobserved function $m(.)$.

The ratio κ_k in equation (5.7) has a natural interpretation in terms of *power compensation*. Assume, for instance, that $\mu_1 > 0$ and $\mu_k < 0$ so that z_1 favors b while z_k serves a. If z_k is increased by some infinitesimal quantity dz_k then $\kappa_k dz_k$ is the increase in z_1 required to offset the change and maintain the same balance of power. Power compensations may be important for welfare analysis, whenever a 'shift of power' has to be compensated. The good news is that even in the most general version of the collective model, they can be directly recovered from observed demands. Furthermore, the proportionality condition (5.3) imposes that the estimation of the power compensation ratio does not depend on the particular commodity chosen. An alternative and important interpretation of this result is that the model always behaves 'as if' there were only one factor, $\bar{\mu}$, influencing the individual's relative powers. Whatever the actual number of distribution factors, they always operate through the index $\bar{\mu}$. Moreover, this index is identifiable. What is not identifiable in the general case is the exact impact of the index on the actual Pareto weight; an impact that will in general depend on the level of total expenditures.

5.2.3 Private consumption.

Although useful, recovery of the Pareto weight up to a strictly monotone function that also depends on total expenditure is far short of what is needed for some important purposes. Is it possible to recover more? To achieve this, we need either better data or more theory restrictions. As an example of the latter, consider the particular but useful case in which all commodities are privately consumed and preferences are either egoistic or caring. As we have seen in chapter 4, efficiency is then equivalent to the existence of a sharing rule in which a receives $\rho(x, \mathbf{z})$ and b receives

$(x - \rho(x, \mathbf{z}))$. Individual a solves:

$$\max v^a(\mathbf{q}^a) \text{ subject to } \mathbf{e}'\mathbf{q}^a = \rho(x, \mathbf{z}) \tag{5.9}$$

and similarly for b. It follows that the household aggregate demand for commodity i takes the form:

$$q_i(x, \mathbf{z}) = q_i^a(\rho(x, \mathbf{z})) + q_i^b(x - \rho(x, \mathbf{z}))$$

where q_i^s is s's demand for good i. The question is: what can be said about q_i^a, q_i^b and ρ from the observation of household demands q_i for $i = 1, ...n$.

Equation (5.7) has an equivalent in this context:

$$\frac{\partial q_i / \partial z_k}{\partial q_i / \partial z_1} = \frac{\partial \rho / \partial z_k}{\partial \rho / \partial z_1} \text{ for all } k \tag{5.10}$$

This result remains valid in the presence of public goods, provided that the sharing rule is taken to be conditional on public goods (as described in subsection 5.2 of Chapter 4). The potential observability of the left hand side of equation (5.10) means that we can recover the sharing rule up to an arbitrary monotone function of total expenditures x. In other words, we can recover some $\bar{\rho}(x, \mathbf{z})$ such that the true sharing rule must be of the form $\rho(x, \mathbf{z}) = G(\bar{\rho}(x, \mathbf{z}), x)$ for some mapping G. And, as above, instead of analyzing the impact of each distribution factor independently, we may just consider the impact of the 'index' $\bar{\rho}$. Consequently we can always consider the case of a unique distribution factor; no loss of generality results.

5.2.4 Assignability.

Up until now we have considered the case where we only observe aggregate household demands. In some cases, we can observe the consumption of a particular good by each partner. That is, for some goods we observe q_i^a and q_i^b. We refer to such a good as being *assignable*. The most widely used example of an assignable good is clothing: in expenditure surveys we always see a distinction made between men and women's clothing. An alternative terminology is that each of the clothing commodities is an *exclusive* good.[4] That is, an exclusive good is one that is consumed by a unique person in the household.

Suppose that we observe the individual consumption of the first good and estimate $\hat{q}_1^a(x, \mathbf{z})$ and $\hat{q}_1^b(x, \mathbf{z})$. Assuming, without loss of generality, that there is only one distribution factor, the collective demands $\tilde{q}_1^s$ are

[4] In general, individual consumptions of an assignable good have the same price, whereas exclusive goods have different prices. The distinction is ineffective in the present context, but will become important when price variations are considered.

related to the observable demands $\hat{q}_1^s$ by:

$$\hat{q}_1^a(x,z) = \tilde{q}_1^a(\rho(x,z)) \tag{5.11}$$
$$\hat{q}_1^b(x,z) = \tilde{q}_1^a(x-\rho(x,z)) \tag{5.12}$$

Thus:

$$\begin{aligned}\frac{\partial \hat{q}_1^a/\partial x}{\partial \hat{q}_1^a/\partial z} &= \frac{\rho_x}{\rho_z}\\ \frac{\partial \hat{q}_1^b/\partial x}{\partial \hat{q}_1^b/\partial z} &= -\frac{1-\rho_x}{\rho_z}\end{aligned} \tag{5.13}$$

Thus the two ratios ρ_x/ρ_z and $(1-\rho_x)/\rho_z$ are identifiable. There is a unique solution to these two equations for (ρ_x,ρ_z) if and only if:

$$\Gamma = \frac{\partial \hat{q}_1^a}{\partial x}\frac{\partial \hat{q}_1^b}{\partial z} - \frac{\partial \hat{q}_1^b}{\partial x}\frac{\partial \hat{q}_1^a}{\partial z} \neq 0 \tag{5.14}$$

If this condition holds, we can identify the partials of ρ:

$$\begin{aligned}\rho_x &= \frac{1}{\Gamma}\frac{\partial \hat{q}_1^a}{\partial x}\frac{\partial \hat{q}_1^b}{\partial z}\\ \rho_z &= \frac{1}{\Gamma}\frac{\partial \hat{q}_1^a}{\partial z}\frac{\partial \hat{q}_1^b}{\partial z}\end{aligned} \tag{5.15}$$

By the result before (5.6), knowing the partials allows us to identify the function itself, up to an additive constant: $\rho = \rho(x,z)+k$. Thus we can learn everything about the sharing rule from observing the assignment of a single good, *except its location.* One good is sufficient because the same Pareto weight function appears in all goods; see equation (5.2). Moreover, new restrictions are generated, since

$$\frac{\partial}{\partial z}\left(\frac{\partial \rho}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial \rho}{\partial z}\right)$$

This provides a test for assignability of any particular good within the collective setting.

Finally, what about the individual Engel curves of the two spouses? First, for any value of the constant k, (5.11) and (5.12) identify individual demands for commodity 1. Consider, now, commodity i; remember that, in general, i is neither exclusive nor assignable. Still, from:

$$\hat{q}_i(x,\mathbf{z}) = \tilde{q}_i^a(\rho(x,\mathbf{z})) + \tilde{q}_i^b(x-\rho(x,\mathbf{z})) \tag{5.16}$$

we have:

$$\begin{aligned}\frac{\partial \hat{q}_i}{\partial x} &= \frac{d\tilde{q}_i^a}{d\rho}\rho_x + \frac{d\tilde{q}_i^b}{d\rho}(1-\rho_x)\\ \frac{\partial \hat{q}_i}{\partial z} &= \left(\frac{d\tilde{q}_i^a}{d\rho} - \frac{d\tilde{q}_i^b}{d\rho}\right)\rho_z\end{aligned} \tag{5.17}$$

Since the left hand side is observed and we have (ρ_x, ρ_z) we invert (so long as $\rho_z \neq 0$) and identify $\tilde{q}_a^i$ and $\tilde{q}_b^i$ up to an additive constant. We conclude that the presence of an assignable good is sufficient to identify (up to additive constant) the sharing rule and individual demands for each commodity, including the non assignable ones.

We thus get a great deal of mileage from the presence of one assignable (or two exclusive) goods. Can we do without? Surprisingly enough, the answer is positive. Bourguignon, Browning and Chiappori (2009) prove the following strong result: if we observe household demand (as a function of total expenditures x and a distribution factor z) for at least three commodities, then in general we can recover individual demands and the sharing rule up to the same additive constants as before *and* (this is the only twist) up to a permutation of a and b.[5] This result arises from equation (5.2) and follows since we have three demands that depend on the one Pareto weight function. For the technical details, see Bourguignon *et al* (2009). The result requires observation of cross partial terms involving x and z; since these are are often difficult to pin down in empirical work, this route for identifying the sharing rule is less robust than using assignability. It is important to note that the identification here does require the existence of at least one distribution factor. Without a distribution factor no information concerning the preferences or the sharing rule can be recovered.

5.3 Observing price responses.

5.3.1 *Testing the collective model*

The basic result

We now turn to the situation in which we observe variation in prices as well as in income and distribution factors. This would be the case, for instance, if we have panel data, or if the cross sectional data exhibit important and *exogenous* fluctuations in prices. Then strong tests are available. Moreover, the model can be proved to be identifiable under reasonably mild exclusion conditions.

Again, we consider a two person household for expositional convenience. Tests of the most general form of the collective model are based on the fundamental SNR1 condition demonstrated in Chapter 4. Namely, the Slutsky matrix S (which can be derived from estimated demand functions) must be of the form:

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[5] Identifiability, here, is only 'generic'. It is indeed possible to construct examples in which it does not hold, but these examples are not robust. For instance, if individual demands and the sharing rule are all linear, identification does not obtain. However, adding quadratic terms is sufficient to guarantee identification except maybe for very specific values of the coefficients.

$$S = \Sigma + R \tag{5.18}$$

where Σ is symmetric, negative and R is of rank at most one.

Direct tests of (5.18) are not straightforward, because the theorem simply says that *there exists* such a decomposition. To construct a testable implication of the symmetry of Σ, consider the matrix M defined by:

$$M = S - S'$$

where S' is the transpose of S. Since Σ is symmetric:

$$M = R - R'$$

and since R is of rank (at most) 1, M is of rank (at most) 2. This property is easy to test, using either standard rank tests or more specific approaches. Note, however, that five commodities (at least) are needed for that purpose. The reason is that neither M nor S are of full rank. Indeed, a standard result of consumer theory, stemming from homogeneity and adding up, states that

$$\boldsymbol{\pi}'S = S\boldsymbol{\pi} = \mathbf{0}$$

where $\boldsymbol{\pi}$ denotes the price vector. It follows that $M\boldsymbol{\pi} = \mathbf{0}$, and M cannot be invertible. Moreover, M is antisymmetric (equal to minus its transpose); hence its rank must be even. With four commodities, M is a 4×4, antisymmetric, non-invertible matrix, so that its rank can never exceed 2 anyway.

Negative semidefiniteness of Σ, on the other hand, can be directly tested on the Slutsky matrix. Indeed, among the eigenvalues of S, one is zero (reflecting non invertibility); among the others, one (at most) can be positive. Therefore, while symmetry of Σ cannot be tested from less than five goods, three are sufficient to test negativeness. In practice, such a test may however not be very powerful. An alternative approach is use revealed preference techniques; following an early discussion in Chiappori (1988), Cherchye, De Rock and Vermeulen (2007) and (2008) provide a complete characterization of the revealed preference approach to collective models.

Distribution factors

Distribution factors can be readily introduced for parametric approaches. Using equation (4.18) in Chapter 4, Browning and Chiappori (1998) prove the following result. Take any distribution factor k, and compute the vector $v' = \left(\frac{\partial \hat{q}_1}{\partial z_k}, ..., \frac{\partial \hat{q}_n}{\partial z_k}\right)$. Then replacing any column (or any row) of M with v should not increase the rank. It is relatively simple to devise an empirical test for this; see Browning and Chiappori (1998) for details.

Some extensions

Finally, a similar investigation has been conducted for other, non-unitary models of household behavior. Lechène and Preston (2009) analyze the demand function stemming from a non cooperative model (involving private provision of the public goods) similar to that discussed in Chapter 4. They show that, again, a decomposition of the type (5.18) holds. However, the rank conditions on the 'deviation' matrix R are different; specifically, Lechène and Preston show that the rank of R can take any value between 1 and the number of public goods in the model. Recently, d'Aspremont and Dos Santos Fereira (2009) have introduced a general framework that provides a continuous link between the cooperative and the non cooperative solutions. In their setting, couples are characterized by a pair of parameters that indicate how 'cooperatively' each agent behaves. Again, they derive a (5.18) decomposition; however, the rank of matrix R can now take values between 1 and twice the number of public goods. On the empirical front, Del Bocca and Flinn (2009) have proposed models in which agents may cooperate at some coordination cost; the decision to cooperate (or not) is then endogeneously derived from the model.

5.3.2 Identifying the collective model

In the presence of price variation, the identifiability problem can be stated in full generality; indeed, when price effects are observable it may be possible to recover individual preferences and demand functions (not only the Engel curves). Clearly, identifying assumptions are necessary; in its most general version (with general preferences $u^a\left(\mathbf{q}^a,\mathbf{q}^b,\mathbf{Q}\right)$ and $u^b\left(\mathbf{q}^a,\mathbf{q}^b,\mathbf{Q}\right)$), there exists a continuum of different structural models generating the same demand function. For instance, Chiappori and Ekeland (2006) show that any function satisfying SNR1 (see equation (5.18)) can be generated as the Pareto efficient demand of a household in which all consumption is public, and also of an (obviously different) household in which all consumption is private. Therefore, we assume in this subsection that preferences are egoistic $\left(u^a\left(\mathbf{q}^a,\mathbf{Q}\right)\text{ and }u^b\left(\mathbf{q}^b,\mathbf{Q}\right)\right)$, although our results have implications for caring preferences as well. We also assume that the econometrician knows which goods are private and which are public.

Even with egoistic preferences, however, the collective structure cannot in general be fully identified from demand data. To give a simple counterexample, assume for a moment that all goods are publicly consumed and consider two pairs of utility functions, $\left(u^a\left(\mathbf{Q}\right),u^b\left(\mathbf{Q}\right)\right)$ and $\left(\tilde{u}^a\left(\mathbf{Q}\right),\tilde{u}^b\left(\mathbf{Q}\right)\right)$ with

$$\begin{aligned}\tilde{u}^a &= F\left(u^a,u^b\right)\\ \tilde{u}^b &= G\left(u^a,u^b\right)\end{aligned}$$

for two arbitrary, increasing functions F and G. It is easy to check that any allocation that is Pareto efficient for $\left(\tilde{u}^a, \tilde{u}^b\right)$ must be Pareto efficient for $\left(u^a, u^b\right)$ as well; otherwise one could increase u^a and u^b without violating the budget constraint, but this would increase $\tilde{u}^a$ and $\tilde{u}^b$, a contradiction. It follows that any demand that can be rationalized by $\left(\tilde{u}^a, \tilde{u}^b\right)$ can also be rationalized by $\left(u^a, u^b\right)$ (of course, with different Pareto weights), so that the two structures are empirically indistinguishable. Since F and G are arbitrary, we are facing a large degree of indeterminacy.

A negative result of this type has a simple meaning: additional identifying hypotheses are required. If there are at least four commodities, then Chiappori and Ekeland (2009) prove the following results.

- If for each household member there is a commodity that this member does not consume and is consumed by at least one other member, then generically one can exactly recover the collective indirect utility function[6] of each member (up to an increasing transform). For any cardinalization of these utility functions, Pareto weights can be recovered. If there are only two persons in the household then this exclusion restriction is equivalent to an *exclusivity* condition that each member has one good that only they consume; with at least three members, exclusion is weaker than exclusivity.

- If all commodities are publicly consumed, identifying collective indirect utility functions is equivalent to identifying individual utilities. With private consumptions, on the contrary, any given pair of collective indirect utilities is compatible with a continuum of combinations of individual utilities and (conditional) sharing rules. However, all these combinations are welfare equivalent, in the sense that they generate the same welfare conclusions. For instance, if a given reform is found to increase the welfare of a while decreasing that of b under a specific combination of individual utilities, the same conclusion will hold for all combinations.

- Finally, if there is at least one distribution factor, the exclusivity restriction can be relaxed and identifiability obtains with one *assignable* good only.

In the literature the traditional choice for exclusive goods for husband and wife is men and women's clothing respectively. There is a subtle but important difference between the notion of exclusivity and that of assignability. In both cases, we observe consumptions at the individual level. But exclusive goods have different prices, whereas under assignability we observe individual consumptions of the same good - so there is only one price. Therefore, when considering clothing as two exclusive goods we have to assume they have different prices. In practice prices for men and women's

[6] See section 2.2 of chapter 4 for the definition of the collective indirect utility function.

clothing tend to be very colinear and we have to treat clothing as an assignable good.

Two remarks are in order at that point. First, the identifiability result just presented is, by nature, non parametric, in the sense that it does not rely on the choice of a specific functional form for either preferences or Pareto weights.[7] Under an explicitly parametric approach, stronger identification results may obtain; for instance, it may be the case that one exclusive good only is sufficient to identify all the relevant parameters. Clearly, these additional properties are due to the specific functional form under consideration. Second, the result is generic, in the sense that it holds for 'almost all' structures. An interesting remark is that (non-generic) exceptions include the case in which Pareto weights are constant; in such a case, the collective indirect utilities are not identifiable in general.[8] To see why, simply note that, in that case, the household maximizes a collective utility of the form:

$$U\left(\mathbf{q}^a, \mathbf{q}^b, \mathbf{Q}\right) = \mu u^a\left(\mathbf{q}^a, \mathbf{Q}\right) + u^b\left(\mathbf{q}^b, \mathbf{Q}\right) \tag{5.19}$$

under budget constraint; remember that here μ is a constant. Standard results in consumer theory guarantee that we can recover U from observed (household) demand. However, for any given U there exists a continuum of u^a and u^b such that (5.19) is satisfied. For instance, take any such u^a and u^b that are strongly increasing and concave, pick up any smooth function ϕ, and define $\bar{u}^a$ and $\bar{u}^b$ by:

$$\begin{aligned}\bar{u}^a\left(\mathbf{q}^a, \mathbf{Q}\right) &= u^a\left(\mathbf{q}^a, \mathbf{Q}\right) + \varepsilon\phi\left(\mathbf{Q}\right)\\ \bar{u}^b\left(\mathbf{q}^b, \mathbf{Q}\right) &= u^b\left(\mathbf{q}^b, \mathbf{Q}\right) - \mu\varepsilon\phi\left(\mathbf{Q}\right)\end{aligned}$$

Then $\mu\bar{u}^a + \bar{u}^b = U$ and (5.19) is satisfied; moreover, on any compact set, $\bar{u}^a$ and $\bar{u}^b$ are concave and increasing for ε small enough.

Ironically, the case of a constant Pareto weight corresponds to the Samuelson justification of the unitary setting, in which a single, price-independent welfare index is maximized. From an identification viewpoint, adopting a unitary framework is thus a very inappropriate choice, since *it rules out the identification of individual welfares.*

Our general conclusion is that welfare relevant structure is indeed identifiable in general, provided that one can observe one exclusive consumption

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[7] This notion of 'non parametric', which is used for instance by econometricians, should be carefully distinguished from the perspective based on revealed preferences - which, unfortunately, is also often called 'non parametric'. In a nutshell, the revealed preferences approach does not require the observability of a demand *function*, but only of a finite number of points; it then describes relationship that must be satisfied for the points to be compatible with the model under consideration. This view will be described in Subsection 5.3.4.

[8] This case is 'non generic' in the sense that in the set of continuous functions, constant functions are non-generic.

per member (or one overall with a distribution factor). However, identifiability fails to obtain in a context in which the household behaves as a single decision maker.

5.3.3 *A simple example*

The previous results can be illustrated by the following example, directly borrowed from Chiappori and Ekeland (2009).Consider individual preferences of the LES type:

$$U^s(q^s, Q) = \sum_{i=1}^{n} \alpha_i^s \log\left(q_i^s - c_i^s\right) + \sum_{j=n+1}^{N} \alpha_j^s \log\left(Q_j - C_j\right), \quad s = a, b$$

where the parameters α_i^s are normalized by the condition $\sum_{i=1}^{N} \alpha_i^s = 1$ for all s, whereas the parameters c_i^s and C_j are unconstrained. Here, commodities 1 to n are private while commodities $n+1$ to N are public. Also, given the LES form, it is convenient to assume that the household maximizes the weighted sum $\mu U^a + (1-\mu)\, U^b$, where the Pareto weight μ has the simple, linear form:

$$\mu = \mu^0 + \mu^x x + \mu^z z, \quad s = a, b$$

Household demand

The group solves the program:

$$\max\left(\mu^0 + \mu^x x + \mu^z z\right)\left(\sum_{i=1}^{n} \alpha_i^a \log\left(q_i^a - c_i^a\right) + \sum_{j=n+1}^{N} \alpha_j^a \log\left(Q_j - C_j\right)\right)$$
$$+\left(1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right)\left(\sum_{i=1}^{n} a_i^b \log\left(q_i^b - c_i^b\right) + \sum_{j=n+1}^{N} \alpha_j^b \log\left(Q_j - C_j\right)\right)$$

under the budget constraint:

$$\mathbf{p}'\left(\mathbf{q}^a + \mathbf{q}^b\right) + \mathbf{P}'\mathbf{Q} = x$$

where one price has been normalized to 1. Individual demands for private goods are given by:

$$\begin{aligned}
p_i q_i^a &= p_i c_i^a + \alpha_i^a\left(\mu^0 + \mu^x x + \mu^z z\right)\left(x - \sum_{i,s} p_i c_i^s - \sum_j P_j C_j\right) \\
p_i q_i^b &= p_i c_i^b + \alpha_i^b\left[1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right]\left(x - \sum_{i,s} p_i c_i^s - \sum_j P_j C_j\right)
\end{aligned}$$

generating the aggregate demand:

$$p_i q_i = p_i c_i + \left[\alpha_i^a \left(\mu^0 + \mu^x x + \mu^z z\right) + \alpha_i^b \left(1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right)\right] Y \quad (5.20)$$

and for public goods:

$$P_j Q_j = P_j C_j + \left[\alpha_j^a \left(\mu^0 + \mu^x x + \mu^z z\right) + \alpha_j^b \left(1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right)\right] Y$$

Here, $c_i = c_i^a + c_i^b$ and $Y = \left(x - \sum_{i,s} p_i c_i^s - \sum_j P_j C_j\right)$. The household demand is thus a direct generalization of the standard LES, with additional quadratic terms in x^2 and cross terms in xp_i and xP_j, plus terms involving the distribution factor z; one can readily check that it does not satisfy Slutsky symmetry in general, although it does satisfy SNR1.

A first remark is that c_i^a and c_i^b cannot be individually identified from group demand, since the latter only involves their sum c_i. As discussed above, this indeterminacy is however welfare irrelevant, because the collective indirect utilities of the wife and the husband are, up to an additive constant:

$$\begin{aligned} W^a(p, P, x, z) &= \log Y + \log \left(\mu^0 + \mu^x x + \mu^z z\right) - \sum_i \alpha_i^a \log p_i - \sum_j \alpha_j^a \log P_j \\ W^b(p, P, x, z) &= \log Y + \log \left(1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right) - \sum_i \alpha_i^b \log p_i - \sum_j \alpha_j^b \log P_j \end{aligned}$$

which does not depend on the c_i^s. Secondly, the form of aggregate demands is such that private and public goods have exactly the same structure. We therefore simplify our notations by defining

$$\begin{aligned} &\xi_i = q_i \quad \text{for } i \leq n,\ \xi_i = Q_i \quad \text{for } n < i \leq N \\ &\quad \text{and similarly} \\ &\gamma_i = c_i \quad \text{for } i \leq n,\ \gamma_i = C_i \quad \text{for } n < i \leq N \\ &\pi_i = p_i \quad \text{for } i \leq n,\ \ \pi_i = P_i \quad \ \text{for } n < i \leq N \end{aligned}$$

so that the group demand has the simple form:

$$\pi_i \xi_i = \pi_i \gamma_i + \left[\alpha_i^a \left(\mu^0 + \mu^x x + \mu^z z\right) + \alpha_i^b \left(1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right)\right] Y \quad (5.21)$$

leading to collective indirect utilities of the form:

$$\begin{aligned} W^a(p, P, x, z) &= \log Y + \log \left(\mu^0 + \mu^x x + \mu^z z\right) - \sum_i \alpha_i^a \log \pi_i \\ W^b(p, P, x, z) &= \log Y + \log \left(1 - \left(\mu^0 + \mu^x x + \mu^z z\right)\right) - \sum_i \alpha_i^b \log \pi_i \end{aligned}$$

It is clear, on this form, that the distinction between private and public goods can be ignored. This illustrates an important remark: while the *ex* *ante* knowledge of the public versus private nature of each good is necessary for the identifiability result to hold in general, for many parametric forms it is actually not needed.

Identifiability

The general case

The question, now, is whether the empirical estimation of the form (5.21) allows to recover the relevant parameters - namely, the α_i^s, the γ^i, and the μ^α. We start by rewriting (5.21) as:

$$\pi_i \xi_i = \pi_i \gamma_i + \left(\alpha_i^b + \left(\alpha_i^a - \alpha_i^b\right)\mu^0 + \left(\alpha_i^a - \alpha_i^b\right)\left(\mu^x x + \mu^z z\right)\right)\left(x - \sum_m \pi_m \gamma^m\right) \tag{5.22}$$

The right hand side of (5.22) can in principle be econometrically identified; we can thus recover the coefficients of the variables, namely x, x^2, xz, the π_m and the products $x\pi_m$ and $z\pi_m$. For any i and any $m \neq i$, the ratio of the coefficient of x by that of π_m gives γ^m; the γ^m are therefore vastly overidentified. However, the remaining coefficients are identifiable only up to an arbitrary choice of two of them. Indeed, an empirical estimation of the right hand side of (E_i') can only recover for each j the respective coefficients of x, x^2 and xz, that is the three expressions $K_x^j = \alpha_j^b + \left(\alpha_j^a - \alpha_j^b\right)\mu^0$, $K_{xx}^j = \left(\alpha_j^a - \alpha_j^b\right)\mu^x$ and $K_{xz}^j = \left(\alpha_j^a - \alpha_j^b\right)\mu^z$. Now, pick up two arbitrary values for μ^0 and μ^x, with $\mu^x \neq 0$. The last two expressions give $\left(\alpha_j^a - \alpha_j^b\right)$ and μ^z; the first gives α_j^b therefore α_j^a.

As expected, a continuum of different models generate the same aggregate demand. Moreover, these differences are welfare relevant, in the sense that the individual welfare gains of a given reform (say, a change in prices and incomes) will be evaluated differently by different models; in practice, the collective indirect utilities recovered above are not invariant across the various structural models compatible with a given aggregate demand.

A unitary version of the model obtains when the Pareto weights are constant: $\mu^x = \mu^z = 0$. Then $K_{xz}^j = 0$ for all j (since distribution factors cannot matter[9]), and $K_{xx}^j = 0$ for all j (demand must be linear in x, since a quadratic term would violate Slutsky). We are left with $K_x^j = \alpha_j^b + \left(\alpha_j^a - \alpha_j^b\right)\mu^0$, and it is obviously impossible to identify independently α_j^a, α_j^b and μ^0; as expected, the unitary framework is not identifiable.

Identification under exclusion

We now show that in the non-unitary version of the collective framework, an exclusion assumption per member is sufficient to exactly recover all the

[9] For a discussion of the role of distribution factor in a unitary context, see Browning, Chiappori and Lechene (2006).

coefficients. Assume, indeed, that member a does not consume commodity 1 and member b does not consume commodity 2; that is, $\alpha_1^a = \alpha_2^b = 0$. Then equation (E_1') gives:

$$\alpha_1^b \left(1 - \mu^0\right) = K_x^1, \; -\alpha_1^b \mu^x = K_{xx}^1, \; -\alpha_1^b \mu^z = K_{xz}^1$$

while (E_2') gives:

$$\alpha_2^a \mu^0 = K_x^2, \; \alpha_2^a \mu^x = K_{xx}^2, \; \alpha_2^a \mu^z = K_{xz}^2$$

Combining the first two equations of each block and assuming $\mu^x \neq 0$, we get:

$$\frac{1 - \mu^0}{\mu^x} = -\frac{K_x^1}{K_{xx}^1} \text{ and } \frac{\mu^0}{\mu^x} = \frac{K_x^2}{K_{xx}^2}$$

therefore, assuming $K_x^2 K_{xx}^1 - K_x^1 K_{xx}^2 \neq 0$

$$\frac{1 - \mu^0}{\mu^0} = -\frac{K_x^1 K_{xx}^2}{K_x^2 K_{xx}^1} \text{ and } \mu^0 = \frac{K_x^2 K_{xx}^1}{K_x^2 K_{xx}^1 - K_x^1 K_{xx}^2}$$

It follows that

$$\mu^x = \frac{K_{xx}^2}{K_x^2} \mu^0 = \frac{K_{xx}^2 K_{xx}^1}{K_x^2 K_{xx}^1 - K_x^1 K_{xx}^2}$$

and all other coefficients can be computed as above. It follows that the collective indirect utility of each member can be exactly recovered, which allows for unambiguous welfare statements. As mentioned above, identifiability is only generic in the sense that it requires $K_x^2 K_{xx}^1 - K_x^1 K_{xx}^2 \neq 0$. Clearly, the set of parameters values violating this condition is of zero measure.

Finally, it is important to note that this conclusion requires $\mu^x \neq 0$; in particular, *it does not hold true* in the unitary version, in which $\mu^x = \mu^z = 0$. Indeed, the same exclusion restrictions as above only allow to recover $\alpha_1^b \left(1 - \mu^0\right) = K_x^1$ and $\alpha_2^a \mu^0 = K_x^2$; this is not sufficient to identify μ^0, let alone the α_j^i for $j \geq 3$. This confirms that the unitary version of the model is not identified even under the exclusivity assumptions that guarantee generic identifiability in the general version.

5.3.4 The revealed preference approach

Up until now we have considered analysis that posits that we can estimate smooth demands and test for the generalized Slutsky conditions for integrability. An alternative approach to empirical demand analysis that has gained ground in the last few years is the *revealed preference* (RP) approach that derives from Afriat (1967) and Varian (1982). This style of analysis explicitly recognizes that we only ever have a finite set of observations on prices and quantities which cannot be used to directly construct smooth demand functions without auxiliary assumptions. The revealed preference approach instead identifies linear inequality conditions on the finite data set that characterize rational behavior. The most attractive feature of the Afriat-Varian approach is that no functional form assumptions are imposed. Moreover powerful numerical methods are available to implement the RP tests. The drawback of the RP approach is that even when the data satisfy the RP conditions, we can only set identify preferences; see Blundell *et al* (2008).

Generalizing the unitary model RP conditions to the collective setting was first achieved in Chiappori (1988) for a specific version of the collective model. The conditions for the general model have been established in Cherchye, De Rock and Vermeulen (2007), (2009a) and Cherchye, De Rock, Sabbe, and Vermeulen (2008); these papers provide a complete characterization of the collective model in a revealed preference context. This requires several significant extensions to the RP approach for the unitary model. In particular, these authors allow for non-convex preferences and develop novel (integer programming) methods since the linear programming techniques that work for the unitary model are not applicable for the collective model. The tests for 'collective rationality' require finding individual utility levels, individual marginal utilities of money (implying Pareto weights) and individual assignments for private goods and Lindahl prices for public goods. As in the unitary model, these methods can only set identify the preferences of the household members and the Pareto weight. Cherchye, De Rock and Vermeulen (2009b) apply these methods to a Russian expenditure panel.

5.4 The case of labor supply

5.4.1 Egoistic preferences and private consumption

A large part of the empirical literature on household behavior is devoted to labor supply. The theory has been presented in Section 4 of Chapter 4; here we concentrate on the empirical implications. Most empirical works consider the simple setting with egoistic preferences and private consumption; see subsection 4.2 of Chapter 4. In this framework, results have been established by Chiappori (1988, 1992) and Chiappori, Fortin and Lacroix (2002). Regarding testability, strong implications can be derived, even in this simple setting. Even more remarkable is the fact that the observation of individual labor supplies, as functions of wages, non labor income and distribution factors, allows us to identify the sharing rule up to an additive constant. We start from the two leisure demand equations:

$$l^a\left(w^a,w^b,y,z\right) = \tilde{l}^a\left(w^a,\rho\left(w^a,w^b,y,z\right)\right) \tag{5.23}$$

$$l^b\left(w^a,w^b,y,z\right) = \tilde{l}^b\left(w^b,y-\rho\left(w^a,w^b,y,z\right)\right) \tag{5.24}$$

where $\tilde{l}^a$ denotes the Marshallian demand for leisure by person a, y is full income and $\rho(.)$ is a's share of full income. We assume that both partners shares are increasing in full income, $0 < \partial\rho/\partial y < 1$, and that the distribution factor is 'meaningful', $\partial\rho/\partial z \neq 0$.

Taking derivatives through (5.23):

$$\begin{aligned}
\frac{\partial l^a}{\partial w^b} &= \frac{\partial \tilde{l}^a}{\partial \rho}\frac{\partial \rho}{\partial w^b} \\
\frac{\partial l^a}{\partial y} &= \frac{\partial \tilde{l}^a}{\partial \rho}\frac{\partial \rho}{\partial y} \\
\frac{\partial l^a}{\partial z} &= \frac{\partial \tilde{l}^a}{\partial \rho}\frac{\partial \rho}{\partial z}
\end{aligned} \tag{5.25}$$

so that:

$$\frac{\partial l^a/\partial z}{\partial l^a/\partial y} = \frac{\partial\rho/\partial z}{\partial\rho/\partial y} \tag{5.26}$$

Similarly for b:

$$\begin{aligned}
\frac{\partial l^b}{\partial w^a} &= -\frac{\partial \tilde{l}^b}{\partial y^b}\frac{\partial \rho}{\partial w^a} \\
\frac{\partial l^b}{\partial y} &= \frac{\partial \tilde{l}^b}{\partial y^b}\left(1 - \frac{\partial \rho}{\partial y}\right) \\
\frac{\partial l^b}{\partial z} &= -\frac{\partial \tilde{l}^b}{\partial y^b}\frac{\partial \rho}{\partial z}
\end{aligned} \tag{5.27}$$

so that:

$$\frac{\partial l^b/\partial z}{\partial l^b/\partial y} = -\frac{\partial\rho/\partial z}{1 - \partial\rho/\partial y} \tag{5.28}$$

For notational simplicity, let F^s denote the fraction $\frac{\partial l^s/\partial z}{\partial l^s/\partial y}$ for $s = a, b$; note that F^s can in principle be observed (or estimated) as a function of $\left(w^a, w^b, y, z\right)$, and that $F^a = F^b$ would imply

$$\frac{\partial\rho/\partial z}{\partial\rho/\partial y} = -\frac{\partial\rho/\partial z}{1 - \partial\rho/\partial y}$$

which is impossible if $\partial\rho/\partial z \neq 0$.

Now, (5.26) and (5.28) can be solved in $\partial\rho/\partial z$ and $\partial\rho/\partial y$ (since $F^b \neq F^a$):

$$\begin{aligned}
\frac{\partial \rho}{\partial y} &= \frac{F^b}{F^b - F^a} \\
\frac{\partial \rho}{\partial z} &= \frac{F^a F^b}{F^b - F^a}
\end{aligned}$$

We thus conclude that the partials of ρ with respect to income and distribution factor are identifiable.

Finally, the first two equations of (5.25) and of (5.27) give respectively:

$$\begin{aligned}\frac{\partial\rho}{\partial w^b} &= \frac{\partial l^a/\partial w^b}{\partial l^a/\partial y}\frac{\partial\rho}{\partial y} = \frac{\partial l^a/\partial w^b}{\partial l^a/\partial y}\frac{F^b}{F^b-F^a} \text{ and}\\ \frac{\partial\rho}{\partial w^a} &= \frac{\partial l^b/\partial w^a}{\partial l^b/\partial y}\left(1-\frac{\partial\rho}{\partial y}\right) = -\frac{\partial l^a/\partial w^b}{\partial l^a/\partial y}\frac{F^a}{F^b-F^a}\end{aligned} \tag{5.29}$$

The conclusion is thus that *all partial derivatives of the sharing rule can be exactly recovered* from the observation of the two labor supply functions. From the sole observation of labor supplies, one can recover the impact of wages, non labor income and distribution factors on the sharing rule. Finally, the cross derivative restrictions generate additional testable predictions. The reader may realize that this conclusion is by no means surprising; indeed, it stems from the general results presented in the previous subsection.

The sharing rule itself is identified up to an additive constant; that constant cannot be identified unless either all commodities are assignable or individual preferences are known (for instance, from data on singles). To see why, take labor supply functions l^a and l^b that satisfy (5.23) and (5.24) for some sharing rule ρ and some Marshallian demands $\tilde{l}^s$ derived from individual utilities $u^s, s = a, b$. Now, for some constant K, define ρ_K, u^a_K and u^b_K by:

$$\begin{aligned}\rho_K\left(w^a, w^b, y, z\right) &= \rho\left(w^a, w^b, y, z\right) + K\\ u^a_K\left(l^a, C^a\right) &= u^a\left(l^a, C^a - K\right)\\ u^b_K\left(l^b, C^b\right) &= u^b\left(l^b, C^b + K\right)\end{aligned}$$

It is easy to check that the Marshallian demands derived from ρ_K, u^a_K and u^b_K satisfy (5.23) and (5.24). The intuition is illustrated in Figure 5.1 in the case of a. Switching from ρ and u^a to ρ_K and u^a_K does two things. First, the sharing rule, therefore the intercept of the budget constraint, is shifted downward by K; second, all indifference curves are also shifted downward by the same amount. When only labor supply (on the horizontal axis) is observable, these models are empirically indistinguishable.

Note, however, that the models are also welfare equivalent (that is, the constant is 'irrelevant'), in the sense defined in section 3.3 of chapter 4: changing the constant affects neither the comparative statics nor the welfare analysis derived from the model. Technically, the collective indirect utility of each member is the same in both models; one can readily check that the two models generate the same *level* of utility for each spouse. In the end, the optimal identification strategy depends on the question under consideration. If one want to formulate welfare judgments, collective indirect utilities are sufficient, and they can be recovered without additional

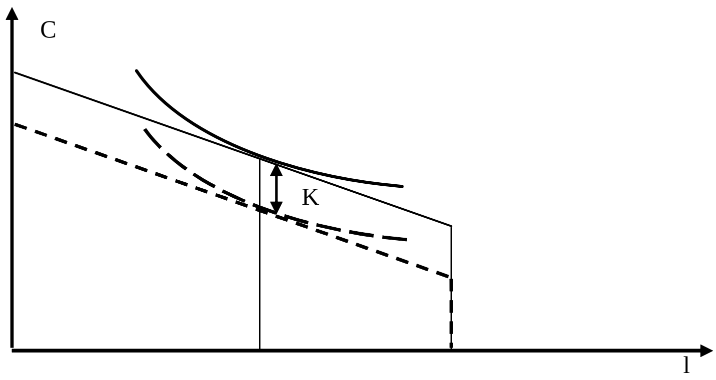

FIGURE 5.1. A Shift in the Sharing Rule

assumptions. If, on the other hand, the focus is on intrahousehold inequality, the basic model can identify the *changes* affecting intrahousehold inequality, but not its initial *level*; therefore additional assumptions may be needed. For instance, some empirical works assume that preferences are unchanged by marriage, therefore can be identified from the labor supply of singles; then the constant can also be recovered.

Finally, one should not conclude from the previous derivation that the presence of a distribution factor is needed for identifiability. This is actually not the case. The observation of individual labor supplies, as functions of wages and non labor income, are 'generically' sufficient to recover the sharing rule up to an additive constant (Chiappori 1988, 1992). However, identification is only generic in that case; moreover, it is arguably less robust, since it involves second derivatives of the labor supply functions.

5.4.2 Extensions

The model has been extended in various directions. First, while the assumption of a unique, Hicksian composite consumption good is standard in the labor supply literature, the model can address a more general framework. Chiappori (2011) consider a model with two leisures and many consumption goods that are privately (but not exclusively) consumed by the members. The context is cross-sectional, in the sense that there is variation in wages but not in prices. He shows that if one distribution factor (at least) is available, then it is possible to identify (again up to additive constants) not only the sharing rule but also the individual demands for all private commodities, as functions of wages and non-labor income. It follows that in a collective model of consumption and labor supply estimated on cross sectional data, it is possible to recover the income and wage elasticities of individual demands for each good.

Secondly, the computations above rely on the assumption that labor supply is a continuous variable. This may fail to hold for two reasons. First, in some households one member may elect not to participate; in that case, the person's labor supply is at a corner solution equal to zero. Secondly, the structure of labor markets may put constraints on the number of hours supplied by individuals. For instance, the choice may be only between working part time, full time or not at all; then labor supply should rather be modelled as a discrete variable. Extensions of the previous model to such situations have been studied by Blundell *et al* (2007) and Donni (2007).

Although very convenient, this framework has its limitations. The privateness assumption has been criticized on two grounds. First, while some consumptions are indeed private, others are not. Children expenditures are a typical example of public goods within the household. Blundell, Chiappori and Meghir (2005) analyze a model similar to the previous one but for the consumption good, which is taken to be public. They show that, again, the model is identifiable from the observation of labor supply behavior. They show how their approach can be extended to household production under various specifications. A second criticism concerns the private nature of individual leisure. It could indeed be argued that leisure is, to some extent, publicly consumed; after all, the utility I derive from my own free time may be higher when my spouse is available as well. The general insight, here, is that a model in which both members' leisure enter each individual utility is still identifiable, provided that some other commodities are exclusive (this is a consequence of the general identifiability results described in Section 2). Fong and Zhang (2001) analyze a framework in which leisure is partly private and partly public; they show that one assignable good is sufficient for identification in the presence of a distribution factor.

Finally, a standard problem with traditional models of labor supply is the implicit assumption that time is divided between market work and leisure - so that any moment not spend working of a wage tends to be assimilated with leisure. This, of course, disregards domestic production, and may result in misleading evaluations. For instance, if a given reform is found to reduce female market labor supply, we may conclude that it increases her leisure, hence her utility, whereas the actual outcome is more domestic work (and ultimately less leisure) for the wife. Donni (2008) shows, however, that the direction of the mistake depends on the properties of the domestic production function. To take an extreme example, consider the case in which the latter is additively separable; that is, when t^s denotes the time spend on domestic production by agent s, then the outcome is:

$$C = f^a\left(t^a\right) + f^b\left(t^b\right)$$

Assuming that the domestic good is marketable with price p, first order conditions require that:

$$f'^{s}\left(t^{s}\right)=\frac{w_{s}}{p}$$

which implies that the time spent on domestic production by s only depends on their wage (and on the price of the domestic good). It follows that any welfare judgment that ignores domestic production is in fact unbiased - that is, a reform that is found to increase the wife's welfare when ignoring domestic production has the same impact even when domestic production is taken into account and conversely. This conclusion, however, does not hold when the productivity of the wife's domestic work depends on the husband's. In the latter case, it becomes necessary to estimate a model that explicitly allows for domestic production - which requires in general time use data; for a short list of such works, see subsection 5.5.3.

5.5 Empirical evidence.

5.5.1 Evidence against the unitary model.

As we have seen, there are two strands to testing for the unitary model: the Slutsky conditions and independence of behavior from distribution factors. Regarding the former, Slutsky symmetry is often rejected on household expenditure survey data. Rejections of Slutsky symmetry may be due to many factors other than a failure of the unitary assumption. For example, we might have the wrong functional form or an inappropriate grouping of goods or be wrongly assuming separability from housing and durables or accounting for latent heterogeneity inappropriately and so on. A widely cited piece of evidence that the unitary assumption itself is problematic is from Browning and Chiappori (1998) who model commodity demands using Canadian data. Using a QAIDS formulation, they test for symmetry for three sub-samples: single women, single men and couples with no children. An important finding is that Slutsky symmetry is *not* rejected for single women or single men, while it is (very strongly) for couples. Since most of underlying modelling assumptions are the same across the three strata, this suggests that it is the unitary assumption that is the problem. These findings have been replicated by Kapan (2009) using Turkish data; and Vermeulen (2005) obtains similar results for labor supply.

Although suggestive, the rejection of Slutsky symmetry would not, by itself, warrant abandoning the unitary assumption. Much more convincing are the next set of tests we discuss. The second principle implication of the unitary model is that possible distribution factors do not have any significant impact on the household choice variable being considered. Unlike the test for the Slutsky conditions, such tests can be conducted whether or not we have price variation. Table 5.1 gives a partial listing of distribution factors that have been considered in the literature. Below we discuss the validity of these factors. The most widely used distribution factor for this is some measure of relative incomes, earnings or wages. Such tests are often called tests of 'income pooling': only household income matters for choice outcomes and not the source of the income.[10] As we have seen, Becker explicitly introduced the RKT to justify income pooling. Tests for the exclusion of other distribution factors constitute a generalization of income pooling.

	Distribution factor
1	Relative income
2	Relative wages
3	Relative unearned income
4	Relative age
5	Relative education
6	Local sex ratio
7	Household income
8	Background family factors
9	Control of land
10	Previous children
11	Reported influence within household
12	Married or cohabiting
13	Divorce laws
14	Alimonies
15	Single parent benefits
16	Gender of a benefit's recipient

TABLE 5.1. Distribution factors

Bruce (1989) provides a listing of the research on low income countries documenting tensions within households about the use of household resources. Strauss *et al* (2000) present an exhaustive list of tests for income pooling for low income countries up to their publication date. Table 5.2 lists some of the studies that have considered non-unitary models.[11] As can be seen from this Table, a wide variety of outcomes and distribution factors have been considered for many different countries. The most widely used distribution factor is relative income (the 'income pooling' test). All of the cited papers find a significant role for the distribution factors that should not affect the outcomes in a unitary model. For instance, an early and influential paper by Thomas (1990), based on Brazilian cross-sectional

[10] Income pooling is a necessary condition for a unitary model but not a sufficient condition. In particular, income pooling can hold locally if we have a noncooperative voluntary contributions game; see section 4 of chapter 3.

[11] This listing is by no means exhaustive and tends to focus on results from high income countries.

data, finds that the relative share of non labor income coming from the wife has a very significant impact on the health status of children within the household.

This unanimity may be somewhat misleading; our impression is that there is a strong publication bias against not finding an effect. That is, editors may not be interested in papers that confirm a conventional view by finding an insignificant effect. Nonetheless, the evidence seems overwhelming: a principal implication of the unitary model is rejected on a wide set of data sets for a wide range of outcomes.

Reference	Outcome	Country	Df's
Anderson & Baland (2002)	Participation in a rosca	Kenya	1
Aronsson *et al* (2001)	Leisure demand	Sweden	2,3,4,5,6
Attanasio & Lechene (2002)	Commodity demands;	Mexico	1
	influence on various decisions		
Barmby & Smith (2001)	labor supplies	Denmark, UK	2
Bayudan (2006)	Female labor supply	Philippines	2, 11
Bourguignon *et al* (1993)	Commodity demands	France	1
Browning (1995)	Saving	Canada	1
Browning & Bonke (2006)	Commodity demands	Denmark	1,7,8,10
Browning & Gørtz (2006)	Commodity demands, leisures	Denmark	2,4,7
Browning *et al* (1994)	Demand for clothing	Canada	1,4,7
Browning & Chiappori (1998)	Commodity demands	Canada	1,4
Chiappori *et al* (2002)	labor supplies	US	6
Couprie (2007)	labor supply and leisure	UK	2,3
Donni (2007)	labor supplies, demands	France	1,7
Duflo (2003)	Child health	South Africa	1
Ermisch & Pronzato (2006)	Child support payments	UK	1
Fortin & Lacroix (1997)	Joint labor supply	Canada	1,2
Haddad & Hoddinott (1994)	Child health	Cote D'Ivoire	1
Hoddinott & Haddad (1995)	Food, alcohol and tobacco	Cote D'Ivoire	1
Lundberg, *et al* (1997)	Clothing demands	UK	3
Oreffice (2008)	Labor supply	US	1
Phipps & Burton (1998)	Commodity demands	Canada	1
Schultz (1990)	labor supplies and fertility	Thailand	3
Thomas (1990)	Child health	Brazil	3
Udry (1996)	Farm production	Burkina Faso	9
Vermeulen (2005)	labor supplies	Netherlands	3,4,12
Ward-Batts (2008)	Household demands	UK	3

TABLE 5.2. Empirical collective studies

Even these results may not be fully conclusive, however, because these rejections may in many cases have other explanations than a failure of the unitary assumption. For example, consider a unitary demand model in which the relative (labor or non labor) earnings of the two partners do *not* affect demand behavior directly. Suppose, however, that there is unobserved heterogeneity in tastes between husbands and wives and this heterogeneity is correlated with heterogeneity in earnings. For example, suppose the relative preference for clothing between a husband and wife is correlated with their relative tastes for work. Then we would find that the demand for clothing (conditional on prices, total expenditure and preference factors) will be correlated with relative earnings, with higher earners having relatively more clothing expenditure than their partner. In this case, a finding that relative clothing demands are partially correlated with relative earnings is spurious in the sense that it is due to inadequate control for heterogeneity rather than a failure of the unitary assumption. Attempts to find instruments to wash out this spurious correlation have not been notably successful: it has proven impossible to find observables that are correlated with, say, relative earnings but not with demand heterogeneity.[12] Similarly, Thomas's findings might simply reflect the fact that some women are more willing to invest over the long term than others; such women would be likely to spend more on children, and also to have saved more in the past, hence to receive more non labor income today. Such a mechanism does not rely on a shift in powers triggered by the wife's larger relative contribution to total income, but only on unobserved heterogeneity between women; as such, it is fully compatible with a unitary representation.

However, several recent papers provide strong evidence concerning income pooling that can hardly be attributed to heterogeneity biases. Lundberg *et al* (1997) present quasi-experimental evidence based on a reform of the UK child public support system in April 1977. Prior to that time families with children received a child tax allowance and a taxable child allowance. This effectively meant that the child benefits were paid to the higher earner, mostly the father. After April 1977, the old scheme was dropped in favor of a non-taxable child benefit which is paid directly to the mother. This re-allocation of income within the household can reasonably be treated as exogenous to the affected households. Moreover, the child benefit was a sizable transfer (equal to 8% of male earnings for a two child household). Thus we have a large, exogenous 'treatment' which can be used to assess the importance of the distribution of income within the household. The major confounding factor is that the reform was not revenue neutral for all households with children and some saw a substantial rise in net household income. LPW use UK Family Expenditure Survey cross-section

[12] Luo (2002) estimates a demand system explicitly allowing for uncorrelated heterogeneity and finds that the BC results for Slutsky symmetry hold up

data from before and after the change to gauge the effect of the reform on assignable expenditures. They focus attention on the ratio of expenditures on children's clothing and women's clothing, both relative to men's clothing. Their findings are unequivocal: both ratios rose significantly after the reform.[13] Another strong rejection is provided by Duflo (2003), who analyzes a reform of the South African social pension program for elderly that extended the benefits to a large, previously not covered black population. Due to eligibility criteria, the coverage is not universal; in some households, in particular one only of the grand parents receives the benefit. Duflo uses a difference of difference approach based on the demographics of the siblings to control for selection in eligibility. She shows that the recipient's gender - a typical distribution factor - is of considerable importance for the impact of the transfers on children's health: a payment to the grandfather has no significant effect, whereas the same amount paid to the grandmother results in a huge improvement in the health status of girls in the family. These contributions and several others (including a subsequent analysis on micro data for all goods by Ward-Batts (2008)) very convincingly suggest that income pooling is indeed strongly rejected on real data.

5.5.2 Evidence on the collective model.

Although the evidence against the unitary model in specific contexts is not as robust as widely believed, it does add up and most researchers in the field now seem to agree that any reasonable model should account for spouses having different preferences and for the intrahousehold distribution of 'powers' to matter for behavior. Evidence against the unitary model does not, however, necessarily constitute evidence for the collective model. Unfortunately it has turned out to be difficult to devise powerful tests for the collective model. This is because such tests must rely either on a test of the quasi-Slutsky condition or the proportionality restriction on distribution factors; see subsection 5.2.1. As regards the $SNR1$ restriction (see equation (5.18)), we need price variation and at least five goods to reject symmetry. This largely restricts our ability to test for $SRN1$ in the labor supply context, although tests based on more specific assumptions - for example, exclusivity of leisure - or on different approaches - typically revealed preferences - are indeed feasible (see below). Tests based on proportionality are in general easier to implement, but they still require at least two unequivocal distribution factors.

Among the few attempts to take $SRN1$ to the data are Browning and

[13] A re-analysis of the Lundberg *et al* episode by Hotchkiss (2005) suggests that it may not be valid. The point at issue is that women in childless couples also appeared to increase their clothing expenditure in the same period. Ward-Batts (2008) convincingly contests this finding: the Hotchkiss timing is not consistent and Ward-Batts uses micro data rather than the grouped data of Lundberg *et al* and Hotchkiss.

Chiappori (1998), Dauphin *et al.* (2009) and Kapan (2009). These works share common features: they both estimate a demand system, using a well known and flexible functional form (QUAIDS) that nests both the unitary and the collective settings as specific cases (the former being itself nested within the latter). While the data sets are different (a specific feature of the Turkish data considered by Kapan is the presence of important and largely exogenous variations in relative prices, due to high inflation over the period), they reach similar conclusions; for instance, when testing Slutsky and SNR1 on three subsamples - single males, single females, and couples, they all fail to reject the unitary version on singles; it on couples, they very strongly reject the unitary version, but not SNR1. In addition, the contributions provide interesting insights on various specific aspects of intrahousehold decision processes. Both Browning and Chiappori and Dauphin *et al.* provide additional tests using distribution factors, which tend to support the collective model. Kapan finds that while most Turkish families do not behave as if there was a single decision maker, a notable exception is provided by traditional, rural households, for whom the unitary version is not rejected. Finally, both Kapan and Dauphin *et al.* find that older children (above 16) do play a role in the decision process.

The validity of proportionality tests, on the other hand, depends crucially on an *a priori* division of demographic and environmental factors between preference factors and distribution factors (a variable can be both). Typical candidate preference factors include household composition, the age of one of the spouses, the ownership of a car or a house, region of residence etc.. Typical distribution factors are listed in Table 5.1. A general concern is that the household specific variables could be correlated with constraints or preferences which would invalidate them as distribution factors; societal variables are less susceptible to this problem. Fortunately, as we have shown above (see subsection 5.2.1) we only need one unequivocal distribution factor to credibly test for proportionality. To illustrate, suppose we construct an index quantifying the extent to which laws governing divorce favor women, and we take that index as a unequivocal distribution factor. If the index is 'significant' in the choice equations, we can then test for proportionality for other candidate distribution factors. In theory, we could simply take all of the factors that satisfy the proportionality tests as distribution factors and assign other 'significant' variables as preference factors. In practice, this may not be appealing if the factor that fails the proportionality test is unlikely to be a preference factor. For example, if the situation on the marriage market (as measured for instance by the local sex ratio) impacts on demand behavior but fails the proportionality test, we would be very reluctant to designate it a preference factor. Rather, this would cast doubt on our original choice of an unequivocal distribution factor (or the collective model itself!).

There is no evidence against the collective model in the papers listed in Table 5.1. There is, however, alternative evidence against the efficiency assumption of a different sort. The most convincing evidence of inefficient outcomes is Udry (1996). This is a different style of test than SNR1 and the relevance of distribution factors. Udry uses information on household production.

To sum up: there is considerable evidence against the unitary model and some evidence in favor of the collective model.[14] What is singularly lacking in the literature are tests for the collective model against other non-unitary models for high income countries. This is part reflects the lack of non-collective models that can be taken to the data.

5.5.3 Estimating the collective model

Many of the works mentioned above go beyond testing the collective model; insofar as the predictions are not rejected, they often propose an estimation of the structural components of the model. Although this field is still largely in construction, we may briefly summarize some findings obtained so far.

Demand studies

Some of the works mentioned above go beyond testing the collective model; when the predictions are not rejected, they often propose estimation of the structural components of the model. Although this field is still largely in construction, we may briefly summarize some findings obtained so far.

Many of the papers listed in Table 5.2 use demand data alone to test for the collective model. Only three of them go beyond testing and impose the collective model restrictions and then estimate the sharing rule and how it depends on distribution factors. The first paper to do this was Browning *et al* (1994). These authors use Canadian Family Expenditure Survey data on men and women's clothing to test for the collective model restrictions and to identify the determinants of the sharing rule. Although they have price data they absorb prices into year/region dummies and treat the data as cross-sectional. Thus the 'no price variation' analysis of section 5.2 is appropriate. They only consider singles and married couples who are in fulltime employment. The distribution factors they find significant are the difference in ages and the relative earnings of the two partners; they also allow that total expenditure on nondurables and services enters the sharing rule. They address directly the problem that variations in relative earnings may be spuriously correlated with spending on clothing (higher paid jobs might require relatively more expensive clothing) by testing whether singles

[14] A notable exception to the latter are the results for efficient risk sharing in low income countries; see, for example, Dercon and Krishnan (2000), Dubois and Ligon (2005), Duflo and Udry (2003), Goldstein (2002), Ligon (2002). These tests, however, are based on specific models that crucially involve specific asusmptions regarding commitment; their discussion is therefore postponed until chapter 6 which deals with dynamic issues.

have clothing demands that depend on earnings. They find that for single men and single women, earnings do not impact on clothing demand once we take account of total expenditure. It is important to note that this does not imply that clothing demand is separable from labor supply (it is not) since they condition on both partners being in fulltime work and effectively test for whether wages affect preferences. Given the finding for singles, relative earnings are a reasonable candidate for being a distribution factor for couples. As discussed in section 5.2 we cannot generally identify the location of the sharing rule, so Browning *et al* simply set it equal to one half (at the median of total expenditure) if the two partners have the same age and earnings. They find that differences in earnings have a highly significant but quantitatively small impact on sharing: going from the wife having 25% of total earnings to 75% of total earnings shifts the sharing rule by 2.3 percentage points. Differences in age are similar with significant but small effects: going from being 10 years younger than her husband to being 10 years older raises the wife's share by two percentage points. Conversely, total expenditure (taken as a proxy for lifetime wealth) is less statistically significant but with a large effect: a 60% increase in total expenditure increases the wife's share by 12%. This suggests that wives in high wealth households have a higher share of nondurable expenditure.

Browning and Bonke (2009) use a supplement to the Danish Household Expenditure Surveys for 1999 to 2005. This supplement (designed by the authors) takes the form of respondents recording for every expenditure in a conventional expenditure diary for whom the item was bought: 'mainly for the household', 'for the husband', 'for the wife', 'for the children' and 'outside the household'. This is the first time that such information has been collected in a representative survey in a high income country. Another notable feature of these data is that they contain a richer set of potential distribution factors than most expenditure data sets. For example, questions were asked on the length of the current partnership; the labor force participation of the mothers of the husband and wife when they were 14 and the marital and fertility histories of the two partners. Since all expenditures are allocated in these data, a sharing rule can be constructed for each household. This allows for the identification of the location of the sharing rule as well as its dependence on distribution factors. These authors find that the mean of the sharing rule is very close to one half (at the mean of the data).[15] This equality of the mean total expenditures for the two partners masks that the sharing rule in different households varies widely. For example the first and third quantiles for the wife's share are 0.31 and 0.68 so that close to half of households have one partner receiving twice as

[15] This equality of total assigned expenditures is not reflected in the expenditures on individual goods. For example, the individual allocations show that, in mean, wives spend more on clothing but less on alcohol and tobacco than their husbands.

much as the other. Some of this variation can be attributed to observable differences in distribution factors but most of it is 'latent' heterogeneity.

Some of the significant distribution factors in Browning and Bonke (2009) are familiar from earlier studies; for example, if the wife has a higher share of gross income then she has a higher share of total expenditure. On the other hand, these authors do not find a significant role for the difference in age nor for total expenditure. Of more interest (because they have never been used in this context before) are the family and individual background variables. The two highly significant variables here are on whether the husband's mother was in full-time employment when he was 14 and whether the partners have children from before the partnership. A husband having grown up in a household in which his mother was in full-time employment increases his share of expenditure. This is consistent with the theory model in which such men make desirable husbands (perhaps because they contribute more in housework) and hence do better in any match than an otherwise similar male who does not have this background. The other finding is less easy to rationalize. If either the husband or the wife has a previous child then the wife's share is lower. Thus a women who has had a previous child and is married to a man who has also had a previous child receives a share of total expenditure that is about nine percentage points lower than an otherwise comparable women in which neither partner has children from before the marriage. This is a very large effect which defies easy rationalization.

Browning, Chiappori and Lewbel (2009) also present identification results and estimates of the location of the sharing rule. These are based on making the strong assumptions that the preferences of singles and married people are the same and that only the household technology changes at marriage. This allows them to identify the location of the sharing rule as well as its dependence on distribution factors. Differences between the demands of singles and couples are picked up by a Barten style technology (see section 2 of chapter 2). For example, 'transport' is largely a public good whereas 'food at home' is largely private. The data used is the same as in Browning *et al* (1994) with the important difference that explicit account is taken of price variations across time and over regions. The distribution factors are very similar to those used in Browning *et al* (1994): the wife's share in total gross income, the difference in age between husband and wife, a home-ownership dummy and household total expenditure. The point estimate for the sharing rule (at the mean of the distribution factors) is 0.65; this is much higher than found in any other study. Mechanically it arises since the budget shares of couples are more similar to those of single women than to the budget shares of single men; this suggests that some relaxing of the unchanging preferences assumption is called for in future work. Having the allocations of total expenditure to each partner allows us to calculate budget shares for husbands and wives; see Table 5.3. Wives have higher budget shares for clothing, personal services and recreation whereas hus-

Model	Budget shares (×100)	
	Wife	Husband
Food at home	13.9	20.7
Restaurants	9.9	12.8
Clothing	16.3	7.1
Alcohol and tobacco	6.3	11.8
Transport	22.0	27.6
Personal services	15.2	12.1
Recreation	16.4	7.8

TABLE 5.3. Budget shares for husbands and wives

bands have higher budget shares for food inside and outside the home, alcohol and tobacco and transport. Where comparisons can be made, this is similar to the Danish data discussed in the previous paragraph.

The results presented here on the location and determinants of the sharing rule do not sit together comfortably. This partly reflects the fact that potential distribution factors differ widely across different data sets and the excluded distribution factors are correlated with the included ones. For example, only one study can take account of the impact of previous children but this is correlated with the difference in age between the partners. More fundamentally, there is no coherent theory of the sharing rule. Without such a theory a 'kitchen sink' approach is adopted in which whatever variables are available in a particular data set are included as distribution factors (if they are not obviously preference or constraint factors) with limited explicit concern for biases due to endogeneity (a particular worry for income shares), omitted distribution factors or correlated latent heterogeneity. Equally worrying is the widespread assumption that private assignable goods are separable from public goods (see Donni (2009)). It is clear that much remains to be done and that 'much' probably requires better data than we have had available until now.

Labor supply

The first empirical estimations of a collective model of labor supply are due to Fortin and Lacroix (1997) and Chiappori, Fortin and Lacroix (2002). Using data from the 1988 PSID, the latter analyze the total number of hours worked each year by single males, single females and couples, concentrating exclusively on couples without children in which both spouses work. They consider two distribution factors, namely the state of the market for marriage, as summarized by the sex ratio computed by age and race at the state level, and the legislation governing divorce, summarized by an aggregate index with the convention that a larger value indicates laws that are more favorable to women. Their main findings can be summarized as follows:

- The distribution factors have a significant impact on both labor supplies. The signs are as predicted by the theory; that is, a higher sex ratio (denoting a smaller percentage of women on the marriage market), as well as divorce laws more favorable to women, reduce the wife's labor supply and increase the husband's, suggesting a transfer of resources to the wife. Interestingly, these effects are not present for singles; divorce laws do not impact singles' labor supplies in a significant way, whereas the sex ratio has no effect on the labor supply of single males and *increases* the labor supply of single women. Finally, the authors do not reject the prediction from the collective model that the impacts of the two factors on the two labor supplies should proportional.

- The corresponding transfers can be evaluated, since the sharing rule is identified up to an additive constant. A one percentage point increase in the sex ratio (representing roughly one standard deviation from the mean) is found to result in an annual transfer to the wife of more than $2,000, or about 5% of the average household income. Likewise, a one point increase in the Divorce Laws Index (which varies from 1 to 4, with a mean at 2.8) induces husbands to transfer and additional $4,300 to their wives. Both estimates are statistically significant at conventional levels.

- In addition, one can recover the impact of wages and non labor incomes on the sharing rule. For instance, a one dollar increase in the wife's wage rate (which is equivalent to an annual increase of about $1,750 in her labor income, at the mean of hours worked by women) translates into more income being transferred to her husband. At sample mean, the transfer amounts to more than $1,500, although this effect is not precisely estimated. Also, a one dollar increase in the husband's wage rate (equivalent to an annual increase of $2,240 in his labor income) translates into $600 being transferred to his wife, although again this effect is imprecisely estimated. Finally, a one dollar increase in household nonlabor income will increase the wife's nonlabor income by 70 cents; that non labor income goes mostly to the wife on average is actually a common finding of most empirical studies based on the collective framework.

- Finally, wage elasticities can be computed in two ways. A direct estimation gives a positive, significant elasticity for women, close to 0.2, while men's wage elasticities are very small and not statistically significant. The structural model also allows us to estimate the 'true' own-wage elasticities of individual labor supplies, taking into account the impact of wages on the sharing of nonlabor income. Both women's and men's elasticities are significant but smaller than those reported previously - reflecting the fact that a marginal increase in either spouse's wage rate reduces their share of the nonlabor income, which in turn increases their labor supply through an income effect. Indeed, both men's and women's labor supply elasticities with respect to nonlabor income are negative and significant.

Recent empirical developments involving cooperative models of labor supply include Donni (2003), which generalizes the standard approach to corner solutions and non-linear budget constraints, and Blundell, Chiappori, Magnac and Meghir (2007), who consider a model in which female labor supply is continuous whereas male labor supply is discrete; they show that the sharing rule can equally be recovered in this case. Moreau and Donni (2002) also introduce distribution factors, applied to French data, and take into account the non-linearity of taxation. Other empirical analyzes include Bloemen (2009), Clark, Couprie and Sofer (2004) and Vermeulen (2005) on Dutch, British and Belgian data respectively.

In a series of recently published papers, several authors apply the collective model to welfare issues, including the impact of changes in the tax/benefit system, in different European countries. The basic methodology, as described in Vermeulen *et al* (2006), presents interesting features. One is its scope: the approach addresses standard problems of welfare analysis of labor supply, such as non linear taxation, non convex budget sets and discrete participation decisions, within a collective framework. In addition, individual preferences are more general than in the standard collective model of labor supply (Chiappori 1988, 1992) in the sense that they allow for interactions between individual leisures (that is, the marginal utility of a spouse's leisure is a function of the other spouse's labor supply). Since individual leisures are treated as public goods, the standard identification results do not apply. The identification strategy relies on a different assumption - namely, that the 'direct' trade-off between individual leisure and consumption (disregarding the impact of the spouse's leisure) is identical for singles and married individuals, and can therefore be directly estimated from the labor supply of singles; of course the additional, 'external' effect of one spouse's leisure on the other's utility can only be estimated from the sample of married couples. This approach allows to calibrate a collective model that can then be used for welfare analysis. Myck *et al.* (2006) uses this framework to analyze the impact of a recent welfare reform in the UK, namely the introduction of the Working Families' Tax Credit (WFTC). In particular, they consider two hypothetical versions of the reform: one in which the recipient remains the main carer (as for the previous Family Credit), and another in which the benefit is paid to the main earner. The model allows to predict the impact of each version on the spouses' respective Pareto weights, and the corresponding labor supply responses; they conclude that, indeed, the two versions have different impact on individual labor supplies and ultimately welfares. Similar studies have been undertaken in various countries, including Belgium, France, Germany, Italy and Spain; the findings are summarized in Myck *et al.* (2006). Finally, Beninger *et al.* (2006) provide a systematic comparison of the evaluations of tax policy reforms made within the unitary or the collective approaches respectively. They show, in particular, that the unitary version tends to overestimate male (and underestimate female) labor supply responses vis a vis the collective counterpart; moreover, for a significant fraction of households, a tax reform that appears to be Pareto improving in the collective setting is found to reduce household utility in the unitary version - a possibility that had already be mentioned by the theoretical literature but had not received an empirical confirmation so far.

Another interesting analysis is provided by Lise and Seitz (2009), who study consumption inequality in the UK from 1968 to 2001. The main findings of the paper is that ignoring consumption inequality within the household produces misleading estimates of inequality. Using a rich version of the collective model that allows for public consumption and caring preferences, they reach to important conclusions. First, the standard analysis of inequality, based on adult equivalence scales and the implicit assumption of equal sharing of consumption within the household, underestimates the level of cross sectional consumption inequality in 1968 by 50%; the reason being that large differences in the earnings of husbands and wives translate into large intrahousehold inequality in consumption. Second, the considerable and well known rise in inequality between household during the 80s was largely offset by a drastic reduction in *intrahousehold* inequality, due to changes in female labor supply. As a result, inequality *between individual*, once (properly) computed by taking into account changes in intrahousehold allocation, turns out to be practically the same in 2000 as in 1970 - a conclusion that sharply contrasts with standard studies. Other works on intrahousehold inequality include Kalugina, Radchenko and Sofer (2009a, b) and Lacroix and Radchenko (2009).

Natural experiments can provide a rich source of applications for the collective approach to labor supply. Kapan (2009) studies the impact of a change in UK divorce laws in 2000, whereby the allocation of wealth, initially based on a principle of separate ownership of assets, shifted to 'the yardstick of equal division'. A change of this kind is a typical distribution factor; however, because of its discrete nature, the analysis cannot rely on the same technique as Chiappori, Fortin and Lacroix (2002). Kapan shows how the estimation strategy can be adapted to take advantage of discrete distribution factors. He finds that, indeed, the shift resulted in an additional transfer to women, at least when their wealth was smaller than their husband's; in turn, this reallocation had a significant impact of labor supplies and individual welfares.

Finally, models involving domestic productions have been empirically analyzed in a number of contributions. For example, Apps and Rees (1996), Rapoport, Sofer and Solaz (2004) estimate the canonical model with Australian, French and Dutch data, respectively, whereas Couprie (2007) and van Klaveren, van Praag and Maassen van den Brink (2008) consider models where the domestic good is public and present empirical results on various data sets.

5.5.4 Concluding remarks.

The empirical evidence reviewed in this chapter employs a largely static framework. Although intertemporal separability gives a justification for such an approach, dynamic issues do arise. One such issue is the stability over time of the Pareto weight (or sharing rule) which requires a coherent theory of the evolution of the Pareto weight. Another issue is human capital formation which necessarily introduces dynamics into the labor supply decisions. A third issue is accounting for the formation and dissolution of partnerships which is tightly bound up with the individual gains from a marriage. These are discussed in the next chapter and, in greater depth, in the second half of the book. For example, chapter 8 presents the implications for the within household distribution of the gains from marriage if we embed the couple in a society in which agents choose to match partly on the share they receive in equilibrium. Chapter 9 takes up the issue of human capital formation. Chapters 10 and 11 considers models with marriage and divorce.

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6

Uncertainty and Dynamics in the Collective model

The models developed in the previous chapters were essentially static and were constructed under the (implicit) assumption of perfect certainty. As discussed in chapter 2, such a setting omits one of the most important roles of marriage - namely, helping to palliate imperfections in the insurance and credit markets by sharing various risks and more generally by transferring resources both across periods and across states of the world. Risk sharing is an important potential gain from marriage: individuals who face idiosyncratic income risk have an obvious incentive to mutually provide insurance. In practice, a risk sharing scheme involves intrahousehold transfers that alleviate the impact of shocks affecting spouses; as a result, individual consumptions within a couple may be less responsive to idiosyncratic income shocks than it would be if the persons were single. Not only are such risk-sharing mechanisms between risk averse agents welfare improving, but they allow the household to invest into higher risk/higher return activities; as such, they may also increase total (expected) income and wealth in the long run. For instance, the wife may be able to afford the risk involved in creating her own business because of the insurance implicitly offered by her husband's less risky income stream.

Another, and closely related form of consumption smoothing stems from intrafamily credit relationship: even in the absence of a perfect credit market, a spouse can consume early a fraction of her future income thanks to the resources coming from her partner. Again, intrahousehold credit may in turn enable agents to take advantage of profitable investment opportunities that would be out of the reach of a single person.

While intertemporal and risk sharing agreements play a key role in economic life in general and in marriage in particular, they also raise specific difficulties. The main issue relates to the agents' ability to credibly commit to specific future behavior. Both types of deals typically require that some agents reduce their consumption in either some future period or some possible states of the world. This ability to commit may however not be guaranteed. In some case, it is even absent (or severely limited); these are cases in which the final agreement typically fails to be fully efficient, at least in the *ex ante* sense.[1]

The theoretical analysis underpinning these issues leads to fascinating empirical questions. Again, these can be formulated in terms of testability and identifiability. When, and how, is it possible to test the assumption of perfect commitment, and more generally of *ex ante* efficiency? And to what extent is it possible to recover the underlying structure - namely individual preferences (here, aversions to risk and/or fluctuations) and the decision process (here, the Pareto weights) from observed behavior? These questions - and others - are analyzed in the present chapter.

6.1 Is commitment possible?

We start with a brief discussion of the commitment issue. As discussed above, credit implies repayment, and the very reason why a formal credit market may fail to be available (say, non contractible investments) may result in enforcement problems even between spouses. As the usual cliche goes, a woman will be hesitant to support her husband through medical school if she expects him to break the marriage and marry a young nurse when he finishes (this is a standard example of the hold-up problem). Similarly, risk sharing requires possibly important transfers between spouses; which enforcement devices can guarantee that these transfers will actually take place when needed is a natural question. In subsequent sections we shall consider conventional economic analyzes of the commitment problem as they relate to the family. In the remainder of this section we consider possible commitment mechanisms that are specific to the family.

From a game-theoretic perspective, marriage is a typical example of repeated interactions between the same players; we know that cooperation is easier to support in such contexts.[2] This suggests that, in many case, cooperation is a natural assumption. Still, the agents' ability to commit is probably not unbounded. Love may fade away; fidelity is not always limitless; commitment is often constrained by specific legal restrictions (for

[1] A second problem is information: in general, efficient trade is much easier to implement in a context of symmetric information. Asymmetric information, however, is probably less problematic in households than in other types of relationship (say, between employers and employees or insurers and insurees), because the very nature of the relationship often implies deep mutual knowledge and improved monitoring ability.

[2] Del Boca and Flinn (2009) formulate a repeated game for time use that determines the amount of market work and housework that husbands and wives perform. Their preferred model is a cooperative model with a noncooperative breakdown point. They have a repeated game with a trigger strategy for adopting the inefficient non-cooperative outcome if the discount is too small. The value of the threshold discount factor they estimate to trigger noncooperative behavior is 0.52which implies that 94% of households behave cooperatively.

instance, agents cannot legally commit not to divorce).[3] And while the repeated interaction argument for efficiency is convincing in many contexts, it may not apply to some important decisions that are made only exceptionally; moving to a different location and different jobs is a standard example, as argued by Lundberg and Pollak (2003).

A crucial aspects of lack of commitment is that, beyond restraining efficiency in the *ex ante* sense, it may also imply *ex post* inefficiencies. The intuition is that whenever the parties realize the current agreement will be renegotiated in the future, they have strong incentives to invest now into building up their future bargaining position. Such an investment is in general inefficient from the family's viewpoint, because it uses current resources without increasing future (aggregate) income. For instance, spouses may both invest in education, although specialization would be the efficient choice, because a high reservation wage is a crucial asset for the bargaining game that will be played later.[4]

Love and all these things

How can commitment be achieved when the repeated interaction argument does not hold? Many solutions can actually be observed. First, actual contracts can be (and actually are) signed between spouses. Prenuptial agreements typically specify the spouses' obligations both during marriage and in case of divorce; in particular, some provisions may directly address the hold-up problem. To come back to the previous example, a woman will less be hesitant fund her husband's training if their prenuptial agreement stipulates that she will receive, in case of divorce, a large fraction of his (future) income. Contract theory actually suggests that even if long term agreements are not feasible, efficiency can in general be reached through a sequence of shorter contracts that are regularly renegotiated (see for instance Rey and Salanié (1990).[5] Still, even though a private, premarital agreement may help alleviating the limits to commitments (say, by making divorce very expensive for one of the parties), renegotiation proofness may be an issue, especially if divorce has been made costly for both spouses; furthermore, in some countries courts are free to alter *ex post* the terms of premarital agreements. At any rate, some crucially important intrahousehold issues may hardly be contractible.

[3] Of course, *moral* or *religious* commitment not to divorce do exist, although they may not be globally prevalent.

[4] See Konrad and Lommerud (2000) and Brossolet (1993).

[5] In practice, prenuptial agreements are not common (although they are more frequently observed in second marriages). However, this may simply indicate that, although easily feasible, they are rarely needed, possibly because existing enforcement mechanisms (love, trust, repeated interactions) are in general sufficient. Indeed, writing an explicit contract that lists all contingencies may in fact "crowd out" the emotional bonds and diminish the role of the initial spark of blind trust that is associated with love.

Alternative enforcement mechanisms can however be implemented. Religious or ethical factors may be important; in many faiths (and in several social groups), a person's word should never be broken. Love, affection and mutual respect are obviously present in most marriages, and provide powerful incentives to honoring one's pledge and keeping one's promises. Browning (2009) has recently provided a formalization of such a mechanism. The model is developed in the specific context of the location decision model of Mincer (1978) and Lundberg and Pollak (2003) but has wider application. In the location model a couple, a and b, are presented with an opportunity to increase their joint income if they move to another location. Either partner can veto the move. The problem arises when the move shifts power within the household toward one partner (partner b, say); then the other partner (a) will veto the move if she is worse off after the move. Promises by partner b are not incentive compatible since a does not have any credible punishment threat.[6] Particular commitment mechanisms may be available in this location decision model. For example, suppose there is a large indivisible choice that can be taken at the time of moving; choosing a new house is the obvious example. If this choice has a large element of irreversibility then partner b can defer to a on this choice and make the move more attractive. At some point, however, commitment devices such as this may be exhausted without persuading a that the move is worthwhile. Now assume that spouses are caring, in the usual sense that their partner's utility enter their preferences. Browning (2009) suggests that if one partner exercises too aggressively their new found bargaining power then the other partner feels betrayed and loses some regard (or love) for them. The important element is that this loss of love (by a in this case) is out of the control of the affected partner; in this sense, this is betrayal. Thus the threat is credible. In a model with mutual love, this 'punishment' is often sufficient to deter a partner from exercising their full bargaining power if the move takes place.

To formalize, consider a married couple a and b. Income, which is normalised to unity if they do not move, is divided between them so that a receives x for private consumption and b receives $1-x$. There are no public goods. Each person has the same strictly increasing, strictly concave felicity function, so that:

$$u^a = u(x), \ u^b = u(1-x) \tag{6.1}$$

Each person also cares for the other with individual utility functions given

[6] We neglect the option in which they divorce and the husband moves to the new location. Mincer (1978) explicitly considers this.

by:

$$\begin{aligned} W^a &= u^a + \delta^a u^b \\ &= u(x) + \delta^a u(1-x) \end{aligned} \tag{6.2}$$

$$\begin{aligned} W^b &= \delta^b u^a + u^b \\ &= \delta^b u(x) + u(1-x) \end{aligned} \tag{6.3}$$

where $\delta^s \geq 0$ is person s's caring for the other person, with $\delta^a \delta^b < 1$ (see chapter 3). We assume that the caring parameters are constant and outside the control of either partner. Rather than choosing an explicit game form to choose x, we simply assume that there is some (collective) procedure that leads the household to behave as though it maximizes the function:

$$\begin{aligned} W &= W^a + \mu W^b \\ &= \left(1 + \mu\delta^b\right) u(x) + \left(\delta^a + \mu\right) u(1-x) \end{aligned} \tag{6.4}$$

As discussed in chapter 4, caring modifies the Pareto weight for b to an effective value of $\left(\delta^a + \mu\right) / \left(1 + \mu\delta^b\right)$.

Now suppose there is a (moving) decision that costlessly increases household income from unity to $y > 1$. If this is the only effect then, of course, both partners would agree to move. However, we also assume that the decision increases b's Pareto weight to $\mu(1+m)$ where $m \geq 0$. In this case there is a reservation income $y^*(m)$ such that person a will veto the move if and only if $y < y^*(m)$. In such a case there will be unrealized potential Pareto gains. Now allow that the husband can choose whether or not to exercise his new found power if they do move. If he does *not* exercise his new power then the household utility function is given by:

$$W = \left(1 + \mu\delta^b\right) u(x) + \left(\delta^a + \mu\right) u(y-x) \tag{6.5}$$

which obviously dominates (6.4). Of course, a simple statement that "I promise to set $m = 0$" has no credibility. Suppose, however, that if such a promise is made and then broken, then the wife feels betrayed. In this case her love for her husband falls from δ^a to $\delta^a(1-\sigma)$ where $\sigma \in [0,1]$. The fall in her caring for him is taken to be out of her control, so that a has an automatic and hence credible punishment for b choosing to take advantage of his improved position. If they move and the husband exercises his new power the household utility function is given by:

$$W = \left(1 + \mu(1+m)\delta^b\right) u(x) + \left(\delta^a(1-\sigma) + (1+m)\mu\right) u(y-x) \tag{6.6}$$

If the husband's implicit Pareto weight is less in this case than in (6.5) then he will not betray his wife. In the simple case in which he does not care for her ($\delta^b = 0$) this will be the case if:

$$\begin{aligned} (\delta^a + \mu) &\geq (\delta^a (1 - \sigma) + (1 + m)\,\mu) \\ &\Leftrightarrow \delta^a \sigma \geq m\mu \end{aligned} \tag{6.7}$$

That is, there will be a move with no betrayal if δ^a and σ are sufficiently large relative to m and μ. For example, a husband who lacks power (and hence relies on his wife's caring for resources) or has a small increase in power (so that $m\mu$ is small) will be less likely to betray; and the same holds if his wife cares a lot for him (δ^a) and she feels the betrayal strongly (σ close to unity).

Psychological games

A different but related analysis is provided by Dufwenberg (2002), who uses "psychological games" to discuss commitment in a family context. The basic idea, due to Geanakoplos *et al* (1989), is that the utility payoffs of married partners depend not only on their actions and the consequences in terms of income or consumption but also on the beliefs that the spouses may have on these actions and consequences. The basic assumption is that the stronger is the belief of a spouse that their partner will act in a particular manner, the more costly it is for that partner to deviate and disappoint their spouse. This consideration can be interpreted as guilt. A crucial restriction of the model is that, in equilibrium, beliefs should be consistent with the actions. Dufwenberg (2002) uses this idea in a context in which one partner (the wife) extends credit to the other spouse. For instance, the wife may work when the husband is in school, expecting to be repaid in the form of a share from the increase in family income (see Chapter 2). But such a repayment will occur only if the husband stays in the marriage, which may not be the case if he is unwilling to share the increase in his earning power with his wife and walks away from the marriage.

Specifically, consider again the two period model discussed in Chapter 2. There is no borrowing or lending and investment in schooling is lumpy. In the absence of investment in schooling, each spouse has labor income of 1 each period. There is also a possibility to acquire some education; if a person does so then their earnings are zero in the first period and 4 in the second period. We assume that preferences are such that in each period each person requires a consumption of $\frac{1}{2}$ for survival and utility is linear in consumption otherwise. This implies that without borrowing, no person alone can undertake the investment, while marriage enables the couple to finance the schooling investment of one partner. We assume that consumption in each period is divided equally between the two partners if they are together and that if they are divorced then each receives their own income. Finally, suppose that each partner receives a non monetary gain from companionship of $\theta = 0.5$ for each period they are together. The lifetime payoff if neither educate is $(2 + 2\theta) = 3$ for each of them. Since both have the same return to education, for ease of exposition we shall assume that they only consider the husband taking education.[7] If he does educate and they stay together then each receives a total of $(3+2\theta)=4$ over the two periods. There is thus a potential mutual gain for both of them if the investment is undertaken and marriage continues. However, if the husband educates and then divorces, he receives a payoff of 4 in the second period and if he stays he receives only 3 $(=2.5+\theta)$. Thus, without commitment, he would leave in the second period[8] and the wife will then be left with a lifetime utility of 2 which is less than she would have in the absence of investment, 3. Therefore, the wife would not agree to finance her husband's education in the first period. The basic dilemma is illustrated in Figure 6.1, where the payoffs for the wife are at the top of each final node and the payoffs for the husband are at the bottom. The only equilibrium in this case is that the wife does not support her husband, the husband does not invest in schooling and stays in the marriage so that the family ends up in an inefficient equilibrium. However, Dufwenberg (2002) then shows that if one adds guilt as a consideration, an efficient equilibrium with consistent beliefs can exist. In particular, suppose that the husband's payoff following divorce is $4-\gamma\tau$ where τ is the belief of the husband at the beginning of period 2 about the beliefs that his wife formed at the time of marriage, about the probability that her husband will stay in the marriage following her investment and γ is a fixed parameter. Then, if $\gamma>2$, the husband chooses to stay in the marriage, the wife agrees to support her husband to invest and efficiency is attained. To show the existence of consistent beliefs that support this equilibrium, consider the special case in which $\gamma=2$. Suppose that the wife actually invests, as we assume for this equilibrium. Then, she reveals to her husband that she expects to get a life time utility of at least 3 following this choice, which means that her belief, τ' about the probability that the husband would stay is such that $1+\tau'4>3$, implying $\tau'\geq\frac{1}{2}$. Knowing that, the husband's belief τ about her belief that he stays exceeds $\frac{1}{2}$. Therefore, his payoff upon leaving in the second period $4-2\tau$ is less or equal to his payoff if he stays, 3. Thus for any γ strictly above 2, he stays. In short, given that the wife has shown great trust in him, as indicated by her choice to support him, and given that he cares a great deal about that, as indicated by the large value of γ, the husband will feel more guilty about disappointing her and will in fact stay in the marriage, justifying his wife's initial beliefs. The husband on, his part, avoids all feelings of guilt and efficient investment will be attained. A

[7] The issue of what happens if the two have different returns to education is one that deserves more attention.

[8] Note that if the match quality is high enough then he will not divorce even if he educates. In the numerical example this will be the case if $\theta>1.5$. In this case there is no need for commitment. This is analogous to the result concerning match quality and children.

happy marriage indeed.

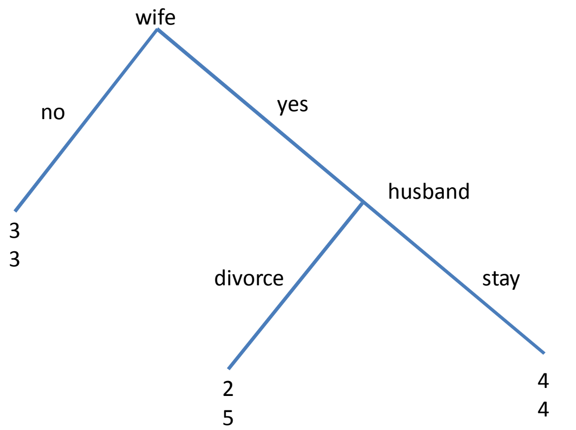

FIGURE 6.1. Game tree for investment in education

Somewhat different considerations arise when we look at 'end game' situations in which the spouse has no chance to reciprocate. A sad real example of this sort is when the husband has Altzheimer's and his wife takes care of him for several (long) years, expecting no repayment from him whatsoever as he does not even know her. Here, the proper assumption appears to be that she believes that he would have done for her the same thing had the roles been reversed. Unfortunately, the consistency of such beliefs is impossible to verify. Another possibility is that she cares about him and about her children that care about him to the extent that caring for the sick husband in fact gives her satisfaction. In either case, some emotional considerations must be introduced to justify such cases of unselfish behavior in families.

The commitment issue is complex. In the end, whether agents are able to implement and enforce a sufficient level of commitment to achieve *ex ante* efficiency is an empirical issue. Our task, therefore, is to develop conceptual tools that allow a precise modeling of these problems, and empirical tests that enable us to decide whether, and to what extent, the lack of commitment is an important problem for household economics.

6.2 Modeling commitment

6.2.1 Full commitment

Fortunately enough, the tools developed in the previous chapters can readily be extended to modeling the commitment issues. We start from the full commitment benchmark. The formal translation is very simple: under full commitment, *Pareto weights remain constant,* either over periods or over states of the world (or both). To see why, consider for instance the risk sharing framework with two agents. Assume that there exists S states of the world, with respective probabilities $\pi_1, ..., \pi_S$ (with $\sum_s \pi_s = 1$); let y_s^a denote member a's income is state s. Similarly, let $\mathbf{p}_s$ (resp. $\mathbf{P}_s$) be the price vector for private (public) goods in state s, and $\mathbf{q}_s^a$ (resp. $\mathbf{Q}_s$) the vector of private consumption by member a (the vector of household public consumption). An allocation is *ex ante efficient* if it solves a program of the type:

$$\begin{aligned} &\max_{\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b} \sum_s \pi_s u^a\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) \\ &\text{subject to } \mathbf{P}_s' \mathbf{Q}_s + \mathbf{p}_s'\left(\mathbf{q}_s^a + \mathbf{q}_s^b\right) \leq \left(y_s^a + y_s^b\right) \text{ for all } s \\ &\text{and } \sum_s \pi_s u^b\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) \geq \bar{u}^b \end{aligned} \tag{6.8}$$

for some $\bar{u}^b$. As in chapter 3, if μ denotes the Lagrange multiplier of the last constraint, this program is equivalent to:

$$\begin{aligned} &\max_{\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b} \sum_s \pi_s u^a\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) + \mu \sum_s \pi_s u^b\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) \\ &\text{subject to } \mathbf{P}_s' \mathbf{Q}_s + \mathbf{p}_s'\left(\mathbf{q}_s^a + \mathbf{q}_s^b\right) \leq \left(y_s^a + y_s^b\right) \text{ for all } s \end{aligned}$$

or:

$$\begin{aligned} &\max_{\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b} \sum_s \pi_s \left[u^a\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) + \mu u^b\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right)\right] \\ &\text{subject to } \mathbf{P}_s' \mathbf{Q}_s + \mathbf{p}_s'\left(\mathbf{q}_s^a + \mathbf{q}_s^b\right) \leq \left(y_s^a + y_s^b\right) \text{ for all } s \end{aligned} \tag{6.9}$$

This form shows two things. First, for any state s, the allocation contingent on the realization of this state, $\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right)$, maximizes the weighted sum of utilities $u^a\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) + \mu u^b\left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right)$ under a resource constraint. As such, it is efficient in the *ex post* sense: there is no alternative allocation $\left(\bar{\mathbf{Q}}_s, \bar{\mathbf{q}}_s^a, \bar{\mathbf{q}}_s^b\right)$ that would improve both agents' welfare *in state s*. Secondly, the weight μ is the same across states of the world. This guarantees *ex ante* efficiency: there is no alternative allocation

$$\left[\left(\bar{\mathbf{Q}}_1, \bar{\mathbf{q}}_1^a, \bar{\mathbf{q}}_1^b\right), ..., \left(\bar{\mathbf{Q}}_S, \bar{\mathbf{q}}_S^a, \bar{\mathbf{q}}_S^b\right)\right]$$

that would improve both agents' welfare *in expected utility terms* - which is exactly the meaning of programs (6.8) and (6.9).

Finally, note that the intertemporal version of the problem obtains simply by replacing the state of the world index s by a time index t and the probability π_s of state s with a discount factor - say, δ^t.

6.2.2 Constraints on commitment

Limits to commitment can generally be translated into additional constraints in the previous programs. To take a simple example, assume that in each state of the world, one member - say b - has some alternative option that he cannot commit not to use. Technically, in each state s, there is some lower bound $\bar{u}_s^b$ for b's utility; here, $\bar{u}_s^b$ is simply the utility that b would derive from his fallback option. This constraint obviously reduces the couple's ability to share risk. Indeed, it may well be the case that, in some states, efficient risk sharing would require b's welfare to go below this limit. However, a contract involving such a low utility level in some states is not implementable, because it would require from b more commitment than what is actually available.

The technical translation of these ideas is straightforward. Introducing the new constraint into program (6.8) gives:

$$\max_{\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b} \sum_s \pi_s u^a \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right)$$

$$\text{subject to } \mathbf{P}_s'\mathbf{Q}_s + \mathbf{p}_s' \left(\mathbf{q}_s^a + \mathbf{q}_s^b\right) \leq \left(y_s^a + y_s^b\right) \text{ for all } s, \tag{6.10}$$

$$\sum_s \pi_s u^b \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) \geq \bar{u}^b$$

$$\text{and } u^b \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) \geq \bar{u}_s^b \text{ for all } s \tag{6.11}$$

Let μ_s denote the Lagrange multiplier of constraint (C_s); the program can be rewritten as:

$$\max_{\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b} \sum_s \pi_s u^a \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) + \sum_s \left(\mu\pi_s + \mu_s\right) u^b \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right)$$
$$\text{subject to } \mathbf{P}_s'\mathbf{Q}_s + \mathbf{p}_s' \left(\mathbf{q}_s^a + \mathbf{q}_s^b\right) \leq \left(y_s^a + y_s^b\right) \text{ for all } s$$

or equivalently:

$$\max_{\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b} \sum_s \pi_s \left[u^a \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right) + \left(\mu + \frac{\mu_s}{\pi_s}\right) u^b \left(\mathbf{Q}_s, \mathbf{q}_s^a, \mathbf{q}_s^b\right)\right] \tag{6.12}$$
$$\text{subject to } \mathbf{P}_s'\mathbf{Q}_s + \mathbf{p}_s' \left(\mathbf{q}_s^a + \mathbf{q}_s^b\right) \leq \left(y_s^a + y_s^b\right) \text{ for all } s$$

Here, *ex post* efficiency still obtains: in each state s, the household maximizes the weighted sum $u^a + \left(\mu + \frac{\mu_s}{\pi_s}\right) u^b$. However, the weight is no longer constant; in any state s in which constraint (6.11) is binding, implying that $\mu_s > 0$, b's weight is increased by μ_s/π_s. Intuitively, since b's utility cannot go below the fallback value $\bar{u}_s^b$, the constrained agreement inflates b's Pareto weight in these states by whichever amount is necessary to make b just indifferent between the contract and his fallback option. Obviously, this new contract is not efficient in the *ex ante* sense; it is only second best efficient, in the sense that no alternative contract can do better for both spouses without violating the constraints on commitment.

6.2.3 Endogenous Pareto weights

Finally, assume as in Basu (2006), that the fallback utility $\bar{u}_s^b$ is *endogenous*, in the sense that it is affected by some decision made by the agents. For instance, $\bar{u}_s^b$ depends on the wage b would receive on the labor market, which itself is positively related to previous labor supply (say, because of human capital accumulation via on the job training). Now, in the earlier periods b works for two different reasons. One is the usual trade-off between leisure and consumption: labor supply generates an income that can be spent on consumption goods. The second motive is the impact of current labor supply on future bargaining power; by working today, an agent can improve her fallback option tomorrow, therefore be able to attract a larger share of household resources during the renegotiation that will take place then, to the expenses of her spouse. The first motive is fully compatible with (static) efficiency; the second is not, and results in overprovision of labor with respect to the optimum level.

We can capture this idea in a simple, intertemporal version of the previous framework. Namely, consider a two-period model with two agents and two commodities, and assume for simplicity that agents are egoistic:

$$\max_{\mathbf{q}_t^a, \mathbf{q}_t^b} \sum_{t=1}^{2} \delta^{t-1} u^a\left(\mathbf{q}_t^a\right)$$

$$\text{subject to } \mathbf{p}_t'\left(\mathbf{q}_t^a + \mathbf{q}_t^b\right) \leq \left(y_t^a + y_t^b\right) \text{ for } t = 1, 2, \tag{6.13}$$

$$\sum_{t=1}^{2} \delta^{t-1} u^b\left(\mathbf{q}_t^b\right) \geq \bar{u}^b$$

$$\text{and } u^b\left(\mathbf{q}_2^b\right) \geq \bar{u}_2^b \tag{6.14}$$

where $\mathbf{q}_t^X = \left(q_{1,t}^X, q_{2,t}^X\right), X = a, b$; note that we assume away external financial markets by imposing a resource constraint at each period. Assume, moreover, that the fallback option $\bar{u}_2^b$ of b in period 2 is a decreasing function of $q_{1,t}^b$; a natural interpretation, suggested above, is that commodity 1 is leisure, and that supplying labor at a given period increases future potential wages, hence the person's bargaining position. Now the Lagrange multiplier of (6.14), denoted μ_2, is also a function of $q_{1,t}^b$. The program becomes:

$$\max_{\mathbf{q}_t^a,\mathbf{q}_t^b} \sum_{t=1}^{2} \delta^{t-1} u^a\left(\mathbf{q}_t^a\right) + \mu \sum_{t=1}^{2} \delta^{t-1} u^b\left(\mathbf{q}_t^b\right) + \mu_2\left(q_{1,t}^b\right) u^b\left(\mathbf{q}_2^a, \mathbf{q}_2^b\right)$$
$$\text{subject to } \mathbf{p}_t'\left(\mathbf{q}_t^a + \mathbf{q}_t^b\right) \leq \left(y_t^a + y_t^b\right) \text{ for } t = 1,2$$

or equivalently:

$$\max_{\mathbf{q}_t^a,\mathbf{q}_t^b} \left[u^a\left(\mathbf{q}_1^a\right) + \mu u^b\left(\mathbf{q}_1^b\right)\right] + \delta\left[u^a\left(\mathbf{q}_2^a\right) + \mu u^b\left(\mathbf{q}_2^b\right)\right] + \mu_2\left(q_{1,t}^b\right) u^b\left(\mathbf{q}_2^b\right)$$
$$\text{subject to } \mathbf{p}_t'\left(\mathbf{q}_t^a + \mathbf{q}_t^b\right) \leq \left(y_t^a + y_t^b\right) \text{ for } t = 1,2.$$

The first order conditions for $q_{1,1}^b$ are:

$$\mu \frac{\partial u^b\left(\mathbf{q}_1^b\right)}{\partial q_{1,1}^b} = \lambda p_{1,t} - u^b\left(\mathbf{q}_2^b\right) \frac{d\mu_2\left(q_{1,t}^b\right)}{dq_{1,t}^b}$$

which does not coincide with the standard condition for static efficiency because of the last term. Since the latter is positive, the marginal utility of leisure is above the optimum, reflecting under-consumption of leisure (or oversupply of labor). In other words, both spouses would benefit from an agreement to reduce both labor supplies while leaving Pareto weights unchanged.

6.3 Efficient risk sharing in a static context

6.3.1 The collective model under uncertainty

Ex ante and *ex post* efficiency

We can now discuss in a more precise way the theoretical and empirical issues linked with uncertainty and risk sharing. For that purpose, we specialize the general framework sketched above by assuming that consumptions are private, and agents have egoistic preferences. We first analyze a one-commodity model; then we consider an extension to a multi-commodity world.

We consider a model in which two risk averse agents, a and b, share income risks through specific agreements. There are N commodities and S states of the world, which realize with respective probabilities $(\pi_1, ..., \pi_S)$. Agent X ($X = a, b$) receives in each state s some income y_s^X, and consumes a vector $c_s^X = \left(c_{s,1}^X, ..., c_{s,N}^X\right)$; let $p_s = (p_{s,1}, ..., p_{s,N})$ denote the price vector in state s. Agents are expected utility maximizers, and we assume that their respective Von Neumann-Morgenstern utilities are strictly concave, that is that agents are strictly risk averse.

The efficiency assumption can now take two forms. Ex post efficiency requires that, *in each state s of the world*, the allocation of consumption is efficient in the usual, static sense: no alternative allocation could improve both utilities at the same cost. That is, the vector $c_s = \left(c_s^a, c_s^b\right)$ solves:

$$\max u^a \left(c_s^a\right) \tag{6.15}$$

under the constraints:

$$\begin{aligned} u^b \left(c_s^b\right) &\geq \bar{u}_s^b \\ \sum_i p_{i,s} \left(c_{i,s}^a + c_{i,s}^b\right) &= y_s^a + y_s^b = y_s \end{aligned}$$

As before, we may denote by μ_s the Lagrange multiplier of the first constraint; then the program is equivalent to:

$$\max u^a \left(c_s^a\right) + \mu_s u^b \left(c_s^b\right)$$

under the resource constraint. The key remark is that, in this program, the Pareto weight μ_s of member b may depend on s. Ex post efficiency requires static efficiency *in each state*, but imposes no restrictions on behavior *across states.*

Ex ante efficiency requires, in addition, that the allocation of resources across states is efficient, in the sense that no state-contingent exchange can improve both agents' expected utilities. Note that, now, welfare is computed *ex ante*, in expected utility terms. Formally, the vector $c = (c_1, ..., c_S)$ is efficient if it solves a program of the type:

$$\max \sum_s \pi_s u^a \left(c_s^a\right) \tag{6.16}$$

under the constraints:

$$\sum_s \pi_s u^b \left(c_s^b\right) \geq \bar{u}^b \tag{6.17}$$

$$\sum_i p_{i,s} \left(c_{i,s}^a + c_{i,s}^b\right) = y_s^a + y_s^b = y_s,\ s = 1, ..., S \tag{6.18}$$

Equivalently, if μ denotes the Lagrange multiplier of the first constraint, the program is equivalent to:

$$\max \sum_s \pi_s u^a \left(c_s^a\right) + \mu \sum_s \pi_s u^b \left(c_s^b\right) = \sum_s \pi_s \left[u^a \left(c_s^a\right) + \mu u^b \left(c_s^b\right)\right]$$

under the resource constraint (6.18).

One can readily see that any solution to this program also solves (6.15) for $\mu_s = \mu$. But *ex ante* efficiency generates an additional constraint - namely, *the Pareto weight μ should be the same across states.* A consequence of this requirement is precisely that risk is shared efficiently between agents.

The sharing rule as a risk sharing mechanism

We now further specify the model by assuming that prices do not vary:

$$p_s = p, \; s = 1, ..., S$$

Let V^X denote the indirect utility of agent X. For any *ex post* efficient allocation, let ρ_s^X denote the total expenditure of agent X in state s:

$$\rho_s^X = \sum_i p_i c_{s,i}^X$$

Here as above, ρ^X is the *sharing rule* that governs the allocation of household resources between members. Obviously, we have that $\rho_s^a + \rho_s^b = y_s^a + y_s^b = y_s$. If we denote $\rho_s = \rho_s^a$, then $\rho_s^b = y_s - \rho_s$. Program (6.16) becomes:

$$W(y_1, ..., y_S; \mu) = \max_{\rho_1, ..., \rho_S} \sum_s \pi_s V^a(\rho_s) + \mu \sum_s \pi_s V^b(y_s - \rho_s) \tag{6.19}$$

In particular, in the absence of price fluctuations, the risk sharing problem is one-dimensional: agents transfer one ‘commodity’ (here dollars) across states, since they are able to trade it for others commodities on markets once the state of the world has been realized, in an *ex post* efficient manner.

When is a unitary representation acceptable?

The value of the previous program, $W(y_1, ..., y_S; \mu)$, describes the household's attitude towards risk. For instance, an income profile $(y_1, ..., y_S)$ is preferred over some alternative $(y_1', ..., y_S')$ if and only if $W(y_1, ..., y_S; \mu) \geq W(y_1', ..., y_S'; \mu)$. Note, however, that preferences in general depend on the Pareto weight μ. That is, it is usually the case that profile $(y_1, ..., y_S)$ may be preferred over $(y_1', ..., y_S')$ for some values of μ but not for others. In that sense, W cannot be seen as a unitary household utility: the ranking over income profiles induced by W varies with the intrahousehold distribution of powers (as summarized by μ), which in turns depends on other aspects (*ex ante* distributions, individual reservation utilities,...).

A natural question is whether exceptions can be found, in which the household's preferences over income profiles would *not* depend on the member's respective powers. A simple example can convince us that, indeed, such exceptions exist. Assume, for instance, that both VNM utilities are logarithmic:

$$V^a(x) = V^b(x) = \log x$$

Then (6.19) can be written as:

$$\max_{\rho_1, ..., \rho_S} \sum_s \pi_s \log(\rho_s) + \mu \sum_s \pi_s \log(y_s - \rho_s) \tag{6.20}$$

First order conditions give

$$\frac{\pi_s}{\rho_s} = \frac{\mu \pi_s}{y_s - \rho_s}$$

therefore

$$\rho_s = \frac{y_s}{1 + \mu}$$

Plugging into (6.20), we have that:

$$\begin{aligned} W(y_1, ..., y_S; \mu) &= \sum_s \pi_s \log \left(\frac{y_s}{1+\mu} \right) + \mu \sum_s \pi_s \log \left(\frac{\mu y_s}{1+\mu} \right) \\ &= \sum_s \pi_s \left[\left(\log \frac{1}{1+\mu} + \log y_s \right) + \mu \left(\log \frac{\mu}{1+\mu} + \log y_s \right) \right] \\ &= k(\mu) + 2 \sum_s \pi_s \log y_s \end{aligned}$$

where

$$k(\mu) = \log \frac{1}{1+\mu} + \mu \log \frac{\mu}{1+\mu}$$

and we see that maximizing W is equivalent to maximizing $\sum_s \pi_s \log y_s$, which does not depend on μ. In other words, the household's behavior under uncertainty is equivalent to that of a *representative agent*, whose VNM utility, $V(x) = \log x$, is moreover the same as that of the individual members. Equivalently, the unitary approach - which assumes that the household behaves as if there was a single decision maker - is actually valid in that case.

How robust is this result? Under which general conditions is the unitary approach, based on a representative agent, a valid representation of household behavior under risk? Mazzocco (2004) shows that one condition is necessary and sufficient; namely, individual utilities must belong to the ISHARA class. Here, ISHARA stands for 'Identically Shaped Harmonic Absolute Risk Aversion', which imposes two properties:

- individual VNM utilities are of the harmonic absolute risk aversion (HARA) type, characterized by the fact that the index of absolute risk aversion, $-u''(x)/u'(x)$, is an harmonic function of income:

$$-\frac{u''(x)}{u'(x)} = \frac{1}{\gamma x + c}$$

  For $\gamma = 0$, we have the standard, constant absolute risk aversion (CARA). For $\gamma = 1$, we have an immediate generalization of the log form just discussed:

$$u^i(x) = \log\left(c^i + x\right)$$

for some constants $c^i, i = a, b$. Finally, for $\gamma \neq 0$ and $\gamma \neq 1$, we have:

$$u^i(x) = \frac{\left(c^i + \gamma^i x\right)^{1-1/\gamma^i}}{1 - 1/\gamma^i}$$

for some constants c^i and $\gamma^i, i = a, b$.

- moreover, the 'shape' coefficients γ must be equal:

$$\gamma^a = \gamma^b$$

The intuition of this result is that in the ISHARA case, the sharing rule that solves (6.19) is an affine function of realized income. Note that ISHARA is not simply a property of each utility independently: the second requirement imposes a compatibility restriction between them. That said, CARA utilities always belong to the ISHARA class, even if their coefficients of absolute risk aversion are different (that's because they correspond to $\gamma^a = \gamma^b = 0$). On the other hand, constant relative risk aversion (CRRA) utilities, which correspond to $c^a = c^b = 0$, are ISHARA if and only if the coefficient of relative risk aversion, equal to the shape parameter γ^i in that case, is identical for all members (it was equal to one for both spouses in our log example).

6.3.2 Efficient risk sharing in a one-commodity world

Characterizing efficient risk sharing

We now characterize *ex ante* efficient allocations. We start with the case in which prices do not vary; as seen above, we can then model efficient risk sharing in a one commodity context. A sharing rule ρ shares risk efficiently if it solves a program of the form:

$$\max_{\boldsymbol{\rho}} \sum_s \pi_s \left[u^a\left(\rho\left(y_s^a, y_s^b\right)\right) + \mu u^b\left(y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right)\right]$$

for some Pareto weight μ. The first order condition gives:

$$u'^a\left(\rho\left(y_s^a, y_s^b\right)\right) = \mu . u'^b\left(y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right)$$

or equivalently:

$$\frac{u'^a(\rho_s)}{u'^b(y_s - \rho_s)} = \mu \text{ for each } s \tag{6.21}$$

where $y_s = y_s^a + y_s^b$ and $\rho_s = \rho\left(y_s^a, y_s^b\right)$.

This relationship has a striking property; namely, since μ is constant, the left hand side does not depend on the state of the world. This is a standard characterization of efficient risk sharing: *the ratio of marginal utilities of income of the agents remains constant across states of the world.*

The intuition for this property is easy to grasp. Assume there exists two states s and s' such that the equality does not hold - say:

$$\frac{u'^a(\rho_s)/d\rho}{u'^b(y_s-\rho_s)/d\rho} < \frac{u'^a(\rho_{s'})/d\rho}{u'^b(y_{s'}-\rho_{s'})/d\rho}$$

Then there exists some k such that

$$\frac{\pi_s u'^a(\rho_s)/d\rho}{\pi_{s'} u'^a(\rho_{s'})/d\rho} < k < \frac{\pi_s u'^b(y_s-\rho_s)/d\rho}{\pi_{s'} u'^b(y_{s'}-\rho_{s'})/d\rho}$$

But now, both agents can marginally improve their welfare by some additional trade. Indeed, if a pays some small amount ε to b in state s but receives $k\varepsilon$ in state s', a's welfare changes by

$$dW^a = -\pi_s u'^a(\rho_s)\varepsilon + \pi_{s'} u'^a(\rho_{s'}) k\varepsilon > 0$$

while for b

$$dW^b = \pi_s u'^b(y_s-\rho_s)\varepsilon - \pi_{s'} u'^b(y_{s'}-\rho_{s'}) k\varepsilon > 0$$

and both parties gain from that trade, contradicting the fact that the initial allocation was Pareto efficient.

The sharing rule ρ is thus a solution of equation (6.21), which can be rewritten as:

$$u'^a(\rho) = \mu u'^b(y_s-\rho) \tag{6.22}$$

where $\rho = \rho\left(y_s^a, y_s^b\right)$. Since the equation depends on the weight μ, there exists a continuum of efficient risk sharing rules, indexed by the parameter μ; the larger this parameter, the more favorable the rule is to member b.

As an illustration, assume that agents have Constant Absolute Risk Aversion (CARA) preferences with respective absolute risk aversions equal to α and β for a and b respectively:

$$u^a(x) = -\exp(-\alpha x),\ u^b(x) = -\exp(-\beta x)$$

Then the previous equation becomes:

$$\alpha \exp\left[-\alpha\rho\left(y_s^a, y_s^b\right)\right] = \mu\beta \exp\left[-\beta\left(y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right)\right]$$

which gives

$$\rho\left(y_s^a, y_s^b\right) = \frac{\beta}{\alpha+\beta}\left(y_s^a + y_s^b\right) - \frac{1}{\alpha+\beta}\log\left(\frac{\mu\beta}{\alpha}\right)$$

We see that CARA preferences lead to a linear sharing rule, with slope $\beta/(\alpha+\beta)$; the intercept depends on the Pareto weight μ.

Similarly, if both spouses exhibit Constant Relative Risk Aversion (CRRA) with identical relative risk aversion γ, then:

$$u^a(x) = u^b(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

and the equation is:

$$\left[\rho\left(y_s^a, y_s^b\right)\right]^{-\gamma} = \mu\left[y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right]^{-\gamma}$$

which gives

$$\rho\left(y_s^a, y_s^b\right) = k\left(y_s^a + y_s^b\right) \tag{6.23}$$

where

$$k = \frac{\mu^{-\frac{1}{\gamma}}}{1+\mu^{-\frac{1}{\gamma}}} \tag{6.24}$$

Therefore, with identical CRRA preferences, each spouse consumes a fixed fraction of total consumption, the fraction depending on the Pareto weight μ. Note that, in both examples, ρ only depends on the sum $y_s = y_s^a + y_s^b$, and

$$0 \leq \rho'(y_s) \leq 1$$

Properties of efficient sharing rules

While the previous forms are obviously specific to the CARA and CRRA cases, the two properties just mentioned are actually general.

Proposition 6.1 *For any efficient risk sharing agreement, the sharing rule ρ is a function of aggregate income only:*

$$\rho\left(y_s^a, y_s^b\right) = \bar{\rho}\left(y_s^a + y_s^b\right) = \bar{\rho}(y_s)$$

Moreover,

$$0 \leq \bar{\rho}' \leq 1$$

Proof. *Note, first, that the right hand side of equation (6.22) is increasing in ρ, while the left hand side is decreasing; therefore the solution in ρ must be unique. Now, take two pairs $\left(y_s^a, y_s^b\right)$ and $\left(\bar{y}_s^a, \bar{y}_s^b\right)$ such that $y_s^a + y_s^b = \bar{y}_s^a + \bar{y}_s^b$. Equation (6.22) is the same for both pairs, therefore its solution must be the same, which proves the first statement. Finally, differentiating (6.22) with respect to y_s gives:*

$$\frac{u''^a(\bar{\rho})}{u'^a(\bar{\rho})}\bar{\rho}' = \frac{u''^b(y_s-\bar{\rho})}{u'^b(y_s-\bar{\rho})}(1-\bar{\rho}') \tag{6.25}$$

and finally:

$$\bar{\rho}'(y_s) = \frac{-\frac{u''^b(y_s-\bar{\rho})}{u'(y_s-\bar{\rho})}}{-\frac{u''^a(\bar{\rho})}{u'^a(\bar{\rho})} - \frac{u''^b(y_s-\bar{\rho})}{u'^b(y_s-\bar{\rho})}} \tag{6.26}$$

which belongs to the interval $[0,1]$*. Note, moreover, that* $0 < \bar{\rho}'(y_s) < 1$ *unless one of the agents is (locally) risk neutral.* ■

The first statement in Proposition 6.1 is often called the *mutuality principle*. It states that when risk is shared efficiently, an agent's consumption is not affected by the idiosyncratic realization of her income; only shocks affecting aggregate resources (here, total income y_s) matter. It has been used to test for efficient risk sharing, although the precise test is much more complex than it may seem - we shall come back to this aspect below.

Formula (6.26) is quite interesting in itself. It can be rewritten as:

$$\bar{\rho}'(y_s) = \frac{-\frac{u'^a(\bar{\rho})}{u''^a(\bar{\rho})}}{-\frac{u'^a(\bar{\rho})}{u''^a(\bar{\rho})} - \frac{u'^b(y_s-\bar{\rho})}{u''^b(y_s-\bar{\rho})}} \tag{6.27}$$

The ratio $-\frac{u'^a(\bar{\rho})}{u''^a(\bar{\rho})}$ is called the *risk tolerance* of A; it is the inverse of A's risk aversion. Condition (6.27) states that the marginal risk is allocated between the agents in proportion of their respective risk tolerances. To put it differently, assume the household's total income fluctuates by one (additional) dollar. The fraction of this one dollar fluctuation born by agent a is proportional to a's risk tolerance. To take an extreme case, if a was infinitely risk averse - that is, her risk tolerance was nil - then $\bar{\rho}' = 0$ and her share would remain constant: all the risk would be born by b.

It can actually be showed that the two conditions expressed by Proposition 6.1 are also sufficient. That is, take any sharing rule ρ satisfying them. Then one can find two utility functions u^a and u^b such that ρ shares risk efficiently between a and b.[9]

6.3.3 *Efficient risk sharing in a multi-commodity context: an introduction*

Regarding risk sharing, a multi commodity context is much more complex than the one-dimensional world just described. The key insight is that consumption decisions also depend on the relative prices of the various available commodities, and that typically these prices fluctuate as well. Surprisingly enough, sharing price risk is quite different from sharing income risk. A precise investigation would be outside the scope of the present volume; instead, we simply provide a short example.[10]

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[9] The exact result is even slightly stronger; it states that for any ρ satisfying the conditions and any increasing, strictly concave utility u^A, one can find some increasing, strictly concave utility u^B such that ρ shares risk efficiently between A and B (see Chiappori, Samphantharak, Schulhofer-Wohl and Townsend 2010 for a precise statement).

[10] The reader is referred to Chiappori, Townsend and Yamada (2008) for a precise analysis. The following example is also borrowed from this article.

Consider a two agent household, with two commodities - one labor supply and an aggregate consumption good. Assume, moreover, that agent b is risk neutral and only consumes, while agent a consumes, supplies labor and is risk averse (with respect to income risk). Formally, using Cobb-Douglas preferences:

$$U^a\left(c^a, l^a\right) = \frac{\left(l^a c^a\right)^{1-\gamma}}{1-\gamma} \text{ and } U^b\left(c^b\right) = c^b$$

with $\gamma > 1/2$. Finally, the household faces a linear budget constraint; let w_a denote 2's wages, and y (total) non labor income.

Since agent b is risk neutral, one may expect that she will bear all the risk. However, in the presence of wage fluctuations, it is *not* the case that agent a's consumption, labor supply or even utility will remain constant. Indeed, *ex ante* efficiency implies *ex post* efficiency, which in turn requires that the labor supply and consumption of a vary with his wage:

$$l^a = \frac{\rho + w_a T}{2w_a}, c^a = \frac{\rho + w_a T}{2}$$

where ρ is the sharing rule. The indirect utility of a is therefore:

$$V^a\left(\rho, w_a\right) = \frac{2^{\gamma-1}}{1-\gamma}\left(\rho + w_a T\right)^{2-2\gamma} w_a^{-(1-\gamma)}$$

while that of b is simply $V^b\left(y-\rho\right) = y - \rho$.

Now, let's see how *ex ante* efficiency restricts the sharing rule. Assume there exists S states of the world, and let $w_{a,s}, y_s$ and ρ_s denote wage, non labor income and the sharing rule in state s. Efficient risk sharing requires solving the program:

$$\max_{\rho} \sum_s \pi_s \left[V^a\left(\rho_s, w_{a,s}\right) + \mu V^b\left(y_s - \rho_s\right)\right]$$

leading to the first order condition:

$$\frac{\partial V^a\left(\rho_s, w_{a,s}\right)}{\partial \rho_s} = \mu \frac{\partial V^b\left(y_s - \rho_s\right)}{\partial \rho_s}$$

In words, efficient risk sharing requires that the ratio of marginal utilities of income remains constant - a direct generalization of the previous results. Given the risk neutrality assumption for agent b, this boils down to the marginal utility of income of agent a remaining constant:

$$\frac{\partial V^a}{\partial \rho} = 2^{\gamma}\left(\rho + w_a T\right)^{1-2\gamma} w_a^{-(1-\gamma)} = K$$

which gives

$$\rho = 2K'.w_a^{\frac{1-\gamma}{1-2\gamma}} - w_a T$$

where K' is a constant depending on the respective Pareto weights. In the end:

$$l^a = K'.w_a^{\frac{-\gamma}{2\gamma-1}}, c^a = K'.w_a^{-\frac{1-\gamma}{2\gamma-1}}$$

and the indirect utility is of the form:

$$V^a = K''.w_a^{-\frac{1-\gamma}{2\gamma-1}}$$

for some constant K''. As expected, a is sheltered from non labor income risk by his risk sharing agreement with b. However, his consumption, labor supply and welfare fluctuate with his wage. The intuition is that that agents respond to price (or wage) variations by adjusting their demand (here labor supply) behavior in an optimal way. The maximization implicit in this process, in turn, introduces an element of convexity into the picture.[11]

6.3.4 Econometric issues

Distributions versus realizations

We now come back to the simpler, one-commodity framework. As expressed by Proposition 6.1, efficient risk sharing schemes satisfy the mutuality principle, which is a form of income pooling: the sharing rules depends only on total income, not on the agent's respective contributions y^a and y^b *per se*. This result may sound surprising; after all, income pooling is a standard implication of the unitary setting which is typically not valid in the collective framework; moreover, it is regularly rejected empirically.

The answer to this apparent puzzle relies on the crucial distinction between the (*ex post*) *realization* and the (*ex ante*) *distribution* of income shocks. When risk is shared efficiently, income realizations are pooled: my consumption should not suffer from my own bad luck, insofar as it does not affect aggregate resources. On the other hand, there exists a continuum of efficient allocations of resources, indexed by some Pareto weights; different weights correspond to different (contingent) consumptions. The Pareto weights, in turn, depend on the *ex ante* situations of the agents; for instance, if a has a much larger *expected* income, one can expect that her Pareto weight will be larger than b's, resulting in a higher level of consumption. In other words, the pooling property does not apply to expected incomes, and in general to any feature (variance, skewness,...) of the probability distributions of individual income streams. The main intuition of the collective model is therefore maintained: power (as summarized by Pareto weights) matters for behavior - the nuance being that under efficient risk

[11] Generally, the ability of risk neutral agents to adjust actions *after* the state is observed induces a "risk loving" ingredient, whereby higher price variation is preferred, and which may counterweight the agent's risk aversion.

sharing it is the distribution of income, instead of its realization, that (may) affect individual powers.

In practice, however, this raises a difficult econometric issue. Testing for efficient risk sharing requires checking whether observed behavior satisfies the mutuality principle, that is pooling of income realization. However, by the previous argument, this requires being able to control for distributions, hence to distinguish between ex post realizations and ex ante distributions. On cross-sectional data, this is impossible.

It follows that cross-sectional tests of efficient risk sharing are plagued with misspecification problems. For instance, some (naive) tests of efficient risk sharing that can be found in the literature rely on a simple idea: since individual consumption should not respond to idiosyncratic income shocks (but only to aggregate ones), one may, on cross sectional data, regress individual consumption (or more specifically marginal utility of individual consumption) on (i) indicators of aggregate shocks (for example, aggregate income or consumption), and (ii) individual incomes. According to this logic, a statistically significant impact of individual income on individual consumption, controlling for aggregate shocks, should indicate inefficient risk sharing.

Unfortunately, the previous argument suggests that in the presence of heterogeneous income processes, a test of this type is just incorrect. To get an intuitive grasp of the problem, assume that two agents a and b share risk efficiently. However, the *ex ante* distributions of their respective incomes are very different. a's income is almost constant; on the contrary, b may be hit by a strong, negative income shock. In practice, one may expect that this asymmetry will be reflected in the respective Pareto weights; since b desperately needs insurance against the negative shock, he will be willing to accept a lower weight, resulting in lower expected consumption than a, as a compensation for the coverage provided by a.

Consider, now, a large economy consisting of many independent clones of a and b; assume for simplicity that, by the law of large numbers, aggregate resources do not vary. By the mutuality principle, efficient risk sharing implies that individual consumptions should be constant as well; and since a agents have more weight, their consumption will always be larger than that of b agents. Assume now than an econometrician analyzes a cross section of this economy. The econometrician will observe two features. One is that some agents (the 'unlucky' b's) have a very low income, while others (the lucky b's and all the a's) have a high one. Secondly, the low income agents also exhibit, *on average,* lower consumption levels than the others (since they consume as much as the lucky b's but less than all the a's). Technically, any cross sectional regression will find a positive and significant correlation between individual incomes and consumptions, which seems to reject efficient risk sharing - despite the fact that the mutuality principle is in fact perfectly satisfied, and risk sharing is actually fully efficient. The key remark, here, is that the rejection is spurious and due to a misspecification of the model. Technically, income is found to matter only because income *realizations* capture (or are proxies for) specific features of income *distributions* that influence Pareto weights.

A simple solution

We now discuss a specific way of solving the problem. It relies on the availability of (short) panel data, and on two additional assumptions. One is that agent's preferences exhibit Constant Relative Risk Aversion (CRRA), a functional form that is standard in this literature. In practice:

$$u^a(x) = \frac{x^{1-\alpha}}{1-\alpha}, u^b(x) = \frac{x^{1-\beta}}{1-\beta}$$

The second, much stronger assumption is that risk aversion is identical across agents, implying $\alpha = \beta$ in the previous form.

We have seen above (in equations 6.23 and 6.24) that under these assumptions, the efficiency condition (6.22) leads to a sharing rule that is linear in income, the coefficient depending on the Pareto weights. Taking logs:

$$\begin{aligned} \log c^a &= \log \rho = \log \left(\frac{\mu^{-\frac{1}{\alpha}}}{1+\mu^{-\frac{1}{\alpha}}} \right) + \log y, \text{ and} \\ \log c^b &= \log \left(\frac{1}{1+\mu^{-\frac{1}{\alpha}}} \right) + \log y \end{aligned}$$

Assume, now, that agents are observed for at least two periods. We can compute the difference between log consumptions in two successive periods, and thus eliminate the Pareto weights; we get:

$$\Delta \log c^a = \Delta \log c^b = \Delta \log y$$

In words, a given variation, in percentage, of aggregate income should generate equal percentage variations in all individual consumptions.[12]

Of course, this simplicity comes at a cost - namely, the assumption that individuals have identical preferences: one can readily check that with different risk aversion, the sharing rule is not linear, and differencing log consumptions does not eliminate Pareto weights. Assuming homogeneous risk aversions is difficult for two reasons. First, all empirical studies suggest that the cross sectional variance of risk aversion in the population is huge. Second, even if we assume that agents match to share risk (so that a sample of people belonging to the same risk sharing group is not representative of the

[12] This prediction is easy to test even on short panels - see for instance Altonji *et al* (1992) and Duflo and Udry (2004); incidentally, it is usually rejected. See Mazzocco and Saini (2006) for a precise discussion.

general population), theory[13] suggests that the matching should actually be *negative assortative* (that is, more risk averse agents should be matched with less risk averse ones) - so that heterogeneity should be, if anything, *larger* within risk sharing groups than in the general population.[14]

Finally, can we test for efficient risk sharing without this assumption? The answer is yes; such a test is developed for instance in Chiappori, Townsend and Yamada (2008) and in Chiappori, Samphantharak, Schulhofer-Wohl and Townsend (2010). However, it requires long panels - since one must be able to disentangle the respective impacts of income distributions and realizations.

6.4 Intertemporal Behavior

6.4.1 The unitary approach: Euler equations at the household level

We now extend the model to take into account the dynamics of the relationships under consideration. Throughout this section, we assume that preferences are time separable and of the expected utility type. The first contributions extending the collective model to an intertemporal setting are due to Mazzocco (2004, 2007); our presentation follows his approach. Throughout this section, the household consists of two egoistic agents who live for T periods. In each period $t \in \{1, ..., T\}$, let y_t^i denote the income of member i.

We start with the case of a unique commodity which is privately consumed; c_t^i denotes member i's consumption at date t and p_t is the corresponding price. The household can save by using a risk-free asset; let s_t denotes the net level of (aggregate) savings at date t, and R_t its gross return. Note that, in general, y_t^i, s_t and c_t^i are random variables

We start with the standard representation of household dynamics, based on a unitary framework. Assume, therefore, that there exists a utility function $u\left(c^a, c^b\right)$ representing the household's preferences. The program de-

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[13] See, for instance Chiappori and Reny (2007).

[14] An alternative test relies on the assumption that agents have CARA preferences. Then, as seen above, the sharing rule is an affine function, in which only the intercept depends on Pareto weights (the slope is determined by respective risk tolerances). It follows that variations in *levels* of individual consumptions are proportional to variations in total income, the coefficient being independent of Pareto weights. The very nice feature of this solution, adopted for instance by Townsend (1994), is that it is compatible with any level of heterogeneity in risk aversion. Its main drawback is that the CARA assumption is largely counterfactual; empirical evidence suggests that *absolute* risk aversion decreases with wealth.

scribing dynamic choices is:

$$\max E_0 \left(\sum_t \beta^t u\left(c_t^a, c_t^b\right) \right)$$

under the constraint

$$p_t\left(c_t^a + c_t^b\right) + s_t = y_t^a + y_t^b + R_t s_{t-1}, \quad t = 0, ..., T$$

Here, E_0 denotes the expectation taken at date 0, and β is the household's discount factor. Note that if borrowing is excluded, we must add the constraint $s_t \geq 0$.

Using a standard result by Hicks, we can define household utility as a function of total household consumption; technically, the function U is defined by:

$$U(c) = \max\left\{u\left(c^a, c^b\right) \text{ such that } c^a + c^b = c\right\}$$

and the program becomes:

$$\max E_0 \left(\sum_t \beta^t U(c_t) \right)$$

under the constraint

$$p_t c_t + s_t = y_t^a + y_t^b + R_t s_{t-1}$$

The first order conditions give the well-known Euler equations:

$$\frac{U'(c_t)}{p_t} = \beta E_t \left[\frac{U'(c_{t+1})}{p_{t+1}} R_{t+1} \right] \tag{6.28}$$

In words, the marginal utility of each dollar consumed today equals, in expectation, β times the marginal utility of R_{t+1} dollars consumed tomorrow; one cannot therefore increase utility by marginally altering the savings.

In practice, many articles test the empirical validity of these household Euler equations using general samples, including both couples and singles (see Browning and Lusardi 1995 for an early survey); most of the time, the conditions are rejected. Interestingly, however, Mazzocco (2004) estimates the same standard household Euler equations *separately* for couples and for singles. Using the CEX and the Panel Study of Income Dynamics (PSID), he finds that the conditions are rejected for couples, but *not for singles*. This seems to suggest that the rejection obtained in most articles may not be due to technical issues (for example, non separability of labor supply), but more fundamentally to a misrepresentation of household decision processes.

6.4.2 Collective Euler equations under ex ante efficiency

Household consumption

We now consider a collective version of the model. Keeping for the moment the single commodity assumption, we now assume that agents have their own preferences and discount factors. The Pareto program is therefore:

$$\max (1-\mu) E_0 \left(\sum_t (\beta^a)^t u^a (c_t^a) \right) + \mu E_0 \left(\sum_t \left(\beta^b\right)^t u^b \left(c_t^b\right) \right)$$

under the same constraints as above. First order conditions give:

$$\begin{aligned} \frac{u'^a (c_t^a)}{p_t} &= \beta^a E_t \left[\frac{u'^a \left(c_{t+1}^a\right)}{p_{t+1}} R_{t+1} \right] \\ \frac{u'^b \left(c_t^b\right)}{p_t} &= \beta^b E_t \left[\frac{u'^b \left(c_{t+1}^b\right)}{p_{t+1}} R_{t+1} \right] \end{aligned} \tag{6.29}$$

which are the *individual* Euler equations. In addition, individual consumptions at each period must be such that:

$$\frac{(\beta^a)^t}{\left(\beta^b\right)^t} \frac{u'^a (c_t^a)}{u'^b \left(c_t^b\right)} = \frac{\mu}{1-\mu} \tag{6.30}$$

The right hand side does not depend on t: the ratio of discounted marginal utilities of income of the two spouses must be constant through time. This implies, in particular, that

$$\frac{u'^a (c_t^a)}{u'^b \left(c_t^b\right)} = \frac{\mu}{1-\mu} \frac{\left(\beta^b\right)^t}{(\beta^a)^t}$$

If, for instance, a is more patient than b, in the sense that $\beta^a > \beta^b$, then the ratio u'^a/u'^b declines with time, because a postpones a larger fraction of her consumption than b.

An important remark is that if individual consumptions satisfy (6.29), then typically the aggregate consumption process $c_t = c_t^a + c_t^b$ does *not* satisfy an individual Euler equation like (6.28), except in one particular case, namely ISHARA utilities and identical discount factors. For instance, assume, following Mazzocco (2004), that individuals have utilities of the CRRA form:

$$u^X (c) = \frac{c^{1-\gamma^X}}{1-\gamma^X}, \quad X = a, b$$

and that, moreover, $\beta^a = \beta^b = \beta$. Then (6.29) becomes:

$$\begin{aligned} c_t^a &= \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1}^a \right)^{-\gamma^a} \right] \right\}^{-1/\gamma^a} \\ c_t^b &= \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1}^b \right)^{-\gamma^b} \right] \right\}^{-1/\gamma^b} \end{aligned} \tag{6.31}$$

If $\gamma^a = \gamma^b$ (the ISHARA case), one can readily see that the ratio c_{t+1}^a / c_{t+1}^b is constant across states of the world; therefore

$$c_{t+1}^a = k c_{t+1},\ c_{t+1}^b = (1-k)\, c_{t+1}$$

for some constant k. It follows that:

$$\begin{aligned} c_t^a &= \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(k c_{t+1} \right)^{-\gamma} \right] \right\}^{-1/\gamma} \\ &= k \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1} \right)^{-\gamma} \right] \right\}^{-1/\gamma} \end{aligned} \tag{6.32}$$

and by the same token

$$c_t^b = (1-k) \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1} \right)^{-\gamma} \right] \right\}^{-1/\gamma}$$

so that finally:

$$c_t = c_t^a + c_t^b = \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1} \right)^{-\gamma} \right] \right\}^{-1/\gamma} \tag{6.33}$$

and aggregate consumption satisfies an individual Euler equation: the household behaves as a single.

However, in the (general) case $\gamma^a \neq \gamma^b$, Mazzocco shows that this result no longer holds, and household aggregate consumption does not satisfy a Euler equation even though each individual consumption does. In particular, testing the Euler conditions on aggregate household consumption should lead to a rejection even when all the necessary assumptions (efficiency, no credit constraints, ...) are fulfilled.

Individual consumption and labor supply

The previous, negative result is not really surprising: it simply stresses once more than groups, in general, do not behave as single individuals. What then? Well, if individual consumptions are observable, conditions (6.29) and (6.30) are readily testable using the standard approach. Most of the time, however, only aggregate consumption is observed. Then a less restrictive framework is needed. In particular, one may relax the single commodity assumption. Take, for instance, a standard model of labor supply, in which each agent consumes two commodities, namely leisure and a consumption good. The collective model suggests that individual consumptions can be recovered (up to additive constants - see chapters 4 and 5). Then tests of the Euler equation family can be performed.

As an illustration, Mazzocco (2007) studies a dynamic version of the collective model introduced in chapter 4. The individual Euler equations become, with obvious notations:

$$\begin{aligned} \frac{\partial u^i\left(c_t^i, l_t^i\right)/\partial c}{p_t} &= \beta^i E_t\left[\frac{\partial u^i\left(c_{t+1}^i, l_{t+1}^i\right)/\partial c}{p_{t+1}} R_{t+1}\right] \\ \frac{\partial u^i\left(c_t^i, l_t^i\right)/\partial l}{w_t^i} &= \beta^i E_t\left[\frac{\partial u^i\left(c_{t+1}^i, l_{t+1}^i\right)/\partial l}{w_{t+1}^i} R_{t+1}\right] \end{aligned} \tag{6.34}$$

for $i = a, b$. In particular, since individual labor supplies are observable, these equations can be estimated.

6.4.3 The ex ante inefficiency case

What, now, if the commitment assumption is not valid? We have seen above that this case has a simple, technical translation in the collective framework - namely, the Pareto weights are not constant. A first remark, due to Mazzocco (2007), is that even in the ISHARA case, aggregate consumption no longer satisfies the martingale property (6.33). Indeed, let μ_t denote the Pareto weight of b in period t, and assume for the moment that μ_t does not depend on the agent's previous consumption decisions. We first have that

$$c_t^a + c_t^b = \left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}^a\right)^{-\gamma^a}\right]\right\}^{-1/\gamma^a} + \left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}^b\right)^{-\gamma^b}\right]\right\}^{-1/\gamma^b} \tag{6.35}$$

Moreover,

$$\frac{u'^a\left(c_t^a\right)}{u'^b\left(c_t^b\right)} = \frac{\mu_t}{1-\mu_t}\left(\frac{\beta^b}{\beta^a}\right)^t \tag{6.36}$$

for all t, which for ISHARA ($\gamma^a = \gamma^b = \gamma$) preferences becomes

$$\left(\frac{c_t^a}{c_t^b}\right)^{-\gamma} = \frac{\mu_t}{1-\mu_t}\left(\frac{\beta^b}{\beta^a}\right)^t$$

If μ_t is not constant, neither is the ratio c_t^a/c_t^b. A result by Hardy, Littlewood and Polya (1952) implies that whenever the ratio x/y is not constant, then for all probability distributions on x and y:

$$\left\{E_t\left[(x+y)^{-\gamma}\right]\right\}^{-1/\gamma} > \left\{E_t\left[x^{-\gamma}\right]\right\}^{-1/\gamma} + \left\{E_t\left[y^{-\gamma}\right]\right\}^{-1/\gamma}$$

which directly implies that:

$$\left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}\right)^{-\gamma}\right]\right\}^{-1/\gamma} > c_t$$

In words, the (marginal utility of) aggregate consumption now follows a *supermartingale*.

Regarding now *individual* consumptions, one can readily check that equations (6.29) become:

$$\begin{aligned}
\left(1-\mu_t\right)\frac{u'^a\left(c_t^a\right)}{p_t} &= \left(1-\mu_{t+1}\right)\beta^a E_t\left[\frac{u'^a\left(c_{t+1}^a\right)}{p_{t+1}}R_{t+1}\right] \\
\mu_t\frac{u'^b\left(c_t^b\right)}{p_t} &= \mu_{t+1}\beta^b E_t\left[\frac{u'^b\left(c_{t+1}^b\right)}{p_{t+1}}R_{t+1}\right]
\end{aligned} \tag{6.37}$$

or equivalently:

$$\begin{aligned}
E_t\left[\frac{u'^a\left(c_{t+1}^a\right)}{u'^a\left(c_t^a\right)}\frac{p_t R_{t+1}}{p_{t+1}}\right] &= \frac{1}{\beta^a}\frac{1-\mu_t}{1-\mu_{t+1}} \\
E_t\left[\frac{u'^b\left(c_{t+1}^b\right)}{u'^b\left(c_t^b\right)}\frac{p_t R_{t+1}}{p_{t+1}}\right] &= \frac{1}{\beta^b}\frac{\mu_t}{\mu_{t+1}}
\end{aligned} \tag{6.38}$$

In words: under full commitment, the left hand side expressions should be constant, while they may vary in the general case. A first implication, therefore, is that whenever individual consumptions are observable, then the commitment assumption is testable. Moreover, we know that (6.36) holds for each t. These relations imply that μ_t is identifiable from the data. That is, if Pareto weights vary, it is possible to identify their variations, which can help characterizing the type of additional constraint that hampers full commitment.

Finally, individual consumptions are not observed in general, but individual labor supplies typically are; the same tests can therefore be performed using labor supplies as indicated above. Again, the reader is referred to Mazzocco (2007) for precise statements and empirical implementations. In particular, Mazzocco finds that both the unitary and the collective model with commitment are rejected, whereas the collective model without commitment is not. This finding suggests that while static efficiency may be expected to hold in general, dynamic (*ex ante*) efficiency may be more problematic.

6.4.4 Conclusion

The previous results suggest several conclusions. One is that the collective approach provides a simple generalization of the standard, 'unitary' approach to dynamic household behavior. Empirically, this generalization seems to work significantly better than the unitary framework. For instance, a well-known result in the consumption literature is that household Euler equations display excess sensitivity to income shocks. The two main explanations are the existence of borrowing constraints and non-separability between consumption and leisure. However, the findings in Mazzocco (2007) indicate that cross-sectional and longitudinal variations in relative decision power explain a significant part of the excess sensitivity of consumption growth to income shocks. Such variations, besides being interesting per se, are therefore crucial to understanding the dynamics of household consumption. A second conclusion is that the commitment issue is a crucial dimension of this dynamics; a couple in which agents can credibly commit on the long run will exhibit behavioral patterns that are highly specific. Thirdly, it is possible to develop models that, in their most general form, can capture both the 'collective' dimensions of household relationships and the limits affecting the spouse's ability to commit. The unitary model *and* the full efficiency version of the collective approach are nested within this general framework, and can be tested against it.

6.5 Divorce

6.5.1 The basic model

Among the limits affecting the spouses' ability, an obvious one is the possibility of divorce. Although divorce is, in many respects, an ancient institution, it is now more widespread than ever, at least in Western countries. Chiappori, Iyigun and Weiss (2008) indicate for instance that in 2001, among American women then in their 50s, no less than 39% had divorced at least once (and 26% had married at least twice); the numbers for men are slightly higher (respectively 41% and 31%). Similar patterns can be observed in Europe (see chapter 1). Moreover, in most developed countries unilateral divorce has been adopted as the legal norm. This implies that any spouse may divorce if (s)he will. In practice, therefore, divorce introduces a constraint on intertemporal allocations within the couple; that is, at any period, spouses must receive each within marriage at least as much as they would get if they were divorced.

Clearly, modeling divorce - and more generally household formation and dissolution - is an important aspect of family economics. For that purpose, a unitary representation is probably not the best tool, because it is essential to distinguish *individual* utilities within the couple. If each spouse is characterized, both before and after marriage, by a single utility, while the couple itself is represented by a third utility with little or no link with the previous ones, modeling divorce (or marriage for that matter) becomes very difficult and largely *ad hoc* . Even if the couple's preferences are closely related to individual utilities, for instance through a welfare function a la Samuelson, one would like to investigate the impact of external conditions (such as wages, the tax-benefit system or the situation on the marriage market) on the decision process leading to divorce; again, embedding the analysis within the black box of a unitary setting does not help clarifying these issues.

In what follows, we show how the collective approach provides a useful framework for modeling household formation and dissolution. Two ingredients are crucial for this task. One is the presence of *economic gains* from marriage. A typical example is the presence of public goods, as we have extensively discussed in the previous chapters. Alternative sources of marital gains include risk sharing or intertemporal consumption smoothing, along the lines sketched in the previous sections. At any rate, we must first recognize that forming a couple is often efficient from the pure economic perspective.

A second ingredient is the existence of *non-pecuniary benefits* to marriage. These 'benefits' can be interpreted in various ways: they may represent love, companionship, or other aspects. The key feature, in any case, is that these benefits are match-specific (in that sense, they are an indicator of the 'quality' of the match under consideration) and they cannot be exactly predicted *ex ante*; on the contrary, we shall assume that they are revealed with some lag (and may in general be different for the two spouses). The basic mechanism is that a poor realization of the non pecuniary benefits may trigger divorce, either because agents hope to remarry (and, so to speak, 'take a new draw' from the distribution of match quality), or because the match is so unsatisfactory that the spouses would be better off as singles, even at the cost of forgoing the economic gains from marriage. The existence of a trade-off between the economic surplus generated by marriage and the poor realization of non economic benefits plays a central role in most models of divorce.

More specifically, we shall consider a collective framework in which couples may consume both private and public goods, and marriage generates a non-pecuniary benefit. In principle, this benefit can enter individual utilities in an arbitrary manner. In what follows, however, we concentrate on a particular and especially tractable version of the model, initially due to Weiss and Willis (1993, 1997), in which the non monetary gain is additive; that is, the utility of each spouse is of the form

$$U^i = u^i\left(q^i, Q\right) + \theta^i, \ \ i = a, b$$

where $q^i = \left(q_1^i, ..., q_n^i\right)$ is the vector of private consumption of agent i, $Q = (Q_1, ..., Q_N)$ is the vector of household public consumption, and θ^i is the non monetary gain of i. In particular, while the total utility does depend on the non monetary components θ^i, the marginal rates of substitution between consumption goods does not, which simplifies the analysis.

For any couple, the pair $\left(\theta^a, \theta^b\right)$ of match qualities is drawn from a given distribution Φ. In general, any correlation between θ^a and θ^b is possible. Some models introduce an additional restriction by assuming that the quality of the match is the same for both spouses - that is, $\theta^a = \theta^b$.

To keep things simple, we present the model in a two periods framework. In period one, agents marry and consume. At the end of the period, the quality of the match is revealed, and agents decide whether to remain married or split. If they do not divorce, they consume during the second period, and in addition enjoy the same non monetary gain as before. If they split, we assume for the moment that they remain single for the rest of the period, and that they privately consume the (previously) public goods.[15] The prices of the commodities will not play a role in what follows; we may, for simplicity, normalize them to unity.

Finally, let y^a and y^b denote the agents' respective *initial* incomes, which they receive at the beginning of each period; and to simplify, we assume no savings and borrowing. In case of divorce, the couple's total income, $y^a + y^b$, is split between the ex-spouses. The rule governing this division leads to an allocation in which a receives some $D^a\left(y^a, y^b\right)$ and b receives $D^b\left(y^a, y^b\right) = y^a + y^b - D^a\left(y^a, y^b\right)$. For instance, if incomes are considered to be private property of each spouse, then $D^i\left(y^a, y^b\right) = y^i, i = a, b$, whereas an equal distribution rule would lead to $D^a\left(y^a, y^b\right) = D^b\left(y^a, y^b\right) = \left(y^a + y^b\right)/2$. A natural interpretation is that the rule $D = \left(D^a, D^b\right)$ is exogenous and imposed by law; however, while an agent cannot be forced to transfer to the ex-spouse more than the legal amount D, he may freely elect to do so, and will in some cases (see next subsection). An alternative approach considers divorce contracts as endogenous, for instance in a risk sharing perspective.[16]

We may now analyze the couple's divorce decision. First, the second period utility of agent i if divorced is simply $V^i\left(D^i\left(y^a, y^b\right)\right)$ (where, as before, V^i is agent i's indirect utility). If, on the other hand, the spouses remain married, then they choose some efficient allocation; as usual, their consumption plan therefore solves a program of the type:

$$\max u^a\left(q^a, Q\right) + \theta^a$$

under the constraints:

$$\begin{aligned} \sum_j \left(q_j^a + q_j^b\right) + \sum_k Q_k &= y^a + y^b \\ u^b\left(q^b, Q\right) + \theta^b &\geq \bar{u}^b \end{aligned}$$

[15] Some commodities may remain public even after divorce; children expenditures are a typical example. For a detailed investigation, see Chiappori *et al* (2007).

[16] See for instance Chiappori and Weiss (2009).

where $\bar{u}^b$ is a constant. Let $\left(\bar{q}^a, \bar{q}^b, \bar{Q}\right)$ denote the solution to this program, and $\bar{u}^a = u^a\left(q^a, Q\right) + \theta^a$ the corresponding utility for a. Note that both are functions of $\bar{u}^b$; we note therefore $\bar{u}^a\left(\bar{u}^b\right)$. Let $\mathcal{P}_M$ denote the Pareto set if married, that is the set of utilities $\left(u^a, u^b\right)$ such that $u^a \leq \bar{u}^a\left(u^b\right)$; in words, any pair of utilities in $\mathcal{P}_M$ can be reached by the couple if they remain married.

Then we are in one of the following two situations:

- either the reservation point $\left(V^a\left(D^a\left(y^a, y^b\right)\right), V^b\left(D^b\left(y^a, y^b\right)\right)\right)$, representing the pair of individual utilities reachable through divorce, belongs to the Pareto set if married, $\mathcal{P}_M$. Then there exists a second period distribution of income which is preferred over divorce by *both* spouses. The efficiency assumption implies that this opportunity will be taken, and the model predicts that the marriage will continue.
- or, alternatively, $\left(V^a\left(D^a\left(y^a, y^b\right)\right), V^b\left(D^b\left(y^a, y^b\right)\right)\right)$ is outside $\mathcal{P}_M$. Then the marriage cannot continue, because any second period allocation of resources the spouses may choose will be such that one spouse at least would be better off as a single; therefore, divorce must follow.

The model thus provides a precise description of the divorce decision; namely, divorce takes place whenever it is the efficient decision under the constraint that agents cannot receive less than their reservation utility $V^i\left(D^i\left(y^a, y^b\right)\right), i = a, b$.

Some remarks are in order at this point. First, the argument presented above assumes that divorce is unilateral, in the sense that each partner is free to terminate the marriage and obtain divorce, even if the spouse does not agree. An alternative setting requires mutual consent - that is, divorce cannot occur unless *both* spouses agree. An old question of family economics is whether a shift from mutual consent to unilateral has an impact on divorce rates; we shall consider that question in the next subsection.

Secondly, the fact that spouses *may* disagree about divorce - that is, a spouse may ask for divorce against the partner's will - does not imply that they *will*. In the setting just presented, a partner who would consider divorce may sometimes be 'bribed back' into marriage by her spouse, through an adequate redistribution of income. Only when such a redistribution cannot take place, because the cost to the other partner would exceed the benefits of remaining married, will divorce occur. In that sense, there is not disagreement about divorce in this model; simply, divorce sometimes comes out as the best solution available.

A third remark is that, ultimately, divorce is triggered by the realization of the match quality parameters $\left(\theta^a, \theta^b\right)$. Large values of the θs inflate the Pareto frontier, making it more likely to contain the divorce threat point; conversely, poor realizations contract it, and divorce becomes probable.

Formally, it is easy to check that the divorce decision is monotonic in the θ's, in the sense that if a couple remains married for some realization $\left(\bar{\theta}^a, \bar{\theta}^b\right)$, then they also do for any $\left(\theta^a, \theta^b\right)$ such that $\theta^i \geq \bar{\theta}^i$, $i = a, b$; and conversely, if they divorce for some $\left(\bar{\theta}^a, \bar{\theta}^b\right)$, so do they for any $\left(\theta^a, \theta^b\right)$ such that $\theta^i \leq \bar{\theta}^i$, $i = a, b$. In general, there exists a divorce frontier, namely a decreasing function ϕ such that the coupe divorce if and only if $\theta^a < \phi\left(\theta^b\right)$. Note, however, that for a 'neutral' realization $\theta^a = \theta^b = 0$, the couples always remains married, because of the marital gains arising from the presence of public consumption; negative shocks are required for a marriage to end.

Finally, how is the model modified when divorced agents are allowed to remarry? The basic principle remains valid - that is, agents (efficiently) divorce if no point within the Pareto frontier if married can provide both agents with the same expected utility as if single. The latter value is however more difficult to compute, because it now includes the probability of finding a new mate multiplied by the utility the ex spouse will get in their new marriage. In other words, one need to predict which particular allocation of resources and welfare will prevail in newly formed couples - a task that requires a more complete investigation of the equilibrium forces governing the (re)marriage market. We shall come back to this issue in the second part of the book.

6.5.2 Divorce under transferable utility and the Becker-Coase theorem

The TU framework

We now further investigate the divorce model under an additional assumption - namely, that utility is transferable between spouses, both during and after marriage.[17] Technically, we first assume that preferences of married individuals are of the *generalized quasi-linear* (GQL) form (see Bergstrom, 1989).

$$u_m^i\left(q^i, Q\right) = F\left(Q\right) q_1^i + G_m^i\left(Q, q_{-1}^i\right) + \theta^i,\ i = a, b \tag{6.39}$$

where $q_{-1}^i = \left(q_2^i, ...q_n^i\right)$. Here, the functions F and $G_m^i,\ i = a, b$, are positive, increasing, concave functions such that $F(0) = 1$ and $G_m^i(0) = 0$.

Secondly, we assume that preferences if single take the *strictly quasi-linear* form:

$$u_s^i\left(q^i, Q\right) = q_i^1 + G_s^i\left(Q, q_{-1}^i\right),\ i = a, b, \tag{6.40}$$

where again the $G_s^i,\ i = a, b$, are increasing concave functions, with $G_s^i\left(0\right) = 0$. Because of quasi-linearity, the optimal consumptions of public goods and

[17] The material presented in this subsection is borrowed from Chiappori, Iyigun and Weiss (2007).

private goods other than good 1 are given by the conditions:

$$\frac{\partial G_s^i\left(Q, q_{-1}^i\right)}{\partial Q^j} = 1, \quad 1 \leq j \leq N \qquad \text{and} \qquad \frac{\partial G_s^i\left(Q, q_{-1}^i\right)}{\partial q_{ki}^i} = 1, \quad 2 \leq k \leq n \; .$$

Neither these conditions nor the optimal levels of all private and public consumptions (except for good 1) depend on income. Let these optimal levels be denoted $\left(\bar{Q}, \bar{q}_{-1}^i\right)$. To simplify notations, we choose units such that $G_s^i\left(\bar{Q}, \bar{q}_{-1}^i\right) = \sum_{j=1}^N \bar{Q}_j + \sum_{k=2}^n \bar{q}_k^i,\ i = a, b$. Then, the indirect utility of a single person equals his or her income.

Now, consider a man with income y^b married with a woman with income y^a. There is a unique efficient level for the consumption of each of the public goods and each of the private goods 2 to n. Moreover, these levels depend only on the total income of the partners, $y = y^a + y^b$. If we define

$$\eta\left(y\right) = \max_{\left(Q, q_{-1}^a, q_{-1}^b\right)} \left\{ \begin{array}{c} F\left(Q\right)\left[y - \sum_{j=1}^N Q_j + \sum_{k=2}^n \left(q_k^a + q_k^b\right)\right] \\ + G_m^a\left(Q, q_{-1}^a\right) + G_m^b\left(Q, q_{-1}^b\right) \end{array} \right\}$$

then the Pareto frontier is given by

$$u_m^a + u_m^b = \ \eta\left(y\right) + \theta^a + \theta^b, \tag{6.41}$$

Here, u_m^a and u_m^b are the attainable utility levels that can be implemented by the allocations of the private good q_1 between the two spouses, given the efficient consumption levels of all other goods. The Pareto frontier is a straight line with slope -1: utility is transferable between spouses (see chapter 3). Assuming, as is standard, that the optimal public consumptions are such that $F\left(Q\right)$ is increasing in Q, we see that $\eta(y)$ is *increasing and convex* in y.[18] Moreover, $\eta\left(0\right) = 0$ and $\eta'\left(0\right) = F\left(0\right) = 1$. Since η is convex, this implies that $\eta\left(y\right) > y$ and $\eta'\left(y\right) > 1$ for all $y > 0$.

Finally, if divorce takes place, the post-divorce utility of agent i is:

$$V_s^i\left(D^i\left(y^a, y^b\right)\right) = D^i\left(y^a, y^b\right) \tag{6.42}$$

In particular, we see that

$$V_s^a + V_s^b = D^a\left(y^a, y^b\right) + D^b\left(y^a, y^b\right) = y^a + y^b = y \tag{6.43}$$

In this framework, the divorce decision takes a particularly simple form. Indeed, agents divorce if and only if the point $\left(V_s^a, V_s^b\right)$ is outside the Pareto set when married. Given (6.41), this occurs when the sum $V_s^a + V_s^b$ is larger

[18] By the envelope theorem, the derivative $\eta'\left(y\right)$ is equal to $F\left(Q\right)$. Therefore, η is increasing in y and, if $F\left(Q\right)$ is increasing in y as well, then η is convex. Note that a sufficient (but by no means necessary) condition is that public consumptions are all normal.

than $\eta(y) + \theta^a + \theta^b$. Using (6.43), we conclude that divorce takes place whenever:

$$\eta(y) + \theta^a + \theta^b < y$$

or equivalently:

$$\theta^a + \theta^b < y - \eta(y) \tag{6.44}$$

Condition (6.44) has a simple, probabilistic translation; namely, the probability that a couple with total income y divorces is simply

$$P = \Pr\left(\theta^a + \theta^b < y - \eta(y)\right) = \Phi\left(y - \eta(y)\right)$$

where Φ is the cumulative distribution function of $\theta^a + \theta^b$. As expected, the threshold $\bar{\theta} = y - \eta(y)$ is negative and decreases with income: wealthier couples are less likely to divorce, because they receive larger economic gains from marriage. Note also that the divorce decision only depends on the realization of the sum $\theta^a + \theta^b$: under transferable utility, a poor realization of θ for one spouse can always be compensated by a transfer from the partner.

The Becker-Coase theorem

This result has several consequences. One is that *the divorce decision does not depend on the law governing post divorce income allocation*; indeed, condition (6.44) above is independent of the rule D.

Moreover, let us compare the two dominant legal systems governing divorce, namely unilateral divorce and mutual consent. One can readily see that in both cases, agents divorce if and only if condition (6.44) is satisfied. The result is obvious under unilateral divorce, because condition (6.44) implies that no intrahousehold resource allocation can provide both agents with at least as much as their utility if single. The case of mutual consent is slightly more complex, because even when condition (6.44) is satisfied, the post divorce allocation D may be such that one member, say a, strictly loses from divorce (of course, (6.44) then requires that her spouse, b, strictly gains). But then b may bribe a into divorcing by offering a post divorce allocation that is more favorable to a than D^a. Of course, the price for b is that he will receive less than D^b. But condition (6.44) precisely state that this is still better for b than remaining married.

We can therefore conclude that *the laws governing divorce have no impact on divorce probability.* This neutrality result, initially established by Becker in a slightly less general framework, is in fact a natural consequence of the well-known Coase (1960) theorem, stating that under transferable utility, the allocation of the surplus stemming from a decision has no impact on the decision taken. This does *not* mean that divorce laws are irrelevant, but simply that they only influence the distribution of welfare between

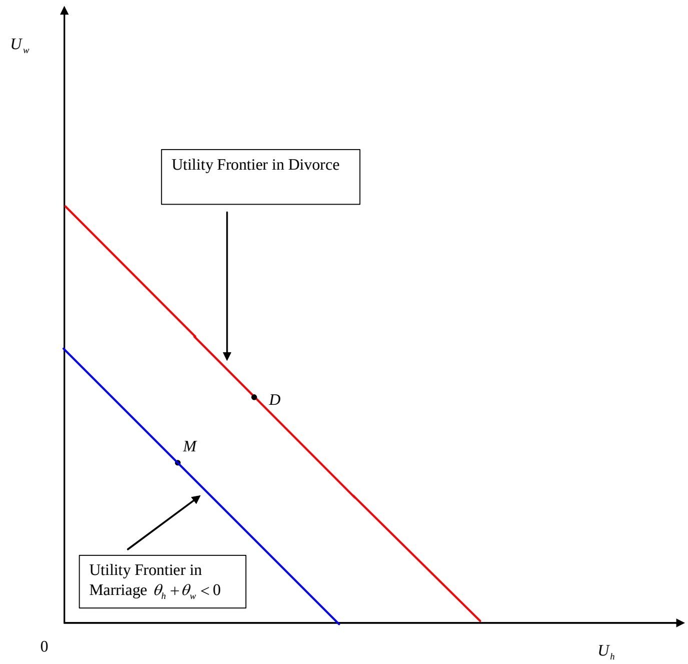

FIGURE 6.2. Pareto frontiers in marriage and divorce, no public goods

the spouses, both in marriage and after divorce - not the divorce decision itself.[19]

The corresponding intuition is easy to grasp from Figure 6.2. Under transferable utility, both the Pareto frontier when married and the Pareto frontier when divorced are straight line with slope -1. Therefore, they cannot intersect; one Pareto set must be included within the other. The optimal divorce decision simply picks up the larger Pareto set. What legal dispositions can do is vary the post divorce allocation along the post divorce Pareto frontier. But if the latter is located within the Pareto set when married, there always exist a particular redistribution of marital surplus that will make both spouses better off than divorce; if, conversely, it is located outside, then whatever the planned allocation of resources within the couple, it is always possible to redistribute income after divorce in such a way that both agents prefer separation.

Finally, it is important to understand the assumptions that are needed for the Becker-Coase theorem to hold. Chiappori, Iyigun and Weiss 2007 (from now on CIW) show that there are three. One is that utility is transferable within marriage (which, in our setting, justifies the GQL form taken for

[19] A recent attempt to test this theoretical prediction is Wolfers (2006).

utilities when married). A second requirement is that utility be transferable *after divorce*; here, we have therefore assumed quasi-linear preferences for singles. Finally, the slopes of the two Pareto frontiers (before and after divorce) must be equal.

While these requirements are indeed satisfied in the example just given, they are in fact quite unlikely to hold in reality. For instance, the assumption of quasilinear preferences if single is totally *ad hoc* . Assume, on the contrary, that preferences if single have the same general form as when married - that is, that:

$$u_s^i\left(q^i,Q\right)=F_s^i\left(Q\right)q_1^i+G_s^i\left(Q,q_{-1}^i\right)+\theta^i,\ i=a,b$$

The question, now, is whether commodity Q, which was publicly consumed when the couple was married, remains public after divorce. In many cases, it does not; for instance, housing typically stops being jointly consumed after the separation. CIW show that, in that case, the second requirement is not satisfied in general. In other situations, the commodity remains public, in the sense that it still enters both ex-spouse's utilities; this is the case for children consumption, for instance. However, the utility adults derive from children's well being may well change after divorce, especially for the parent who does not have full custody. Technically, the F_s^i function is now different between spouses, which violates either the second or the third requirement. All in all, CIW argue that, in general, these requirements are unlikely to be fulfilled - therefore that the Becker-Coase result is unlikely to hold.

An important implication is that the claim, frequently encountered in the literature, that the Becker-Coase theorem is a consequence of the efficiency assumption is incorrect. Whenever any of the CIW requirements is violated, the neutrality result does not hold true. Then the general model developed in the previous subsection, which only assumes efficient behavior (including for divorce decisions), remains valid; but one can find situations in which couples would split under unilateral divorce but not under mutual consent - and also, more surprisingly, cases in which this intuition is reversed, in the sense that divorce occurs under mutual consent but not under unilateral divorce. Figure 6.3, borrowed from Clark (1999) and CIW, illustrates the latter case. With mutual consent, each partner has a "property right" on the allocation within marriage, represented by M. This point is contained in the divorce frontier and both partners can be made better of by renegotiating the divorce settlement and leaving the marriage. In contrast, with unilateral divorce partners have property rights on their divorce allocation, represented by D. This point, however, is contained within the marriage frontier and the partners can find an allocation within marriage that will sustain the marriage.

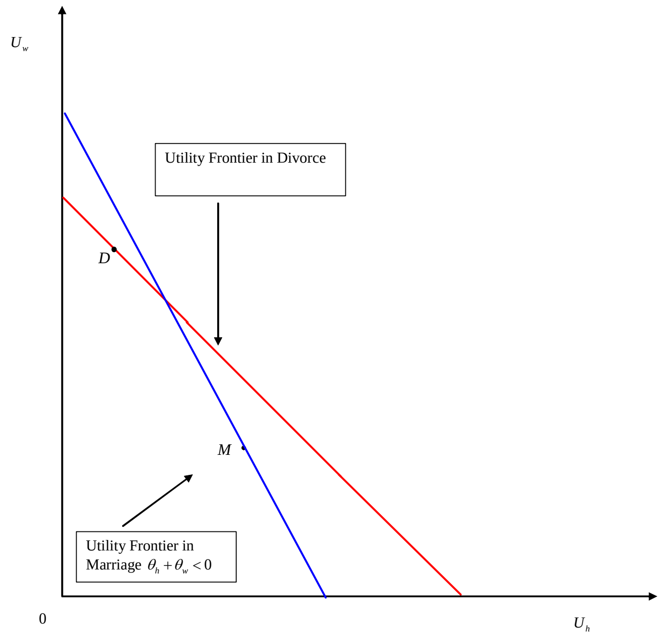

FIGURE 6.3. Divorce without transferable utility

6.5.3 Divorce and investment in children: a non transferable utility example

Endogenous divorce raises some particular contracting issues that do not arise when divorce is exogenous. This is particularly true when we take into account marriage specific investments, such as children - who are (at least partially) 'specific' in the sense that the welfare derived by the parents from the presence of children is often reduced upon divorce (that is, parents suffer a 'capital loss' upon divorce). This kind of problem usually motivates post divorce transfers in the form of child support that will be discussed at length in Chapter 11. Here we wish to examine the role of these post divorce transfers on the investment in children when they are young. To highlight their role, we shall now discuss an extreme case in which such transfers are not possible *within marriage,* because all goods that a couple consumes are public; therefore post divorce transfers are the only feasible transfers between the spouses.

Agents live two periods. Marriage takes place at the beginning of the first period and each marriage produces one child. Caring for the child requires an investment of time by both parents in the first period and the outcome (child quality) is enjoyed in the second period. The household production function for child quality is

$$Q = \sqrt{(1+t^a)(1+t^b)} \tag{6.45}$$

where t^a and t^b are are the proportions of available time spent on child care by a and b, respectively. The time constrains are

$$\begin{aligned} 0 &\leq t^a \leq 1 \\ 0 &\leq t^b \leq 1 \end{aligned} \tag{6.46}$$

The opportunity cost of the time spent with children in the first period is market work. In the second period there is no need to spend time on children and both spouses work full time. However the wage in the second period of life depends on the amount of market work in the first period. We normalize the first period wage of a to 1 and assume that $w^b < 1$. We further assume that the second period wages are directly proportional to the first period labor supply - that is, they are equal to $\gamma(1-t^a)$ and $\gamma w^b(1-t^b)$ for a and b respectively, where $\gamma > 1$. Effectively, this means that incomes in the two period are proportional, which simplifies the analysis considerably.

The utility that parents derive from the child (or child quality) depends on whether or not the parents live together. If the parents stay married, their utility from quality is $\alpha \ln Q$, but if the parents separate, their utility from child quality is reduced to $(1-\delta)\alpha \ln Q$, where $0 < \delta < 1$. The utility parents depends on the child quality, on their consumption of goods q and if married, the quality of their match, θ, that is revealed only after one period of marriage.

If the partners are married, the utility of both partners is

$$u_m = \ln q + \alpha \ln Q + \theta \tag{6.47}$$

Divorce may occur if the realized value (revealed at the beginning of the second period) is sufficiently low. Following divorce, the utilities of the former spouses are

$$u_d = \ln q_d^i + (1-\delta)\alpha \ln Q, \;\; i = a, b \tag{6.48}$$

where q_d^i denotes the post divorce consumption of the two spouses. Note that we assume here that when a couple is married *all* good are public. The only way to influence the division of the gains from marriage is through transfer in the aftermath of divorce. As we shall show, such transfers can influence the investment in children during marriage and probability of divorce.

As in the previous subsection, we continue to assume no borrowing or lending. Then,

$$\begin{aligned} q_1 &= w^a(1-t^a) + w^b(1-t^b) \\ q_2 &= \gamma w^a(1-t^a) + \gamma w^b(1-t^b) = \gamma q_1 \end{aligned} \tag{6.49}$$

where q_1 denotes the joint consumption in the first period, while q_2 is the joint consumption if the partners remain married or the sum of their private consumptions if they separate. Thus, the allocation of time in the first period determines the consumption available to the parents each period as well as the quality of the child that they enjoy in the second period. The only issue then is how is this allocation determined.

A necessary condition for an efficient allocation of time is that the cost of producing child quality, in terms of the foregone earnings of the couple during the two periods of life, should be minimized. In the this example, these costs are

$$C(Q) = (1+\gamma)(w^a t^a + w^b t^b) \tag{6.50}$$

and cost minimization takes a simple form. In particular if there is an interior solution and both partners contribute time to the child[20] then we must have

$$w^b(1+t^b) = 1 + t^a \tag{6.51}$$

Whether or not an interior solution arrises, efficiency requires that the low wage person, b, should contribute more time to the child and the question is if and how such *unequal* contribution can be implemented. The answer depends on the contracting options that the couple have. We shall assume here that the partners can always commit, at the time of marriage, on some post divorce allocation of resources, provided that it falls within some legal bounds. The justification for this assumption is that the event of separation and the resources available upon separation can be verified so that contracts contingent on these variables can be enforced by law. Denoting by β the share received by the low wage person, b, the post divorce consumption levels are

$$\begin{aligned} q_d^a &= (1-\beta)[\gamma w^a(1-t^a) + \gamma w^b(1-t^b)] \\ q_d^b &= \beta[\gamma w^a(1-t^a) + \gamma w^b(1-t^b)]. \end{aligned} \tag{6.52}$$

It is more difficult, however, to verify the time allocation and in particular time spent on children, and we shall allow for the possibility that partners cannot commit at the time of marriage on how much time they will spend with the child.

Following the realization of θ at the beginning of the second period, and given the predetermined quality of children and divorce contract, marriage will continue if

$$\alpha \ln Q + \ln q_2 + \theta \geq (1-\delta)\alpha \ln Q + \max\{(\alpha \ln q_d^a, \alpha \ln q_d^b) \tag{6.53}$$

[20] The efficiency requirements include regions in which only one person contributes. These regions depend on the desire for children relative the wages of the two spouse. If $\alpha < 1$ the mother will work only at home and the father only in the market. To allow for an interior solution, we assume that $2 > \alpha > 1$. Then for $\alpha w^b > 1$, both partners work part time at home and part time in the market.

and dissolve otherwise. This rule holds because, by assumption, utility is not transferable within marriage and each partner is free to walk away from the marriage. Clearly, the person who can attain higher consumption outside marriage will trigger the divorce.

Examining equation (6.52), we see that if b receives a higher share of family resources upon divorce, $\beta > \frac{1}{2}$, he will trigger the divorce and divorce occurs if

$$\theta < -\delta\alpha \ln Q + a \ln \beta)$$

If a obtains the larger share, $\beta < \frac{1}{2}$, she will trigger the divorce an divorce occurs if

$$\theta < -\delta \ln \alpha Q + \alpha \ln(1-\beta)$$

Finally, with equal sharing divorce occurs if

$$\theta < -\delta\alpha \ln Q - \alpha \ln 2$$

The probability of divorce is, therefore,

$$\text{Prob}(\text{divorce}) = \begin{cases} F(-\delta\alpha \ln Q + \ln(1-\beta)) & \text{if} \quad \beta \leq \frac{1}{2} \\ F(-\delta\alpha \ln Q - \ln 2) & \text{if} \quad \beta = \frac{1}{2} \\ F(-\delta\alpha \ln Q + \ln \beta) & \text{if} \quad \beta \geq \frac{1}{2} \end{cases} \tag{6.54}$$

where $F(.)$ is the cumulative distribution of θ. We assume that this distribution is symmetric with zero mean. We see that a high child quality, Q, and high loss of child quality upon divorce, δ, generate higher gains from continued marriage and reduce the probability of divorce. A negative shock to θ is required to initiate a divorce, because of the cost associated with reduced child quality, represented here by the term $\delta\alpha \ln Q$, and loss of the utility gains from joint consumption, which depends on the allocation of resources upon divorce ($\ln(1-\beta)$) if $\beta \leq \frac{1}{2}$ or $\ln \beta$ if $\beta \geq \frac{1}{2}$).

At this point we can already make three observations:

- An increase in child quality reduces the probability of divorce.
- For a given child quality, Q, the lowest probability of divorce is attained when $\beta = \frac{1}{2}$.
- For $\beta \neq \frac{1}{2}$, divorce is inefficient in the sense that the spouse who triggers the divorce does not internalize the reduced welfare of the spouse who is left behind and would rather stay married for at least some range of θ's below the trigger. Note that the contrast to the results in the previous section, where divorce was efficient and the probability;of divorce was independent of the division of income in the aftermath of divorce. The Becker-Coase theorem does not hold when transfers within marriage are not feasible.

We now turn to the determination of the investment in children in the first period. We first consider the benchmark case of equal sharing, with $\beta = \frac{1}{2}$. Defining the trigger value for divorce as

$$\theta^* = -\delta\alpha \ln Q - \ln 2, \tag{6.55}$$

the expected utility of each of the two partners is then

$$\begin{aligned} E(u) &= \ln q_1 + (1 - F(\theta^*)[\alpha \ln Q + \ln q_2] + \int_{\theta^*}^{\infty} \theta f(\theta) d\theta \\ &\quad + F(\theta^*[(1-\delta)\alpha \ln Q + \ln \frac{q_2}{2}] \\ &= \ln q_1 + \ln q_2 + \alpha \ln Q + \int_{\theta^*}^{\infty} \theta f(\theta) d\theta + F(\theta^*)\theta^* \end{aligned}$$

Maximizing $E(u)$ with respect to t^a and t^b, respectively, we obtain the first order conditions for an interior solution

$$\frac{1}{q_1} + \frac{\gamma}{q_2} = [1 - \delta F(\theta^*] \frac{\alpha}{2(1+t^a)} \tag{6.56}$$

$$\frac{w^b}{q_1} + \frac{w^b \gamma}{q_2} = [1 - \delta F(\theta^*)] \frac{\alpha}{2(1+t^b)} \tag{6.57}$$

The interpretation of these two conditions is transparent. For each spouse, the couple equates the expected marginal gain in terms of child quality, associated with an increase in the time investment, to the marginal costs in terms of forgone consumption of the parents in the two periods. The two conditions together imply condition (6.55) which means that, under equal division, efficiency is maintained. Importantly, there is no need for the partners to commit on the time spent with the child because the Nash equilibrium that arises under non cooperation satisfies exactly the *same* conditions. That is, in equilibrium, each spouse, including the low wage person who is called upon to supply more hours, would do it from selfish reasons, provided that the other spouse supplies the efficient quantity of time.

The situation is quite different if the partners choose ex-ante an unequal division but cannot commit on the allocation of time. For concreteness, consider the case that in which the low wage person, b, is the husband and he receives a lower share of family resources, $\beta < \frac{1}{2}$. Now each spouse will maximize their *own* payoff functions. Let the new trigger function be

$$\hat{\theta} = -\delta\alpha \ln Q + \ln(1-\beta). \tag{6.58}$$

Then, the choice of t^a as a function t^b is determined by the maximization with respect to t^a of

$$E(u^a) = \ln q_1 + \ln q_2 + \alpha \ln Q + \int_{\hat{\theta}}^{\infty} \theta f(\theta) d\theta + F(\hat{\theta})(\hat{\theta}), \tag{6.59}$$

with the first order condition

$$\frac{1}{q_1} + \frac{\gamma}{q_2} = [1 - \delta F(\hat{\theta}] \frac{\alpha}{2(1+t^a)}. \tag{6.60}$$

Similarly, the choice of t^b as a function t^a is determined by the maximization with respect to t^b of

$$E(u^b) = \ln q_1 + \ln q_2 + \alpha \ln Q + \int_{\hat{\theta}}^{\infty} \theta f(\theta) d\theta + F(\hat{\theta})\hat{\theta} + F(\hat{\theta})[\ln \beta - \ln(1-\beta], \tag{6.61}$$

with the first order condition

$$\frac{w^b}{q_1} + \frac{\gamma w^b}{q_2} = [1 - \delta F(\hat{\theta}) + f(\hat{\theta}) \ln \frac{\beta}{1-\beta}] \frac{\alpha}{2(1+t^b)}. \tag{6.62}$$

We see that the expected marginal reward from exerting effort is smaller to the husband (note that for $\beta < \frac{1}{2}$, $\ln \frac{\beta}{1-\beta} < 0$). The husband takes into account her lower consumption, and thus higher marginal utility from consumption, following divorce. He responds by shifting additional time in the first period into work so that his future wage will be higher. This defensive investment in market work by the husband causes an inefficient time allocation. Examining conditions (6.60) and (6.61), we see that the requirement for cost minimization is *not* satisfied.

When partners cannot commit on the allocation of time, commitments made at the time of marriage should adjust. One may assume that the husband has a higher bargaining power at the time of marriage, because of his higher wage and thus higher consumption as single. However, it makes sense for the husband to give up some of his power, which will raise the "pie" available during marriage that he and his the wife enjoy equally.

Returning now to the case of equal division and efficient allocation of time, we can provide some further analysis of the investment decision. Using the efficiency conditions (and constant returns to scale) we have that, in an interior solution,

$$Q = \sqrt{w^b}(1+t^b). \tag{6.63}$$

We also have that

$$\begin{aligned} q_1 &= 1 + w^b - t^a - w^b t^b \\ &= 2(1+w^b) - 2\sqrt{w^b} Q \end{aligned} \tag{6.64}$$

We can, therefore, rewrite condition (6.60) in the form

$$\frac{2\sqrt{w^b}}{1+w^b-\sqrt{w^b}Q} = [1-\delta F(-\delta\alpha \ln Q - \ln 2]\frac{\alpha}{Q}. \tag{6.65}$$

Condition (6.65) then determines the desired child quality and we can then use the efficiency conditions to trace back the implied allocation of time. The left hand side of (6.65) represents the marginal disutility (associated with lost consumption) and unambiguously rises with Q. However, the right hand side of (6.65), which represents the expected marginal utility from having children in the second period, involves two conflicting effects: A higher level of child quality reduces the marginal utility from children and also reduces the probability of divorce. Therefore, the marginal expected utility can either rise or fall and the outcome depends on the shape of the *hazard* associated with the distribution of quality match $F(\theta)$. Specifically,

$$\begin{aligned}
&\frac{d}{dQ}\alpha[\frac{1-\delta F(-\delta \ln Q - \ln 2)}{Q}] \\
&= \alpha[\frac{(\delta^2 f(-\delta \ln Q - \ln 2) - (1-\delta F(-\delta \ln Q - \ln 2))}{Q^2}]
\end{aligned}$$

which is negative if.

$$\delta^2 \frac{f(-\delta \ln Q - \ln 2)}{1-F(-\delta \ln Q - \ln 2)} < 1.$$

This condition is satisfied, for instance, for the normal distribution if $\sigma \geq 1$, because then the hazard is an increasing function and its value at zero is $\sqrt{\frac{2}{\pi}\frac{1}{\sigma}}$.

Assuming that the expected marginal utility from children declines with Q, it is easy to see that the investment in children is reduced in response to increasing risk, represented here by a mean preserving increase in the spread of the *shocks* to match quality. However, that the expected marginal utility from children declines with Q, the investment in children may rise in order to stabilize the marriage. In either case, the "efficient" family responds to such change in circumstances in a way which is optimal for *both* spouses, without having equality in action. What is required, of course, is for both partners to have equal interest in the outcome and, given that all goods in marriage are public, such harmony can be achieved by a binding commitments of an equal division of resources upon divorce. This considerations can go part of the way in explaining the prevalence of equal divisions following divorce. A binding commitment is required, because, *ex post*, after divorce has occurred and the investments have been made, there is no incentive for further transfers. However at the time of marriage, a spouse with a higher income may be willing to commit on a post divorce transfer in order to induce the seemingly less powerful spouse to invest in children.

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Part II

Equilibrium Models of the Marriage Market

7

Matching on the Marriage Market: Theory

Individuals in society have many potential partners. This situation creates competition over the potential gains from marriage. In modern societies, explicit price mechanisms are not observed. Nevertheless, the assignment of partners and the sharing of the gains from marriage can be analyzed within a market framework. The main insight of this approach is that the decision to form and maintain a particular union depends on the whole range of opportunities and not only on the merits of the specific match. However, the absence of explicit prices raises important informational issues. There are two main issues distinguishing the approaches used in the matching literature. The first issue concerns the information structure and the second relates to the extent of transferability of resources among agents with different attributes. Specifically, models based on frictionless matching assume that perfect and costless information about potential matches is available to all participants; the resulting choices exclusively reflect the interaction of individual preferences. Such models may belong to several classes, depending on whether or not compensating transfers are allowed to take place between individuals and, if so, at what 'exchange rate'. Still, they all rely on a specific equilibrium concept, namely stability. Formally, we say that a matching is *stable* if:

(i) There is no married person who would rather be single.

(ii) There are no two (married or unmarried) persons who prefer to form a new union.

The interest in stable marriage assignments arises from the presumption that in a frictionless world, a marriage structure which fails to satisfy (i) and (ii) either will not form or will not survive.

Models based on frictionless matching are studied in the next three Sections. An alternative approach emphasizes the role of frictions in the matching process; in these models, based on search theory, information is limited and it takes time to find a suitable match. The corresponding framework will be discussed in the last Section.

7.1 Stable Matching without transfers: the Gale-Shapley Algorithm

We begin our analysis of the marriage market assuming that there are no frictions - that is, that each man and woman knows the potential gains from marrying any potential mate. Marriage can be viewed as a voluntary matching of males and females, allowing for the possibility of staying single. We consider here only monogamic marriages so that each person can have at most one spouse of the opposite sex. These assignments can be presented by matrices with 0/1 entries depending upon whether or not male i is married to female j. Since we consider only monogamic marriages, there is at most one nonzero entry in each column and row. An illustration of such a representation is shown in Example 7.1. In this example there are 4 men and 3 women, where man 1 is married to woman 3, man 2 is married to woman 1, man 3 is single and man 4 is married to woman 2:

Example 7.1

	1	*2*	*3*
1	0	0	1
2	1	0	0
3	0	0	0
4	0	1	0

We first study matching when agents cannot make transfers between each other. We thus assume that a marriage generates an outcome for each partner that is fully determined by the individual traits of the partners; this outcome cannot be modified by one partner compensating the other for his or her deficient traits. Although somewhat extreme, this assumption captures situations where, because of public goods and social norms that regulate within family allocations, there is limited scope for transfers, so that the success of a marriage mainly depends on the attributes of the partners. However, an undesired marriage can be avoided or replaced by a better one. Although there is no scope for trade within marriage, there is margin for trade across couples.

Let there be a given, finite number of men, M, and a given, finite number of women, N. We designate a particular man by i and a particular woman by j. Assume that each man has a preference ranking over all women and each woman has a preference ordering over all men. Such preferences can be represented by a $M \times N$ bi-matrix with a pair of utility payoffs, (u_{ij}, v_{ij}) in each cell. For a given j, the entries v_{ij} describe the preference ordering of woman j over all feasible males, $i = 1, 2...M$. Similarly, for a given i, the entries u_{ij} describe the preference ordering of man i over all feasible women $j = 1, 2...N$. We may incorporate the ranking of the single state by adding a column and a row to the matrix, denoting the utility levels of single men and women by u_{i0} and v_{0j}, respectively. The preferences of men and women are datum for the analysis. However, the representations of these preferences by the utility payoffs are only unique up to monotone transformations. An illustration of such a representation with 4 men and 3 women is shown in Example 7.2:

Example 7.2

	1	*2*	*3*	*0*
1	u_{11}, v_{11}	u_{12}, v_{12}	u_{13}, v_{13}	u_{10}
2	u_{21}, v_{21}	u_{22}, v_{22}	u_{23}, v_{23}	u_{20}
3	u_{31}, v_{31}	u_{32}, v_{32}	u_{33}, v_{33}	u_{30}
4	u_{41}, v_{41}	u_{42}, v_{42}	u_{43}, v_{43}	u_{40}
0	v_{01}	v_{02}	v_{03}	

Gale and Shapley (1962) were the first to demonstrate that a stable matching always exists, and suggested an algorithm which generates a stable outcome in a finite number of steps. For simplicity, we assume here that all rankings are strict. To begin, let each man propose marriage to his most favored woman. A woman rejects any offer which is worse than the single state, and if she gets more than one offer she rejects all the dominated offers; the non rejected proposal is put on hold ('engagement'). At the second step, each man who is not currently engaged proposes to the woman that he prefers most among those women who have not rejected him. Women will reject all dominated offers, including the ones on hold. This mechanism is repeated until no male is rejected; then the process stops. Convergence is ensured by the fact that no woman is approached more than once by the same man; since the number of men and women is finite, this requirement implies that the process will stop in finite time. The process must yield a stable assignment because women can hold all previous offers. So if there is some pair not married to each other it is only because either the man did not propose (implying that he found a better mate or preferred staying single) or that he did and was rejected (implying that the potential wife had found a better mate or preferred staying single).

The stable assignment that is realized in the way just described need not be unique. For instance, a different stable assignment may be obtained if women make the offers and men can reject or hold them. Comparing these stable assignments, it can be shown that if all men and women have strict preferences, the stable matching obtained when men (women) make the proposal is weakly preferred by *all* men (women). This remarkable result shows that social norms of courting can have a large impact on matching patterns (see Roth and Sotomayor 1990, ch. 2).

As an example, let there be 3 men and 3 women and consider the matrix of utility payoff in Example 7.3 (setting the value of being single to zero for all agents):

Example 7.3

$$\begin{array}{cc|ccc} & & & \text{Women} & \\ & & \mathit{1} & \mathit{2} & \mathit{3} \\ \hline & \mathit{1} & 3,2 & 2,6 & 1,1 \\ \text{Men} & \mathit{2} & 4,3 & 7,2 & 2,4 \\ & \mathit{3} & 1,1 & 2,1 & 0,0 \end{array}$$

Note that, in this case, preferences diverge among men; man 1 ranks woman 1 above women 2 and 3, while men 2 and 3 both put woman 2 at the top of their ranking. Similarly, there is disagreement among women; man 1 is the most attractive match for woman 2, while women 1 and 3 both consider man 2 as the best match. There is also a lack of reciprocity; man 1 would rather marry woman 1 but, alas, she would rather marry man 2.

As a consequence there are two possible stable assignments, depending on whether men or women move first. If men move first, man 1 proposes to woman 1, and men 2 and 3 both propose to woman 2, who rejects man 3, but keeps man 2. In the second round, man 3 proposes to woman 1 who rejects him. In the last round, man 3 proposes to woman 3 and is not rejected so that the procedure ends up with the outcome emphasized in bold letters in the matrix below:

$$\begin{array}{cc|ccc} & & & \text{Women} & \\ & & \mathit{1} & \mathit{2} & \mathit{3} \\ \hline & \mathit{1} & \mathbf{3},\mathbf{2} & 2,6 & 1,1 \\ \text{Men} & \mathit{2} & 4,3 & \mathbf{7},\mathbf{2} & 2,4 \\ & \mathit{3} & 1,1 & 2,1 & \mathbf{0},\mathbf{0} \end{array}$$

One can check directly that this assignment is stable. Men 1 and 2 obtain their best option and do not wish to change spouse, while man 3 cannot find a better match who is willing to marry him.

Now, if women move first, woman 2 proposes to man 1 and women 1 and 3 both propose to man 2, who rejects woman 3, but keeps woman 1. In the second round, woman 3 proposes to man 1 who rejects her. In the last round, woman 3 proposes to man 3 and is not rejected so that the procedure ends up with the outcome emphasized in bold letters in the matrix below:

$$\begin{array}{cc|ccc} & & & \text{Women} & \\ & & \mathit{1} & \mathit{2} & \mathit{3} \\ \hline & \mathit{1} & 3,2 & \mathbf{2},\mathbf{6} & 1,1 \\ \text{Men} & \mathit{2} & \mathbf{4},\mathbf{3} & 7,2 & 2,4 \\ & \mathit{3} & 1,1 & 2,1 & \mathbf{0},\mathbf{0} \end{array}$$

Again, one can check directly that this assignment is stable. Women 1 and 2 obtain their best option and do not wish to change spouse, while woman 3 cannot find a better match who is willing to marry her. It is seen that the first assignment, in which men move first is better for all men (except for man 3 who is indifferent) and the second assignment, in which women move first is better for all women (except for woman 3 who is indifferent).

A very special case arises if women and men can be ranked by a single male trait x and a single female trait y. This assumption introduces a strong commonality in preferences, whereby all men agree on the ranking of all women and vice versa. Specifically, let us rank males and females by their marital endowment (that is, $x_{i+1} > x_i$ and $y_{j+1} > y_j$), and let us assume that there exists a "household output function" $h(x_i, y_j)$ that specifies the marital output as a function of the attributes of the two partners.[1] This output is then consumed jointly as a public good, or shared between the partners in some rigid fashion (equally for instance) in all marriages. A natural question is: Who marries whom? Would a stable assignment associate a male with a high marital endowment to a female with high marital endowment (what is called *positive assortative mating*)? Or, to the contrary, will a highly endowed male be matched with a low endowment female (*negative assortative mating*)? The answer obviously depends on the properties of the function $h(x, y)$. It is easy to show that if $h(x, y)$ is strictly increasing in both traits, the unique stable assignment is one with perfect positive assortative mating. To see that, suppose that men propose first. In the first round, all men will propose to the woman with the highest female attribute and she will reject all offers but the one from the best man. In the second round, all remaining men will propose to the second best woman and she will reject all but the second best man and so on. The situation when women propose first is identical. Symmetrically, if the male and female traits have opposing effects on output, the unique stable assignment is one with perfect negative assortative mating.

An interesting extension arises when the relevant features of spouses are not immediately revealed, which may cause a delay in marriage. Bergstrom and Bagnoli (1993) consider a matching with asymmetric information in a two period model. They assume that the female trait is immediately revealed but the male trait is revealed later. The equilibrium that emerges is such all women marry in the first period. Men that know their high quality will delay their marriage and low quality males will marry early but to low quality women. The more desirable females marry successful older males. Thus, the model can explain the prevalent pattern if matching by age, whereby the bride is typically younger than the groom.

In addition to the identification of stable assignments, one can use the Gale-Shapley algorithm to obtain simple comparative static results. Allow-

[1] This "household output function" should be distinguished from the standard household production function described in the previous sections, which take the attributes of the spouses as fixed. Here we are interested in a reduced form that depends only on attributes after all relevant activities have been chosen so as to achieve intrahousehold efficiency.

ing for unequal numbers of men and women, it can be shown that a change in the sex ratio has the anticipated effect. An increase in the number of women increases the welfare of men and harms some women. The same result holds in many to one assignment.

7.2 Stable Matching with transferable utilities: the Becker-Shapley-Shubik model

7.2.1 The basic framework

The properties of the previous model heavily depend on the assumption that transfers are impossible, so that a person cannot 'compensate' a potential partner for marrying him or her despite some negative traits. In practice, this assumption is hard to maintain. Whenever one commodity at least is privately consumed, a spouse can reduce her private consumption to the partner's benefit, which de facto implements a compensation. We now consider the opposite polar case in which not only transfers are feasible, but there is a medium of exchange that allows partners to transfer resources between them *at a fixed rate of exchange*; that is, we assume that utilities are transferable (see Chapter 3).

Instead of introducing two *exogenous* matrices $u = (u_{ij})$ and $v = (v_{ij})$ as in the case of non-transferable utility, we now consider a unique output matrix with entries ζ_{ij} which specifies the total output of each marriage. Given the assumption of transferable utility, this total output can be divided between the two partners. We denote the utility payoff of the husband by u_{ij} and the utility payoff of the wife by v_{ij}. Thus, by definition, if i and j form a match we have

$$u_{ij} + v_{ij} = \zeta_{ij} \tag{7.1}$$

Note, however, the key difference with the previous section with no transfers: while matrices u and v were then *given* (as part of the statement of the problem), they are now *endogenous* (and part of its solution, since they are determined at equilibrium - or, here, at the stable matching).

As before, we are interested only in *stable* matching. The question is: for a given matrix $\zeta = (\zeta_{ij})$, which are the stable assignments, and what are the corresponding allocations of output (or *imputations*) within each marriage. Note that the question is, in a sense, more difficult than in the case with no transfers, since the distribution of output between members is now endogenous and has to be determined in equilibrium. Still, it is relatively easy to apply the criteria for stability in the case of transferable utility. Specifically, one can show that *a stable assignment must maximize total output over all possible assignments.* It is this simple and powerful result that makes the assumption of transferable utility attractive in matching models.

Two examples

To understand this result, consider first the simplest possible case: let there be two people of each sex. Assuming that marriage dominates the single state (that is if any two individuals remain unattached they can gain by forming a union), there are two possible assignments: Man 1 marries woman 1 and man 2 marries woman 2, or man 1 is married to woman 2 and man 2 is married to woman 1. In testing for stability we treat the potential marital outputs ζ_{ij} as given and the divisions u_{ij} and v_{ij} as variables. Suppose, now, that the assignment in which man 1 marries woman 2 and man 2 marries woman 1 (the off diagonal assignment) is stable. Then, the following inequalities must hold:

$$u_{12} + v_{21} \geq \zeta_{11} \tag{7.2}$$

$$u_{21} + v_{12} \geq \zeta_{22} \tag{7.3}$$

If the first inequality fails to hold then male 1 and female 1, who are currently not married to each other, can form a union with a division of utilities which will improve upon their current situations, defined by u_{12} and v_{21}. If the second inequality does not hold then man 2 and woman 2, who are presently not married to each other, can form a union and divide utilities so as to improve over the current values u_{21} and v_{12}. From equation 1 we have $\zeta_{12} = u_{12} + v_{12}$ and $\zeta_{21} = u_{21} + v_{21}$ so that equation (7.2) can be rewritten as

$$\zeta_{12} - v_{12} + \zeta_{21} - u_{21} \geq \zeta_{11}. \tag{7.4}$$

Adding conditions (7.4) and (7.3) we obtain

$$\zeta_{12} + \zeta_{21} \geq \zeta_{11} + \zeta_{22} \tag{7.5}$$

By a similar argument, an assignment along the main diagonal will be stable only if (7.5) is reversed. Condition (7.5) is not only necessary but also sufficient for stability of the off diagonal assignment. For if it is satisfied we can find values of u and v such that (7.2) and (7.3) hold. Such imputations support the stability of the assignment since it is then impossible for both partners to gain from reassignment.

To illustrate the implications of the transferable utility assumption and the implied maximization of *aggregate* marital output, let us consider a second example. There are 3 men and 3 women and consider the matrix of marital outputs below:

Example 7.4

		Women		
		1	*2*	*3*
Men	*1*	5	8	2
	2	7	9	6
	3	2	3	0

Notice that the entries in this matrix are just the sums of the two terms in Example 7.3 discussed above. In this regard, non transferable utility can be thought of as a special case of transferable utility, where the division of the output in each marriage is predetermined and cannot be modified by transfers between spouses. For instance, if each partner receives half of the marital output in *any* potential marriage, the Gale Shapley algorithm yields the unique stable outcome, which is on the diagonal of this matrix. In contrast, with transferable utility, the unique assignment that maximizes aggregate marital output, indicated by the bold numbers in the matrix below, is *not* on the diagonal. This assignment yields aggregate output of 16, compared with an aggregate output of 14 on the diagonal.

		Women		
		1	*2*	*3*
Men	*1*	5	**8**	2
	2	7	9	**6**
	3	**2**	3	0

Though all men would obtain the highest marital output with woman 2, and all women would obtain the highest output with man 2 (implying that ζ_{22} is the largest entry in the marital output matrix 7.4), the best man and the best woman are *not* married to each other. With transfers, the assignment on the diagonal is no longer stable, because if couple $1,1$ and couple $2,2$ exchange partners, there is an aggregate gain of 1 unit of the transferable good. Then man 1 can, despite his lower contribution to the marital output, bid away the best woman by offering her a larger amount of private consumption and still be better off than in the initial match with woman 1. Similarly, woman 1 can bid away the best man by offering him a larger share of private consumption and still be better off than in the initial match with man 1. The higher *aggregate* output achievable when man 2 and woman 2 are *not* married to each other implies that, for any division of the marital output of 9 that these partners can obtain together, at least one of the partners can be made better off in an alternative marriage.

Stable matching with a finite number of agents

Let us now consider the general assignment problem with M males and N females. Let ζ_{ij} denote the total output of a marriage between male i and female j, and let ζ_{i0} (resp. ζ_{0j}) be the utility that person i (resp. person j) receives as single (with $\zeta_{00} = 0$ by notational convention). Then the difference $z_{ij} = \zeta_{ij} - \zeta_{i0} - \zeta_{0j}$ is the marital *surplus* that male i and female j generate by marrying each other.

We define assignment indicators, a_{ij}, such that $a_{ij} = 1$ if and only if i is married with j and $a_{ij} = 0$ otherwise. We also define $a_{i0} = 1$ if and only if i is single, and similarly $a_{0j} = 1$ if and only if j is single. Then, following Gale (1960, chapters 1 and 5) and Shapley and Shubik (1972), we may describe the stable assignment as a solution to an integer linear programming problem:

$$\max_{a_{ij}} \sum_{i=0}^{M} \sum_{j=0}^{N} a_{ij} \zeta_{ij} \tag{7.6}$$

subject to $a_{ij} \geq 0$ and

$$\sum_{j=0}^{N} a_{ij} = 1, \; i = 1, 2., .M, \tag{7.7}$$

$$\sum_{i=0}^{M} a_{ij} = 1, \; j = 1, 2., .N. \tag{7.8}$$

A first remark is that since $a_{0j} = 1 - \sum_{i=1}^{M} a_{ij}$ and $a_{i0} = 1 - \sum_{j=1}^{N} a_{ij}$ the program can be rewritten as

$$\max_{a_{ij}} \sum_{i=1}^{M} \sum_{j=1}^{N} a_{ij} \left(\zeta_{ij} - \zeta_{i0} - \zeta_{0j} \right) + C = \max_{a_{ij}} \sum_{i=1}^{M} \sum_{j=1}^{N} a_{ij} z_{ij} + C \tag{7.9}$$

subject to

$$\sum_{j=1}^{N} a_{ij} \leq 1, \; i = 1, 2., .M, \tag{7.10}$$

$$\sum_{i=1}^{M} a_{ij} \leq 1, .j = 1, 2., .N, \tag{7.11}$$

where $C = \sum_{i=1}^{M} \zeta_{i0} + \sum_{j=1}^{N} \zeta_{0j}$ is the aggregate utility of singles. Therefore, the maximization of aggregate marital output over all possible assignments is equivalent to the maximization of aggregate surplus and, without loss of generality, we can normalize the individual utilities by setting $\zeta_{i0} = \zeta_{0j} = 0$ for all i and j.

Secondly, one can actually assume that in the problem above, the a_{ij} can be real numbers in the $(M-1)$-dimensional simplex (instead of constraining them to be integers). Intuitively, a_{ij} can then be interpreted as the probability that Mr. i marries Mrs. j. Note, however, that given the linearity of the structure, one solution at least to this generalized problem is anyway attained with all a_{ij} being either zero or one.

The basic remark, at that point, is that the program thus defined is a standard, linear programming problem; that is, we want to find a vector

(a_{ij}) that maximizes the linear objective (7.6) (or (7.9)) subject to the linear constraints (7.7) and (7.8) (resp. (7.10) and (7.11)). We can therefore use the standard tools of linear programming - specifically, duality theory. Associated with the maximization of aggregate surplus which determines the assignment is a dual cost minimization problem that determines the set of possible divisions of the surplus. Specifically, one can define a dual variable u_i for each constraint (7.10) and a dual variable v_j for each constraint (7.11); the dual program is then:

$$\min_{u_i,v_j}\left(\sum_{i=1}^{M} u_i + \sum_{j=1}^{N} v_j\right) \tag{7.12}$$

subject to

$$\begin{array}{rcl} u_i + v_j & \geq & z_{ij},\ i \in \{1,...,M\},\ j \in \{1,...,N\} \\ u_i & \geq & 0,\ v_j \geq 0. \end{array} \tag{7.13}$$

The optimal values of u_i and v_j can be interpreted as shadow prices of the constraints in the original maximization problem (the primal). Thus, $u_i+v_j = z_{ij}$ if a marriage is formed and $u_i+v_j \geq z_{ij}$ otherwise.[2] This result is referred in the literature as the complementarity slackness condition, see for instance Gale (1978). It has a very simple interpretation. Any man i is a resource that be can allocated to any woman, but only one woman, in society. Similarly woman j is a resource that can be allocated to any man in society, but only one man. The shadow price of each constraint in (7.10) describes the social cost of moving a particular man (woman) away from the pool of singles, where he (she) is a potential match for others. The sum of these costs $u_i + v_j$ is the social cost of removing man i and woman j from the pool while z_{ij} is the social gain. Thus if $u_i + v_j > z_{ij}$, the costs exceed the gains and the particular marriage would not form. However, if a marriage is formed then $u_i + v_j = z_{ij}$ and each person's share in the resulting surplus equals their opportunity cost in alternative matches.

The crucial implication of all this is that the shadow price u_i is simply the share of the surplus that Mr.i will receive at the stable matching (and similarly for v_j); consequently, conditions (7.13) are nothing else than the stability conditions, stating that if i and j are not matched at the stable matching, then it must be the case that the surplus they would generate if matched together (that is z_{ij}) is not sufficient to increase *both* utilities above their current level!

These results have a nice interpretation in terms of decentralization of the stable matching. Indeed, a stable assignment can be supported (implemented) by a reservation utility vector, whereby male i enters the market

[2] Conversely, a_{ij} can be seen as the dual variable for constraint (7.13). In particular, if $a_{ij} > 0$, then the constraint must be binding, implying that $u_i + v_j = z_{ij}$.

with a reservation utility u_i and is selected by the woman that gains the highest surplus $z_{ij} - u_i$ from marrying him. Similarly, woman j enters with a reservation utility v_j and is selected by the man who has the highest gain $z_{ij} - v_j$ from marrying her. In equilibrium, each agent receives a share in marital surplus that equals his\her reservation utility. In a sense, u_i and v_j can be thought of as the 'price' that must be paid to marry Mr. i or Mrs. j; each agent maximizes his/her welfare taking as given this 'price' vector.

It is important to note that the informational requirements for implementing a stable assignment with transferable utility is quite different than for the Gale-Shapley no transfer case. For the latter, we only require that each person can rank the members of the opposite sex. With transferable utility, the planner needs to know the surplus values of all possible matches and agents should each know the share of the surplus that they would receive with any potential spouse.

In general, there is a whole set of values for u_i , v_j that support a stable assignment. While the issues related to the distribution of surplus will be discussed in the next Chapter, we present in the table below three (of many) such imputations, denoted by a, b and c, for the stable assignment in example 7.4.

Imputation		a				b				c		
		W		M		W		M		W		M
Individual shares	v_1	2	u_1	3	v_1	2	u_1	4	v_1	1	u_1	5
	v_2	5	u_2	5	v_2	4	u_2	5.5	v_2	3	u_2	6
	v_3	1	u_3	0	v_3	0.5	u_3	0	v_3	0	u_3	1

The reader can readily check that each of these imputations supports a stable match.

Extension: continuum of agents

Finally, although the previous argument is presented in a finite setting, it is fully general, and applies to continuous models as well. From a general perspective, we only need that the set of men and the set of women, denoted X and Y, be complete, separable metric spaces equipped with Borel probability measures F and G; note that no restriction is imposed on the dimension of these spaces (it may even be infinite). The surplus function $h(x, y)$ is only assumed to be upper semi-continuous. The problem can be stated as follows: find a measure Φ on $X \times Y$ such that:

- The marginals of Φ on X and Y are F and G, respectively.
- The measure Φ solves $\max_\Phi \int_{X\times Y} h(x, y)\, d\Phi(x, y)$, where the max is taken over the set of measures satisfying the previous conditions.

A complete analysis of this problem is outside the scope of this book; the reader is referred to Chiappori, McCann and Neishem (2010) or Ekeland

(2010) for recent presentations. Let us just mention that the existence of a stable match obtains in general; this comes from the fact that the linear optimization problem does have a solution under very general assumptions.

7.2.2 Assortative mating

The basic result

Suppose now that each male is endowed with a single characteristic, x, and each female is endowed with a single characteristic, y, which positively affects the family's output. When can we expect the stable assignments to exhibit either positive or negative assortative mating? Again, the answer is quite different from the no transfer case. It follows in the present case from the observation that a stable assignment must maximize the aggregate marital output (or surplus) over all possible assignments.

Specifically, let, as above,

$$\zeta_{ij} = h(x_i, y_j) \tag{7.14}$$

be the household output function that specifies the marital output as a function of the attributes of the two partners. We say that a function $h(x_i, y_j)$ is *super modular* if $x' > x$ and $y\,' > y$ always imply that

$$h(x', y') + h(x, y) \geq h(x', y) + h(x, y'), \tag{7.15}$$

and it is *sub modular* if inequality (7.15) is always reversed. This definition captures the idea of complementarity and substitution as usually understood. Rewriting (7.15) in the form

$$h(x', y') - h(x', y) \geq h(x, y') - h(x, y), \tag{7.16}$$

we see that the requirement is that the contribution to marital output of a given increase in the female attribute rises with the level at which the male trait is held fixed. By a similar rearrangement, the impact of a given increase in the male's attribute rises in the female's attribute. Note also that if h is twice differentiable then h is super (sub) modular if the second cross derivative h_{yx} is always positive (negative).[3] The condition that h_{yx}

[3] Indeed, for any given (x, y) define

$$H(x', y') = h(x', y') + h(x, y) - h(x', y) - h(x, y').$$

Then

$$H_{x'}(x', y') = h_{x'}(x', y') - h_{x'}(x', y)$$

which is positive for $y\prime > y$ if $h_{xy} \geq 0$ (since h_x is then increasing in f). Similarly,

$$H_{y'}(x', y') = h_{y'}(x', y') - h_{y'}(x, y') \geq 0$$

for $x\prime > x$ if $h_{xy} \geq 0$. Hence H is increasing in its arguments, and $H(x, y) = 0$; we conclude that $H(x', y') \geq 0$ whenever $x\prime > x$ and $y\prime > y$ and $h_{xy} \geq 0$.

is monotonic is sometimes called the *single crossing* or the *Spence-Mirrlees* condition; indeed, a similar condition is crucial in contract theory, signalling models (a la Spence) and optimal taxation (a la Mirrlees).

The basic result is that complementarity (substitution) in traits must lead to a positive (negative) assortative mating; otherwise aggregate output is not maximized. Assuming that $h(x, y)$ is increasing in x and y, we obtain that in the case of positive assortative mating, the best man marries the best woman, and if there are more women than men the women with low female quality remain single. If there is negative assortative mating, the best man marries the worst woman among the married women but if there are more women than men, it is the women with the lower female attributes who remain single. (see Appendix) In other words, who marries whom depends on second order derivatives of $h(x, y)$ but who remains single depends on the first order derivatives of $h(x, y)$. If there is no interaction in traits and the marginal contribution of each agent is the same in all marriages, any assignment is (weakly) stable and it does not matter who marries whom, because whichever way we arrange the marriages the aggregate output of all marriages remains the same.

We may explain these results intuitively by referring again to the basic idea of a stable assignment. Complementarity (substitution) implies that males with high x will be willing to pay *marginally* more (less) for the female attribute. Thus, if x stands for money and y stands for beauty, the wealthy men will be matched with the pretty women if and only if their (marginal) willingness to pay for beauty is higher. If there is negative interaction between money and beauty, the most wealthy man will *not* marry the most pretty woman, because whichever way they divide their gains from marriage, either he is bid away by a less pretty woman or she is bid away by a poorer man.

This result is in a sharp contrast to the non transferable case, where monotonicity in traits is sufficient to determine the outcome.[4] The consequence is that assortative (negative or positive) mating is more prevalent in the absence of transfers, because it is impossible for agents with less desirable traits to compensate their spouses through a larger share of the marital output (see Becker, 1991, ch. 4 and Becker and Murphy, ch. 12). The sad message for the econometrician is that, based on the same information, namely the household production function, one can get very different outcomes depending on the ability to compensate within households, a feature that we usually cannot directly observe. But, conversely, it also means that one can in principle test one model against the other (since they have

[4] However, monotonicity may fail to hold when super modularity holds. A potentially important case is when preferences are single peaked in the attribute of the spouse. In such cases, we can have assortative mating in the sense that married partners have similar traits, but individuals with extreme traits may fail to marry. The interested reader may consider the case in which the marital surplus is given by $g - (x - y)^2$.

different implications); we will discuss such tests later on.

Finally, the impact of traits on the value of being single does not affect these considerations, because the welfare of each person as single depends only on his own traits. Therefore, in the aggregate, the output that individuals obtain as singles is independent of the assignment. Although the value of being single does matter to the question who marries, it does not affect who marries whom, in equilibrium.

Examples

In many models, the surplus function takes a specific form. Namely, the two traits x and y can often be interpreted as the spouses' respective incomes. Following the collective approach described in the previous Chapters, we may assume that a couple consisting of a husband with income x and a wife with income y will make Pareto efficient decisions; then it behaves as if it was maximizing a weighted sum of individual utilities, subject to a budget constraint. The important remark is that the constraint only depends on the sum of individual incomes. Then the Pareto frontier - or in our specific case the value of the surplus function $h(x, y)$ which defines it - only depends on the sum $(x + y)$;[5] that is:

$$h(x, y) = \bar{h}(x + y)$$

The various properties described above take a particular form in this context. For instance, the second cross derivative h_{xy} is here equal to the second derivative $\bar{h}''$. It follows that we have assortative matching if $\bar{h}$ is convex, and negative assortative matching if $\bar{h}$ is concave. The interpretation is as above: a convex $\bar{h}$ means that an additional dollar in income is more profitable for wealthier people - meaning that wealthier husbands are willing to bid more aggressively for a rich wife than their poorer competitors. Conversely, if $\bar{h}$ is concave then the marginal dollar has more value for poorer husbands, who will outbid the richer ones.

In models of this type, the TU assumption actually tends to generate convex output functions, hence assortative matching. To see why, consider a simple model of transferable utility in the presence of a public good. Preferences take the form

$$u_i = c_i g(q) + f_i(q), \tag{7.17}$$

where c and q denote private and public consumption, respectively. The Pareto frontier is then

$$u_a + u_b = h(Y) = \max_q [(Y - q)g(q) + f(q)],$$

[5] Of course, while the Pareto set only depends on total income, the *location* of the point ultimately chosen on the Pareto frontier depends on individual incomes - or more specifically on the location of each spouse's income within the corresponding income distribution. These issues will be analyzed in the next Chapter.

where $f(q) = f_a(q) + f_b(q)$ and $Y = x + y$. By the envelope theorem, $h'(Y) = g(q)$ and therefore,

$$h''(Y) = g'(q)\frac{dq}{dY} = \frac{-(g'(q))^2}{(y-q)g''(q) - 2g'(q) + f''(q)} > 0.$$

since the denominator must be negative for the second order conditions for a maximum to hold. Hence, if there is an interior solution for q, the household production function is *convex* in family income, Y, implying that the two incomes x and y must be complements.

As an illustration, recall the examples discussed in sections 2.1 and 2.2 of Chapter 2. In section 2.1, we considered the case in which the spouses pool their (fixed) incomes and share a public good and individual preferences were of the form $u_i = c_i.q$, compatible with (7.17). If we now rank men and women by their incomes we have a situation in which the household production function is $h(x, y) = \frac{(x+y)^2}{4}$. This is a convex function of total income; there is a positive interaction everywhere, leading to assortative sorting.

In contrast, in section 2.2, we considered a case in which division of labor has led to marital output given by $\max(w_i, w_j)$, which is *not* a function of total income. Here, we obtain negative assortative mating. This holds because a high wage person is more useful to a low wage person, as indicated by the submodularity of $h(x, y) = \max(x, y)$.[6] For instance, if man i has wage i and woman j has wage j, the output matrix for the 3 by 3 case is:

Example 7.5

		Women		
		1	2	3
Men	1	1	*2*	**3**
	2	2	**2**	*3*
	3	**3**	3	3

implying three stable assignments; the opposite diagonal (in bold), one close to it in which couples $(1, 3)$ and $(2, 2)$ exchange partners (emphasized), and a symmetric one in which couples $(3, 1)$ and $(2, 2)$ exchange partners. The assignment also depends on the location of the wage distribution for each gender. As an extreme case, let the worst woman have a higher wage

[6] For all $x' \geq x$ and $y' \geq y$, we have $\max(x', y') + \max(x, y) \leq \max(x', y) + \max(x, y')$ .Going over the six possible orders of four numbers x,x', y, y' satisfying $x' \geq x$ and $y' \geq y$, we see that

$$\begin{array}{lcl}
x' \geq x \geq y' \geq y & \Rightarrow & x' + x \leq x' + x, \\
x' \geq y' \geq x \geq f & \Rightarrow & x' + x \leq x' + y', \\
x' \geq y' \geq y \geq x & \Rightarrow & x' + y \leq x' + y', \\
y' \geq y \geq x' \geq x & \Rightarrow & y' + y \leq y + y', \\
y' \geq x' \geq y \geq x & \Rightarrow & y' + y \leq x' + y', \\
y' \geq x' \geq x \geq y & \Rightarrow & y' + x \leq x' + y'.
\end{array}$$

than the best man. Then in all marriages the female wage determines the outcome and all assignments are equally good.

Note, finally, that in the absence of any interaction, we have $h(x, y) = x + y$; this describes a situation where the two spouses simply pool their incomes and consume only private goods. Since the output is a linear function of both incomes, any assignment of men to women is stable. It is interesting that although the assignment is completely indeterminate, the set of imputations shrinks substantially and is given by

$$\begin{aligned} v_i &= x_i + p, \\ u_j &= y_j - p, \end{aligned} \tag{7.18}$$

for some fixed p. Thus, in the absence of interaction in traits, the *same* transfer p occurs in *all* marriages and we may interpret it as a common bride price or dowry, depending on whether p is positive or negative in equilibrium.[7] As we shall show in the next Chapter, if there is interaction in traits, this single price is replaced by an intrahousehold allocation rule that depends on the attributes of both partners.

7.2.3 Matching with a continuum of agents

The discussion above shows that a crucial feature of the problem is the interaction in the traits that the two partners bring into marriage. We shall for the time being focus here on situations where income is the only marital trait and individual incomes are complement in the household output function - that is, $h(x, y)$ is super modular, or $h_{xy}(x, y) > 0$. Moreover, we assume here that there exists a *continuum* of men, with a total mass normalized to 1, and a *continuum* of women, with a total mass denoted r. We allow different income distributions for men and women; specifically, male

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[7] Consider any two couples, (i, j) and (r, s), in a stable assignment. Then, using the duality results,

$$\begin{aligned} u_i + v_j &= x_i + y_j \\ u_r + v_s &= x_r + y_s, \end{aligned}$$

because the imputations for married couples exhaust the marital output. Also, because couples (i, s) and (r, j) are not married to each other

$$\begin{aligned} u_i + v_s &\geq x_i + y_s \\ u_r + v_j &\geq x_r + y_j \end{aligned}$$

But none of these inequalities can be strict, because their sum must equal to the sum of the equalities above. It then follows that in *all* marriages on *any* stable assignment

$$\begin{aligned} u_i - u_r &= x_i - x_r \\ v_j - v_s &= y_j - y_s, \end{aligned}$$

which is equivalent to (7.18).

incomes x are distributed on $[0,1]$ according to some distribution F and female incomes y are distributed on $[0,1]$ according to some distribution G.

The assumed positive interaction implies a positive assortative matching. Therefore, if a man with income x is married to a woman with income y, then the set of men with incomes above x must have the same measure as the set of women with incomes above y. Thus, for all x and y in the set of married couples,

$$1 - F(x) = r(1 - G(y)). \tag{7.19}$$

Hence,

$$x = \Phi[1 - r(1 - G(y))] \doteq \phi(y), \tag{7.20}$$

where $\Phi = F^{-1}$, or equivalently,

$$y = \Psi\left[1 - \frac{1}{r}(1 - F(x))\right] \doteq \psi(x), \tag{7.21}$$

where $\Psi = G^{-1}$ and $\psi = \phi^{-1}$; note that both ϕ and ψ are increasing.

All men and women are married if there is an equal measure of men and women, $r = 1$. All women are married if there is scarcity of women, $r < 1$, implying that men with income x less than $x_0 = \Phi(1 - r)$ remain single. All men are married if there is scarcity of men, $r > 1$, implying that women with income y less than $y_0 = \Psi(1 - 1/r)$ remain single. If $r > 1$, then the function $y = \psi(x)$ determines the income of the wife for each man with income x in the interval $[0,1]$. Similarly, if $r < 1$, then the function $x = \phi(y)$ determines the husband's income of each woman with income y in the interval $[0,1]$. We shall refer to these functions as the *matching functions* and to the resulting assignment as the *assignment profile.*

In Figure 7.1 we show the matching function $\psi(x)$ for the case in which x is distributed uniformly on $[0,1]$, y is distributed uniformly on $[0,\sigma]$, $\sigma < 1$ and $r > 1$. Applying (7.19) and solving

$$1 - x = r(1 - \frac{y}{\sigma}),$$

we obtain

$$\psi(x) = \frac{\sigma}{r}(r - 1 + x).$$

We see that women with incomes y such that $y \leq y_0 = \frac{\sigma}{r}(r-1)$ remain single. Women with incomes in the range $[y_0, y'] = [\frac{\sigma}{r}(r-1, \frac{\sigma}{r}(r-1+x')]$ marry men with incomes in the range $[0, x\prime]$. Finally, women with incomes in the range $[y'', \sigma] = [\frac{\sigma}{r}(r-1+x''), \sigma]$ marry men with incomes in the range $[x'', 1]$. Thus women with higher incomes marry men with higher incomes. Note the equality in the measures of women and men in these intervals, as indicated by the areas of the corresponding rectangles. For instance the rectangular with base $x\prime$ and height 1 has the same area as the rectangular

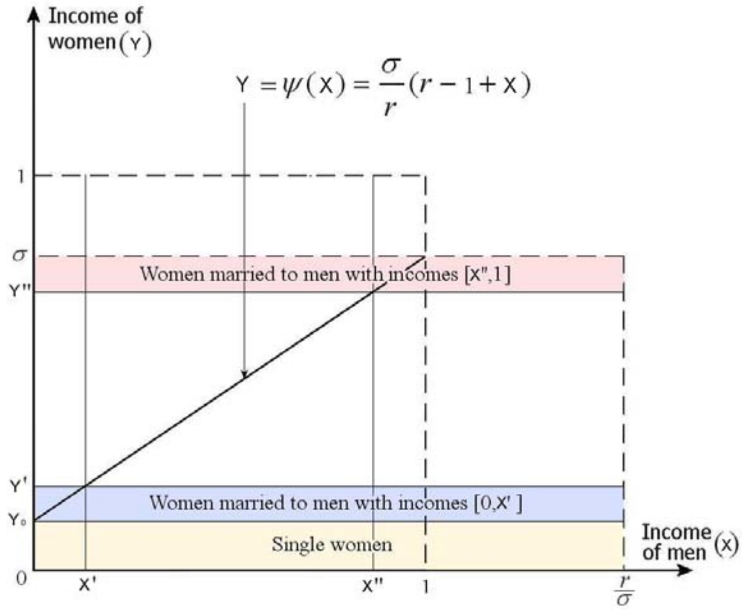

FIGURE 7.1. Positive Assortative Mating

with base $\frac{r}{\sigma}$ and height $\frac{\sigma}{r}(r-1+x')-\frac{\sigma}{r}(r-1)$. Such equality of measures must hold throughout the assignment profile.

The slope of each matching function is related to the *local scarcity* of men relative to women. Men are locally scarce if there are more women than men at the assigned incomes $(\phi(y), y) = (x, \psi(x))$. Or, equivalently, if an increase in the husband's income is associated with a smaller increase in the income of the matched wife. That is,

$$\begin{aligned} \frac{dx}{dy} &= \phi'(y) = r\frac{g(y)}{f(\phi(y))} > 1, \\ \frac{dy}{dx} &= \psi'(x) = \frac{1}{r}\frac{f(x)}{g(\psi(x))} < 1. \end{aligned} \tag{7.22}$$

Men are locally abundant if these inequalities are reversed.

7.2.4 Multidimensional matching

The previous discussion explicitly refers to a one-dimensional framework. Assortative matching is harder to define when several dimensions (or several traits) are involved; moreover, conditions like supermodularity or single-crossing do not have an obvious extension to a multidimensional setting. Still, they can be generalized; again, the reader is referred to Chiappori, McCann and Neishem (2010) or Ekeland (2010) for recent presentations. The main insights can briefly be described as follows. Assume that X and Y are finite dimensional. Then:

1. The Spence-Mirrlees condition generalizes as follows: if $\partial^x h(x_0, y)$ denotes the *superdifferential* of h in x at (x_0, y), then for almost all x_0, $\partial^x h(x_0, y_1)$ is disjoint from $\partial^x s(x_0, y_2)$ for all $y_1 \neq y_2$ in Y. This is the 'twisted buyer' condition in Chiappori, McCann and Neishem 2010.

2. If the 'twisted buyer' condition is satisfied, then the optimal match is unique; in addition, it is *pure*, in the sense that the support of the optimal measure Φ is born by the graph of some function $y = \phi(x)$; that is, for any x there exists exactly one y such that x is matched with y with probability one.

3. There exists a relaxation of the 'twisted buyer' condition (called the 'semi-twist') that guarantees uniqueness but not purity.

The notion of 'superdifferential' generalizes the standard idea of a linear tangent subspace to non differentiable functions. If h is differentiable, as is the case is most economic applications, then $\partial^x h(x_0, y)$ is simply the linear tangent (in x) subspace to h at (x_0, y), and the condition states that for almost all x_0, there exists a one to one correspondence between y and

$\partial^x h(x_0, y)$. Note that if X and Y are one-dimensional, then $\partial^x h(x_0, y)$ is fully defined by the partial $\partial h/\partial x(x_0, y)$, and the condition simply requires that $\partial h/\partial x(x_0, y)$ be strictly monotonic in y - that is, the sign of $\partial^2 h/\partial x \partial y$ be constant, the standard single-crossing condition, Similarly, if X and Y are one-dimensional, then purity imposes a one-to-one matching relationship between x and y; if this matching is continuous, it has to be monotonic, that is matching must be either positive or negative assortative (in that sense, purity is a generalization of assortativeness to multi-dimensional settings).

In general, purity rules out situations in which a subset of agents (with a positive measure) randomize between several, equivalent matches. Such situations may be frequent in practice; in particular, Chiappori, McCann and Neishem (2010) show that they are likely to occur when agents are located on an Hotelling-type circle. Finally, only recently have empirical models of multidimensional matching been developed; the main reference, here, is Galichon and Salanié (2009).

7.3 Matching with general utilities

In the previous two sections, the matching process is studied in specific and somewhat extreme settings: either transfers cannot take place at all, or they can be made at a constant exchange rate (so that reducing a member's utility by one 'unit' always increases the spouse's utility by one unit as well). We now consider the general case, in which although transfers are feasible, there is no commodity that allows the partners to transfer utilities at a fixed rate of exchange. Then the utility frontier is no longer linear and it is impossible to summarize the marital output from a match by a single number. In this more general framework, stability is defined in the same manner as before, that is, an assignment is stable if no pair who is currently not married can marry and choose an allocation of family resources that yields a result which is better for both of them than under the existing assignment and associated payoffs. Observe that the assignment and payoffs are simultaneously restricted by this definition. However, it is no longer true that aggregate marital output must be maximized - actually, such an 'aggregate output' is not even defined in that case. Mathematically, the matching model is *no longer* equivalent to an optimization problem.

Still, it is in principle possible to simultaneously solve for the stable assignment and the associated distribution(s) of surplus. The interested reader is referred to Roth and Sotomayer (1990, ch. 6), Crawford (1991), Chiappori and Reny (2006) and Legros and Newman (2007). To give a quick idea of how the general problem can be approached with a continuum of agents, let us assume, as above, that each agent is characterized by one trait, and let's assume that this trait is income (assumed to be exogenous).

Male income is denoted by x and female income is denoted by y. We no longer assume transferable utility; hence the Pareto frontier for a couple has the general form

$$u = H(x, y, v) \tag{7.23}$$

with $H(0, 0, v) = 0$ for all v.

As above, if a man with income x remains single, his utility is given by $H(x, 0, 0)$ and if a woman of income y remains single her utility is the solution to the equation $H(0, y, v) = 0$. By definition, $H(x, y, v)$ is decreasing in v; we assume that it is increasing in x and y, that is that a higher income, be it male's or female's, tends to expand the Pareto frontier. Also, we still consider a continuum of men, whose incomes x are distributed on $[0, 1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0, 1]$ according to some distribution G; let r denote the measure of women.

Finally, let us *assume* for the moment that an equilibrium matching exists and that it is assortative. Existence can be proved under mild conditions using a variant of the Gale-Shapley algorithm; see Crawford (1991), Chiappori and Reny (2006). Regarding assortativeness, necessary conditions will be derived below. Under assortative matching, the 'matching functions' ϕ and ψ are defined exactly as above (eq. 7.19 to 7.21).

Let $u(x)$ (resp. $v(y)$) denote the utility level reached by Mr. x (Mrs. y) at the stable assignment. Then it must be the case that

$$u(x) \geq H(x, y, v(y))$$

for all y, with an equality for $y = \psi(x)$. As above, this equation simply translates stability: if it was violated for some x and y, a marriage between these two persons would allow to strictly increase *both* utilities. Hence:

$$u(x) = \max_y H(x, y, v(y))$$

and we know that the maximum is actually reached for $y = \psi(x)$. First order conditions imply that

$$\frac{\partial H}{\partial y}(\phi(y), y, v(y)) + v'(y) \frac{\partial H}{\partial v}(\phi(y), y, v(y)) = 0. \tag{7.24}$$

while *second* order conditions for maximization are

$$\frac{\partial}{\partial y}\left(\frac{\partial H}{\partial y}(\phi(y), y, v(y)) + v'(y) \frac{\partial H}{\partial v}(\phi(y), y, v(y))\right) \leq 0 \quad \forall y. \tag{7.25}$$

This expression may be quite difficult to exploit. Fortunately, it can be simplified using a standard trick. The first order condition can be written as:

$$F(y, \phi(y)) = 0 \quad \forall y$$

where

$$F(y,x)=\frac{\partial H}{\partial y}(x,y,v(y))+v'(y)\frac{\partial H}{\partial v}(x,y,v(y)). \tag{7.26}$$

Differentiating:

$$\frac{\partial F}{\partial y}+\frac{\partial F}{\partial x}\phi'(y)=0 \quad \forall y,$$

which implies that

$$\frac{\partial F}{\partial y}\leq 0 \text{ if and only if } \frac{\partial F}{\partial x}\phi'(y)\geq 0.$$

The second order conditions can hence be written as:

$$\left(\frac{\partial^2 H}{\partial x\partial y}(\phi(y),y,v(y))+v'(y)\frac{\partial^2 H}{\partial x\partial v}(\phi(y),y,v(y))\right)\phi'(y)\geq 0 \ \ \forall y. \tag{7.27}$$

Here, assortative matching is equivalent to $\phi'(y)\geq 0$; this holds if

$$\frac{\partial^2 H}{\partial x\partial y}(\phi(y),y,v(y))+v'(y)\frac{\partial^2 H}{\partial x\partial v}(\phi(y),y,v(y))\geq 0 \ \ \forall y. \tag{7.28}$$

Since $v'(y)\geq 0$, a sufficient (although obviously not necessary) condition is that

$$\frac{\partial^2 H}{\partial x\partial y}(\phi(y),y,v(y))\geq 0 \text{ and } \frac{\partial^2 H}{\partial x\partial v}(\phi(y),y,v(y))\geq 0. \tag{7.29}$$

One can readily see how this generalizes the transferable utility case. Indeed, TU implies that $H(x,y,v(y))=h(x,y)-v(y)$. Then $\frac{\partial^2 H}{\partial x\partial v}=0$ and the condition boils down to the standard requirement that $\frac{\partial^2 H}{\partial x\partial y}=\frac{\partial^2 h}{\partial x\partial y}\geq 0$. General utilities introduces the additional requirement that the cross derivative $\frac{\partial^2 H}{\partial x\partial v}$ should also be positive (or at least 'not too negative'). Geometrically, take some point on the Pareto frontier, corresponding to some female utility v, and increase x - which, by assumption, expands the Pareto set, hence shifts the frontier to the North East (see Figure 7.2). The condition then means that at the point corresponding to the same value v on the new frontier, the slope is less steep than at the initial point. For instance, a homothetic expansion of the Pareto set will typically satisfy this requirement.

The intuition is that whether matching is assortative depends not only on the way total surplus changes with individual traits (namely, the usual idea that the marginal contribution of the husband's income increases with the wife's income, a property that is captured by the condition $\frac{\partial^2 H}{\partial x\partial y}\geq 0$), but also on how the 'compensation technology' works at various income levels.

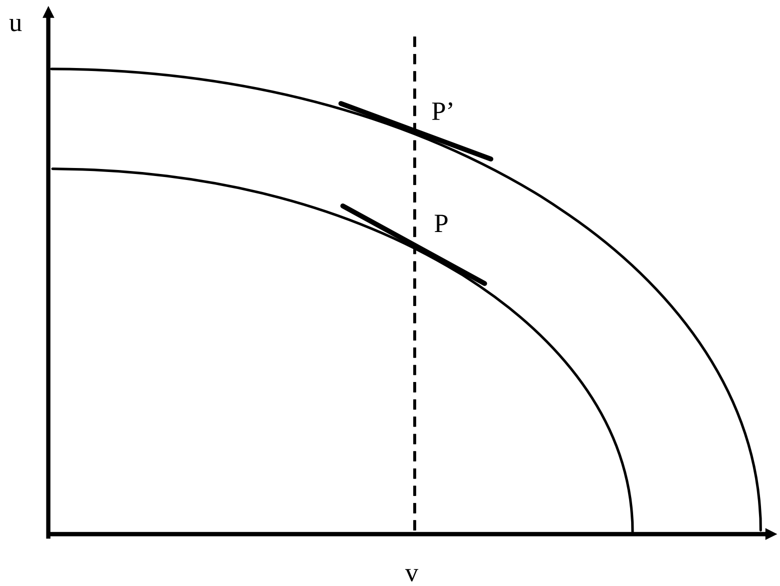

FIGURE 7.2. The slope condition

With general utilities, while the technology for transferring *income* remains obviously linear, the cost (in terms of husband's utility) of transferring *utility* to the wife varies with incomes. The second condition implies that, keeping the wife's utility level fixed, a larger income alleviates the cost (in terms of husband's utility) of providing an additional unit of utility to the wife. Then wealthy males have a double motivation for bidding aggressively for wealthy women: they benefit more from winning, and their 'bidding costs' are lower. They will thus systematically win. Note, however, that when the two partials $\frac{\partial^2 H}{\partial x \partial y}$ and $\frac{\partial^2 H}{\partial x \partial v}$ have opposite signs, the two aspects - benefits from winning and cost of bidding - vary with income in opposite directions. Assume, for instance, that $\frac{\partial^2 H}{\partial x \partial y} \geq 0$ but $\frac{\partial^2 H}{\partial x \partial v} \leq 0$. Then the outcome is uncertain because while wealthy males still value wealthy females more than poor males do, they are handicapped by their higher cost of bidding.

7.4 Search

We now turn to the alternative approach that stresses that in real life the matching process is characterized by scarcity of information about potential matches. The participants in the process must therefore spend time and money to locate their best options, and the set of potential partners they actually meet is partially random. The realized distribution of matches and the division of the gains from each marriage are therefore determined in an equilibrium which is influenced by the costs of search and the search policies of other participants.

7.4.1 The basic framework

The main ingredients of the search model are as follows. There is a random process which creates meetings between members of society of the opposite sex. When a meeting occurs the partners compare their characteristics and evaluate their potential gains from marriage. Each partner anticipates their share in the joint marital output. If the gains for both partners from forming the union exceed their expected gain from continued search then these partners marry. Otherwise, they depart and wait for the next meeting to occur (see Mortensen, 1988).

We assume that meetings occur according to a Poisson process. That is, the waiting times between successive meetings are iid exponential variables with mean $1/\lambda$. Within a short period h, there is a probability of a meeting given by $\lambda h + o(h)$ and a probability of no meeting given by $1 - \lambda h + o(h)$, where, $o(h)/h$ converges to zero as h approaches zero. The arrival rate λ is influenced by the actions of the participants in the marriage market. Specifically, imagine an equal number of identical males and females, say N, searching for a mate. Let s_i denote the "search intensity" (that is number of meetings per period) initiated by a particular male. If all females search at the same intensity s_f , they will generate Ns_f contacts per period distributed randomly across all males. In this case, the probability that male i will make a contact with some female, during a short interval, h, is $(s_f + s_i)h$. If all males search at a rate s_m and all females at a rate s_f then the rate of meetings between agents of opposite sex is

$$\lambda = s_m + s_f. \tag{7.30}$$

The key aspect in (7.30) is that activities on both sides of the market determine the occurrence of meetings. A limitation of the linear meeting technology is that the number of searchers, N, has no effect on the arrival rate λ. Each participant who searches actively and initiates meetings must bear a monetary search cost given by $c_i(s)$, $i = m, f$, where we allow the costs of search to differ by sex. The total and the marginal costs of search increase as search intensity increases. When a meeting occurs the marital output (quality of match) that the partners can generate together is a random variable, z, drawn from some fixed distribution, $F(z)$. Having observed z, the couple decides whether to marry or not. With transferable utility, the decision to marry is based on the total output that can be generated by the couple within marriage relative to the expected total output if search continues. Hence, a marriage occurs if and only if

$$z \geq v_m + v_f, \tag{7.31}$$

where, v_m and v_f denote the value of continued search for the male and female partners, respectively. These values depend, in equilibrium, on the search intensity that will be chosen if the marriage does not take place. Specifically, for $i = m, f$,

$$rv_i = \underset{s}{Max}\{(s + s_j) \int_{v_m+v_f}^{\infty} (w_i(z) - v_i)dF(z) - c_i(s)\}, \tag{7.32}$$

where r is the instantaneous interest rate and $w_i(z)$ denote the shares of the gains of marital output that male and female partners expect. By definition,

$$w_m(z) + w_f(z) = z. \tag{7.33}$$

Equation (7.32) can be derived by using the following standard argument. Let h be a short time interval. Then, the Bellman equation for dynamic programming is

$$\begin{aligned} v_i \;=\; & \underset{s}{Max} e^{-rh}\{\lambda h(s + s_j)[p(z \geq v_m + v_f)(E_z(Max(v_i, w_i(z)|z \geq v_m + v_f)] \\ & +[1 - \lambda h(s + s_j)]v_i\} + o(h). \end{aligned}$$

Note that, due to the stationarity of the Poisson process and the infinite horizon, v_i and and $w_i(z)$ do not depend on time. Approximating $e^{-rh} \simeq 1-rh$, cancelling terms that do not depend on h and rearranging, we obtain:

$$\begin{aligned} \underset{s}{Max}\{\lambda h(s + s_j)[p(z &\geq v_m + v_f)E_z(Max(v_i, w_i(z)|z \geq v_m + v_f) - v_i] \\ +[1 - \lambda h(s + s_j)]v_i - rh^2\lambda(s + s_j)[(p(z &\geq v_m + v_f)E_z(Max(v_i, w_i(z)|z \geq v_m + v_f) - v_i]\} \\ +o(h) &= rhv_i. \end{aligned}$$

Dividing both sides of this equation by h, we obtain (7.32) as the limit when h approach zero.

Equation (7.32) states that the value of being an unattached player arises from the option to sample from offers which arrive at a rate $s + s_f$ and are accepted only if (7.31) holds. Each accepted offer yields a surplus of $w_i(z)-v_i$ for partner i. Integration over all acceptable offers yields expected gain from search. Since each participant controls his own intensity of search, he will choose the level of s that maximizes his value in the unattached state. Therefore, with identical individuals in each gender,

$$\int_{v_m+v_f}^{\infty} (w_i(z) - v_i)dF(z) = c_i'(s), \; i = m, f. \tag{7.34}$$

The marginal benefits from search, the left hand side of (7.34), depend on the share that a person of type i expects in prospective marriages. As

$w_i(z)$ rises, holding z constant, he or she searches more intensely. Hence, the equilibrium outcome depends on the allocation rules that are adopted. The literature examined two types of allocation rules. One class of allocation rules relies on Nash's axioms and stipulates

$$w_i(z) = v_i + \gamma_i(z - v_m - v_f), \tag{7.35}$$

where, $\gamma_i \geq 0$ and $\gamma_m + \gamma_f = 1$, $i = m, f$. The parameter γ_i allows for asymmetry in the bilateral bargaining between the sexes due to preferences or social norms. The crucial aspect of this assumption, however, is that outside options, reflected in the market determined values of v_m and v_f, influence the shares within marriage.

Wolinsky (1987) points out that a threat to walk out on a potentially profitable partnership is not credible. Rather than walking away, the partners exchange offers. When an offer is rejected, the partners search for an outside opportunity that would provide more than the expected gains from an agreement within the current marriage. Hence, during the bargaining process the search intensity of each partner is determined by

$$\int_y^\infty (w_i(x) - w_i(y))dF(x) = c_i'(s), \;\; i = m, f, \tag{7.36}$$

where, y is the quality of the current marriage and $w_i(y)$ is the expected share in the current marriage if an agreement is reached. Since $y \geq v_m + v_f$ and $w_i(y) \geq v_i$, a person who searches for better alternatives during a bargaining process will search less intensely and can expect lower gains than an unattached person. The threat of each partner is now influenced by two factors: The value of his outside opportunities (that is, the value of being single), which enters only through the possibility that the other partner will get a better offer and leave; The value of continued search during the bargaining process, including the option of leaving when an outside offer (whose value exceeds the value of potential agreement) arrives. Therefore, the threat points, v_i , in (7.35) must be replaced by a weighted average of the value of remaining without a partner and the value of continued search during the bargaining (the weights are the probabilities of these events). Given these modified threat points, the parameters γ_i that determine the shares depend on the respective discount rates of the two partners and the probabilities of their exit from the bargaining process. The logic behind this type of formula, due to Rubinstein (1982), is that each person must be indifferent between accepting the current offer of his partner or rejecting it, searching for a better offer and, if none is received, return to make a counter offer that the partner will accept.

Given a specification of the share formulae, one can solve for the equilibrium levels of search intensities and the values of being unattached. For instance, if the shares are determined by (7.35) and γ_i is known, then equations (7.34) and (7.35) determine unique values for s_m, s_f, v_m, and v_f. Because of the linear meeting technology, these equilibrium values are independent of the number of searchers. Observe that although the share formulae depend on institutional considerations, the actual share of marital output that each partner receives depends on market forces and is determined endogenously in equilibrium.

We can close the model by solving for the equilibrium number of unattached participants relative to the population. Suppose that each period a new flow of unattached persons is added to the population and the same flow of married individuals exit. To maintain a steady state, this flow must equal the flow of new attachments that are formed from the current stock of unattached. The rate of transition into marriage is given by the product of the meeting rate λ and the acceptance rate $1 - F(z_0)$, where z_0 is the reservation quality of match. Using (7.30) and (7.31), we obtain

$$u(s_m + s_f)(1 - F(v_m + v_f)) = e \tag{7.37}$$

where, u is the endogenous, steady state, rate of non-attachment and e is the exogenous constant rate of entry and exit.

The meeting technology considered thus far has the unsatisfactory feature that attached persons "do not participate in the game". A possible extension is to allow matched persons to consider offers from chance meetings initiated by the unattached, while maintaining the assumption that married people do not search. In this case divorce becomes an additional option. If an unattached person finds a married person who belongs to a marriage of quality z and together they can form a marriage of quality y then a divorce will be triggered if $y > z$. The search strategies will now depend on the relative numbers of attached and unattached persons. Specifically, (7.32) is replaced by

$$\begin{aligned} rv_i &= \underset{s}{Max}\{u(s+s_j)\int_{v_m+v_f}^{\infty}(w_i(z)-v_i)dF(z) + \\ &\quad (1-u)s\int_{v_m+v_f}^{\infty}\int_{y}^{\infty}(w_i(z)-w_i(y)-v_i)dG(z)dF(y) - c_i(s)\} \end{aligned} \tag{7.38}$$

where $G(z)$ is the distribution of quality of matched couples. The second term in equation (7.38) is derived from the following argument. Suppose i is a male and he meets a married woman who together with her current husband has marital output y. Together with i, the marital output would be z, where $z \geq y$. The threat point of this woman in the bargaining with man i is what she would receive from her current husband when she threatens to leave him, which is $y - v_f$. Thus, the total surplus of the new marriage is $z - (y - v_m) - v_m$. Hence, following bargaining, man i will receive in the new marriage $v_m + \gamma_m(z - y) = v_m + w_i(z) - w_i(y)$.See Mortensen (1988).

Observe that the expected returns from meeting an attached person are lower than those of meeting with an unmarried one. Therefore, the higher is the aggregate rate of non-attachment the higher are the private returns for search.

Assuming that partners are ex-ante identical, the search models outlined above do not address the question who shall marry whom. Instead, they shift attention to the fact that in the process of searching for a mate there is always a segment of the population which remains unmatched, not because they prefer the single state but because matching takes time. A natural follow up to this observation is the question whether or not there is "too much" search. Clearly, the mere existence of waiting time for marriage does not imply inefficiency since time is used productively to find superior matches. However, the informational structure causes externalities which may lead to inefficiency. One type of externality arises because in deciding on search intensity participants ignore the higher chance for meetings that others enjoy. This suggests that search is deficient. However, in the extended model which allows for divorce there is an additional externality operating in the opposite direction. When two unattached individuals reject a match opportunity with $z < v_m + v_f$, they ignore the benefits that arise to other couples from a higher non attachment rate. Thus, as in a related literature on unemployment, it is not possible to determine whether there is too much or too little non attachment.

An important aspect of equation (7.38) is the two way feedback between individual decisions and market outcomes. The larger is the proportion of the unattached the more profitable is search and each unattached person will be more choosy, further increasing the number of unattached. As emphasized by Diamond (1982) such reinforcing feedbacks can lead to multiplicity of equilibria. For instance, the higher is the aggregate divorce rate the more likely it is that each couple will divorce. Therefore, some societies can be locked into an equilibrium with a low aggregate divorce rate while others will settle on a high divorce rate. There are some additional features which characterize search for a mate and can be incorporated into the analysis. First, as noted by Mortensen (1988), the quality of marriage is revealed only gradually. Moreover, each partner may have private information which is useful for predicting the future match quality (see Bergstrom-Bagnoli, 1993). Second, as noted by Oppenheimer (1988), the offer distribution of potential matches varies systematically with age, as the number and quality of available matches change and the information about a person's suitability for marriage sharpens. Finally, meetings are not really completely random. Unattached individuals select jobs, schools and leisure activities so as to affect the chances of meeting a qualified person of the opposite sex (see Goldin, 2006).

7.4.2 Search and Assortative Mating

Models of search add realism to the assignment model, because they provide an explicit description of the sorting process that happens in real time. Following Burdett and Coles (1999), consider the following model with non transferable utility whereby if man m marries woman f, he gets f and she gets m. Assume a continuum of men, whose traits m are distributed on $[0, \bar{m}]$ according to some distribution F, and a continuum of women, whose traits f are distributed on $\left[0, \bar{f}\right]$ according to some distribution G. To bring in the frictions, assume that men and women meet according to a Poisson process with parameter λ. Upon meeting, each partner decides whether to accept the match or to continue the search. Marriage occurs only if both partners accept each other. A match that is formed cannot be broken. To ensure the stationary of the decision problem, we assume a fixed and equal number of infinitely lived men and women.

Each man chooses an acceptance policy that determines which women to accept. Similarly, each woman chooses an acceptance policy that determines which men to accept. These policies are characterized by reservation values, R, such that all potential partners with a trait exceeding R are accepted and all others are rejected. The reservation value that each person chooses depends on his\her trait. In particular, agents at the top of the distribution of each gender can be choosier because they know that they will be accepted by most people on the other side of the market and hence continued search is more valuable for them. Formally,

$$\begin{aligned} R_m &= b_m + \frac{\lambda \mu_m}{r} \int_{R_m}^{\bar{f}} (f - R_m) dG_m(f), \\ R_f &= b_f + \frac{\lambda \mu_f}{r} \int_{R_f}^{\bar{m}} (m - R_f) dF_f(m), \end{aligned} \tag{7.39}$$

where, the flow of benefits as single, b, the proportion of meetings that end in marriage, μ, and the distribution of "offers" if marriage occurs, all depend on the trait of the person as indicated by the m and f subscripts. The common discount factor, r, represents the costs of waiting.

In equilibrium, the reservation values of all agents must be a best response against each other, yielding a (stationary) Nash equilibrium. The equilibrium that emerges is an approximation of the perfect positive assortative mating that would be reached without frictions. Using the Gale-Shapley algorithm to identify the stable outcome, we recall that, in the absence of frictions, this model generates a positive assortative mating. Thus, if men move first, all men will propose to the best woman and she will keep only the best man and reject all others. All rejected men will propose to the second best woman and she will accept the best of these and reject all others and so on. This outcome will also emerge here if the cost of waiting is low or frictions are not important, because λ is high. However, if frictions are relevant and waiting is costly, agents will compromise. In particular, the "best" woman and the "best" man will adopt the policies

$$\begin{aligned} R_{\bar{m}} &= b_{\bar{m}} + \frac{\lambda}{r}\int_{R_{\bar{m}}}^{\bar{f}} (f - R_{\bar{m}})dG_m(f), \\ R_{\bar{f}} &= b_{\bar{f}} + \frac{\lambda}{r}\int_{R_{\bar{f}}}^{\bar{m}} (m - R_{\bar{f}})dF_f(m). \end{aligned} \tag{7.40}$$

Thus, the best man accepts some women who are inferior to the best woman and the best woman accepts some men who are inferior to the best man, because one bird at hand is better than two birds on the tree.

The assumption that the rankings of men and women are based on a single trait, introduces a strong commonality in preferences, whereby all men agree on the ranking of all women and vice versa. Because all individuals of the opposite sex accept the best woman and all women accept the best man, μ is set to 1 in equation (7.40) and the distribution of offers equals the distribution of types in the population. Moreover, if the best man accepts all women with f in the range $[R_{\bar{m}}, \bar{f}]$ then all men who are inferior in quality will also accept such women. But this means that all women in the range $[R_{\bar{m}}, \bar{f}]$ are sure that all men accept them and therefore will have the *same* reservation value, $R_{\bar{f}}$, which in turn implies that all men in the range $[R_{\bar{f}}, \bar{m}]$ will have the same reservation value, $R_{\bar{m}}$. These considerations lead to a *class structure* with a finite number of distinct classes in which individuals marry each other. Having identified the upper class we can then examine the considerations of the top man and woman in the rest of the population. These individuals will face $\mu < 1$ and a *truncated* distribution of offers that, in principle, can be calculated to yield the reservation values for these two types and all other individuals in their group, forming the second class. Proceeding in this manner to the bottom, it is possible to determine all classes.

With frictions, there is still a tendency to positive (negative) assortative mating based on the interactions in traits. If the traits are complements, individuals of either sex with a higher endowment will adopt a more selective reservation policy and will be matched, on the average, with a highly endowed person of the opposite sex. However, with sufficient friction, it is also possible to have negative assortative mating under complementarity. The reason for this result is that, because of the low frequency of meetings and costs of waiting, agents in a search market tend to compromise. Therefore, males with low m, expect some women with high f to accept them, and if the gain from such a match is large enough, they will reject all women with low f and wait until a high f woman arrives.

The class structure result reflects the strong assumption that the utility that each partner obtains from the marriage depends only on the trait of the *other* spouse, so that there is no interaction in the household production function between the traits of the two spouses. In general, there will be some mingling of low and high income individuals, but the pattern of a positive assortative mating is sustained, provided that the complementarity in traits is large enough to motivate continued search for the "right" spouse. Smith (2006) provides a (symmetric) generalization of the problem where if man m marries woman f he receives the utility payoff $v = \pi(m, f)$ and she receives the utility payoff $u = \pi(f, m)$. It is assumed that this function is increasing in its second argument, $\pi_2(x, y) > 0$, so that all men prefer a woman with a higher f and all women prefer a man with a higher m, but individuals can differ in the intensity of their ordering.[8] He then shows that a sufficient condition for positive assortative mating, in the sense of a higher likelihood that a rich person will have a rich spouse, is that $log\,(\pi(m, f))$ be super modular. That is, $m > m\prime$ and $f > f\prime$ imply that

$$\pi(m, f)\pi(m', f') > \pi(m, f')\pi(m', f). \tag{7.41}$$

The reason for such a condition is that one needs sufficiently strong complementarity to prevent the high types from accepting low types, due to impatience.

Surprisingly, the assumption of transferable utility loses some of its edge in the presence of frictions. In particular, it is no longer true that the assignment is determined by the maximization of the aggregate marital output of all potential marriages. To see why, consider the following output matrix:

Example 7.6

		Women		
		1	*2*	*3*
Men	*1*	4	1	0
	2	1	0	1
	3	0	1	4

where aggregate output is maximized on the main diagonal. With frictions, this assignment is in general *not* stable, because man 2 and woman 2 will prefer continued search to marriage that yield, 0, even if the value of being single is 0. The reason is that they can marry other men and women with whom they can obtain 1, who might be willing to marry them if the arrival rate of offers is low or the cost of waiting is high.

Generally speaking, the nature of the assignment problem changes, because of the need to consider the cost of time spent in search, as well as the

[8] Intensity is a meaningful concept because, given the risky environment, agents are endowed with a Von Neumann Morgenstern utility function that is unique up to a linear transformation.

benefits from matching. An additional complication, relative to the case of non-transferable utility, is the presence of rents. As we have seen, when meetings are random, and agents adopt reservation polices for accepted matches, the realized match will generally exceed the outside options of the married partner so that the rules for dividing the rents enter into the analysis. As a consequence, one generally needs stronger conditions to guarantee assortative matching. Shimer and Smith (2000) provide an analysis of the degree of complementarity that must hold to guarantee positive assortative mating if rents are divided equally in all marriages. Positive assortative mating, in the sense that a high m male is more likely to match with a high f female (on the average) requires, in addition to the supermodularity of $h(m, f)$, the supermodularity of the logs of its partial derivatives and the log of the cross derivative $h_{mf}(m, f)$. This means that the simple predictions of the frictionless model carry over only under restrictive assumptions. For instance, $h(m, f) = \frac{(m+f)^2}{4}$, which, as we have shown, arises naturally in the presence of public goods, does *not* satisfy these requirements.[9]

7.5 Bargaining In Marriage (BIM)

As we have just seen, search models with random and intermittent meetings provide a natural framework to deal with rents and bargaining over rents in the marriage market. However, if marriage specific capital, such as children, is generated during marriage, then rents and bargaining can arise even without uncertainty and frictions. As is well known from models of specific human capital (see Becker (1993 ch. 3)) the accumulation of capital that is useful only in a particular relation partly insulates from competition the division of the gain from marriage. There is, therefore, a scope for bargaining over such rents.

It has been recently pointed out by Lundberg and Pollak (2009) that if the division resulting from bargaining in marriage is fully anticipated prior to marriage and if, in addition, binding contracts cannot be made at marriage, then the assignment into marriage must be based on the Gale Shapley algorithm. Specifically, Lundberg and Pollak contrast their 'BIM' (Bargaining In Marriage) framework with the standard, 'BAMM' (Binding Agreements on the Marriage Market) model, which is one of the possible foundation of the Becker-Shapley-Shubik construct. In a BIM world, any promise I may make before marriage can (and therefore will) be reneged

[9] Specifically, the partial derivatives $\frac{m+f}{2}$ are not log super modular because $m > m\prime$ and $f > f\prime$ imply that

$$(m + f)(m' + f') < (m + f')(m' + f).$$

upon minutes after the ceremony; there is just no way spouses can commit beforehand on their future behavior. Moreover, 'upfront' payments, whereby an individual transfers some money, commodities or property rights to the potential spouse *conditional on marriage*, are also excluded. Then the intrahousehold allocation of welfare will be decided after marriage, irrespective of the commitment made before. Marriage decision will therefore take the outcome of this yet-to-come decision process as given, and we are back in a non transferable utility setting in which each partner's share of the surplus is fixed and cannot be altered by transfers decided *ex ante*.

This result is an outcome of the assumed inability to credibly bid a person prior to marriage either by payments up-front or by short term commitments. This argument raises some important modeling issues about the working of the marriage market. A first remark is that it is not clear why premarital contracting is assumed away. Historically, contracts specifying what one brings into marriage and what the husband and wife take away upon divorce were universal (see Anderson, 2007). In modern societies prenuptial contracts still exist, although they are less prevalent. One possibility is that formal contracting and the associated enumeration of contingencies would "crowd out" the emotional trust on which the partners rely. This argument, however, has somewhat ambiguous implications, because the mere existence of such emotional trust seems to imply the existence of at least some minimum level of 'emotional commitment' - an idea that has been formalized by Browning (2009). Another important issue is verification. Typically it is difficult for the courts to verify the division of consumption or work within families. It must however be emphasized that commitment on intrahousehold allocation is *not* needed to implement a BAMM solution. Any transfer that (i) is decided *ex ante*, that is before marriage, and (ii) can be used to alter the spouse's respective bargaining positions after marriage, can do the trick. For instance, if the husband can, at (or just before) marriage, sign a legally enforceable contract specifying the transfers that would occur in case of separation, then we are back to a BAMM framework: I can now 'bid' my wife by offering her a very advantageous contract, because *even if we do not ultimately divorce,* the additional bargaining power provided to her by the *ex ante* contract will allow her to get a larger share of household resources - and is therefore equivalent to an *ex post* cash transfer. An even more striking example is the 'payment for marriage' situation, in which the husband can transfer a predetermined amount to his wife upon marriage (say, by offering her an expensive ring, or putting the couple's residence under her name, or even writing a check). Again, the size of the transfer can be used in the bidding process, and the relevant concept is again BAMM. Conversely, the BIM framework basically requires that no *ex ante* contract can ever be signed, and no conditional payment can ever be made.

A second concern is that even if we accept the total absence of commitment, Gale-Shapley still need not be the relevant equilibrium concept. To see why, consider the extreme situation in which marriage can be done and undone at very low cost. Then at any moment of marital life, each spouse has many close substitutes on the market, and the intrahousehold allocation will typically reflect this fact. Although, technically, this is not a BAMM situation (no binding agreement can be signed by assumption), the relevant concept is still the TU model a la Becker-Shapley-Shubik, because each spouse receives exactly her/his reservation value and the latter is fully determined by market equilibrium forces (at least when the number of potential spouses is 'large enough'). In other words, even in the extreme no transfer/no commitment case, the BIM framework applies only insofar as marriage decision can only be reversed at some cost, and only within the limits defined by this cost.

It is clear, in practice, that entry into marriage is a major decision that can be reversed only at some cost. However, as in any modeling choice, "realism" of the assumptions is not the only concern. It is also important to have a tractable model that allows one to predict the marriage market outcomes under *varying* conditions. In this regard, the presence of transaction costs is quite problematic. To see this, consider again our example 7.3. Suppose that a new woman, 4, unexpectedly enters a marriage market that has been in one of the two equilibria discussed in section 7.1. Let the new payoffs matrix be as below:

Example 7.3*a*

		Women			
		1	*2*	*3*	*4*
Men	*1*	$3,2$	$2,6$	$1,1$	$2,1$
	2	$4,3$	$7,2$	$2,4$	$5,4$
	3	$1,1$	$2,1$	$0,0$	$.5,.5$

By assumption, woman 4 is preferred to woman 3 by all men and one would expect that in the new assignment woman 3 will become single. Suppose, however, that all existing couples bear a transaction cost of 0.75. Then it is easy to see that if the original equilibrium was the one in which men moved first, no man will marry woman 4 and she will remain single. In contrast, if the original equilibrium was the one in which women moved first then man 2 will take woman 4 and his ex-wife (woman 1) will first propose to man 1 who will reject her and then to man 3 who will accept her, so that woman 3 will become single. Thus, in general, it is impossible to predict what would happen when a new player enters the market, without knowing the bargaining outcomes in all marriages, the potential bargaining outcome that the entrant will have with all potential existing partners and the relational capital accumulated in all existing marriages. Such information is never available to the observer. In contrast, the Becker-Shapley-Shubik framework can predict the outcome very easily, using only information about the place of the new woman in the income distribution of women and the form of the household production function that specifies the within couple interaction between men and women of different attributes.

Given the different implications of alternative models of the marriage market, it seems prudent to consider several alternatives, depending on the application. In subsequent chapters we shall apply search models to analyze marriage and divorce when match quality is uncertain, and we shall apply the standard assignment model to discuss the determination of the division of gains from marriage when men and women differ in their attributes.

Appendix: Supermodularity, Submodularity and Assortative Matching

This appendix proves that complementarity (substitution) in traits must lead to a positive (negative) assortative mating; otherwise aggregate output is not maximized. Assuming that $h(x, y)$ is increasing in x and y we also obtain that in the case of positive assortative mating, the best man marries the best woman, and if there are more women than men the women with low female quality remain single. If there is negative assortative mating, the best man marries the worst woman among the married women but if there are more women than men, it is the women with the lower female attributes who remain single.

Super modularity

Let $\bar{x}$ and $\bar{y}$ denote the endowments of the "best" man and woman and suppose that they are *not* married to each other and instead man $\bar{x}$ marries some woman whose female attribute is $y' < \bar{y}$ and woman $\bar{y}$ marries some man whose attribute is $x' < \bar{x}$. Then stability of these matches requires the existence of divisions such that

$$\begin{aligned} u_{\bar{x},y'} + v_{x',\bar{y}} &\geq h(\bar{x}, \bar{y}), \\ u_{x',\bar{y}} + v_{\bar{x},y} &\geq h(x', y'), \\ u_{\bar{x},y\prime} + v_{\bar{x},y\prime} &= h(\bar{x}, y\prime), \\ u_{x',\bar{y}} + v_{x',\bar{y}} &= h(x', \bar{y}), \end{aligned}$$

which implies that

$$h(\bar{x}, y\prime) + h(x', \bar{y}) \geq h(x', y) + h(\bar{x}, \bar{y})$$

and contradicts (strict) super modularity. Thus complementarity implies that the best man must marry the best woman. Eliminating this couple, and restricting attention to the next best pair, we see that it must marry too and so on. If there are more women than men, Then there must be some woman y_m such that all women with lesser quality are single.

Sub modularity

Suppose again that there are more women than men. Then, in this case too, there must be some woman y_m such that all women with lesser quality are single. Otherwise, there must be a married woman with a lower quality than some single woman, which under monotonicity implies that aggregate output is not maximized. However, now man $\bar{x}$ and woman y_m must marry each other. If they are not married to each other and, instead, man $\bar{x}$ marries some woman whose female attribute is $y' > y_m$ and woman y_m marries some man whose attribute is $x' < \bar{x}$, then stability of these matches requires the existence of surplus allocations such that

$$
\begin{aligned}
u_{\bar{x},y\prime} + v_{x',y_m} &\geq h(\bar{x}, y_m),\\
u_{x',y_m} + v_{\bar{x},y} &\geq h(x', y'),\\
u_{\bar{x},y'} + v_{\bar{x},y\prime} &= h(\bar{x}, y\prime),\\
u_{x'y_m} + v_{x',y_m} &= h(x', y_m),
\end{aligned}
$$

which implies that

$$h(\bar{x}, y\prime) + h(x', y_m) \geq h(x', y') + h(\bar{x}, y_m)$$

and contradicts (strict) sub modularity. Eliminating this couple, and restricting attention to the next best pair, that is the second best man among men and the second worst woman among all married women must marry too, and so on.

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8

Sharing the gains from marriage

In this chapter, we discuss in more detail the determination of the division of the marital surplus and how it responds to market conditions. If each couple is considered in isolation, then, in principle, any efficient outcome is possible, and one has to use bargaining arguments to determine the allocation. On the contrary, the stability of the assignments restricts the possible divisions because of the ability to replace one spouse by another one. The options for such substitution depend on the distributions of the marital relevant attributes in the populations of the men and women to be matched. In the present chapter, we precisely ask how the marriage market influences the outcome in the ideal, frictionless case discussed previously. Although the division within marriage is not always fully determined, some qualitative properties of the division can be derived from information on the joint distribution of male and female characteristics together with a specification of the household production function.

As before, we discuss separately the cases of discrete and continuous distributions. The general intuition goes as follows. In the discrete case, competition puts bounds on individual shares but does not completely determine them; this is because on the marriage market, each potential spouse has only a finite number of 'competitors', none of which is a perfect substitute - so some elements of 'bilateral monopoly' persist. In the continuous case, however, competition between potential spouses tends to be perfect, leading to an exact determination of the 'prices' - that is, in our case, budget shares. In addition to the standard case of transferable utility, we also consider the more general case in which the exchange rate of the spouses' utilities varies along the Pareto frontier. We provide detailed examples that illustrate how changes of the distributions of incomes or tastes of men and women can affect the division of resources within couples. We conclude with a discussion of recent developments in estimating equilibrium models of the marriage market, including the gains from marriage and the division of these gains.

The major insight obtained from the equilibrium analysis is that the sharing of the gains from marriage depends not only on the incomes or preferences of spouses in a given match but also and perhaps mainly on the overall distributions of incomes and preferences in society as a whole. Thus, a redistribution of income via a tax reform can influence the shares of the gains from marriage even if the incomes in particular couple are unaffected. Similarly a legal reform or a technological innovation that makes it easier to prevent pregnancy can influence the division of resources within married couples who chose to have children. In either case, the general equilibrium effects arise from competition with potential spouses outside the given marriage. Obviously, our assumptions regarding the agents' ability to transfer resources within marriage (and to a lesser extent the absence of frictions) are crucial for such indirect effects. It is therefore a challenging research agenda to find how important are these considerations in practice.

8.1 Determination of shares with a finite number of agents

We start with matching between finite male and female populations.[1] As explained in the previous chapter, while the matching pattern (who marries whom) and the associated surplus is generally unique, the allocation of surplus between spouses is not. Typically, there exists, within each couple, a continuum of allocations of welfare that are compatible with the equilibrium conditions. That does *not* mean, however, that the allocation is fully arbitrary. In fact, equilibrium imposes strict bounds on these allocations. Depending on the context, these bounds may be quite large, allowing for considerable leeway in the distribution of surplus, or quite tight, in which case the allocation is practically pinned down, up to minor adjustments, by the equilibrium conditions. We present in this section a general description of these bounds.

8.1.1 The two men - two women case

As an introduction, let us consider a model with only two persons of each gender discussed in Chapter 7. Assume for instance that $z_{12} + z_{21} \geq z_{11} + z_{22}$, implying that the stable match is 'off-diagonal' (man i marries woman $j \neq i$, with $i, j \in \{1, 2\}$). Then all pairs (v_1, v_2) satisfying the inequalities:

$$\begin{aligned} z_{12} - z_{11} &\geq v_2 - v_1 \geq z_{22} - z_{21}, \\ z_{21} &\geq v_1 \geq 0, \\ z_{12} &\geq v_2 \geq 0, \end{aligned} \tag{8.1}$$

yield imputations v_1, v_2, $u_1 = z_{12} - v_{2,}$, $u_2 = z_{21} - v_1$ that support the stable assignment along the opposite diagonal. The shaded area in Figure 8.1 describes all the pairs that satisfy the constraints required for stability expressed in condition (8.1). The figure is drawn for the special case in which woman 2 is more productive than woman 1 in all marriages ($z_{22} >$

[1] Ellana Melnik participated in the derivation of the results of this section.

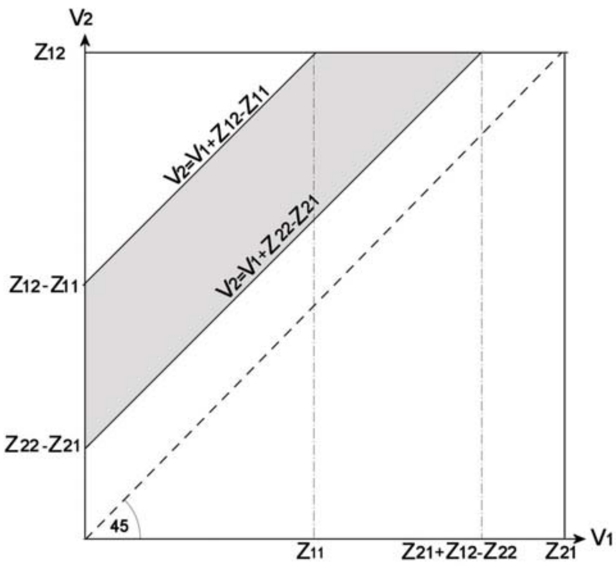

FIGURE 8.1. Imputations when $z_{12} + z_{21} > z_{22} + z_{11}$

$z_{21}, z_{12} > z_{11}$) and symmetry holds, $z_{12} = z_{21}$, implying that man 2 is also more productive than man 1 in all marriages. The main feature here is that the difference $v_2 - v_1$ is bounded between the marginal contributions of replacing woman 1 by woman 2 as spouses of man 1 and man 2. Woman 2 who is matched with man 1 cannot receive in that marriage more than $z_{12} - z_{11} + v_1$, because then her husband would gain from replacing her by woman 1. She would not accept less than $v_1 + z_{22} - z_{21}$, because then she can replace her husband by man 2 offering him to replace his present wife. The assumption that $z_{12} - z_{11} > z_{22} - z_{21}$ implies that man 1 can afford this demand of woman 2, and will therefore "win" her. In this fashion, the marriage market "prices" the different attributes of the two women. Symmetric analysis applies if we would replace (v_1, v_2) with (u_1, u_2).

Similarly, if $z_{12} + z_{21} \leq z_{11} + z_{22}$, implying that the stable match is 'diagonal', then all pairs (v_1, v_2) satisfying the inequalities

$$\begin{aligned} z_{22} - z_{21} &\geq v_2 - v_1 \geq z_{12} - z_{11}, \\ z_{11} &\geq v_1 \geq 0, \\ z_{22} &\geq v_2 \geq 0, \end{aligned} \tag{8.2}$$

yield imputations v_1, v_2, $u_1 = z_{11} - v_1$, $u_2 = z_{22} - v_2$ that support the stable assignment along the diagonal. The shaded area in Figure 8.2 describes all the pairs that satisfy the constraints required for stability expressed

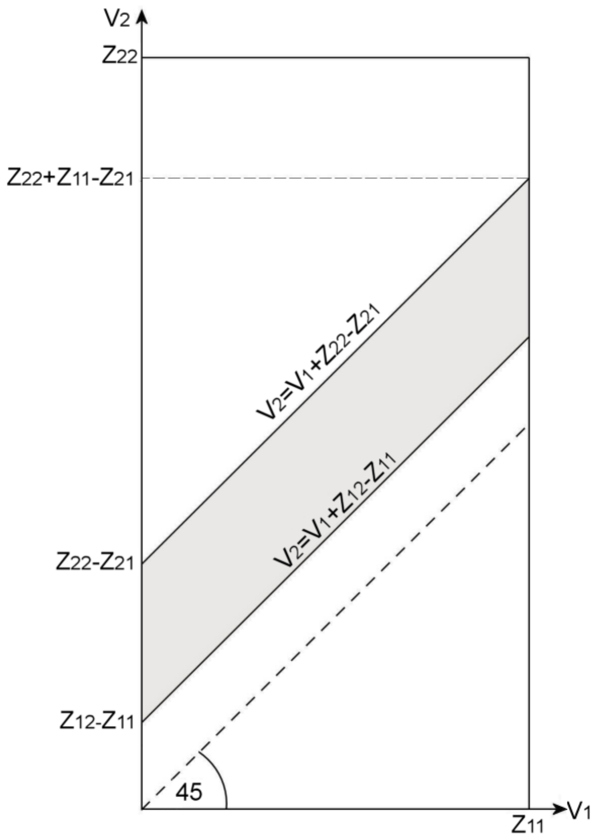

FIGURE 8.2. Imputations when $z_{12} + z_{21} < z_{22} + z_{11}$

in condition (8.2). Again the difference $v_2 - v_1$ is bounded between the marginal contributions of replacing woman 1 by woman 2 as spouses of man 1 and man 2. Because we assume that woman 2 is more attractive than woman 1, she gets a larger part of the surplus in both cases and her share in the surplus is always positive. Woman 1 who is less desirable may get no surplus at all. If she is married to man 1 who is less attractive, she may get the entire surplus. However, if she is married to man 2, he always receives a positive share and she never receives the entire surplus.

The indeterminacy of prices in the marriage market reflects the fact that the "objects traded" are indivisible and have no close substitutes. Therefore, agents may obtain in the stable assignment utility levels that are strictly higher than they would in alternative marriages. When this is true for all alternative marriages it is possible to slightly shift utility between the partners of each marriage, and still maintain all the inequalities of the dual problem, without any effect on the allocation. An interesting feature, noted by Shapley and Shubik (1972), is that in the core (that is, of the set of imputations that support a stable assignment) "the fortunes of all players of the same type rise and fall together". This is seen by the upward tendency of the shaded areas in figures 8.1 and 8.2. In particular, there is a polar division of the surplus that is best for *all* men, and also a polar division that is best for *all* women.

As an illustration, let us come back to Example 7.4 in the previous Chapter. Specifically, consider the table presenting three imputations, denoted by a, b and c; for commodity, the Table is reproduced below. Note that these imputations are arranged in such a manner that the reservation utility of all men rise and those of all women decline.

Imputation		a				b				c		
		W		M		W		M		W		M
Individual shares	v_1	2	u_1	3	v_1	2	u_1	4	v_1	1	u_1	5
	v_2	5	u_2	5	v_2	4	u_2	5.5	v_2	3	u_2	6
	v_3	1	u_3	0	v_3	0.5	u_3	0	v_3	0	u_3	1

In each of these three imputations, individuals who are married to each other receive their reservation utility, which together exhaust the marital input, Thus $v_2 + u_1 = z_{12} = 8$, $u_2 + v_3 = z_{23} = 6$ and $v_1 + u_3 = z_{31} = 2$. For marriages that do not form, the sum of the reservation utilities exceeds or equals the potential marital output. For instance, man 2 and woman 2 are not married to each other, and therefore $v_2 + u_2 \geq z_{22} = 9$. This requirement is strict for imputations a and b and holds as equality for imputation c. Similarly, because man 1 and woman 1 are not married to each other, we must have $v_1 + u_1 \geq z_{11} = 5$. This holds as strict inequality for imputations b and c, and as equality for imputation a. The significance of the equalities is that they indicate the bounds within which it is possible to change prices without any affect on the assignment. Hence, imputation c is the best for men and the worst for women and imputation a is the best for women and the worst for men.

8.1.2 Bounds on levels

The previous insights can readily be extended to a more general setting. We now consider N men and M women and assume that the assignment variables a_{ij} are all either zeros or ones. Start with the dual problem:

$$\min_{u,v} \sum_{i=1}^{N} u_i + \sum_{j=1}^{M} v_j \tag{8.3}$$

subject to

$$u_i + v_j \geq z_{ij} \text{ for } i = 1, 2...N, j = 1, 2...M$$

Denote the solution for individual utilities (or individual 'prices') by $(\hat{u}_i, \hat{v}_j)$. From the results on duality (Gale, 1960, chapter 5) we know that the solution to this problem yields the same value as the solution to the primal problem. That is

$$\sum_{i=1}^{N} \hat{u}_i + \sum_{j=1}^{M} \hat{v}_j = \sum_{i,j} \hat{a}_{ij} z_{ij},$$

where $\hat{a}$ denotes the assignment that solves the primal.

Now compare (8.3) to the dual problem when man N is eliminated:

$$\min_{u,,v} \left(\sum_{i=1}^{N-1} u_i + \sum_{j=1}^{M} v_j \right) \tag{8.4}$$

subject to

$$u_i + v_j \geq z_{ij} \text{ for } i = 1, 2...N-1, j = 1, 2...M$$

Denote the solution for prices by $(\bar{u}_i, \bar{v}_j)$. Again we know that the solution to this problem yields the same value as the solution to the primal problem. That is

$$\sum_{i=1}^{N-1} \bar{u}_i + \sum_{j=1}^{M} \bar{v}_i = \sum_{i,j} \bar{a}_{ij} z_{ij}.$$

where $\bar{a}$ denotes the assignment that solves the primal associated with (8.4).

Notice that the values $(\hat{u}_i, \hat{v}_j)$ chosen in the dual problem (8.3) are *feasible* in the dual problem (8.4). It follows that the minimum attained satisfies

$$\sum_{i=1}^{N-1} \bar{u}_i + \sum_{j=1}^{M} \bar{v}_j \leq \sum_{i=1}^{N-1} \hat{u}_i + \sum_{j=1}^{M} \hat{v}_j \ ,$$

or

$$\sum_{i,j} \bar{a}_{ij} z_{ij} \leq \sum_{i,j} \hat{a}_{ij} z_{ij} - \hat{u}_N \ ,$$

implying that

$$\hat{u}_N \leq \sum_{i,j} \hat{a}_{ij} z_{ij} - \sum_{i,j} \bar{a}_{ij} z_{ij} \ .$$

That is, the upper bound on the utility that man N can get is his *marginal contribution* to the value of the primal program (that is, the difference between the maximand with him and without him). Note that to calculate this upper bound we must know the assignments in *both* cases, when N is excluded and N is included. This is easily done if we assume positive or negative assortative mating. For instance, with positive assortative mating and $M > N$,

$$\sum_{i,j} \hat{a}_{ij} z_{ij} - \sum_{i,j} \bar{a}_{ij} z_{ij} = \sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N-1} z_{i,M-(N-1)+i}\,. \tag{8.5}$$

Similar arguments apply for any man and any woman. Using the bounds for men and women who are married to each other at the stable assignment we can put bounds on the possible divisions of the gains from marriage of the husband and wife in each couple. Thus the husband's share in the couple MN is bounded by

$$\begin{aligned} & z_{N,M} - \left(\sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N} z_{i,M-1-N+i}\right) \\ \leq \quad & \hat{u}_N \leq \\ & \sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N-1} z_{i,M-(N-1)+i} \end{aligned}$$

or

$$\sum_{i=1}^{N} z_{i,M-1-N+i} - \sum_{i=1}^{N-1} z_{i,M-N+i} \leq \hat{u}_N \leq \sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N-1} z_{i,M-(N-1)+i}. \tag{8.6}$$

8.1.3 Bounds on Differences

With positive or negative assortative mating we can also put bounds on the *change* in utilities as one moves along the assignment profile. Let there be a positive assortative mating (that is, the matrix z_{ij} is super modular) and suppose that $M > N$. Then man N is married to woman M, and woman $M-1$ is married to man $N-1$. At a stable assignment

$$\begin{aligned} u_N + v_M &= z_{N,M} \\ u_{N-1} + v_{M-1} &= z_{N-1,M-1} \\ u_N + v_{M-1} &\geq z_{N,M-1} \\ u_{N-1} + v_M &\geq z_{N-1,M} \end{aligned}$$

Eliminating u_N and u_{N-1} and substituting into the inequalities we get

$$\begin{aligned} z_{N,M} - v_M + v_{M-1} &\geq z_{N,M-1} \\ z_{N-1,M-1} - v_{M-1} + v_M &\geq z_{N-1,M} \end{aligned}$$

Hence,

$$z_{N,M} - z_{N,M-1} \geq v_M - v_{M-1} \geq z_{N-1,M} - z_{N-1,M-1}$$

and we get the upper and lower bounds on $v_M - v_{M-1}$. Now we also know that woman $M-2$ and man $N-2$ marry each other. Using the fact that $M-1$ and $N-1$ also marry each other we get by the same argument that

$$\begin{aligned} z_{N-1,M-1} - z_{N-1,M-2} &\geq v_{M-1} - v_{M-2} \\ &\geq z_{N-2,M-1} - z_{N-2,M-2} \end{aligned} \tag{8.7}$$

and so on all the way to the lowest married couple. Because we assume more women than men, $M > N$, woman $M-N+1$ will marry man 1. For this particular couple, we have

$$\begin{aligned} u_1 + v_{M-N+1} &= z_{1,M-N+1} \\ u_1 + v_{M-N} &\geq z_{1,M-N} \\ v_{M-N} &= 0 \end{aligned}$$

The boundary condition is therefore

$$z_{1,M-N+1} - z_{1,M-N} \geq v_{M-N+1} \ .$$

We see that along the stable assignment the prices must form an *increasing* sequence. This is a consequence of complementarity.

When we set the bounds on couple N, M in (8.7), we referred only to couple $N-1, M-1$. However, there are $M-1$ stability constraints, one for each woman that man N is not married to:

$$\begin{aligned} u_N &\geq z_{N,M-1} - v_{M-1} \\ u_N &\geq z_{N,M-2} - v_{M-2}\cdots \end{aligned}$$

and also $N-1$ stability constraints for woman M regarding for each man that she is not married to. We now show that the most binding constraint from all these constraints is the one expressing that man N (woman M) does not marry woman $M-1$ (man $N-1$). That is,

$$z_{N,M-1} - v_{M-1} \geq z_{N,M-2} - v_{M-2} \geq \cdots \geq z_{N,M-N+1} - v_{M-N+1} \geq z_{N,M-N}.$$

Note, first, that if man N does not want to marry woman $M-1$, then he does not want to marry woman $M-2$ either; that is, the stability constraint related to woman $M-1$ is more binding than that related to woman $M-2$. Indeed, we want to show that:

$$z_{N,M-1} - v_{M-1} \geq z_{N,M-2} - v_{M-2}. \tag{8.8}$$

Equation (8.8) can be rewritten as :

$$z_{N,M-1} - (z_{N-1,M-1} - u_{N-1}) \geq z_{N,M-2} - v_{M-2},$$

or

$$u_{N-1} + v_{M-2} \geq z_{N,M-2} - z_{N,M-1} + z_{N-1,M-1}.$$

In the stable assignment $u_{N-1}+v_{M-2} \geq z_{N-1,M-2}$, so it is enough to show that

$$z_{N-1,M-2} \geq z_{N,M-2} - z_{N,M-1} + z_{N-1,M-1}$$

. But, this follows directly from the assumption that z_{ij} is super modular. Therefore, the lower bound woman $M-1$ imposes is higher than the lower bound woman $M-2$ imposes on man N. By the same arguments, we now generally show that woman $M-k'$s constraint is more binding than woman $M-k-1'$s constraint. Now we have,

$$\begin{aligned} z_{N,M-k} - v_{M-k} &\geq z_{N,M-k-1} - v_{M-k-1}, \\ z_{N,M-k} - (z_{N-k,M-k} - u_{N-k}) &\geq z_{N,M-k-1} - v_{M-k-1}, \\ u_{N-k} + v_{M-k-1} &\geq z_{N,M-k-1} - z_{N,M-k} + z_{N-k,M-k}. \end{aligned}$$

Again, we know that in a stable assignment, $u_{N-k}+v_{M-k-1} \geq z_{N-k,M-k-1}$, so it is enough to show that

$$z_{N-k,M-k-1} \geq z_{N,M-k-1} - z_{N,M-k} + z_{N-k,M-k}$$

which follows from the super modularity assumption that requires this condition. Therefore we can say that the lower bound woman $M-k$ imposes is higher than the lower bound woman $M-k-1$ imposes on man N, and finally conclude that the highest lower bound on man $N\prime s$ share is imposed by woman $M-1$. In a very similar way it can be shown that the highest lower bound on woman $M\prime s$ share is imposed by man $N-1$.

8.2 The continuous case

We now consider a continuous distribution of agents, in which equilibrium conditions typically pin down the intrahousehold allocation of welfare. The difference between the continuous case and the discrete case analyzed in the previous section is that, with a continuum of agents and continuous distributions, each agent has a very close substitute. In this case, the upper and lower bounds in 8.6 and 8.7, respectively, approach each other and in the limit coincide.

8.2.1 Basic results

The setting, here, is a slight generalization of the one considered in subsection 7.2.3 of Chapter 7. There exists a continuum of men, whose incomes x are distributed on $[0, 1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0, 1]$ according to some distribution G.[2] The measure of all men in the population is normalized to 1, and the measure of women is denoted by r. Also, we still consider a transferable utility (TU) framework. The innovation is that the 'marital output' is now the sum of two components: an economic output, which is a function $h(x, y)$ of individual incomes, and a fixed non monetary gain from marriage, denoted θ, which is perceived by the spouses in addition to the economic benefits:

$$z(x, y) = h(x, y) + \theta$$

As before, h is assumed to be supermodular.

An *allocation rule* specifies the shares of the wife and husband in every marriage. If $r > 1$ and all men are married, we can index the marriage by the husband's income x (then his spouse's income is $\psi(x)$). The marital output is then $h(x, \psi(x)) + \theta$ and the marital shares are $u(x)$ for the husband and $v(\psi(x))$ for the wife. If $r < 1$ and all women are married, we can index the marriage by the wife's income y (then the husband's income is $\phi(y)$). The marital output is then $h(\phi(y), y) + \theta$ and the marital shares are $u(\phi(y))$ for the husband and $v(y)$ for the wife.

As discussed before, the allocation rule that supports a stable assignment must be such that the implied utilities of the partners satisfy

$$u(x) + v(y) \geq h(x, y) + \theta \quad \forall x, y, \tag{8.9}$$

with equality if the partners are married to each other and inequality if they are not.[3] The utility levels $v(x)$ and $u(y)$ that satisfy (8.9) can be interpreted as the demand prices that men with income x and women with income y require to participate in *any* marriage. Marriages that form are consistent with the demands of both partners and exhaust family resources. Marriages that do not form are those in which resources are insufficient to satisfy the demands of *both* partners.

---

[2] Obviously, the support could be changed to any intervals $[a, A]$ and $[b, B]$ - the only cost being more tetious notations.

[3] Note that by deducting $h(y, 0) + h(0, z)$ from both sides of equation (8.9) it can be written, equivalently, in terms of the surplus that the marriage generates, relative to remaining single. Also, because the values of remaining single are independent of the assignment, the condition for stable assignment can be formulated as maximization of the aggregate surplus.

In particular, (8.9) implies that

$$\begin{aligned} u(x) &= \theta + \max_y (h(x,y) - v(y)), \\ v(y) &= \theta + \max_x (h(x,y) - u(x)). \end{aligned} \tag{8.10}$$

That is, each partner gets the spouse that maximizes his/her "profit" from the partnership, taking into account the reservation utility (the 'price') of any potential spouse. The first order conditions for the maximizations in (8.10) give:

$$\begin{aligned} v'(y) &= h_y(\phi(y), y), \\ u'(x) &= h_x(x, \psi(x)). \end{aligned} \tag{8.11}$$

These equations have an important implication - namely that, as we move across matched couples, the welfare of each partner changes according to the *marginal* contribution of his/her *own* income to the *marital* output, irrespective of the potential impact on the partner whom one marries. The reason for this result is that, with a continuum of agents, there are no rents in the marriage market, because everyone receives roughly what he\she would obtain in the best next alternative.[4] Therefore, a change in marital status as a consequence of a marginal change in income has negligible impact on welfare, and the only gain that one receives is the marginal contribution of one's own trait. Although the change of spouse provides no additional utility, the spouse that one has influences the marginal gain from an increase in own traits, reflecting the interactions between the traits in the production of marital output.

Another important condition that needs to be satisfied in a stable assignment is that, if there are unmarried men, the poorest married man (whose income is denoted x_0) cannot get any surplus from marriage. Similarly, if there are unmarried women, the poorest married woman (whose income is denoted y_0) cannot get any surplus from marriage. Otherwise, the unmarried men or women who are slightly less rich could bid away the marginal match. This condition exploits the assumption that there is a continuum of agents. Hence, if $r < 1$ then $u(x_0) = h(x_0, 0)$ and $v(0) = \theta$. Conversely, if $r > 1$ then $v(y_0) = h(0, y_0)$ and $u(0) = \theta$. If $r = 1$, then any allocation of the gains in the least attractive match with $x = y = 0$ that satisfies $u(0) + v(0) = \theta$ is possible.

This initial disparity between the two spouses is modified as they move up the assignment profile. The main features that influence the evolution of utility differences within couples are the *local scarcity* of males and females at different levels of incomes and the strength of the interaction

[4] The absence of rents must be distinguished from the positive surplus that the marriage creates. A positive surplus, $h(y,z) + \theta > h(y,0) + h(0,z)$, simply means that there are positive gains from marriage, relative to the situation in which both partners become single, but this is rarely the best next alternative.

in traits. Assuming, for instance, that $r > 1$ and all men are married then marriages can be indexed by the husband's income. As one moves across all married couples, the utility of the husbands rises at the rate $\frac{du(x)}{dx} = h_x(x, \psi(x))$, while the utility of their assigned wives rises at the rate $\frac{dv(y)}{dy}\frac{dy}{dx} = h_y(x, \psi(x))\, \psi'(x)$. In this case, if men are everywhere locally scarce (that is, $\psi'(x) < 1$), then the utility of the husband rises faster than the utility of the wife. Conversely, if there are less women than men ($r < 1$) and women are everywhere locally scarce (that is, $\phi'(y) < 1$), the utility of the wife rises faster than the utility of the husband. Intuitively, an overall scarcity of men benefits men at the top of the income distribution to a larger extent because these men are desired by all women; by the same token, an overall scarcity of women benefits the women at the top of the income distribution to a larger extent, because these women are desired by all men.

Integrating the expressions in (8.11) and using the boundary conditions described above, one can obtain a *unique* allocation rule, provided that $r \neq 1$. Basically, one first finds the allocation in the least attractive match, in which the minority type has no income, using the no rent condition. Then, the division in better marriages is determined sequentially, using the condition that along the stable matching profile each partner receives his\her marginal contribution to the marital output. The key remark is that the allocation rule is fully determined by the sex ratio r and the respective income distributions of the two sexes. The incomes of the partners in a particular marriage have no direct impact on the shares of the two partners, because the matching is endogenously determined by the requirements of stable matching.

Technically, therefore, assuming for instance $r > 1$:

$$\begin{aligned} v(y) &= h(0, y_0) + \int_{y_0}^{y} h_y(\phi(t), t)\, dt, \\ u(x) &= \theta + \int_0^x h_x(s, \psi(s))\, ds, \\ y_0 &= \Psi(1 - 1/r), \end{aligned} \tag{8.12}$$

(and analogous conditions can readily be derived for for $r < 1$). If $r = 1$,

$$\begin{aligned} v(y) &= k + \int_0^y h_y(t, \phi(t))dt, \\ u(x) &= k' + \int_0^x h_x(\psi(s), s)ds, \\ k + k' &= \theta. \end{aligned} \tag{8.13}$$

where k and k' are arbitrary.

The first terms in the RHS of equations (8.12) and (8.13) are the utilities of the partners in the match of the lowest quality and the integrals describe the accumulated marginal changes, as we move up the stable assignment profile to marriages with higher incomes. Because of the interaction in traits, the change in the marital contribution depends on the income of the spouse that one gets. Note that marginal increases in x_0 or y_0 have *no* effect on $u(x)$ or $v(y)$, respectively, because the marginal persons with these incomes are just indifferent between marrying and remaining single.

In marriages that involve individuals from the bottom of the male and female income distributions, members of the larger sex group typically have higher income. Thus, if $r > 1$ and all men are married, the men in the lowest quality matches have almost no income, while their wives have strictly positive income. The wife receives her utility as single $h(0, y_0)$ and the husband receives the remaining marital output. If $r = 1$, the allocation in the lowest quality match is indeterminate and, consequently, there is a whole set of possible sharing rules that differ by a constant of integration.

8.2.2 A tractable specification

Let us now slightly generalize our previous approach by assuming that male incomes x are distributed on a support $[a, A]$ according to some distribution F and female incomes y are distributed on a support $[b, B]$ according to some distribution G; the assumption of different supports for men and women is useful for empirical applications. We introduce now a simplifying assumption, namely that the output function h depends only on total family income. That is,

$$h(y, x) = H(y + x), \tag{8.14}$$

with $H(0) = 0$. This assumption makes sense in our transferable utility setting, since under TU a couple behaves as a single decision maker. Note that basically all examples of intrahousehold allocation with TU given in Chapter 3 satisfy this property.

Under this assumption, $h_y(y, x) = h_x(y, x) = H'(y + x)$, and assortative matching requires $h_{xy}(y, x) = H''(y + x) > 0$, so that H is increasing and convex. As above, we let $\psi(x)$ (resp. $\phi(y)$) denote the income of Mr. x's (Mrs. y's) spouse. Finally, we maintain the convention that a single person with income s $(= x, y)$ achieves a utility level $H(s)$.

We are interested in how changes in the sex ratio and the distributions of income of the two sexes affect the allocation rule that is associated with a stable matching. In this analysis, we shall distinguish between two issues: (i) the shape of the allocation rule in a cross section of marriages - that is, how do the shares vary as we move up the assignment profile to couples with higher incomes, and (ii) changes of the allocation rule as parameters of the marriage market, such as the sex ratio or the male and female income distribution, change.

Allocation of marital output: general properties

We start by analyzing the properties of the allocation of marital output between spouses, as described by equations (8.12) and (8.13). In the lowest quality matches, the partner that belongs to the majority group has higher income than the minimum of the corresponding income distribution, but, because of competition with lower income singles, receives no rent, and has the same income as a single. In contrast, the partner that belongs to the minority group receives a rent that equals to the total surplus generated, because there are no lower income singles to compete with. These properties exactly define the allocation of welfare between the spouses.

Under assumption (8.14), equation (8.11) becomes:

$$\frac{du(x)}{dx} = H^{\text{`}}\left(x + \psi\left(x\right)\right), \tag{8.15}$$

$$\frac{dv(y)}{dy} = H^{\text{`}}\left(\phi\left(y\right) + y\right) \tag{8.16}$$

Therefore, for any married couple $\left(x, \psi\left(x\right)\right) = \left(\phi\left(y\right), y\right)$,

$$\frac{du(x)}{dx} = \frac{dv(y)}{dy}.$$

In words: the return, in terms of intrahousehold allocation of marital output, of an additional dollar of income is the same for males and females. This symmetry between genders, however, is *not* maintained when moving from one couple to another, because in general the change in husband's income between the couples does not equal to the change in the wife's income - reflecting the local scarcity of the respective genders, as discussed in Chapter 7.

Integrating (8.15) and (8.16), and assuming for instance that $r > 1$, we have:

$$u\left(x\right) = H\left(a + y_0\right) - H\left(y_0\right) + \theta + \int_a^x H'\left(s + \psi\left(s\right)\right) ds \tag{8.17}$$

$$v\left(y\right) = H\left(y_0\right) + \int_{y_0}^y H'\left(\phi\left(t\right) + t\right) dt \tag{8.18}$$

Again, since women are assumed to be on the long side of the market, the poorest married woman, with income y_0, must be indifferent between marriage and singlehood; all the surplus generated by her marriage, namely $H\left(a + y_0\right) - H\left(y_0\right) - H\left(a\right) + \theta$, goes to the husband, generating a utility $H\left(a + y_0\right) - H\left(y_0\right) + \theta$. Moving up along the income distributions, the allocation evolves as described by (8.15) and (8.16).

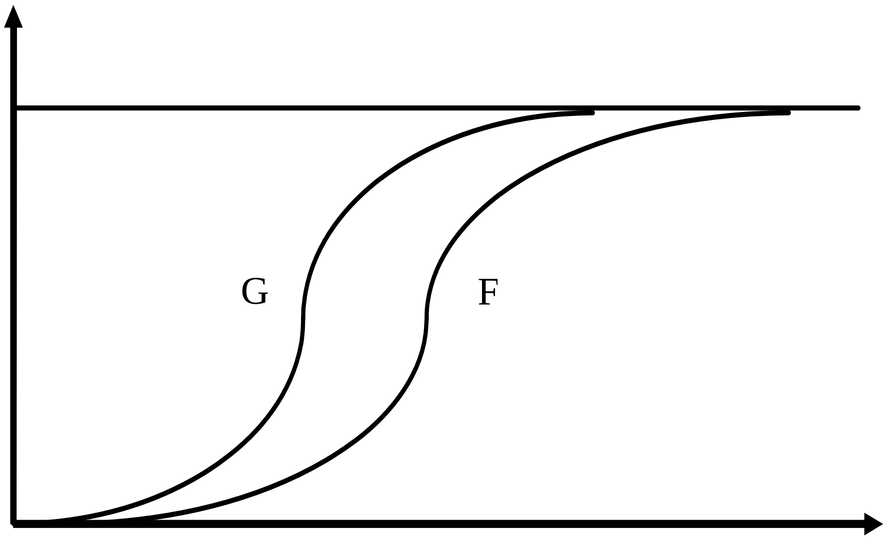

FIGURE 8.3. A linear upward shift

The case $r < 1$ is similar and gives:

$$u(x) = H(x_0) + \int_{x_0}^{x} H'(s + \psi(s))\, ds \tag{8.19}$$

$$v(y) = H(x_0 + b) - H(x_0) + \theta + \int_{b}^{y} H'(\phi(t) + t)\, dt \tag{8.20}$$

Linear shifts of distributions

We now introduce an additional assumption that considerably simplifies the analysis. Specifically, we assume that (i) there are as many men as women ($r = 1$), and (ii) that men's income distribution is a linear upward shift of the income distribution of women. That is,

$$F(t) = G(\alpha t - \beta) \quad \text{for all } t \tag{8.21}$$

for some $\alpha < 1, \beta > 0$. An illustration is provided in Figure 8.3.

This condition is satisfied, for instance, if the income distributions of both men and women are lognormally distributed with parameters (μ_M, σ_M) for males and (μ_F, σ_F) for females, under the condition that $\sigma_M = \sigma_F$ - a form that fits existing data pretty well.[5]

[5] Alternatively, the property is also satisfied if the two income distributions are uniform and the support of the male distribution is $[a, A]$ while the support of the female distribution is $[b, B]$; then $b = \alpha a - \beta$ and $B = \alpha A - \beta$.

The linear shift property implies that, under assortative matching and with populations of equal size, a man with income x is paired with a woman with income $y = \alpha x - \beta$. With the previous notations, therefore, $\phi(y) = (y+\beta)/\alpha$ and $\psi(x) = \alpha x - \beta$. Equations (8.15) and (8.16) then become

$$\frac{du(x)}{dx} = H'((\alpha+1)x - \beta), \tag{8.22}$$

and

$$\frac{dv(y)}{dy} = H'(((\alpha+1)y + \beta)/\alpha), \tag{8.23}$$

yielding upon integration :

$$v(y) = K + \frac{\alpha}{1+\alpha} H(\phi(y) + y) \tag{8.24}$$

and

$$u(x) = K' + \frac{1}{1+\alpha} H(x + \psi(x)) \tag{8.25}$$

where

$$K + K' = \theta.$$

In words, the marriage between Mr. x and Mrs. $y = \psi(x)$ generates a marital output $\theta + H(x + \psi(x))$, which is divided linearly between the spouses. The non monetary part, θ, is distributed between them (he receives K, she receives K') in a way that is not determined by the equilibrium conditions (this is the standard indeterminacy when $r = 1$) but must be the same for all couples (note that K or K' may be negative). Regarding the economic output, however, the allocation rule is particularly simple; he receives some constant share $\alpha/(1+\alpha)$ of it, and she gets the remaining $1/(1+\alpha)$.

8.2.3 Comparative Statics

We now turn to examine the impact of changes in the sex ratio and income distribution.

Increasing the proportion of women

We begin by noting an important feature of the model, namely that *if all marriages yield a strictly positive surplus then the allocation rule has a discontinuity at* $r = 1$. Indeed, examining the expressions in (8.17) and (8.18) we see that if r approaches 1 from above we get in the limit

$$u(x) = H(a+b) - H(b) + \theta + \int_a^x H'(s + \psi(s))\, ds \tag{8.26}$$

$$v(y) = H(b) + \int_b^y H'(\phi(t) + t)\, dt \tag{8.27}$$

while if r approaches 1 from below we get in the limit,

$$u(x) = H(a) + \int_a^x H'(s + \psi(s))\, ds \tag{8.28}$$

$$v(y) = H(a+b) - H(a) + \theta + \int_b^y H'(\phi(t) + t)\, dt \tag{8.29}$$

The marital *surplus* generated by the marriage of lowest income couple, here (a,b), is equal to $H(a+b) - H(a) - H(b) + \theta$. When the two sexes are almost equal in number, a small change in the sex ratio shifts all the surplus to one of the partners in the lowest quality match, the one whose sex is in the minority, and this discontinuity is then transmitted up the matching profile to all participants in the marriage market.[6] This knife-edge property is an undesirable property of the simple model without friction. One can get rid of it either by assuming no rents for couples at the bottom of the distribution, or by limiting our attention to marginal changes in the ranges $r > 1$ or $r < 1$ which do not reverse the sign of these inequalities.

Consider, now, such a change - that is, a *marginal* increase in the proportion of women r that maintains either $r > 1$ or $r < 1$ and assume that the shape of of income distributions of both men and women remain unchanged. From the matching rule $1 - F(x) = r(1 - G(y))$, we see that, as a consequence of such change, any married man with a given income x will now be matched with a woman with a higher y and each woman with a given y, is now matched to a man with a lower x. That is, the matching function $\psi(x)$ shifts upwards and the matching function $\phi(y)$ moves downwards. As we move along a stable assignment profile, the utility of all married men grows with their own income at a higher rate, because $h_x(x, \psi(x)) = H'(x + \psi(x))$ is higher for all x and the utility of all married women grows at slower rate because $h_y(\phi(y), y) = H'(\phi(y) + y)$ is lower for all y. It then follows from (8.15) and (8.16) that the utility of all married men rises and the utility of all married women declines; those who remain single are unaffected. Assuming, for instance, that $r > 1$, we have:

$$\begin{aligned} \phi(y,r) &= \Phi\left[1 - r(1 - G(y))\right] \\ \psi(x,r) &= \Psi\left[1 - \frac{1}{r}(1 - F(x))\right] \\ y_0 &= \Psi(1 - 1/r), \end{aligned}$$

[6] The result that g is the only source of gain from marriage for couples at the bottom of the income distribution reflects the assumptions that $h(0,0) = 0$ and that there is a positive density of the income distribution at zero. In general, participants at the bottom of the income distribution have a positive income, so that the lowest quality match may create a monetary surplus, because of the positive interaction of traits.

and

$$
\begin{aligned}
\frac{\partial \phi (y,r)}{\partial r} &= -\left(1-G(y)\,\Phi'\left[1-r\left(1-G\left(y\right)\right)\right]<0\right. \\
\frac{\partial \psi (x,r)}{\partial r} &= \frac{1}{r^2}\left(1-F\left(x\right)\right)\Psi'\left[1-\frac{1}{r}\left(1-F\left(x\right)\right)\right]>0 \\
\frac{\partial y_0}{\partial r} &= \frac{1}{r^2}\Psi'\left(1-1/r\right)>0.
\end{aligned}
$$

Differentiating (8.17) and (8.18) with respect to r therefore gives:

$$
\begin{aligned}
\frac{\partial u\left(x\right)}{\partial r} &= \left(H'\left(a+y_0\right)-H'\left(y_0\right)\right)\frac{\partial y_0}{\partial r} \\
+\int_a^x H''\left(s+\psi\left(s\right)\right)\frac{\partial \psi\left(s,r\right)}{\partial r}ds &> 0 \\
\frac{\partial v\left(y\right)}{\partial r} &= \left(H'\left(y_0\right)-H^{`}\left(y_0+a\right)\right)\frac{\partial y_0}{\partial r} \\
+\int_{y_0}^y H''\left(\phi\left(t\right)+t\right)\frac{\partial \phi\left(t,r\right)}{\partial r}dt &< 0
\end{aligned}
$$

The case $r<1$ is similar and left to the reader.

We conclude:

A marginal increase in the proportion of women to men in the marriage market, improves (or leaves unchanged) the welfare of all men and reduces (or leaves unchanged) the welfare of all women; the impact is stronger for higher income households.

An important implication of this property is that for any couple, the sex ratio can be used as a *distribution factor* : its variations affect the intra-household allocation of resources without changing neither total income nor the spouses' preferences. The empirical relevance of this remark has been established empirically by several authors. For instance, Chiappori, Fortin and Lacroix (2002), using a collective model of labor supply, find that, other things equal, a one percentage point increase in the sex ratio (defined as the ratio of men to women in the relevant marriage market) induces husbands to transfer some $2,000$ dollars (1988) of income to their spouse (see Chapter 5).

Shifting female income upward

Recalling our assumption that men have the higher income in the sense that their distribution dominates in the first degree the income distribution of women, that is, $F(t)<G(t)$ for $t\in(0,1)$, we now consider a *first degree upward shift* in the distribution of female income, holding the male distribution constant. That is, the proportion of females with incomes

exceeding y rises for all y, so that women become more similar to men in terms of their income, as we observe in practice. Such an upward first order shift in the distribution of female income affects the matching functions in exactly the same way as a marginal increase in the female/male sex ratio. Thus, if all men maintain their income, they all become better off. Similarly, any woman who would maintain her income would become worse off. This remark should however be interpreted with care, because it is obviously impossible for *all* women to maintain their income: when the distribution of female incomes shifts to the right, some (and possibly all) females must have higher income. In particular, those women who maintain their relative rank (quantile) in the distribution will maintain their position in the competition for men, and will be matched with a husband with the same income as before. Such women will be better off, as a consequence of the increase in their own income.

As a special case, consider the linear shift case described above; to keep things simple, assume moreover that $\beta = 0$. Suppose, now, that the income of every woman is inflated by some common factor $k > 1$ and consider a married couple with initial incomes (x, y). After the shift, the partners remain married but the wife's income is boosted to ky while the husband's income remains equal to x. If u_k and v_k denote the new individual utilities, we have from (8.25) and (8.24):

$$\begin{aligned} v_k &= K + \frac{k\alpha}{k\alpha + 1} H\left(ky + x\right) \quad \text{and} \\ u_k &= K' + \frac{1}{k\alpha + 1} H\left(ky + x\right) \end{aligned} \tag{8.30}$$

Differentiating in k around $k = 1$ gives:

$$\begin{aligned} \frac{\partial v_k}{\partial k} &= \frac{\alpha}{\left(\alpha + 1\right)^2} H\left(y + x\right) + \frac{\alpha y}{\alpha + 1} H'\left(y + x\right) \quad \text{and} \\ \frac{\partial u_k}{\partial k} &= -\frac{\alpha}{\left(\alpha + 1\right)^2} H\left(y + x\right) + \frac{y}{\alpha + 1} H'\left(y + x\right) \end{aligned} \tag{8.31}$$

One can readily check that both changes are positive (for the second one, it stems from the convexity of H). We conclude that the shift has two impacts. First, the increase in total income generates some additional surplus (the term in $yH'\left(y + x\right)$), which is shared between spouse in proportion of their respective incomes (that is 1 and α). In addition, a redistribution is triggered by the shift. Specifically, since the wife's share of total income is increased, so is her consumption; the husband therefore transfers to his wife an amount equal to a fraction $\alpha / \left(\alpha + 1\right)^2$ of total surplus. One can readily check that the transfer is proportionally larger for wealthier couples, since the ratio $H\left(y + x\right) / \left(y + x\right)$ increases with $\left(y + x\right)$ due to the convexity of H.

Empirical illustration

It is *a priori* not clear how important is income for matching and how to measure it. Actual incomes are rarely available, and wages are measured with a lot of noise and vary over the life cycle. For an empirical application, we estimate the *predicted hourly wage* of white men and women aged 25-40 in the CPS data and use these predictions as measures of the male and female incomes for this age group.[7] We then obtain the following results (see Figure 8.11):

1. The log normal distribution fits these predictions well.

2. The standard deviations for men's and women's predicted log wages are *similar* and both grow over time,

3. The mean predicted log wages of men are higher than for women but the discrepancy declines over time.

4 Male distributions of predicted log wage dominate in the first degree the female distributions in all years but the gap declines over time.

5. Within couples, there is high positive correlation between the predicted log wages of husbands and wives and this correlation rises over time indicating a high and increasing degree of positive assortative mating.

6. Finally, the ratio of men to women in the CPS sample of whites aged $25-40$ has dropped from 1.045 in 1976 to 0.984 in 2005.

Figure 8.12 shows the male and female income distributions estimated from the CPS data. As seen the cumulative distribution of male incomes is below the cumulative distribution of female in both years but the gap is lower in 2005, indicating a first degree dominance of the male distributions. For both man and women, the cumulative distributions are less steep in 2005, representing the general rise in inequality between 1976 and 2005.

We use this information, together with the assumption that the marital output is given by $h(x,y)=\frac{(x+y)^2}{4}$ (so that the marital surplus from marriage is $\frac{yx}{2}$), to calculate the predicted response of the shares in marital surplus to the observed changes in the male and female income distributions and in the sex ratio between the years 1976-2005. The use of log normal distribution and the specification of $h(x,y)$ allows us to use conditions (8.21) and (8.14) and to calculate the shares using numerical integrations of (8.15) and (8.16). Figures 8.13 and 8.14 show the estimated shares in the marital surplus for men and women in 1976 and 2005. We see that men had a larger estimated share in 1976, while women had the larger share in 2005. Part

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[7] These results are obtained by running regressions with every year for white men and women aged $25-40$ of wages on schooling experience and occupation, excluding self employed. We use up to 53 occupation dummies, which allows for a large variance given schooling and age and also captures a more permanent feature of wages because an occupation tend to relatively stable over the life cycle. For men and women who reported no occupation, we imputed the mode occupation in their schooling group. For men who did not work, we imputed wages conditioned on working and for women we also corrected for selection using the Heckman technique.

of this reversal is due to the narrowing wage gap between men and women and part of it is due to the reduction in the female-male sex ratio over the period.

8.2.4 Taxation

Changes in the income distribution can also arise from a government intervention in the form of taxes and subsidies. For instance, we may consider a linear transfer scheme, such that the after tax (subsidy) income of a person with income s is $\kappa + (1-\tau)\, s$, with $\kappa > 0$ and $0 < \tau < 1$. Let us assume that the scheme is revenue neutral, so that its only impact is to redistribute income between and within couples, and let us for the time being disregard behavioral responses to the tax changes. We have that:

$$\begin{aligned}&\int_0^1 xF(x)\,dx + r\int_0^1 yG(y)\,dy \\ =\;&\int_0^1 (\kappa + (1-\tau)\,x)\,F(x)\,dx + r\int_0^1 (\kappa + (1-\tau)\,y)\,G(y)\,dy \end{aligned} \tag{8.32}$$

and

$$\kappa = \tau \frac{\bar{x} + r\bar{y}}{1+r} \tag{8.33}$$

when $\bar{x} = \int_0^1 xF(x)\,dx$, $\bar{y} = \int_0^1 yG(y)\,dy$ denote average incomes of male and females, respectively, so that $\frac{\bar{x}+r\bar{y}}{1+r}$ is average household income. Here, τ is the taxation rate, and κ is the lump sum subsidy funded by income taxation.

We can think of such an intervention as a change in the household production function from $h(x,y)$ to $\tilde{h}(x,y) = h(\kappa + (1-\tau)\,x, \kappa + (1-\tau)\,y)$. Such a transformation preserves the sign of the cross derivative with respect to the before tax incomes x and y. Therefore, the same pattern of a positive assortative mating is maintained and the matching functions $\psi(x)$ and $\phi(y)$ remain the same. However, the introduction of tax and transfer influences the gains from marriage, which depend on the *after* tax incomes of the partners, and the division of these gains. By construction, a progressive transfer-tax system raises the income of the poor and reduces the income of the rich. Due to positive assortative matching, the progressivity of the program is magnified, because an individual whose after tax income has increased (decreased) is typically assigned to a spouse whose after tax income has increased (decreased). Put differently, the intervention affects the surplus generated by marriage, holding the pre tax incomes fixed. For low income matches, the surplus increases and for high income matches it declines. In addition, the division of the surplus between husbands and wives is affected in general.

When assumption (8.14) holds and only total family income matters, the household production function is modified from $h(x,y) = H(x+y)$ to

$\tilde{h}(x+y) \equiv \tilde{H}(2\kappa + (1-\tau)(x+y))$. Assume, in addition, that male and female income distributions satisfy condition (8.21) so that $\psi(x) = \alpha x - \beta$ and $\phi(y) = \frac{y+\beta}{\alpha}$. Then, for a larger female population ($r > 1$), utilities become:

$$
\begin{aligned}
\tilde{v}(y) &= H(\kappa + (1-\tau) y_0) + \\
&\quad (1-\tau) \int_0^{y+\phi(y)} H'(\kappa + (1-\tau) s) \frac{\alpha ds}{\alpha+1} \\
\tilde{u}(x) &= \theta + (1-\tau) \int_0^{\psi(x)+x} H'(\kappa + (1-\tau) s) \frac{ds}{\alpha+1}
\end{aligned} \tag{8.34}
$$

with $k = \tau \frac{\bar{x}+r\bar{y}}{1+r}$. The impact of a change in the marginal income tax, t, on the utilities of women and men, respectively, is

$$
\begin{aligned}
\frac{\partial \tilde{v}(y)}{\partial t} &= \left(\frac{x+ry}{1+r} - y_0 \right) H'(\alpha + \beta y_0) - \alpha D(x+y) \\
\frac{\partial \tilde{u}(x)}{\partial t} &= -D(x+y),
\end{aligned} \tag{8.35}
$$

where Y denotes total family income and

$$
\begin{aligned}
D(Y) &= \int_0^Y H'(\kappa + (1-\tau) s) \frac{ds}{\alpha+1} \\
&\quad + (1-\tau) \int_0^Y \left(s - \frac{x+ry}{1+r} \right) H''(\kappa + (1-\tau) s) \frac{ds}{\alpha+1}
\end{aligned} \tag{8.36}
$$

The term $D(Y)$ is typically positive for richer households (who therefore lose from the introduction of the tax/benefit system) and negative for poorer ones. In this simple context, the corresponding gain or loss is allocated between husband and wife in respective proportions 1 and α. In addition, since single women are at the bottom of the female income distribution, their utility is typically increased by the tax/benefit scheme (this is the case whenever their income is below the mean). Equilibrium then requires the gain of the marginal woman (that is, of the wealthiest single or poorest married woman) to be forwarded to *all* women in the distribution; hence the term $\left(\frac{x+ry}{1+r} - y_0 \right) H'(\alpha + \beta y_0)$ in equation (8.35) representing the gain of the marginal woman. Note that this boost in income does not go to the poorer spouse, but to the spouse whose population is in excess supply. Should males outnumber females ($r < 1$), they would receive the corresponding benefit. The precise impact of these changes is hard to evaluate in general and we therefore turn to a specific example.

8.2.5 An example

We now provide a simple example in which the shares can be easily calculated. In addition to (8.21) we assume that incomes are uniformly distributed. We use again our example in Chapter 2 with public goods where $h(y,x) = \frac{(y+x)^2}{4}$, which satisfies (8.14). For this example, men and women have the same marginal contribution to marriage, $h_x(y,x) = h_y(y,x) = \frac{y+x}{2}$. Assume that the incomes of men and women are uniformly distributed on $[0,1]$ and $[0,Z]$, respectively, where $Z \leq 1$. If $Z < 1$, then the income distribution of men dominates in a first degree the income distribution of women, because

$$G(t) = \begin{cases} \frac{t}{Z} & if \quad 0 \leq t \leq Z \\ 1 & if \quad Z < t \leq 1 \end{cases} \tag{8.37}$$

exceeds $F(t) = t$, for all t in the interval $(0,1)$. We are also in the 'linear upward shift' case described above, with $\alpha = Z$ and $\beta = 0$.To simplify further, we set $\theta = 0$ so that the lowest quality matches generate no surplus. Therefore, there is no indeterminacy of the allocation rule when $r = 1$ and no discontinuity in the allocation rule.

Under the assumed uniform distributions, the assignment functions are linear and given by

$$x = \phi(y) = 1 - r(1 - \frac{y}{Z}), \tag{8.38}$$

$$y = \psi(x) = \frac{Z}{r}[(r-1) + x]. \tag{8.39}$$

and the local scarcity of men is *constant* and given by $\frac{Z}{r}$. Under the simplifying assumption that $\theta = 0$, the shares of the husband and wife in the marital output can then be rewritten in the form

$$\begin{aligned} v(y) &= \frac{y^2}{4} + \frac{1}{2}\int_{y_0}^{y}[1 - r(1 - \frac{t}{Z})]dt, \\ u(x) &= \frac{x^2}{4} + \frac{1}{2}\int_{x_0}^{x}\frac{Z}{r}[(r-1) + s]ds, \\ y_0 &= \begin{cases} \frac{Z}{r}(r-1) & if \quad r > 1 \\ 0 & if \quad r \leq 1 \end{cases}, \\ x_0 &= \begin{cases} 1 - r & if \quad r < 1 \\ 0 & if \quad r \geq 1 \end{cases}. \end{aligned} \tag{8.40}$$

Notice that $v(y) - \frac{y^2}{4}$ and $u(x) - \frac{x^2}{4}$ are the shares of the husband and wife in the marital *surplus*. Inspecting the integrals in (8.40), we see that the gender in short supply always receives a larger share of the surplus. In contrast, the shares of marital output of husbands and wives depend also on the location of the couple in the income distribution.

If there are more women than men, $r > 1$, the match with the lowest output is the one in which the husband has income $x = 0$, and the wife has income $y_0 = Z\frac{(r-1)}{r}$. His surplus and utility are at this point zero, while she receives the whole marital output $\frac{y_0^2}{4}$, which also equals her utility as single. Because men are always locally scarce, $\frac{r}{Z} > 1$, it follows from (8.40) that their utility must grow along the stable assignment at a faster rate than the utility of their assigned wives. It is readily seen that the husband's share is higher in matches with sufficiently high income. In particular, the best match with $x = 1$ and $y = Z$, yields an output of $\frac{(1+Z)^2}{4}$, of which the husband receives $\frac{1}{4} + \frac{Z}{2} - \frac{Z}{4r}$ and the wife receives $\frac{Z^2}{4} + \frac{Z}{4r}$, which is a smaller share.

If there are more men than women, $r < 1$, the match with the lowest output is the one in which the wife has income $y = 0$, and the husband's income is $x_0 = 1 - r$, and it is now the wife that has the lower utility. The local scarcity parameter can now be higher or lower than 1. If $\frac{r}{Z} > 1$, men are always locally scarce, and it follows from (8.40) that the husband will have a higher share in the output of *all* marriages. If, however, $\frac{r}{Z} < 1$ and women are always locally scarce, then the utility of women grows along the stable assignment profile at a faster rate than the utility of their assigned husbands, and they may eventually overtake them. Indeed, the wife's share in the best match is $\frac{Z}{2} + \frac{Z^2}{4} - \frac{rZ}{4}$ and the husband's share is $\frac{1}{4} + \frac{rZ}{4}$, which is smaller if r is sufficiently small.

This example illustrates clearly the impact of changes in the sex ratio r and the distribution of female income as indexed by Z, on the welfare of women and men. Recall that marginal increases in x_0 or y_0 have *no* effect on $u(x)$ or $v(y)$, respectively. Inspection of the integrands in (8.40), shows that $u(x)$ must increase in r and Z, while $v(y)$ must decrease in r and Z. As we noted above, the result that women are worse off when the mean income of women rises sounds surprising. However, the reason that a woman who maintains her income is worse off when Z rises is that there are more women with income above her, which means that she cannot "afford" anymore a husband with the same income as before. However, any woman who keeps her position in the income distribution, (that is, whose income rose at the same proportion as Z) will obtain a husband with the same x as before the change. Then it can be shown that if $r > 1$, her surplus does not change, and if $r < 1$, her surplus rises.[8] In either case, her welfare must rise, reflecting the rise in her own income. This example can be easily generalized for the case in which there are positive non monetary gains, $\theta > 0$.

The example allows us to examine numerically the impact of a progressive transfer-tax system. Assume that male income is distributed uniformly on $[0, 1]$, while the female income is distributed uniformly on $[0, 75]$. Set $\theta = 0.025$ and $\tau = .7$. Now consider a balanced transfer scheme such that $\kappa(1+r) = (1-\tau)(x+ry)$. We discuss here two separate cases, one in which women are the majority and $r = 1.1$ and the other when women are the minority and $r = 0.9$. In the numerical example, $x = 0.5$ and $y = 0.375$. Thus, for a marginal tax of $\tau = 0.7$, the balanced budget constraint implies that $\kappa = 0.13$ when $r = 1.1$, and $\kappa = 0.132$. when $r = 0.9$.Figures 8.15, 8.16 and Table 8.1 summarize the results.

When women are in the majority, their share is usually less than half but rising in the income of their assigned husband (see Figure 8.15). The tax-subsidy intervention moderates this increase, because in low quality matches, the wife's share is determined by her income, and women with low income gain from the progressive system. When women are in the minority, their share in the marital output declines and the progressive tax system moderates this decline (see Figure 8.16) because in low quality matches, the husband's share is determined by his income, and men with low income gain from the progressive system. The difference in slopes between the two figures reflects the role of the non monetary gains, θ, that are captured by the men when $r > 1$ and by the women when $r < 1$. This effect weakens as one moves to high income couples where the monetary gains become increasingly important.

Table 8.1 provides the numerical values of the shares. In the benchmark: the income of men is uniform on $[0, 1]$, the income of women is uniform on $[0, .75]$, the gain from marriage is $g = 0.025$,the tax rate on income is $\beta = 0.7$ and the implied value of α that balances the budget is $\alpha = 0.1322$ at panel a and $\alpha = 0.1303$ at panel b. We then examine the equilibrium

[8] The surplus of the husband and the surplus of the wife are readily obtained by calculating the integrals in (8.40). For $r \geq 1$, we obtain

$$\begin{aligned} s_h(y) &= u(y) - \frac{y^2}{4} = \frac{Zy^2}{4r} + \frac{1}{2}\frac{Z}{r}(r-1)y, \\ s_w(\psi(y)) &= \frac{\psi(y)\,y}{2} - s_h(y) = \frac{Zy^2}{4r}. \end{aligned}$$

For $r \leq 1$, we obtain

$$\begin{aligned} s_w(z) &= v(z) - \frac{z^2}{4} = \frac{rz^2}{4Z} + \frac{z(1-r)}{2}, \\ s_h(\phi(z)) &= \frac{\phi(z)\,z}{2} - s_w(z) = \frac{rz^2}{4Z}. \end{aligned}$$

shares for some hypothetical couples. Panel a describes the case with more women than men, $r = 1.1$. Then, all men marry and a proportion 0.9091 of the women remains single. The man with the lowest income, 0, is matched with a woman whose income is 0.0682, the man with the mean income, 0.5, is matched with a woman whose income is 0.4091, and the man with the highest income, 1, is matched to the woman with highest income, 0.75. Following the intervention; the after tax income of the man with lowest income rises to 0.1304, and that of his matched wife rises to 0.1781, the after tax income of the average man is reduced to 0.4804 and that of his matched wife rises to 0.3947, while the after tax of the wealthiest man is reduced to 0.8304 and that of his matched wife is reduced to 0.6554. Thus, the tax and transfers scheme reduces inequality both between and within couples.

Although the impact of the intervention on the couples with the average man or average woman is relatively small, some noticeable changes occur at the bottom and the top of the income distribution. At the bottom, the intervention raises the utilities of both men and women but women obtain a larger share of the total utility if $r > 1$ and a *smaller* share if $r < 1$. It seems surprising that a progressive policy that transfers resources to poor women reduces their share in the marital surplus. But when $r < 1$, poor women are married to men who are wealthier than they are, and the intervention makes these men less "useful" to their wives. At the top of the distribution, the intervention lowers substantially the utilities of both men and women but women gain relatively more than men if $r < 1$ and relatively less if $r > 1$. We see that the impact of the tax-subsidy intervention on each spouse reflects three different effects: an increase (decrease) in own income, an increase (decrease) in the spouse's income, and the increase in the incomes of the individuals who are just indifferent between marriage and singlehood. The first two effects influence the marital output that the matched partners can generate together. The third effect reflects the changes in the sharing of this output that are caused by the competition in the marriage market. In order to separate these effects, we examine the impact of the tax for couples for which the intervention does not affect *total* family income, and, therefore, marital output does not change. This comparison is shown in panels c and d of Table 8.1. We see that in both panels the wife gains income relative to the husband. However, when women are in the majority, the wife in such couples loses both in output and surplus terms. In contrast, the wife gains if women are in the minority. This difference can be traced to the impact of the intervention on the lowest quality matches, where the intervention causes a larger gain to the wife than to the husband when women are in the minority, $r < 1$, while the opposite is true when $r > 1$ (see panels a and b). These effects are transmitted along the matching profile to all couples in the marriage market.

The general conclusion that one can draw from these examples is that in a frictionless market, where the shares are determined jointly with the assignments, the simple intuition based on bargaining between two isolated partners fails. For instance, we see in panel *c* that, although family income remains fixed and the wife's share in the total income rises, she ends up with lower share of marital output. In other words, the allocation rule that determines the wife's and husband's utility in a particular marriage, reflects the traits of *all* participants in the marriage markets and, therefore, a change in the income distribution in the economy (society) at large can change the shares within specific marriages in a way that would not be directly predictable from the change in the within-household income distribution.

Table 8.1: Sharing of Marital Output and Surplus

Panel a: Women are the Majority, $r = 1.1$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0	0.0682	no	0.025	0.0012	0.025	0	0.0444	0
0.1304	0.1781	yes	0.0409	0.0079	0.0366	0	0.1625	0
0.5	0.4091	no	0.1472	0.0845	0.0847	0.0426	0.3646	0.3348
0.4804	0.4167	yes	0.1463	0.0798	0.0887	0.0364	0.353	0.2913
1	0.75	no	0.4795	0.3111	0.2295	0.1705	0.3935	0.4261
0.8304	0.6554	yes	0.3548	0.222	0.1825	0.1146	0.3849	0.3858

Panel b: Men are the Majority, $r = 0.9$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0.1	0	no	0.0025	0.025	0	0.025	0.9091	1
0.2022	0.1322	yes	0.0102	0.0427	0	0.0384	0.807	1
0.55	0.375	no	0.1178	0.1211	0.0422	0.0859	0.5069	0.6707
0.5172	0.3947	yes	0.1084	0.1245	0.0415	0.0856	0.5347	0.6735
1	0.75	no	0.4188	0.3719	0.1688	0.2313	0.4704	0.5781
0.8322	0.6572	yes	0.2975	0.2821	0.1243	0.1741	0.4867	0.5834

Panel c: Women are the Majority, $r = 1.1$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0.4762	0.3929	no	0.1366	0.0772	0.0799	0.0387	0.3612	0.3261
0.4637	0.4054	yes	0.139	0.0748	0.0852	0.0338	0.35	0.2837

Panel d: Men are the Majority, $r = 0.9$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0.5264	0.3553	no	0.1072	0.1122	0.0379	0.0806	0.5115	0.6804
0.5007	0.381	yes	0.101	0.1184	0.0383	0.0821	0.5396	0.6819

8.2.6 *Matching on preferences: Roe vs. Wade and female empowerment*

In the matching models presented so far, income is the trait on which people match. But other determinants can also be considered. In a recent paper, Chiappori and Oreffice (2007) use a matching model to analyze the impact of the legalization of abortion on power allocation within couples.[9] In their framework, people differ in their preferences towards children; the corresponding matching patterns - and the resulting allocation of resources - can be studied before and after legalization.

That the legalization of abortion should alter the balance of powers within couples is not surprising;[10] indeed, Oreffice (2007) has provided an empirical study based on the collective approach to household behavior, that confirms the 'empowerment' consequences of Roe vs.. Wade. Still, the *mechanism* by which this empowerment occurs deserves some scrutiny. While it is not hard to convince oneself that *some* women (for example, career-oriented women with little taste for family life) will gain from legalization, whether *all* women will is another matter. A strong objection is that women have heterogeneous preferences for fertility (or different attitudes toward abortion); some do not consider abortion as an option, either for religious and ethical reasons or because they do want children. Whether the legalization will benefit these women as well is not clear. From an economist's perspective, moreover, the new context will affect the matching process on the market for marriage, and in particular the way the surplus generated by marriage is shared between spouses. In principle, such 'general equilibrium' effects could annihilate or even reverse the direct impact of the reform, particularly for these women who are unlikely to derive much direct benefit from it.

Preferences

To investigate these issues, Chiappori and Oreffice consider a model in which a continuum of men and women derive utility from one private composite good c (the price of which is normalized to 1) and from children; Let the dummy variable k denote the presence ($k = 1$) or the absence ($k = 0$) of children in the household. Men have identical, quasi-linear preferences over consumption and children. The utility of single men only depends on their consumption; that is, men cannot derive utility from (and do not share the

[9] The version presented here is a slightly simplified version of the original paper; in particular, we assume here that men have identical preferences, and concentrate on preference heterogeneity among women.

[10] See for instance Héritier (2002).

costs of) out-of-wedlock children, due to the fact that they do not live in the same household. On the other hand, married men's utility is of the form $U_H(c_H, k) = c_H + u_H k$, where the parameter $u_H > 0$ is identical for all men in the economy. Women differ in their preferences toward children. Specifically, female utility functions take the quasi-linear form $U(c, k) = c + uk$. Here, each woman is characterized by the individual-specific taste parameter u, which is distributed according to the density f over the interval $[0, U]$. We assume that any woman (single or married) who wants a child can have one. However, if she plans to have no children, unwanted births may still occur with some probability p, which depends on the available contraceptive technology and the legality of abortion.

The quasilinear structure of the male and female preferences implies that utility is transferable within marriage. For each spouse, the utility depends on the couple's fertility decision and on the share of composite good that he or she receives.

As before, we normalize the mass of men to be 1, and we denote by r the total mass of women on the market; here, we assume that $r > 1$, that is that women are on the long side of the market. Male income is denoted by Y. Women without children have income, y; however, if a woman has children, her income drops to y', with $y' < y$, reflecting both the loss in her earning capacity due to childbearing and the cost of raising the child. Hence a single woman without children consumes her income y; if she decides to have a child (or if an unwanted pregnancy occurs), she also consumes her income (which has dropped to y') and receives a utility u from her child, which is independent of her marital status.

Regarding couples, we assume that $u_H < y - y'$, that is that the gain received by the husband from having a child does not offset by itself the loss in income experienced by the wife. This assumption implies, in our framework, that the couple's decision to have a child or not will also depend on the wife's preferences. Therefore married women must agree with their husband on two issues. One is the fertility decision; that is, they must decide whether to have kids or not, and the decision depends (in particular) on the wife's preferences towards children. The other decision relates to the distribution of resources within the household (that is, the allocation of total income between male and female consumption of the composite good). Both decisions will be ultimately determined by the equilibrium on the market for marriage. Finally, we model the legalization of abortion (and generally the availability of some birth control technology) as an exogenous decrease in the probability p of experiencing an unwanted pregnancy.

Fertility decisions

We first consider the fertility decisions of singles and couples, starting with single individuals. Single men do not make decisions: they consume their income, and get a utility which equals to Y. Single women, on the other hand, will decide to have children if and only if the benefit compensates the income loss, that is if $u \geq y - y'$, leading to a utility which equals $y' + u$. In the alternative case when $u < y - y'$, single the women chooses not to have a child and any pregnancy will be involuntary. As pregnancy occurs with probability p, the expected utility is $y(1-p) + p\,(y' + u)$. In what follows, the threshold $y - y'$ is denoted $\bar{u}$; women whose utility parameter is larger than or equal to $\bar{u}$ will be referred to as 'high' type.

In our transferable utility context, couples maximize their marital surplus. The total benefit, for a couple, of having a child is $u_H + u$, whereas the cost is $y - y'$. It follows that a married couple will plan to have a child if $u \geq y - y' - u_H$ - then total utility is $Y + y' + u_H + u$. The threshold $y - y' - u_H$ is denoted $\underline{u}$; note that $\underline{u} < \bar{u}$. If $u < y - y' - u_H$, only unwanted kids are born, leading to an expected total utility $Y + (1-p)\,y + p\,(y' + u_H + u)$. Women with taste parameter u smaller than $\underline{u}$ will be said to be of 'low' type, while those between $\underline{u}$ and $\bar{u}$ will be called 'intermediate'. To summarize:

- women of 'high' type ($u \geq \bar{u}$) always choose to have a child
- women of 'intermediate' type ($\underline{u} < u < \bar{u}$) choose to have a child only when married
- women of 'low' type ($u \leq \underline{u}$) never choose to have a child

Stable match

We can now derive the properties of the stable match. The key element is provided by Figure 8.4, which plots the maximum utility $\Phi\,(u)$ a man can achieve when marrying a woman of taste u (in other words, $\Phi\,(u)$ denotes his utility if he was to appropriate *all* the surplus generated by marriage). The function Φ is increasing; that is, it is always better (for the husband) to marry a wife with a larger taste coefficient u.

More precisely, women whose parameter u is greater than $\bar{u}$ (the 'high' type), and who would plan to have a child even when single, are the most 'attractive' from the male's perspective. While they differ in taste, this difference is irrelevant from a husband's viewpoint, since they require the same compensation c_H for getting married (namely, to be left with a private consumption which equals their income with a child, y'). Women between $\underline{u}$ and $\bar{u}$ (the 'intermediate' type) come next in males' preferences. They plan to have a child only when married, and the minimum compensation they require is $c_I\,(u) = (y-u)\,(1-p) + py'$. This required compensation decreases with the individual utility u; hence men strictly prefer intermediate women with a higher u. Finally, women with a u smaller than $\underline{u}$ (the 'low' type) never plan to have a child. Again, these women are equivalent from a husband's perspective, since they require the same compensation for getting married, namely their consumption as single, that is $c_L = (1-p)\,y + py'$.

As often in matching models, the properties of the stable match crucially depend on the identity of the marginal spouse (that is, the 'last' married woman). We denote by $u(r)$ the taste parameter of the marginal women (that is, either the 'last' single woman or the 'first' married woman). Technically, $u(r)$ is defined by the fact that the measure of the set of women with a taste parameter larger than $u(r)$ equals the measure of men, which is 1; that is, the value $u(r)$ solves the equation

$$r\int_{u(r)}^{U} f(t)\,dt = 1. \tag{8.41}$$

Competition between women in the marriage market implies that women who generate a larger surplus for their husband are a more desirable match. Hence, whenever a women belonging to the intermediate type is married, then all women with a larger taste parameter are married as well - this is the case depicted in Figure 8.4. The intuition is that women with a larger preference for children have a comparative advantage: the compensation they need from their husband to accept marriage is smaller, because they value highly the prospect of having a child. In general, the identity of this marginal woman depends on the location of $u(r)$ with respect to the two thresholds $\underline{u}$ and $\bar{u}$.

An obvious property of stable matches in this context is that *all males receive the same utility*; indeed, they are assumed identical, and the absence of friction implies that any difference of welfare between males would be competed away. Since the marginal woman is indifferent between being married or single, her husband gets all the surplus generated by the relationship, namely $\Phi(u(r))$. Then all other men receive the same utility. Graphically, this corresponds to the horizontal line going through $\Phi(u(r))$ in Figure 8.4.

A crucial insight, at this point, is the following. Take any woman with a taste parameter u larger than $u(r)$. Then *the difference* $\Phi(u) - \Phi(u(r))$ *represents the surplus received by this woman.*[11] In Figure 8.4, for instance, the surplus received by any woman of 'high' type is depicted by a bold arrow.

Using this geometric intuition, the characterization of the equilibrium is straightforward. Three cases should be distinguished:

- If $1/r \leq \underline{W} = \int_{\bar{u}}^{U} f(t)\,dt$, the excess supply of women is 'large', in the sense that there are less men than high type women. Then $u(r) \geq \bar{u}$, and the marginal married women belongs to the high type. Only (some of) these women are matched. Women of the same type who

[11] If her husband's utility was $\Phi(u)$ he would get all the surplus generated by the marriage. Since his equilibrium utility is only $\Phi(u(r))$, the difference $\Phi(u) - \Phi(u(r))$ represents the part of the surplus appropriated by the wife.

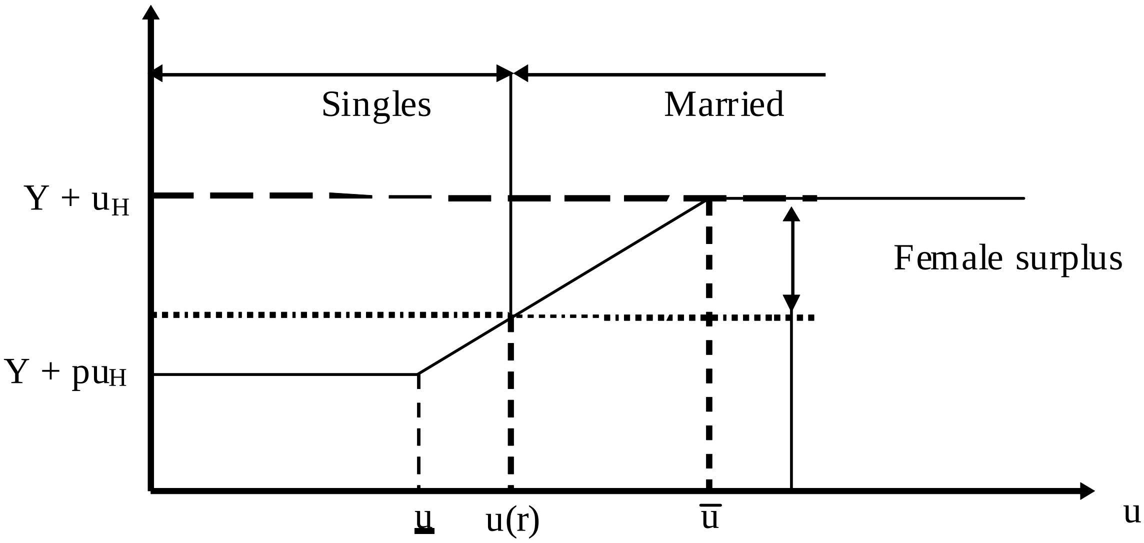

FIGURE 8.4. Maximum husband's utility as a function of the wife's taste - intermediate ESW

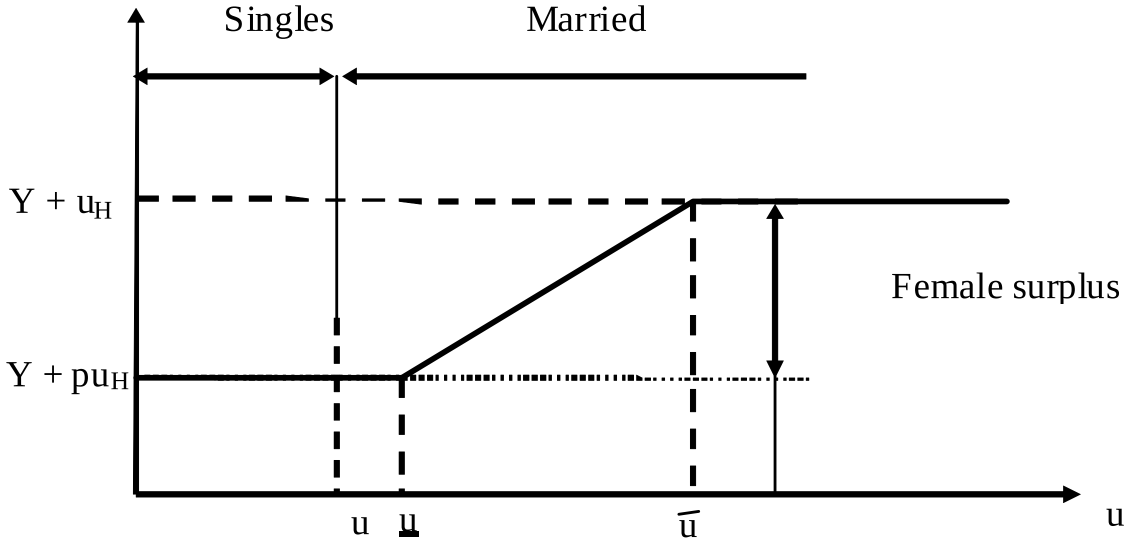

FIGURE 8.5. Maximum husband's utility as a function of the wife's taste - small ESW

remain single decide to have a kid; all other women remain single and decide not to have children (although they may have one involuntarily). Regarding welfare issues, note that, in that case, married women receive no surplus from marriage; their consumption is the same as if single.

- If $\underline{W} < 1/r < \bar{W} = \int_{\underline{u}}^{U} f(t)\,dt$, as depicted in Figure 8.4, the marginal wife belongs to the intermediate type. All married women have a child, and consume the same amount, which is such that the marginal wife is indifferent between getting married and remaining single. All married women (but the marginal one) get a positive surplus from marriage, and high type women receive the maximal surplus.

- Finally, when the excess supply of women is small enough (technically, $1/r \geq \bar{W}$), the marginal wife belongs to the low type (that is $u(r) \leq \underline{u}$ - see Figure 8.5). Her fertility is the same with and without marriage - namely, no planned child. Stability requires that her consumption is also the same, and equals to $(1-p)\,y + py'$. The same conclusion applies to all married, low type women. Other married women belong to the high or intermediate type, hence decide to have a child; their consumption is defined by the fact that men, who are in short supply, must be indifferent between the various potential spouses. Again, this condition generates a positive surplus for all women of high and intermediate types; high type women receive the largest surplus.

The variation in women's utility across the three types of equilibria exhibits interesting patterns. Not surprisingly, women are better off the smaller their excess supply on the market. However, when women's excess supply is either large or small, their welfare does not depend on the size of the imbalance. In the intermediate case, on the contrary, a marginal increase in the number of men continuously reduces the taste parameter $u(r)$ of the marginal woman, which ameliorates the welfare of all married women.

Changes in the birth control technology

We can now come to the main issue, namely the impact of a technological change in birth control that reduces the probability of unwanted pregnancies. A key assumption is that all women (including single) are given free access to the technology; a natural example could be the legalization of abortion that took place in the 1970's.

The situation is depicted in Figure 8.6 (which, for expositional convenience, considers the case in which the risk of unwanted pregnancies goes to zero). The new technology decreases the maximum utility attainable by husbands of low or intermediate type women, resulting in a downwards shift of the graph of the function Φ. This leads to the following conclusions:

Not surprisingly, women who do not want to have a child (either because they belong to the low type or because they are single) benefit from the technology, precisely because unwanted pregnancies become less likely. In the extreme situation in which unwanted pregnancies are eliminated, the monetary gain is thus $p(y - y' - u)$ .More interesting is the fact that women who decide to have a child also benefit from the technology, although to a lesser extent than singles. The intuition is that the intrahousehold distribution of resources is driven by the marginal women; for a small or intermediate excess supply of women, the marginal women is indifferent between getting married and remaining single *without kid*. Her reservation utility is thus improved by the new technology. The nature of a matching game, however, implies that any improvement of the marginal agent's situation must be transmitted to all agents 'above' the marginal one.

In the case of an intermediate excess supply depicted in Figure 8.6, the benefit experienced by all married women, assuming the new technologies drives the risk of unwanted pregnancies to zero, is $p(y - y' - u(r))$ (where, again, $u(r)$ denotes the taste parameter of the marginal married woman). This benefit continuously increases with the number of men M. When the excess supply is small, the gain is pu_H, still smaller than $p(y - y')$ (the

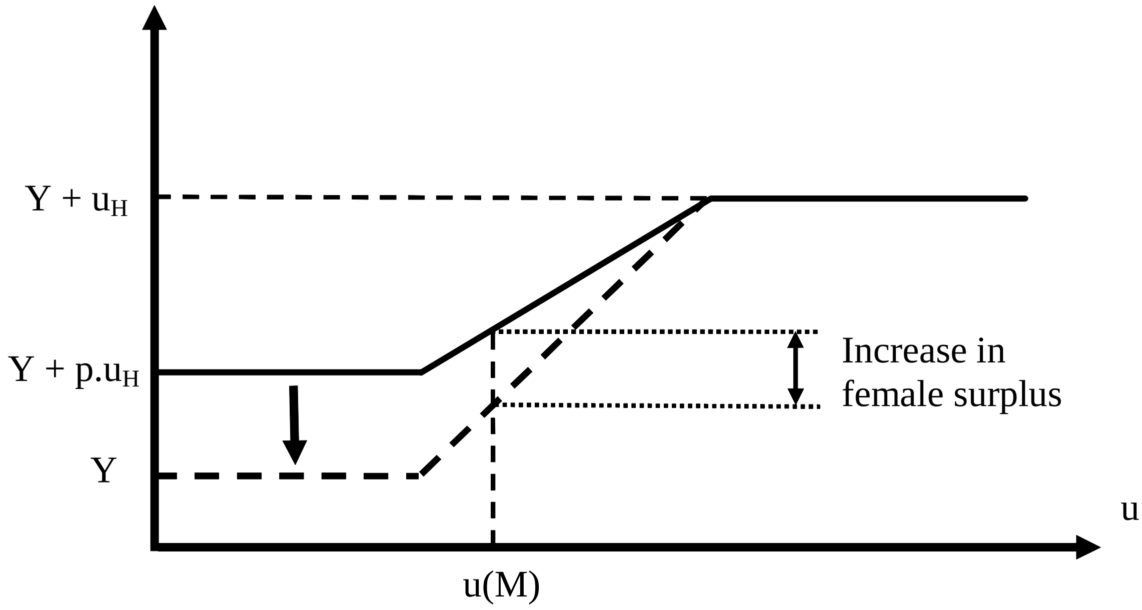

FIGURE 8.6. An Increase in the Risk of Pregnancy

gain for single women) but nevertheless positive. On the other hand, when the excess supply of women is 'large', married women do *not* benefit from the new technology, because the marginal woman does not use it. Hence the consequences of the new technology for married women's welfare are intimately related to the situation that prevails on the marriage market.

Finally, men cannot gain from the introduction of the new technology. When the excess supply of women is large, their utility is not affected. When the excess supply of women is small, so that the marginal wife does not want a child, the total welfare of the household is increased, but so is the reservation utility of the wife; the husband is left with the same consumption, but loses the benefit he would have received from an unwanted birth. The intermediate case is even more spectacular. Here, all marriages result in a child being born, so the total surplus generated by marriage is not affected by the innovation. What changes, however, is the intrahousehold allocation of the surplus. The new technology improves the reservation utility of the marginal woman, hence her share of resources increases. Stability requires this shift to be reproduced in all couples. All in all, the new technology results in a net transfer from the husband to the wife, which equals the expected gain of the marginal single woman, that is $p\,(y - y' - u\,(r))$, without any change on the fertility of married couple (who actually do not use the new technology).

We thus conclude that in our model an improvement in the birth control technology, such as the legalization of abortion, generally increases the welfare of *all* women, including those who want a child and are not interested in the new technology. Note, however, that the mechanism generating this gain is largely indirect. The reason why even married women willing to have a child benefit from the birth control technology is that the latter, by raising the reservation utility of single women, raises the 'price' of all women on the matching market (although this logic fails to apply in situations of 'large' excess supply of women).

An interesting, although somewhat paradoxical implication is that reserving the new technology to married women (as was initially the case for the pill, at least for younger women) would actually *reverse* the empowerment effect. The option of marriage to women with a low taste for children, who are willing to accept a lower compensation from the husband for getting married and gaining access to the new technology, toughens the competition for husbands. Therefore, women of the high or intermediate type are made worse off by the introduction of the new technology. Only women with a very low taste parameter (that is, below the lower marginal value) gain from the innovation. This comparison emphasizes the complex and partly paradoxical welfare impact of a new technology. On the one hand, its effects can go well beyond the individuals who actually use it, or even consider using it. Our model suggests that a major effect of legalizing abortion may have been a shift in the intrahousehold balance of powers and in the resulting allocation of resources, even (and perhaps especially) in couples who were not considering abortion as an option. On the other hand, the new technology benefits all married women only because it is available to singles. A technological improvement which is reserved to married women will have an impact on their fertility, partly because it changes the mechanisms governing selection into marriage. But its impact on women's welfare is largely negative, except for a small fraction of women who choose marriage as an access to the new technology.

8.3 Matching with general utilities

We now switch to the general framework in which we relax the assumption that utility is transferable. The tractability of the transferable utility framework comes at a cost. The most obvious drawback is that under TU, couples behave as singles; in particular, their demand function (that is, the amount spent on each of the public or private commodities) does not depend on the Pareto weights. In other words, changes in male and female income distributions may trigger a reallocation of resources (or more precisely of *one* commodity) between spouses, but under TU, it cannot have income effects, and cannot result in, say, more being spent on children health or education. While this framework may be useful in many contexts, it is clearly too restrictive in other situations.

In this section, we explore the more general framework introduced in Chapter 7, in which utility is not (linearly) transferable. That is, although compensations between spouses are still possible, they need not take place at a constant 'exchange rate': there is no commodity the marginal utility of which is always identical for the spouses. In particular, the matching model is no longer equivalent to a linear optimization problem. The upside is that, now, any change affecting the wife's and husband competitive positions (for example, a change in income distributions) will potentially affect *all* consumptions, including on public goods - which allows for a much richer set of conclusions. The downside is that the derivation of individual shares from the equilibrium or stability conditions is more complex. It remains feasible, however. We first present the general approach to the problem, then we concentrate on a specific and tractable example.

8.3.1 Recovering individual utilities: the general strategy

We use the same framework as in Chapter 7. Male income is denoted by x and female income is denoted by y; the Pareto frontier for a couple has the general form

$$u = H(x, y, v) \tag{8.42}$$

with $H(0, 0, v) = 0$ for all v. If a man with income x remains single, his utility is given by $H(x, 0, 0)$ and if a woman of income y remains single her utility is the solution to the equation $H(0, y, v) = 0$. By definition, $H(x, y, v)$ is decreasing in v; we assume that it is increasing in x and y, that is that a higher income, be it male's or female's, tends to expand the Pareto frontier. Also, we still consider a continuum of men, whose incomes x are distributed on $[0, 1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0, 1]$ according to some distribution G; let r denote the measure of women. Finally, we assume that an equilibrium matching exists and that it is assortative - that is, that the conditions derived in Chapter 7 are satisfied; let $\psi(x)$ (resp. $\phi(y)$) denote the spouse of Mr. x (of Mrs. y).

As previously, the basic remark is that stability requires:

$$u(x) = \max_y H(x, y, v(y))$$

where the maximum is actually reached for $y = \psi(x)$. First order conditions imply that

$$\frac{\partial H}{\partial y}(\phi(y), y, v(y)) + v'(y)\frac{\partial H}{\partial v}(\phi(y), y, v(y)) = 0.$$

or:

$$v'(y) = -\frac{\frac{\partial H}{\partial y}(\phi(y), y, v(y))}{\frac{\partial H}{\partial v}(\phi(y), y, v(y))}. \tag{8.43}$$

Again, we have a differential equation in v. It is more complex than in the TU case, because the right hand side depends on $v(y)$ in a potentially non linear way. Still, under mild regularity conditions, such an equation defines v up to a constant, the value of which can be determined from the condition that the last married person in the 'abundant' side of the market receives no surplus from marriage.

Note, in particular, that from the assumptions made in Chapter 7, we have that:

$$v'(y) = -\frac{\frac{\partial H}{\partial y}(\phi(y), y, v(y))}{\frac{\partial H}{\partial v}(\phi(y), y, v(y))} > 0 \tag{8.44}$$

In words, richer people are always better off. Finally, once v has been computed, the condition

$$u = H(x, \psi(x), v(\psi(x))) \tag{8.45}$$

exactly defines u.

This framework has been applied by Chiappori and Reny (2007), who consider a population of heterogeneous agents with different risk aversions matching to share risks arising from identically distributed random incomes. They show that (i) a stable match always exists, (ii) it is unique, and (iii) it is negative assortative: among married couples, men with lower risk aversion match with more risk averse women and conversely.

8.3.2 *A specific example*

We now present another application due to Chiappori (2009).

Preferences

There is a continuum of males, whose income y is distributed over $[a, A]$ according to some distribution F, and a continuum of females, whose income y' is distributed over $[b, B]$ according to some distribution G. To simplify, we consider the linear shift case, where the matching functions are given by $\phi(y') = (y' + \beta)/\alpha$ and $\psi(y) = \alpha y - \beta$; also, we assume that the number of female is almost equal to, but slightly larger than that of men.[12]

Males have identical preferences, represented by the Cobb-Douglas utility:

$$u_m = c_m Q \tag{8.46}$$

where c_m denotes his consumption of some private, Hicksian composite commodity and commodity Q is publicly consumed within the household;

[12] This last assumption is simply used to pin down the constant in the allocation of marital outcome; it can readily be modified as needed.

all prices are normalized to 1. Similarly, women all share the same preferences, characterized by some minimum level of consumption $\bar{c}$, beyond which private and public consumptions are perfect substitutes:

$$\begin{aligned} u_f(c_f) &= -\infty \text{ if } c_f < \bar{c} \\ &= c_f + Q \text{ if } c_f \geq \bar{c} \end{aligned}$$

In particular, if a woman is single, her income must be at least $\bar{c}$; then her utility equals her income.

An important feature here is that men and women have different preferences: private and public consumption are complements for men and perfect substitutes for women. We shall further assume that household income is always larger than $\bar{c}$; then female utilities are of the quasilinear form $c_f + Q$. In particular, any efficient solution involves $c_f = \bar{c}$, because beyond $\bar{c}$, spending a dollar on private consumption for the wife is inefficient: spent on the public good, the same dollar is as valuable for the wife and strictly better for the husband.

Efficient allocations

We first characterize the set of efficient allocations. An efficient couple solves the program:

$$\max c_m Q \tag{8.47}$$

under the constraints

$$c_m + c_f + Q = y + y' \tag{8.48}$$

$$u_f = c_f + Q \geq U \tag{8.49}$$

where $y + y'$ is household total income and U is some arbitrary utility level. A first remark is that at any efficient allocation, the wife's utility U cannot fall below $((y + y') + \bar{c})/2$. As the wife receives the same consumption $\bar{c}$ in any efficient allocation, her utility varies only with the amount of the public good, Q. Once $\bar{c}$ has been spent, the husband's maximal utility is obtains when he receives his optimal bundle of private and public consumption, namely $Q = c_m = ((y + y') - \bar{c})/2$; this choice generates a wife's utility of $((y + y') + \bar{c})/2$. If $U > ((y + y') + \bar{c})/2$, however, providing her with U requires more resources to be spent on the public good (and less on his private consumption) than what he would choose by himself. Then the constraint (8.49) is binding. Therefore, the Pareto frontier is given by

$$u_m = H((y + y'), u_f) = (u_f - \bar{c})((y + y') - u_f), \tag{8.50}$$

where $u_f \geq \frac{(y+y')+\bar{c}}{2}$. Moreover, one can readily compute the corresponding consumptions; namely, $Q = u_f - \bar{c}$ and $c_m = (y + y') - u_f$. Figure 8.7 displays the Pareto frontier when total total income has been set to $(y + y') = 5$ and the wife's minimal consumption to $\bar{c} = 1$, so that $((y + y') + \bar{c})/2 = 3$.

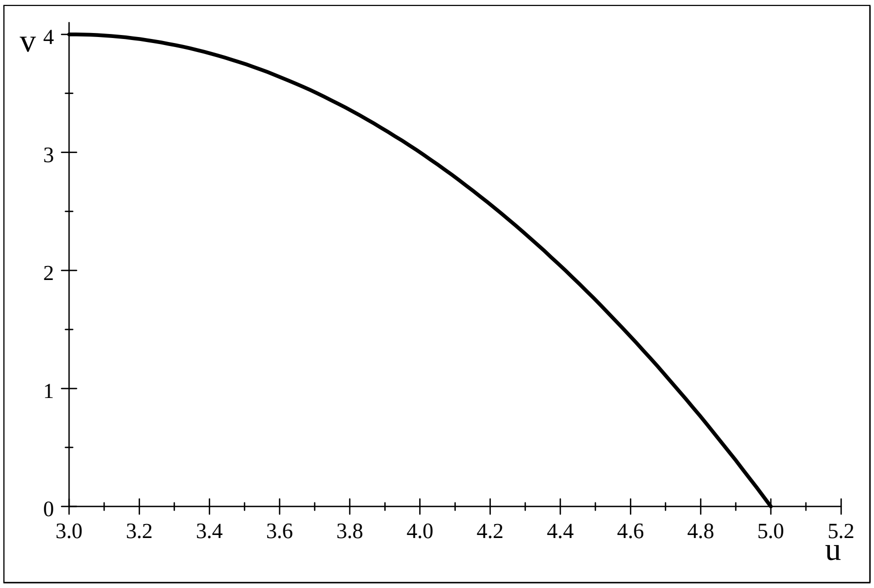

FIGURE 8.7. Pareto frontier

Because of the public consumption, our simple model exhibits what Lundberg and Pollack call 'production dominance'; that is, any single man and any single woman can do better by marrying. To see why, just note that a single man with income y chooses $Q = c_m = y/2$ for a utility of $y^2/4$, while a single woman with income $y' > \bar{c}$ achieves a utility that equals y'. Now, by marrying, they achieve an income $(y + y')$. If $y' \leq y + \bar{c}$, he can achieve $(y + (y' - \bar{c}))^2 /4 > y^2/4$ while she gets $\bar{c} + (y + y')/2 > y'$. If, on the contrary, $y' > y + \bar{c}$ then he can achieve $(y' - \bar{c})\, y > y^2/4$, while she remains at y'. Therefore, in a frictionless model like this one (and without non-monetary gains or costs), either all women or all men marry: singles can only be on one side of the marriage market.

Assortativeness

The Pareto frontier just derived has a particularly tractable form. Indeed, let us analyze the stability conditions along the lines previously described. For $v \geq \frac{(y+y')+\bar{c}}{2}$, we get:

$$\frac{\partial H(y + y', v)}{\partial (y + y')} = v - \bar{c}, \frac{\partial H(y + y', v)}{\partial v} = -(2v - (\bar{c} + (y + y'))) \quad (8.51)$$

implying that

$$\frac{\partial^2 H(y + y', v)}{\partial (y + y')^2} = 0 \text{ and } \frac{\partial^2 H(y + y', v)}{\partial (y + y')\, \partial v} = 1 \quad (8.52)$$

As we have seen in Chapter 7, these conditions are sufficient for the existence of a unique stable match involving assortative matching.

Intrahousehold allocation of welfare

We now turn to the allocation of welfare within the couple. Equation (8.43) becomes:

$$\begin{aligned} v'(y') &= -\frac{\frac{\partial H}{\partial y'}(\phi(y') + y', v(y'))}{\frac{\partial H}{\partial v}(\phi(y'), y', v(y'))} \\ &= \frac{v(y') - \bar{c}}{2v(y') - (\bar{c} + y' + \phi(y'))} \\ &= \frac{\alpha v(y') - \alpha\bar{c}}{2\alpha v(y') - (\alpha+1)y' - (\alpha\bar{c} + \beta)}. \end{aligned} \tag{8.53}$$

Recovering the wife's utility requires solving this differential equation. For that purpose, we may, since v is strictly increasing, define the inverse function ω by:

$$v(y') = v \Leftrightarrow y' = w(v)$$

Then equation 8.53 becomes:

$$\frac{1}{\omega'(v)} = \frac{\alpha v - \alpha\bar{c}}{2\alpha v - (\alpha+1)\omega(v) - (\alpha\bar{c} + \beta)},$$

or

$$\omega'(v) + \frac{(\alpha+1)}{\alpha v - \alpha\bar{c}}\omega(v) = \frac{2\alpha v - (\alpha\bar{c} + \beta)}{\alpha v - \alpha\bar{c}},$$

which is a standard first order, linear differential equation. The general solution is:

$$\omega(v) = K(v - \bar{c})^{-\frac{\alpha+1}{\alpha}} + \frac{2\alpha}{2\alpha+1}v - \frac{\beta + \bar{c}\alpha + 2\alpha\beta}{(\alpha+1)(2\alpha+1)},$$

where K is an integration constant.

To find K, we consider the marginal couple in which the wife receives the lowest female income b and the husband receives the lowest male income $a = (b+\beta)/\alpha$. Since we assumed that the number of women exceeds that of men, the utility of the marginal woman must be at its minimum level, namely $((a+b)+\bar{c})/2$. Thus we have:

$$\omega(b) = K(b - \bar{c})^{-\frac{\alpha+1}{\alpha}} + \frac{2\alpha}{2\alpha+1}b - \frac{\beta + \bar{c}\alpha + 2\alpha\beta}{2\alpha^2 + 3\alpha + 1} = \frac{1}{2}\left(\frac{b+\beta}{\alpha} + b + \bar{c}\right)$$

which yields:

$$K = \left(\frac{1}{2}\left(\frac{b+\beta}{\alpha} + b + \bar{c}\right) - \frac{2\alpha}{2\alpha+1}b + \frac{\beta + \bar{c}\alpha + 2\alpha\beta}{(\alpha+1)(2\alpha+1)}\right)(b - \bar{c})^{\frac{\alpha+1}{\alpha}}$$

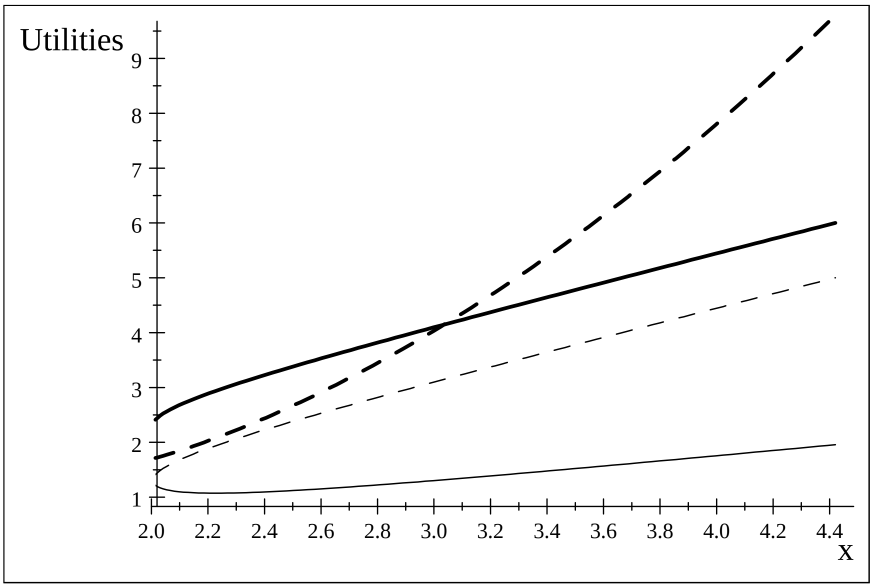

FIGURE 8.8. Husband's and Wife's Utilities, Public Consumption and the Husband's Private Consumption

To illustrate, suppose that $\beta = 0, \alpha = .8, a = 2, b = 1.6, \bar{c} = 1$. Then

$$K = 0.65$$

and

$$\omega(v) = 0.615v + \frac{.65}{(v-1)^{2.25}} - 0.171,$$

while the husband's utility is:

$$\begin{aligned} u &= H(y+y', v) \\ &= (v-\bar{c})(2.25\omega(v) - v) \end{aligned}$$

The resulting utilities are plotted in Figure 8.8. The horizontal line indicates the husband's income y. The wife's utility is represented by the thick line, while the husband's is in dotted and thick. Also, the consumption of the public good $Q = v - \bar{c}$ is represented by dashed line, while the consumption of the husband, c_m, is represented by the thin line.

As one moves up the assignment profile, the total income of the couples and utilities of both husband and wife rise. The consumption of the public good also rises. The private consumption of the wife remains constant at $\bar{c} = 1$, while the private consumption of the husband, c_m, first declines and than rises.

All the comparative statics exercises can be adapted to this general framework. For instance, suppose that we keep $\bar{c} = 1$ and $a = 2$ but shift

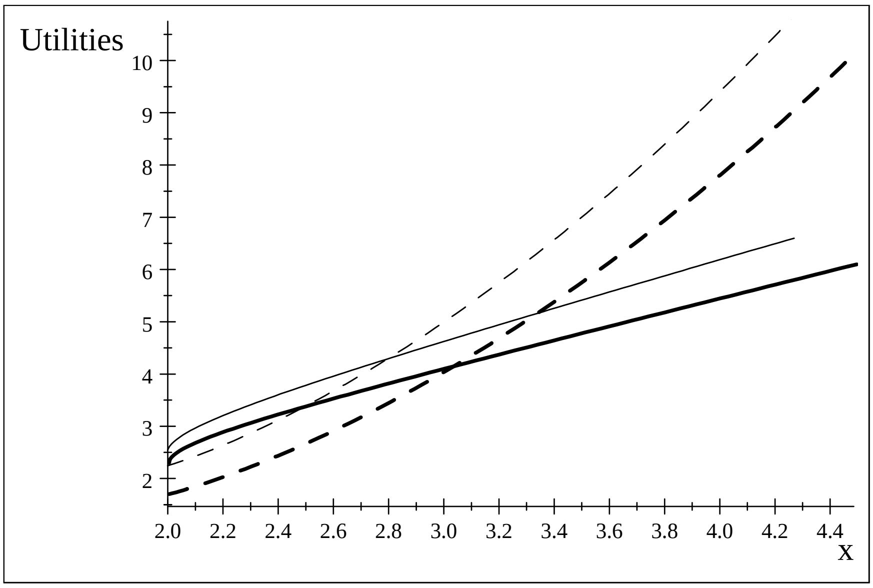

FIGURE 8.9. Impact of a Rightwards Shift in Female Income Distribution on Husband's and Wife's Utilities

the income distribution of women to the right so that $\alpha = 1$ and $b = 2$. Then, $K = 1.12$ so that:

$$\omega(v) = 1.12\,(v-1)^{-2} + \frac{2}{3}v - \frac{1}{6},$$

while his utility is still

$$u = (v-1)\,(2\omega(v) - v)$$

The husband's and the wife's utilities for these two cases are displayed in Figure 8.9, where couples are indexed by male income (which remains invariant). For $\alpha = .8$, we represent, as before, the wife's utility by a thick line and husband's by a dotted and thick line. Thin lines (dashed for males and solid for females) represent u and v when $\alpha = 1$. We see that the shift of the female distribution to the right benefits both men and women. More interesting are the spending patterns. Figure 8.10 displays public (thick) and husband's private (thin) consumptions, both before (solid) and after (dashed) the shift. We see that most of the additional income is spent on the public good; increases in the husband's private consumption are quantitatively small, and tend to shrink with income. In other words, while the husband does benefit from the increase in the wife's income, most of his gain stems from a higher level of public consumption (which actually benefits both partners). We conclude that in this model, unlike the TU case,

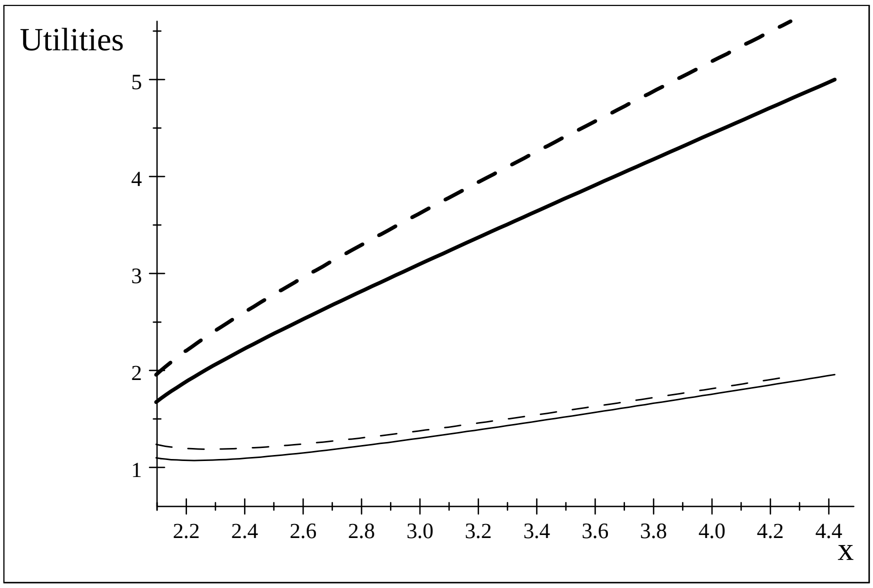

FIGURE 8.10. Impact of a Rightwards Shift in Female Income Distribution on Public Consumption and the Husband's Private Consumption

changes affecting the wife's situation do affect the structure of consumption; moreover, improving the wife's status boost public spending within the couples - a fact that has been abundantly confirmed by empirical investigation, especially if we think of children as a primary example of public consumption (see Chapter 5).

8.4 Matching by Categories

The matching model and the associated allocation rules that we have discussed so far assume some idealized conditions that are not likely to hold in practice. The most common way to make the model more applicable is to introduce frictions and some bargaining over the resulting surplus. There is, however, an alternative modeling choice that goes part of the way towards reality and is based on the recognition that the researcher observes only part of the data that motivates and restricts choices. This is particularly true in marriage markets where explicit market prices do not exist and the division within families of consumption or time is rarely observed. This path has been followed by Choo and Siow (2008) and Chiappori, Salanié and Weiss (2010); the presentation given here follows the latter contribution.

To incorporate unobserved heterogeneity, we consider a case in which the researcher observes marriage patterns within *broad categories,* such as schooling level, race or occupation and observes only some of the individual attributes that distinguish individuals within these classes. That is, in addition to their observed class, individuals are characterized by some *observed* attributes such as income or age and by some idiosyncratic marriage related attributes that are observed by the agents in the marriage market but not by the researcher. We assume that the marital output that is generated by the match of man i, and woman j can be written in the form

$$\zeta_{ij} = z_{I(i)J(j)} + \alpha_{iJ(j)} + \beta_{jI(i)}. \tag{8.54}$$

The first component $z_{I(i)J(j)}$ depends on the *class* of the two partners, the second component $\alpha_{iJ(j)}$ depends on man i and the *class* of woman j and the third component depends on woman j and the *class* of man i. This specification embodies a strong simplifying assumption; the interaction between two married partners is always via their class identity. In particular we do not have a term that depends on both i and j.[13] We further assume that

$$\begin{aligned} \alpha_{iJ(j)} &= a'_{I(i)J(j)}x_i + \varepsilon_{iJ(j)} \\ \beta_{jI(i)} &= b'_{I(i)J(j)}x_j + \varepsilon_{I(i)j} \end{aligned} \tag{8.55}$$

where x_i and x_j are the *observed* attributes of man i and woman j, respectively, a_{IJ} and b_{IJ} are vectors of coefficients that represent the marginal contribution of each male (female) attribute to a marriage between a man of class I and woman of class J. The error terms $\varepsilon_{iJ(j)}$ represent the *unobserved* contribution of man i to a marriage with *any* woman of class J. Similarly, $\varepsilon_{I(i)j}$ represents the contribution of woman j to a marriage with any man of class I.

A basic property of the matching model with transferable utility that we discussed in Chapter 7 is the existence of prices, one for each man, v_i, and one for each woman, u_j, that support a stable outcome. At these prices, the matching is individually optimal for *both* partners in each match. Thus, equilibrium implies that i is matched with j iff

$$\begin{aligned} u_i &= \xi_{ij} - v_j \geq \xi_{ik} - v_k \text{ for all } k, \text{ and } u_i \geq \xi_{i0}, \\ v_j &= \xi_{ij} - u_j \geq \xi_{kj} - u_k \text{ for all } k, \text{ and } v_j \geq \xi_{j0}. \end{aligned} \tag{8.56}$$

Under the special assumptions specified in (8.54) and (8.55), Chiappori, Salanié and Weiss (2010) prove the following Lemma:

Lemma 8.1 *For any stable matching, there exist numbers U_{IJ} and $V_{IJ}, I = 1, ..., M, J = 1, ..., N,$ with the following property: for any matched couple*

[13] This simplifying assumption has been introduced in the context of transferable utility by Choo and Siow (2006). Dagsvik(2002) considers a more general error structure in the context of non transferable utility (e.g. an exogenous sharing rule).

(i, j) *such that* $i \in I$ *and* $j \in J$,

$$\begin{aligned} u_i &= U_{IJ} + \alpha_{iJ} \\ v_j &= V_{IJ} + \beta_{Ij} \end{aligned} \tag{8.57}$$

where

$$U_{IJ} + V_{IJ} = z_{IJ}$$

In words: the differences $u_i - \alpha_{iJ}$ and $v_j - \beta_{jI}$ only depend on the spouses' classes, not on who they are. Note, incidentally, that (8.56) is also valid for singles if we set $U_{I0} = \zeta_{I0}$ and $U_{0J} = \zeta_{0J}$.

The economic interpretation of this result is as follows. The contribution of women j and $j\prime$ who are in the same class J to a marriage with *all* men in class I differ by $\beta_{Ij'} - \beta_{Ij}$. If $v_{j'} - v_j > \beta_{Ij'} - \beta_{Ij}$ no men in I will marry woman $j\prime$ because she is 'too expensive' relative to woman j. Conversely, if $v_{j'} - v_j < \beta_{Ij'} - \beta_{Ij}$ no man in I will marry woman j. Hence, in an equilibrium in which both women j and $j\prime$ find a match with men of the same class I it must be the case that $v_{j'} - v_j = \beta_{Ij'} - \beta_{Ij}$.

To empirically implement these ideas, a first step is to specify the distribution of the unobserved heterogeneity components ε. Given the structure of the model, it is natural to assume that these error terms are identically and independently distributed according to a type 1 extreme value (Gumbel) distribution. We can now write the probability (as viewed by the researcher) that man i marries a woman of a particular class (or remains single) in the familiar multinomial- logit form (see McFadden, 1984)

$$\begin{aligned} & \Pr\left(\, i \in I \ \text{ matched with } j \in J \right) \\ = \; & \frac{\exp\left(U_{I(i)J} + a_{I(i)J} x_i\right)}{\sum_K \exp\left(U_{I(i)K} + a_{I(i)K} x_i\right) + \exp\left(U_{I(i)0} + a_{I0} x_i\right)}, \\ & \Pr\left(i \text{ is single}\right) \\ = \; & \frac{\exp\left(U_{I(i)0} + a_{I(i)0} x_i\right)}{\sum_K \exp\left(U_{I(i)K} + a_{I(i)K} x_i\right) + \exp\left(U_{I(i)0} + a_{I0} x_i\right)}. \end{aligned} \tag{8.58}$$

Analogous expressions hold for women. The terms $U_{I(i)K} + a_{I(i)K} x_i$ represent the systematic part (excluding the unobserved $\varepsilon_{iK(j)}$) of the share that man i receives upon marriage with a woman in class K. The spouse's personal attributes x_j and her idiosyncratic contribution $\varepsilon_{I(i)j}$ have no direct bearing on the probability of marriage, because in equilibrium they are already captured by the unknown constants U_{IK}. Similar remarks apply to the probability of marriage of women. The unknown parameters constants U_{IJ} and V_{IJ} adjust endogenously to satisfy the requirement that the choices of men and women are consistent with each other in the sense of market clearing.

In principle, it is possible to calculate these coefficients directly by solving the market equilibrium (that is the linear programming problem) associated with a stable assignment. More interestingly, one can use data on actual marriage patterns and the observed attributes of participants in a "marriage market" to *estimate* the gains from marriage of these individuals (relative to remaining single).[14] Basically, the preferences for different types of spouses are "revealed" from the choice probabilities of individuals. Taking the simplest case without covariates, we see

$$\begin{aligned} \ln \frac{\Pr(i \in I \text{ is matched with } j \in J)}{\Pr(i \in I \text{ is single})} &= U_{IJ} - U_{I0} \\ \ln \frac{\Pr(j \in J \text{ is matched with } i \in I)}{\Pr(j \in J \text{ is single})} &= U_{IJ} - U_{J0}. \end{aligned} \tag{8.59}$$

Estimating separate multinomial logit for men and women, one can estimate the utilities for each gender in a marriage of each type. Summing the estimated utilities one can recover, for each matching of types (I, J), the systematic output of the marriage ξ_{IJ} (which, under the normalization that being single yields zero utility, equals the total surplus Z_{IJ}). The estimated matrix Z_{IJ} can then be analyzed in terms of the assortative matching that it implies. Of particular interest is whether or not this matrix is supermodular (implying positive assortative mating) or not. As noted by Choo and Siow (2006) and Siow (2009), in the absence of covariates the super modularity of Z_{IJ} is equivalent to the supermodularity of

$$\ln \frac{(\mu(I, J))^2}{\sigma(I)\sigma(J)}$$

where $\mu(I, J)$ is the total number of type (I, J) marriages and $\sigma(I)$ and $\sigma(J)$ are the number of single men and single women, respectively. Such supermodularity requires that for all $I' > I$ and $J' > J$

$$\ln \frac{\mu(I', J')\mu(I, J)}{\mu(I, J')\mu(I', J)} > 0.$$

Siow (2009) uses census data on married couples in the US, where the husband and wife are 32-36 and 31-35 respectively. In each couple, the wife and the husband can belong to one of five possible schooling classes (less than high school, high school, some college, college and college plus). He compares the marriage patterns in the years 1970 and 2000 and finds that in each of the two years strict supermodularity fails to hold as in some of the off diagonal cells, the log odds ratio is negative. Looking at the

[14] Because the probabilities in (8.58) are unaffected by a common proportionality factor, some normalization is required. A common practice is to set the utility from being single to zero for all individuals.

whole matrix, one cannot conclude that there is more positive assortative matching in 2000 than in 1970, although some specific local log odds have increased over time.

Chiappori, Salanié and Weiss (2010) have extended Choo and Siow's 2006 model by assuming that the same determinants of assortative matching operate over a long period of time, during which the distribution of male and female characteristics changes. In practice, their main classification is by education level, and they exploit the remarkable increase in female education over the last decades. In their model, while the surplus generated by the matching of a man in class I with a woman in class J is allowed to vary over time, the supermodular part of the surplus is not; therefore the gains from assortative matching are assumed constant over the period. This assumption generates strong testable predictions; interestingly, they are not rejected by the data. In addition, one can then (over)estimate the model; in particular, the parameters of the surplus function and their drifts can be recovered. From these, it is possible to trace the time changes in the common factors U_{IJ} and V_{IJ} driving the intrahousehold allocation of the surplus, as well as the expected utility of each gender by education level. Note that, as always, this utility is estimated in variations from singlehood; it thus comes *in addition to* any direct benefit affecting all individuals irrespective of their marital status.

This approach has important practical implications. Many theoretical models suggest that education has two types of benefits. One (the so-called 'college premium') is collected on the labor market; it represents the wage differential generated by a college degree, irrespective of a person's marital status. A second, and often omitted aspect is the impact of education on marriage prospects (the 'marital college premium'). An educated person is more likely to marry an educated spouse, resulting in higher household income and surplus; moreover, education typically boost the amount of intra-marital surplus received by the person. This second phenomenon has been recognized by the theoretical literature (see for instance Chiappori, Iyigun and Weiss 2009, and also the next Chapter in this book), but its empirical evaluation has often be perceived as elusive. The approach proposed by Chiappori, Salanié and Weiss exactly addresses this issue. Using CPS data, they show that, indeed, the marital college premium is strong, and that it has significantly increased for women over the last decades - which may help explaining the remarkable growth in female education over the last decades.

Several extensions are currently being pursued. Perhaps the most promising is the explicit modeling of multidimensional matching - recognizing the fact that, ultimately, several factors contribute to the formation of marital surplus, hence to the matching process. The reader is referred to Galichon and Salanié (2010) for a recent and path breaking contribution along these lines.

8.5 Appendix: Extreme Value distributions

We collect here some useful properties of extreme value distributions (See Ben-Akiva and Lerman, 1985, ch. 5 and Johnson *et al.* 1995, ch. 22). The type 1 extreme value distribution for the *maximal* extreme is

$F(x) = e^{-e^{-\frac{x-a}{b}}}$, $b > 0$.

$f(x) = \frac{1}{b} e^{-\frac{x-a}{b}} e^{-e^{-\frac{x-a}{b}}}$.

The moment generating function is $E(e^{tx}) = e^{at}\Gamma(1 - bt)$; the mean is $E(x) = a + kb$, where $k = .57721 = -\Gamma'(1)$ is Euler's constant, the variance is $V(x) = \frac{b^2\pi^2}{6}$, the mode is a and the median is $a - b\log(\log 2) \cong a + .36611b$. Parameter a is thus seen to be a *location* parameter, while b controls the variance. This distribution is sometimes named after E. J. Gumbel and we shall say that $x \backsim G(a, b)$. The distribution is skewed to the right (mean > median > mode). The distribution of the *minimal* extreme is obtained by reversing the sign of x and is skewed to the left.

The *standard form* $G(0, 1)$ has mean k and variance $\frac{\pi^2}{6}$. To get an extreme value with zero mean we can set $a = -kb$ and use $G(-kb, b)$.

The basic properties are the following:

- If $x \backsim G(a, b)$ then $\alpha x + \beta \backsim G(\alpha a + \beta, \alpha b)$.
- If x_1 and x_2 are independent Gumbel variates such that $x_1 \backsim G(a_1, b)$ and $x_2 \backsim G(a_2, b)$ then $x^* = (x_1 - x_2)$ has a logistic distribution, that is,
$$F(x^*) = \frac{1}{1 + e^{\frac{1}{b}(a_2 - a_1 - x^*)}}.$$
- If $x_1, x_{2....}x_n$ are *iid* Gumbel variables with $G(a, b)$ and $v_1, v_{2,..}v_n$ are some constants then
$$Pr\{v_1 + x_1 \geq \max[v_2 + x_2....v_n + x_n]\} = \frac{e^{\frac{v_1}{b}}}{\sum_i e^{\frac{v_i}{b}}}.$$
- If $x_1, x_{2....}x_n$ are independent Gumbel variables with distributions $G(a_i, b)$ then
$$\max(x_1, x_2, ...x_n) \backsim G(b\ln(\sum_i e^{\frac{a_i}{b}}), b).$$
- In particular, if $x_1, x_{2....}x_n$ are *iid* Gumbel variables with $G(a, b)$ and $v_1, v_{2,..}v_n$ are some constants then
$$\begin{aligned} E\{\max[v_1 + x_1, v_2 + x_2....v_n + x_n]\} &= b\ln(\sum_i e^{\frac{v_i + a}{b}}) + kb \\ &= b\ln(\sum_i e^{\frac{v_i}{b}}) + a + kb. \end{aligned}$$

Thus, if $x_1, x_{2....} x_n$ are *iid* Gumbel variables with zero mean then

$$E\{\max[v_1 + x_1, v_2 + x_2....v_n + x_n]\} = b\ln(\sum_i e^{\frac{v_i}{b}}).$$

If we choose one alternative as a benchmark, say alternative 1, and normalize its value to zero, the expected utility relative to this alternative is fully determined by, and inversely related to the probability that the benchmark alternative is selected. The marginal impact of an increase in the value of specific alternative, j, is

$$\frac{\partial E\{\max[v_1 + x_1, v_2 + x_2....v_n + x_n]\}}{\partial v_j} = \frac{e^{\frac{v_j}{b}}}{\sum_i e^{\frac{v_i}{b}}},$$

which is the probability that alternative j will be selected, p_j.

8.6 References

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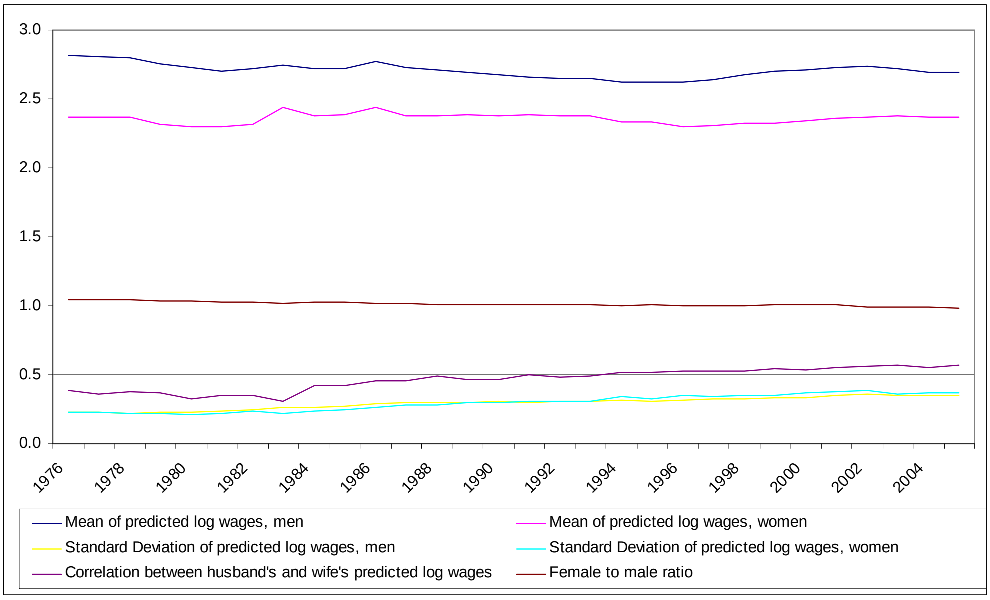

FIGURE 8.11. Parameters of the predicted log wage distribution of men and women and sex ratio, U.S. 1976-2005. Source: Current Population Surveys.

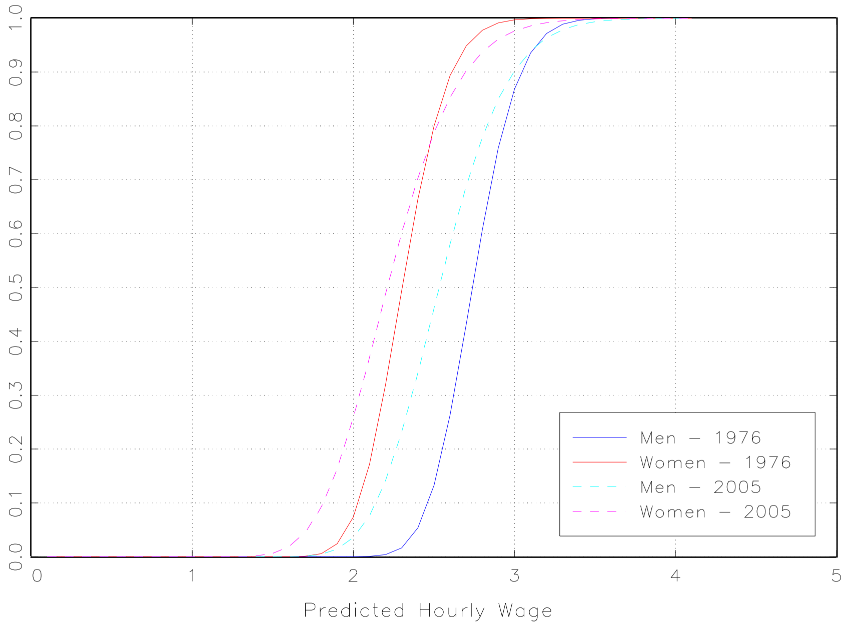

FIGURE 8.12. Cumulative distributions of predicted hourly wages of men and women

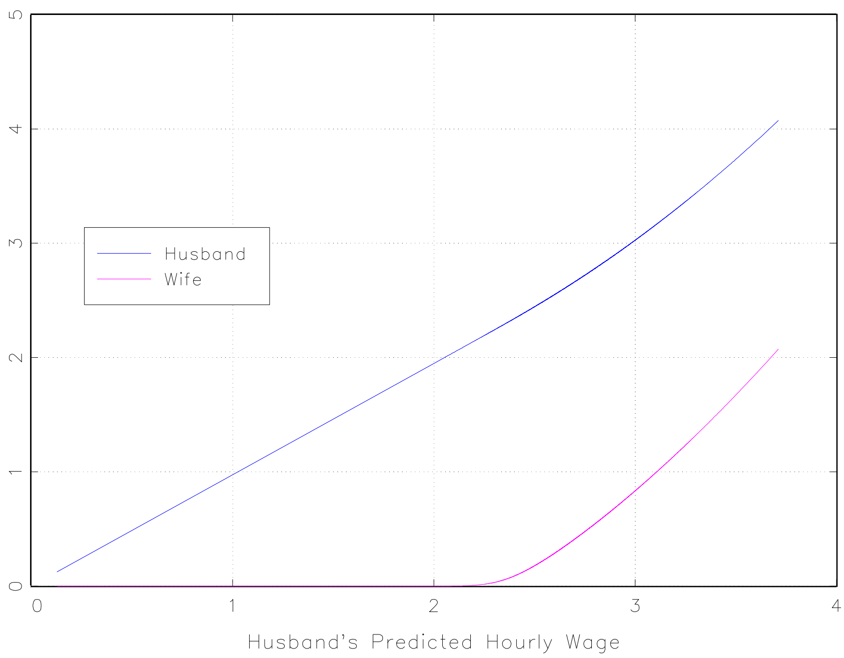

FIGURE 8.13. The surplus of married men and women in 1976, female to male ratio = 1.045

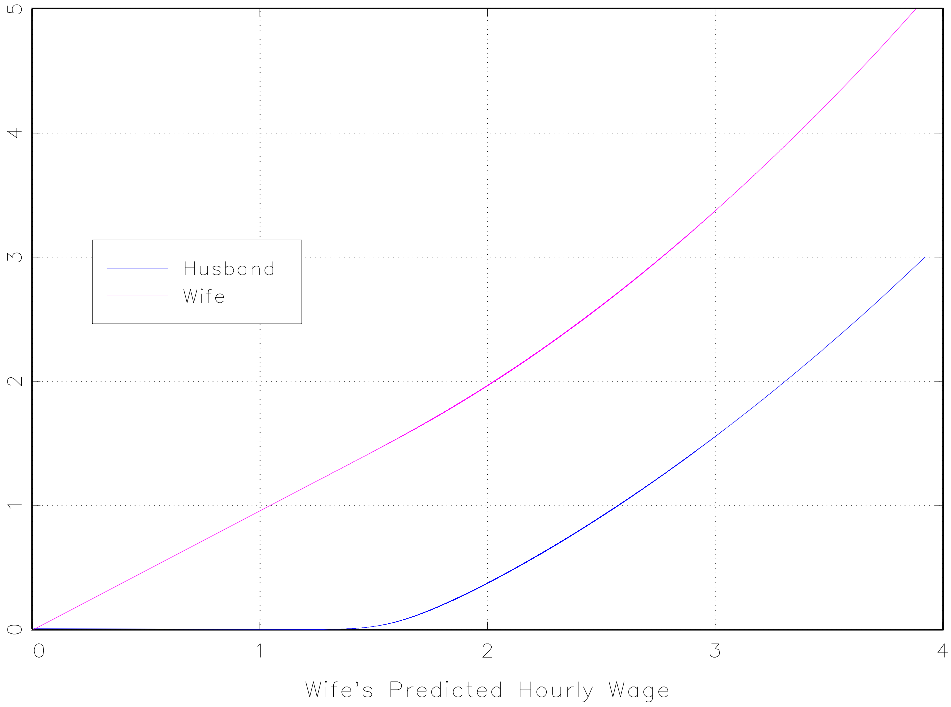

FIGURE 8.14. The surplus of married men and women in 2005, female to male ratio = 0.984

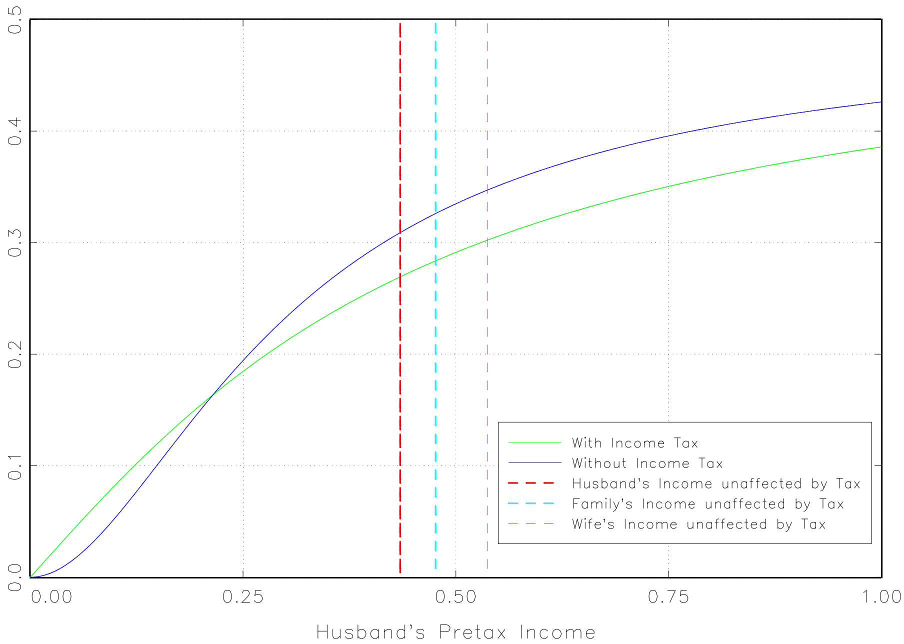

FIGURE 8.15. Wife's relative share in the surplus, women are the majority (r=1.1)

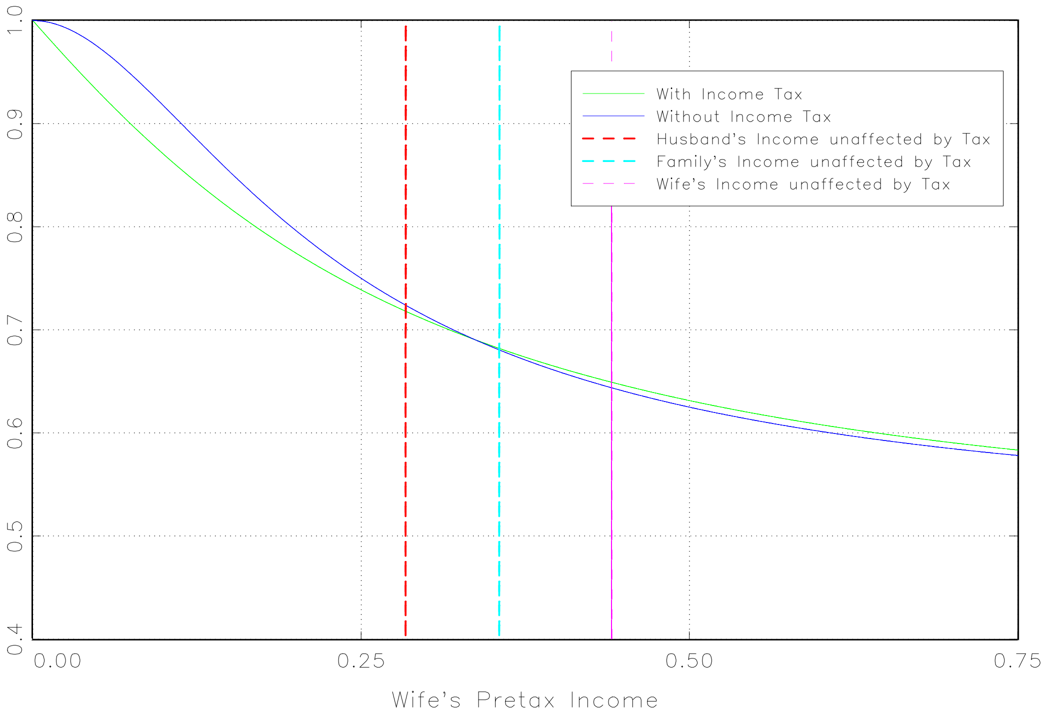

FIGURE 8.16. Wife's relative share in the surplus, women are the majority (r=0.9)

9

Investment in Schooling and the Marriage Market

The purpose of this chapter is to provide a simple equilibrium framework for the joint determination of pre-marital schooling and marriage patterns of men and women.[1] Couples sort according to education and, therefore, changes in the aggregate supply of educated individuals affects who marries whom and the division of the gains from marriage. Unlike other attributes such as race and ethnic background, schooling is an acquired trait that is subject to choice. Acquiring education yields two different returns: First, a higher earning capacity and better job opportunities in the labor market. Second, an improvement in the intra-marital share of the surplus one can extract in the marriage market. Educational attainment influences intra-marital shares by raising the prospects of marriage with an educated spouse and thus raising household income upon marriage, and by affecting the competitive strength outside marriage and the spousal roles within marriage.

The gains from schooling within marriage strongly depend on the decisions of others to acquire schooling. However, since much of schooling happens before marriage, partners cannot coordinate their investments. Rather, men and women make their choices separately, based on the anticipation of marrying a "suitable" spouse with whom schooling investments are expected to generate higher returns. Therefore, an equilibrium framework is required to discuss the interaction between marriage and schooling. Such a framework can address some interesting empirical issues. For instance, it is well documented that the market return to schooling has risen, especially in the second half of the 20th century. Thus, it is not surprising that women's demand for education has risen. What is puzzling, however, is the different response of men and women to the changes in the returns to schooling. Women still receive lower wages in the labor market and spend more time at home than men, although these gaps have narrowed over time. Hence, one could think that women should invest in schooling less than men, because education appears to be less useful for women both at home and in the market. In fact, while women considerably increased their investment in education in the last four decades, men hardly responded to the higher returns to schooling since the 1970s, eventually enabling women

[1] This chapter is based on Chiappori, Iyigun and Weiss (2009).

to overtake them in educational attainment.[2] It has been shown by Chiappori, Iyigun and Weiss (2009) that by introducing marriage market considerations as an additional motivation for investment in schooling one can explain the interrelated investment patterns of women and men.

The returns to pre-marital investments in schooling can be decomposed into two parts: First, higher education raises one's wage rate and increases the payoff from time on the job (the *labor-market* return). Second, it can improve the intra-marital share of the surplus one can extract from marriage (the *marriage-market* return). Educational attainment influences intra-marital spousal allocations directly (due to the fact that education raises household income) and indirectly (by raising the prospects of marriage with an educated spouse and also changing the spousal roles within marriage). In this chapter, we take the labor market returns as given and show how the marriage market returns are determined endogenously together with the proportions of men and women that marry and invest in schooling.

9.1 Is pre-marital investment efficient?

An important issue that we shall address relates to the efficiency of premarital investment. To see where the problem may arise, assume that, after marriage, the spouses' income is used to purchase private and public goods. It follows that, because of the public consumption component, an investment made today will have an external, positive effect on the welfare of the future spouse: if I invest more today, then after my marriage my household will be wealthier and spend more on public consumptions, which will benefit my wife as well. An old argument has it that this external effect will not be taken into account when the investment is done - if only because, at that date, agents probably don't even know who their future spouse will be.[3] That is also true when ex-post bargaining within marriage determines the division of the gains between the two partners. Because each person bears the full cost of his\her investment prior to marriage and receives only part of the gains, there is a potential for under investment. This is known as the "hold-up problem."

Convincing as it may sound, this argument is not robust. Once the matching game is taken into account, it becomes invalid, because the equilibrium conditions imply a full internalization of the externality. This important

[2] Since the late1970s, the returns to schooling have risen steadily for men too. Still, men's college graduation rates have peaked for the cohort born in the mid-1940s (that is, around the mid-1960s). And, after falling for the cohorts that followed, men's college graduation rates have reached a plateau for the most recent cohorts. See Goldin (1997) and Goldin *et al* (2006).

[3] See for instance Bergstrom *et al* (1986) and MacLeod and Malcomson (1993).

result, due to Peters and Siow (2002) and Iyigun and Walsh (2007), can be illustrated on a very simple example. Consider a woman, a, and a man, b, who live for two periods. During the first period, they each receive some income x^s $(s = a, b)$ that they can use for direct consumption or to invest in human capital; therefore $x^s = c^s + i^s$, where c^s denotes consumption and i^s investment. The second period income depends on the investment: $y^s = \phi(i^s)$, where ϕ is increasing and concave. Once married, the couple can spend its total income $y^a + y^b$ on private consumptions q^a and q^b and public consumption Q. Individual utilities have the form:

$$U^s = c^s + q^s Q$$

which satisfies the TU property.

In this very simple setting, one can readily compute the optimal level of investment. Indeed, in our TU framework, efficient allocations solve:

$$\max U^a + U^b = c + qQ$$

where $c = c^a + c^b, q = q^a + q^b$, under the constraint:

$$q + Q = \phi(x^a - c^a) + \phi\left(x^b - c^b\right)$$

In the second period, the optimal consumptions are given by:

$$q = Q = \frac{\phi(x^a - c^a) + \phi\left(x^b - c^b\right)}{2}$$

so the program becomes:

$$\max c^a + c^b + \frac{\left(\phi(x^a - c^a) + \phi\left(x^b - c^b\right)\right)^2}{4}$$

First order conditions give:

$$\frac{\phi(x^a - c^a) + \phi\left(x^b - c^b\right)}{2}\phi'(x^s - c^s) = 1, \ \ s = a, b$$

which implies that $i^a = i^b = i$, where the common level of investment i satisfies:

$$\phi(i)\,\phi'(i) = 1$$

Let us now solve the dynamic game in which agents first *non cooperatively* determine their investments, then match on the marriage market in a frictionless context. Note, first, that once second period incomes have been generated, the output of a couple male with income y^a- female with income y^b is:

$$h\left(y^a, y^b\right) = \frac{\left(y^a + y^b\right)^2}{4}$$

which is supermodular ($h_{y^a y^b} = 1/2 > 0$)

To keep things simple, let us further assume that the model is fully symmetric in gender; that is, for each male there exists exactly one female who has the same income in the initial situation. It is then natural to solve for a symmetric equilibrium, in which a pair of identical individuals of opposite sex invest the same amount and generate the same second period income which put them at the same place in their respective distributions. Supermodularity implies assortative matching, so the two individuals will be matched together. Let $u^s(y^s)$ denote the second period utility of person s at the stable match; from Chapter 8, we know that:

$$\begin{aligned} u'^s(y^s) &= \frac{\partial h\left(y^a, y^b\right)}{\partial y^s} \\ &= \frac{\left(y^a + y^b\right)}{2} = y^s \end{aligned} \tag{9.1}$$

since $y^a = y^b$ by symmetry.

Let us now consider the first period investment decision. Agent s chooses i^s knowing that the second period income $\phi(i^s)$ will, through the matching game, result in a second period utility equal to $u^s(\phi(i^s))$. The first period investment therefore solves:

$$\max_{i^s} x^s - i^s + u^s(\phi(i^s))$$

The first order condition gives:

$$u'^s(\phi(i^s)).\phi'(i^s) = 1$$

and from (9.1):

$$\phi(i^s).\phi'(i^s) = 1$$

which is exactly the condition for efficiency.

Our example clearly relies on a series of strong, simplifying assumptions. Its message, however, is general. The equilibrium condition ((9.1) in our case) precisely states that the marginal gain an individual will receive from a small increase in his trait (here is income) is equal to the marginal impact of the increase over the output generated at the household level. But this is exactly the condition for efficiency. Although part of the consumption is public (which explains the convexity of the output as a function of total income and ultimately the assortative matching), this externality is internalized by the competitive nature of the matching game. My initial investment has actually three benefits: it increases my future income, which will result in more consumption tomorrow; it 'buys' me a better spouse, since second period matching is assortative in income; and it improves the fraction of the marital surplus that I receive. The first effect, by itself, would not be sufficient to induce the efficient level of investment - that is the essence of the externality argument. But the logic of competitive matching requires the three aspects to be considered - and the unambiguous conclusion is that efficiency is restored.

Finally, what about the opposite line of argument, according to which agents actually invest too much? The story goes as follows: since agents compete for the best spouse, a 'rat race' situation follows, whereby all males overinvest in human capital. Well, again, the argument is incorrect in a matching setting in which transfers are feasible between spouses. Indeed, one should take into account not only the 'quality' (here the wealth) of the spouse who will be attracted by a higher second period wealth, but also the 'price' that will have to be paid (in terms of surplus sharing). In a matching game, wealthier spouses come with a higher reservation utility, thus require giving up a larger fraction of the surplus; as illustrated by the previous example, this is exactly sufficient to induce the right investment level. An important remark, however, is that this conclusion would *not* hold in a Gale-Shapley framework, in which transfers are not possible and the spouses' respective gains are exogenously determined (and do not respond to competitive pressures on the marriage market). In such a setting, the 'rat race' effect is much more likely to occur!

The model developed in this chapter assumes a large competitive marriage market without frictions and we shall demonstrate that premarital investments in schooling are efficient.

9.2 The Basic Model

We begin with a benchmark model in which men and women are completely symmetric in their preferences and opportunities. However, by investing in schooling, agents can influence their marriage prospects and labor market opportunities. Competition over mates determines the assignment (that is, who marries whom) and the shares in the marital surplus of men and women with different levels of schooling, depending on the aggregate number of women and men that acquire schooling. In turn, these shares together with the known market wages guide the individual decisions to invest in schooling and to marry. We investigate the rational-expectations equilibrium that arises under such circumstances.

9.2.1 Definitions

When man i and woman j form a union, they generate some aggregate *material output* ζ_{ij} that they can divide between them and the utility of each partner is linear in the share he\she receives (transferable utility). Man i alone can produce ζ_{i0} and woman j alone can produce ζ_{0j}. The *material surplus* of the marriage is defined as

$$z_{ij} = \zeta_{ij} - \zeta_{i0} - \zeta_{0j}. \tag{9.2}$$

In addition, there are emotional gains from marriage and the total *marital surplus* generated by a marriage of man i and woman j is

$$s_{ij} = z_{ij} + \theta_i + \theta_j, \tag{9.3}$$

where θ_i and θ_j represent the non-economic gains of man i and woman j from their marriage.

9.2.2 Assumptions

There are two equally large populations of men and women to be matched.[4] Individuals live for two periods. Each person can choose whether to acquire schooling or not and whether and whom to marry. Investment takes place in the first period of life and marriage in the second period. Investment in schooling is lumpy and takes one period so that a person who invests in schooling works only in the second period, while a person who does not invest works in both periods. To simplify, we assume no credit markets.[5] All individuals with the same schooling and of the same gender earn the same wage rate, but wages may differ by gender. We denote the wage of educated men by w_2^m and the wage of uneducated men by w_1^m, where $w_2^m > w_1^m$. The wage of educated women is denoted by w_2^w and that of uneducated women by w_1^w, where $w_2^w > w_1^w$. Market wages are taken as exogenous and we do not attempt to analyze here the feedbacks from the marriage market and investments in schooling to the labor market. We shall discuss, however, different wage structures.

We denote a particular man by i and a particular woman by j. We represent the schooling level (class) of man i by $I(i)$ where $I(i) = 1$ if i is uneducated and $I(i) = 2$ if he is educated. Similarly, we denote the class of woman j by $J(j)$ where $J(j) = 1$ if j is uneducated and $J(j) = 2$ if she is educated. An important simplifying assumption is that the material surplus generated by a marriage of man i and woman j depends only on the class to which they belong. That is,

$$s_{ij} = z_{I(i)J(j)} + \theta_i + \theta_j. \tag{9.4}$$

We assume that the schooling levels of married partners complement each other so that

$$z_{11} + z_{22} > z_{12} + z_{21}. \tag{9.5}$$

[4] We address the impact of the sex ratio in a separate section below.

[5] Allowing borrowing and lending raises issues such as whether or not one can borrow based on the income of the future spouse and enter marriage in debt.

Except for special cases associated with the presence of children, we assume that the surplus rises with the schooling of both partners. When men and women are viewed symmetrically, we also have $z_{12} = z_{21}$.

The per-period material utilities of man i and woman j as singles also depend on their class, that is $\zeta_{i0} = \zeta_{I(i)0}$ and $\zeta_{0j} = \zeta_{0J(j)}$ and are assumed to increase in $I(i)$ and $J(j)$. Thus, a more educated person has a higher utility as a single. Men and women who acquire no schooling and never marry have life time utilities of $2\zeta_{10}$ and $2\zeta_{01}$, respectively. A person that invests in schooling must give up the first period utility and, if he\she remains single, the life time utilities are ζ_{20} for men and ζ_{02} for women. Thus, the (absolute) return from schooling for never married men and women are $R^m = \zeta_{20} - 2\zeta_{10}$ and $R^w = \zeta_{02} - 2\zeta_{01}$, respectively.[6] The return to schooling of never married individuals depends only on their own market wages and we shall refer to it as the *labor-market* return. However, investment in schooling raises the probability of marriage and those who marry have an additional return from schooling investment in the form of increased share in the material surplus, which we shall refer to as the *marriage-market* return to schooling. In addition to the returns in the labor market or marriage market, investment in schooling is associated with idiosyncratic costs (benefits) denoted by μ_i for men and μ_j for women.

The idiosyncratic preference parameters are assumed to be independent of each other and across individuals. We denote the distributions of θ and μ by $F(\theta)$ and $G(\mu)$ and assume that these distributions are symmetric around their zero means. This specification is rather restrictive because one might expect some correlations between the taste parameters and the observable attributes. For instance, individuals that have a low cost of schooling may also have a high earning capacity and individuals may derive different benefits from marriage depending on the observed quality of their spouses. One may also expect a correlation between the emotional valuations of the marriage by the two spouses. Thus, the model is very basic and intended mainly as an illustration of the possible feedbacks between the marriage market and investment in schooling.

9.2.3 The Marriage Market

Any *stable* assignment of men to women must maximize the *aggregate surplus* over all possible assignments (Shapley and Shubik, 1972).[7] The dual of this linear programming problem posits the existence of non-negative shadow prices associated with the constraints of the primal that each per-

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[6] Because we assume away the credit market, the rate of return from schooling investment depends on consumption decisions and is in utility terms.

[7] Note that the maximization of the aggregate *surplus* is equivalent to the maximization of aggregate *output* because the utilities as singles are independent of the assignment.

son can be either single or married to one spouse. We denote the shadow price of woman j by u_j and the shadow price of man i by v_i. The complementarity slackness conditions require that

$$z_{I(i)J(j)} + \theta_i + \theta_j \leq v_i + u_j, \tag{9.6}$$

with equality if i and j are married and inequality otherwise.

The complementarity slackness conditions are equivalent to

$$\begin{aligned} v_i &= Max\{\underset{j}{Max}[z_{I(i)J(j)} + \theta_i + \theta_j - u_j], 0\} \\ u_j &= Max\{\underset{i}{Max}[z_{I(i)J(j)} + \theta_i + \theta_j - v_i], 0\}, \end{aligned} \tag{9.7}$$

which means that the assignment problem can be *decentralized.* That is, given the shadow prices u_j and v_i, each agent marries a spouse that yields him\her the highest share in the marital surplus. We can then define $\bar{u}_j = u_j + \zeta_{0j}$ and $\bar{v}_i = v_i + \zeta_{i0}$ as the *reservation utility* levels that woman j and man i require to participate in *any marriage.* In equilibrium, a stable assignment is attained and each married person receives his\her reservation utility, while each single man receives ζ_{i0} and each single woman receives ζ_{0j}.

Our specification imposes a restrictive but convenient structure in which the interactions between agents depend on their group affiliation only, that is, their levels of schooling. Assuming that, in equilibrium, at least one person in each class marries, the endogenously-determined shadow prices of man i in $I(i)$ and woman j in $J(j)$ can be written in the form,

$$v_i = Max(V_{I(i)} + \theta_i, 0) \qquad \text{and} \qquad u_j = Max(U_{J(j)} + \theta_j, 0) \tag{9.8}$$

where

$$V_I = \underset{J}{Max}[z_{IJ} - U_J] \qquad \text{and} \qquad U_J = \underset{I}{Max}[z_{IJ} - V_I] \tag{9.9}$$

are the *shares* that the partners receive from the *material surplus* of the marriage (not accounting for the idiosyncratic effects θ_i and θ_j). All agents of a given type receive the *same* share of the material surplus z_{IJ} no matter whom they marry, because all the agents on the other side rank them in the same manner. Any man (woman) of a given type who asks for a higher share than the "going rate" cannot obtain it because he (she) can be replaced by an equivalent alternative.

Although we assume equal numbers of men and women in total, it is possible that the equilibrium numbers of educated men and women will differ. We shall assume throughout that there are some uneducated men who marry uneducated women and some educated men who marry educated women. This means that the equilibrium shares must satisfy

$$U_2 + V_2 = z_{22} \tag{9.10}$$

$$U_1 + V_1 = z_{11} \tag{9.11}$$

We can then classify the possible matching patterns as follows: under *strict* positive assortative mating, educated men marry only educated women and uneducated men marry only uneducated women. Then,

$$U_1 + V_2 \geq z_{21}, \tag{9.12}$$

$$U_2 + V_1 \geq z_{12}. \tag{9.13}$$

If there are more educated men than women among the married, some educated men will marry uneducated women and condition 9.12 also will hold as equality. If there are more educated women than men among the married, equation 9.13 will hold as equality. It is impossible that all four conditions will hold as equalities because this would imply

$$z_{22} + z_{11} = z_{12} + z_{21}, \tag{9.14}$$

which violates assumption 9.5 that the education levels of the spouses are complements. Thus, either educated men marry uneducated women or educated women marry uneducated men but not both.

When types mix and there are more educated men than educated women among the married, conditions 9.10, 9.11 and 9.12 imply

$$\begin{aligned} U_2 - U_1 &= z_{22} - z_{21}, \\ V_2 - V_1 &= z_{21} - z_{11}. \end{aligned} \tag{9.15}$$

If there are more educated women than men among the married, then conditions 9.10, 9.11 and 9.13 imply

$$\begin{aligned} V_2 - V_1 &= z_{22} - z_{12}, \\ U_2 - U_1 &= z_{12} - z_{11}. \end{aligned} \tag{9.16}$$

One may interpret the differences $U_2 - U_1$ and $V_2 - V_1$ as the (additional) *return to schooling in marriage* for women and men, respectively.[8] The quantity $z_{22} - z_{21}$, which reflects the contribution of an educated woman to the material surplus of a marriage with an educated man, provides an *upper bound* on the return that a woman can obtain through marriage, while her contribution to a marriage with an uneducated man, $z_{12} - z_{11}$, provides a *lower bound.* When there are more educated women than men, analogous bounds apply to men. When types mix in the marriage market equilibrium, we see that the side that is in short supply receives the marginal contribution to a marriage with an educated spouse, while the side

[8] The total return from schooling in terms of the output that men receive is R^m if they remain single and $R^m + V_2 - V_1$ if they marry. Similarly, the total return from schooling in terms of the output that women receive is R^w if they remain single and $R^w + U_2 - U_1$ if they marry.

in excess supply receives the marginal contribution to a marriage with an uneducated spouse.

We do not exclude the possibility of negative equilibrium values for some V_I or U_J. This would happen if the marginal person in a class is willing to give up in marriage some of the material output that he\she has as single, provided that the non-monetary benefit from marriage is sufficiently large. Then, all men (women) in that class are also willing to do so and the common factors, V_I or U_J may become negative. However, stability implies that the returns to schooling in marriage, $V_2 - V_1$ and $U_2 - U_1$ are positive in equilibrium, provided that the marital surplus rises with the education of both spouses.

9.2.4 Investment Decisions

We assume rational expectations so that, in equilibrium, individuals know V_I and U_J, which are sufficient statistics for investment decisions. Given these shares and knowledge of their own idiosyncratic preferences for marriage, θ, and costs of schooling, μ, agents know for sure whether or not they will marry in the second period, conditional on their choice of schooling in the first period.

Man i chooses to invest in schooling if

$$\zeta_{20} - \mu_i + Max(V_2 + \theta_i,\ 0) > 2\zeta_{10} + Max(V_1 + \theta_i,\ 0). \tag{9.17}$$

Similarly, woman j chooses to invest in schooling if

$$\xi_{02} - \mu_j + Max(U_2 + \theta_j, 0) > 2\zeta_{01} + Max(U_1 + \theta_j,\ 0). \tag{9.18}$$

Figure 9.1 describes the choices made by different men. Men for whom $\theta < -V_2$ do not marry and invest in schooling if and only if $\mu < R^m \equiv \zeta_{20} - 2\zeta_{10}$. Men for whom $\theta > -V_1$ always marry and they invest in schooling if and only if $\mu < R^m + V_2 - V_1$. Finally, men for whom $-V_2 < \theta < -V_1$ marry if they acquire education and do not marry if they do not invest in schooling. These individuals will acquire education if $\mu < R^m + V_2 + \theta$. In this range, there are two motives for schooling: to raise future earning capacity and to enhance marriage. We shall assume that the variability in θ and μ is large enough to ensure that *all* these regions are non-empty in an equilibrium with positive V_I and U_J. In particular, we assume that, irrespective of marital status, there are some men and women who prefer not to invest in schooling and some men and women who prefer to invest in schooling. That is, $\mu_{\max} > \max[R^m + z_{22} - z_{12}, R^w + z_{22} - z_{21}]$ and $\mu_{\min} < \min[R^m, R^w]$. We shall also assume that $\theta_{\min} < -z_{22}$ so that, irrespective of the education decision, there are some individuals who wish not to marry. Note, finally, that because the support of $F(.)$ extends into the positive range, there are always some educated men and women who marry and some uneducated men and women who marry.

The proportion of men who invest in schooling is

$$G(R^m)F(-V_2)+[1-F(-V_1)]G(R^m+V_2-V_1)+\int_{-V_2}^{-V_1} G(R^m+V_2+\theta)f(\theta)d\theta, \tag{9.19}$$

the proportion of men who marry is

$$[1-F(-V_1)]+\int_{-V_2}^{-V_1} G(R^m+V_2+\theta)f(\theta)d\theta, \tag{9.20}$$

and the proportion of men who invest and marry is

$$[1-F(-V_1)]G(R^m+V_2-V_1)+\int_{-V_2}^{-V_1} G(R^m+V_2+\theta)f(\theta)d\theta. \tag{9.21}$$

The higher are the returns from schooling in the labor market, R^m, and in marriage, V_2-V_1, the higher is the proportion of men who acquire schooling. A common increase in the levels V_2 and V_1 also raises investment because it makes marriage more attractive and schooling obtains an *extra* return within marriage. For the same reason, an increase in the market return R^m raises the proportion of men that marry. Analogous expressions hold for women.

9.2.5 *Equilibrium*

In the marriage market equilibrium, the numbers of men and women who marry must be the same. Using equation 9.20 and applying symmetry, we can write this condition as

$$F(V_1)+\int_{V_1}^{V_2} G(R^m+V_2-\theta)f(\theta)d\theta = F(U_1)+\int_{U_1}^{U_2} G(R^w+U_2-\theta)f(\theta)d\theta. \tag{9.22}$$

Under strictly positive assortative mating, the numbers of men and women in each education group are equal. Given that we impose condition 9.22, it is necessary and sufficient to require that the numbers of men and women who marry but do *not* invest in schooling are the same. Using condition 9.21 and symmetry, we can derive this condition as

$$F(V_1)G(-R^m+V_1-V_2) = F(U_1)G(-R^w+U_1-U_2). \tag{9.23}$$

Together with conditions 9.10 and 9.11, conditions 9.22 and 9.23 yield a system of four equations in four unknowns that are, in principle, solvable.

If there is some mixing of types, equation 9.23 is replaced by an inequality and the shares are determined by the boundary conditions on the returns to schooling within marriage for either men or women, whichever is applicable. If there are more educated men than women among the married, 9.23 becomes

$$F(V_1)G(-R^m + V_1 - V_2) < F(U_1)G(-R^w + U_1 - U_2)$$

and educated women receive their maximal return from marriage while men receive their minimal return so that condition 9.15 holds. Conversely, if there are more educated women than men among the married, we have

$$F(V_1)G(-R^m + V_1 - V_2) > F(U_1)G(-R^w + U_1 - U_2)$$

and educated men receive their maximal return from marriage while educated women receive their minimal return so that condition 9.16 holds. Together with conditions 9.10 and 9.11, we have four equations in four unknowns that are again, in principle, solvable. For a proof of existence and uniqueness see the Appendix.

The two types of solutions are described in Figures 9.2 and 9.3, where we depict the equilibrium conditions in terms of V_1 and V_2 after we eliminate U_1 and U_2 using 9.10 and 9.11. The two positively-sloped and parallel lines in these figures describe the boundaries on the returns to schooling of men within marriage. The negatively-sloped red line describes the combinations of V_1 and V_2 that maintain equality in the numbers of men and women who wish to marry. The positively-sloped blue line describes the combinations of V_1 and V_2 that maintain equality in the numbers of men and women that acquire no schooling and marry. The slopes of these lines are determined by the following considerations: An increase in V_1 (and a reduction in U_1), keeping V_2 and U_2 constant, induces more men and fewer women to prefer marriage. An increase in V_2 holding V_1 constant has a similar effect. Thus, V_1 and V_2 are substitutes in terms of their impact on the incentives of men to marry and U_1 and U_2 are substitutes in terms of their impact on the incentives of women to marry. Therefore, equality in the numbers of men and women who wish to marry can be maintained only if V_2 declines when V_1 rises. At the same time, an increase in V_1 (and a reduction in U_1), keeping V_2 and U_2 constant, increases the number of men that would not invest and marry and reduces the number of women who wish to acquire no schooling and marry. Therefore, equality in the numbers of uneducated men and women who wish to marry can be maintained only if V_2 rises when V_1 rises so that the rates of return to education within marriage are restored.

As long as the model is completely symmetric, that is $R^m = R^w$ and $z_{12} = z_{21}$, the equilibrium is characterized by equal sharing: $V_2 = U_2 = z_{22}/2$ and $U_1 = V_1 = z_{11}/2$. With these shares, men and women have identical investment incentives. Hence, the number of educated (uneducated) men equals the number of educated (uneducated) women, both among the singles and the married. Such a solution is described by point e in Figure 9.2, where the lines satisfying conditions 9.22 and 9.23 intersect. There is a unique symmetric equilibrium. However, with asymmetry, when either $R^m \neq R^w$ or $z_{12} \neq z_{21}$, there may be a mixed equilibrium where the line representing condition 9.22 intersects either the lower or upper bound on $V_2 - V_1$ so that condition 9.23 holds as an inequality. Such a case is illustrated by the point $e\prime$ in Figure 9.3. In this equilibrium, educated men obtain the lower bound on their return to education within marriage, $z_{21} - z_{11}$. The equilibrium point $e\prime$ is on the lower bound and above the blue line satisfying condition 9.23, indicating excess supply of educated men.

The Impact of the Sex Ratio

Although we assume in this chapter an equal numbers of men and women in the population, one can extend the analysis to examine the impact of an uneven sex ratio on the marriage market equilibrium. Let $r \gtreqless 1$ represent the ratio of men to women in the population. Then we modify equations 9.22 and 9.23 as follows, respectively:

$$rF(V_1) + r\int_{V_1}^{V_2} G(R^m + V_2 - \theta)f(\theta)d\theta = F(U_1) + \int_{U_1}^{U_2} G(R^w + U_2 - \theta)f(\theta)d\theta. \tag{9.24}$$

$$rF(V_1)G(-R^m + V_1 - V_2) = F(U_1)G(-R^w + U_1 - U_2). \tag{9.25}$$

Note that, even if $R^m = R^w$ and $z_{12} = z_{21}$, the equilibrium with an uneven sex ratio will not be characterized by equal sharing. For example, if $r > 1$ and there are more men than women in the population, then 9.24 implies that V_2 and U_1 will need to decline and V_1 and U_2 will need to rise to ensure that there are equal numbers of men and women who want to marry. As a result, the marriage-market return for the sex in excess supply (men) will fall and that of the sex in short supply (women) will rise, regardless of whether the marriage market equilibrium is strict or mixed. For r closer to unity, equation 9.25 may still hold, implying a strict sorting equilibrium with equal numbers of educated men and educated women among the married. However, with more uneven sex ratios, equation 9.25 may not hold even if $R^m = R^w$ and $z_{12} = z_{21}$. Then, when $r > 1$ ($r < 1$) there will be a mixed equilibrium where the line representing condition 9.24 intersects the lower (upper) bound on $V_2 - V_1$. In such cases, condition 9.25 will no longer hold as equality.

Efficiency

We can now demonstrate that in our model individuals' pre-marital investments are *efficient*. Consider, first, a mixed equilibrium in which some married men are more educated than their wives and consider a particular couple (i, j) such that the husband is educated and the wife is not. The question is whether by *coordination* this couple could have gained by, for example, changing investments and allowing redistribution between them.

If woman j had gotten educated, the partners together would have gained $\zeta_{22} - \zeta_{21}$ in terms of marital output but the cost of schooling for woman j would have been her forgone earnings in the first period ζ_{01} plus her idiosyncratic non-monetary cost, μ_j. The couple would gain from such a shift only if $\mu_j + \zeta_{01} < \zeta_{22} - \zeta_{21}$ or, equivalently,

$$\mu_j < z_{22} - z_{21} + R^w. \tag{9.26}$$

But, in the assumed marriage market configuration, $z_{22} - z_{21} = U_2 - U_1$ and, by assumption, woman j chose not to invest and marry. Therefore, by 9.17,

$$\mu_j > Max(U_2+\theta_j, 0) - U_1 - \theta_j + R^w \geq U_2 - U_1 + R^w = z_{22} - z_{21} + R^w. \tag{9.27}$$

We thus reach a contradiction, implying that there is no *joint* net gain from such a rearrangement of investment choices. Nor is it profitable from the point of view of the couple that the husband would have refrained from schooling. The couple could gain from such a rearrangement only if the reduction in the costs of the husband's schooling exceeds the lost marital output, $\mu_i + \zeta_{10} > \zeta_{21} - \zeta_{11}$, or equivalently,

$$\mu_i > z_{21} - z_{11} + R^m. \tag{9.28}$$

But, in the assumed marriage market configuration, $z_{21} - z_{11} = V_2 - V_1$ and, by assumption, man i chose to invest and marry. Therefore, by 9.17

$$\mu_i < R^m + V_2 + \theta_i - Max(V_1 + \theta_i,\, 0) \leq V_2 - V_1 + R^m = z_{21} - z_{11} + R^m. \tag{9.29}$$

So, again, we have a contradiction, implying that there is no *joint* net gain from such a rearrangement of investment choices. Similar arguments hold if we consider a mixed equilibrium in which some educated women marry uneducated men.

Next, consider a strictly assortative equilibrium and a married couple (i, j) such that neither spouse is educated. Could this couple have been better off had the partners coordinated their educational investments so that they both had acquired education? This would be profitable if the joint gain $\zeta_{22} - \zeta_{11}$ in terms of marital output exceeds the total costs of the two partners $\zeta_{01} + \zeta_{10} + \mu_j + \mu_i$. That is, if

$$\mu_j + \mu_i < z_{22} - z_{11} + R^m + R^w. \tag{9.30}$$

But, by assumption, man i and woman j married and did not invest, implying that

$$\begin{aligned} \mu_j &> U_2 - U_1 + R^w, \\ \mu_i &> V_2 - V_1 + R^m. \end{aligned} \tag{9.31}$$

By adding up these two inequalities, and using the equilibrium conditions $z_{22} = U_2 + V_2$ and $z_{11} = U_1 + V_1$, we see that it is impossible to satisfy (9.29). Hence, there is no joint gain from such a rearrangement of investments. By similar arguments, there is no joint gain for a couple in which both partners are educated from a coordinated reduction in their investments.

We conclude that the equilibrium shares that individuals expect to receive within marriage induce them to fully internalize the *social* gains from their premarital investments. An important piece of this argument is that the marriage market is large in the sense that individual perturbations in investment do not affect the equilibrium shares. In particular, a single agent cannot tip the market from excess supply to excess demand of educated men or women. This efficiency property of large and frictionless marriage markets has been noted by Cole *et al.* (2001), Felli and Roberts (2002) Peters and Siow (2002) and Iyigun and Walsh (2007). In contrast, markets with frictions or small number of traders are usually characterized by inefficient premarital investments (Lommerud and Vagstad, 2000, Baker and Jacobsen, 2007).[9]

9.3 Gender Differences in the Incentive to Invest

In this section, we discuss differences between women and men that can cause them to invest at different levels. We discuss two possible sources of asymmetry:

- In the labor market, women may receive lower wages than men; this could lower the schooling return for working women.
- In marriage, women may be required to take care of the children; this would lower the schooling return for married women.

Either of the above causes can induce women to invest less in schooling. Therefore, the lower incentives of women to invest can create equilibria

[9] Peters (2007) formulates premarital investments as a Nash game in which agents take as given the actions of others rather than the expected shares (as in a market game). In this case, inefficiency can persist even as the number of agents approaches infinity. The reason is that agents play mixed strategies that impose on other agents the risk of being matched with an uneducated spouse, leading to under-investment in schooling.

with mixing, where educated men are in excess supply and some of them marry less-educated women.

To illustrate these effects we shall perform several comparative statics exercises, starting from a benchmark equilibrium with strictly positive assortative matching, resulting from a complete equality between the sexes in wages and household roles such that $w_1^m = w_1^w = w_1$, $w_2^m = w_2^w = w_2$ and $\tau = 0$.

9.3.1 The Household

We use a rudimentary structural model to trace the impact of different wages and household roles of men and women on the marital output and surplus. We assume that, irrespective of the differences in wages or household roles, men and women have the same preferences given by

$$u = cq + \theta, \tag{9.32}$$

where c is a private good, q is a public good that can be shared if two people marry but is private if they remain single, and θ is the emotional gain from being married (relative to remaining single). The household public good is produced according to a household production function

$$q = e + \gamma t, \tag{9.33}$$

where e denotes purchased market goods, t is time spent working at home and γ is an efficiency parameter that is assumed to be independent of schooling.[10]

This specification implies transferable utility between spouses and allows us to trace the impact of different market wages or household roles on the decisions to invest and marry. Time worked at home is particularly important for parents with children. To simplify, we assume that all married couples have one child and that rearing it requires a specified amount of time $t = \tau$, where τ is a constant such that $0 \leq \tau < 1$. Initially, we shall assume that, due to social norms, all the time provided at home is supplied by the mother. Also, individuals who never marry have no children and for them we set $\tau = 0$.[11]

If man i of class I with wage $w_{I(i)}^m$ marries woman j of class J with wage $w_{J(j)}^w$, their joint income is $w_{I(i)}^m + (1-\tau)w_{J(j)}^w$. Any efficient allocation

[10] A plausible generalization is to allow the mother's schooling level to affect positively child quality. This would be consistent with the findings of Behrman (1997) and Glewwe (1999), for example. However, the qualitative results will be unaffected as long as schooling has a larger effect on market wages than on productivity at home. The fact that educated women participate more in the labor market than uneducated women supports such an assumption.

[11] We make no distinction here between cohabitation and marriage. So either no one cohabits, or, if two individuals cohabit, they behave as a married couple.

of the family resources maximizes the partners' sum of utilities given by $[w^m_{I(i)} + (1-\tau)w^w_{J(j)} - e](e + \tau\gamma) + \theta_i + \theta_j$. In an interior solution with a positive money expenditure on the public good, the maximized material output is

$$\zeta_{ij} = \frac{[w^m_{I(i)} + \tau\gamma + (1-\tau)w^w_{J(j)}]^2}{4}. \tag{9.34}$$

Note that the wages of the husband and wife complement each other in generating marital output, which is a consequence of sharing the public good.[12]

An unmarried man i solves

$$\underset{e_i,c_i}{Max}\ \ c_i e_i \tag{9.35}$$

subject to

$$c_i + e_i = w^m_{I(i)}, \tag{9.36}$$

and his optimal behavior generates a utility level of $\zeta_{i0} = (w^m_{I(i)}/2)^2$. A single woman j solves an analogous problem and obtains $\zeta_{0j} = (w^w_{J(j)}/2)^2$. Therefore, the total *marital surplus* generated by the marriage in the second period is

$$\begin{aligned} s_{ij} &= \frac{[w^m_{I(i)} + \tau\gamma + (1-\tau)w^w_{J(j)}]^2 - (w^m_{I(i)})^2 - (w^w_{J(j)})^2}{4} + \theta_i + \theta_j \\ &\equiv z_{I(i)J(j)} + \theta_i + \theta_j. \end{aligned} \tag{9.37}$$

The surplus of a married couple arises from the fact that married partners jointly consume the public good. If the partners have no children and $\tau = 0$, the gains arise solely from the pecuniary expenditures on the public good. In this case, the surplus function is *symmetric* in the wages of the two spouses. If the couple has a child, however, and the mother takes care of it, then the mother's contribution to the household is a weighted average of her market wage and productivity at home. We assume that $w^w_2 > \gamma > w^w_1$ so that having children is costly for educated women but not for uneducated women. The surplus function in (9.37) maintains complementarity between

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[12] The first-order condition for e is

$$[w^m_{I(i)} + (1-\tau)w^w_{J(j)} - e] - (e + \tau\gamma) \le 0.$$

Hence, $e = [w^m_{I(i)} + (1-\tau)w^w_{J(j)} - \tau\gamma]\ /\ 2$ in an interior solution. The maximized material output in this case is $[w^m_{I(i)} + \tau\gamma + (1-\tau)w^w_{J(j)}]^2\ /\ 4$. If $e = 0$, the maximal material output is $[w^m_{I(i)} + (1-\tau)w^w_{J(j)}]\tau\gamma$, which would imply an additive surplus function, contradicting our assumption of complementarity. A sufficient condition for a positive e is $w^m_1 + (1-\tau)w^w_1 > \tau\gamma$ if the wife works at home and $w^w_1 + (1-\tau)w^m_1 > \tau\gamma$ if the husband works at home. We assume hereafter that these conditions hold.

the wages of the husband and wife, which is a consequence of sharing the public good. However, the assumed asymmetry in household roles between men and women implies that a higher husband's wage always raises the surplus but a higher mother's wage can reduce the surplus. In other words, it may be costly for a high-wage woman to marry and have a child because she must spend time on child care, while if the mother does not marry, her utility as a single remains $w^2_{J(j)}/4$. In addition, it is no longer true that $z_{21} = z_{12}$.[13]

Since we have assumed here that, due to social norms, all the time provided at home is supplied by the mother, all the gains from marriage arise from sharing a public good and the wages of the partners complement each other so that $z_{11} + z_{22} > z_{12} + z_{21}$. In later sections, we discuss endogenous specialization whereby couples act efficiently and the partner with lower wage works at home. For sufficiently low time requirements, that is, τ close to 0, complementarity continues to hold. However, for τ close to 1, the wages of the two partners become substitutes, that is, $z_{11} + z_{22} < z_{12} + z_{21}$, because wage differentials between spouses increase the gain from specialization (see Becker, 1991, ch. 2). Thus, whether couples act efficiently or according to norms influences the equilibrium patterns of assortative mating.[14]

9.3.2 The Impact of the Wage Gap

We are now ready to examine the implications of gender wage differences. The gender difference in wages can be an outcome of discrimination associated, for instance, with fewer opportunities for investment on the job. Such discrimination can reduce or increase the incentives of women to invest, depending on whether discrimination is stronger at the low or high levels of schooling.

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[13] For instance, when the wages of men and women are equal but $\tau > 0$, we have

$$z_{21} - z_{12} = \frac{\tau(w_2 - w_1)}{2}[(1-\tau)\frac{w_2 + w_1}{2} + \tau\gamma] > 0.$$

[14] For fixed household roles, the second cross derivative of the surplus function with respect to wages is positive, implying complementarity. But with endogenous household roles, the relevant measure of complementarity is embedded in the *maximized* marital gains that can change discontinuously as household roles change. Suppose that $w^m_2 > w^w_2 > w^m_1$. Let

$$\begin{aligned} f(\tau) &\equiv 4(z_{11} + z_{22} - z_{12} - z_{21}) \\ &= [w^m_1 + \tau\gamma + (1-\tau)w^w_1]^2 + [w^m_2 + \tau\gamma + (1-\tau)w^w_2]^2 \\ &\quad -[w^w_2 + \tau\gamma + (1-\tau)w^m_1)]^2 - [w^m_2 + \tau\gamma + (1-\tau)w^w_1]^2. \end{aligned}$$

Then, $f(\tau) > 0$ if $\tau = 0$ and $f(\tau) < 0$ if $\tau = 1$, where $\forall\ \tau \in [0, 1]$, $f'(\tau) < 0$.

Define the (relative) wage gap among educated individuals as $d_2 = w_2^w/w_2^m$ and let the gender wage gap between uneducated individuals be $d_1 = w_1^w/w_1^m$. Starting from the benchmark equilibrium with strictly positive assortative mating and equal shares (point e in Figure 9.4), we examine the impact of a difference in the market returns from schooling of women and men. Specifically, we consider an increase in the wage of educated men, w_2^m, combined with a reduction in the wage of educated women, w_2^w, holding the wage of uneducated men at the benchmark value, w_1. To isolate the role of market returns, we assume that the increase in the wage of educated men exactly compensates the reduction in the wage of educated women so that marital output is unaffected and symmetry is maintained.[15] In other words, the change in wages affect directly only the returns as singles, R^m and R^w. For now, we assume that discrimination is uniform across schooling levels so that $d_1 = d_2 \equiv d < 1$ and women have a lower market return from schooling investment than men.[16] Later, we shall discuss a case in which discrimination against educated women is weaker so that $d_1 < d_2 < 1$.

With uniform discrimination, the returns to investment in schooling for never married men and women, respectively, are

$$R^m = z_{20}^m - 2z_{10}^m = (\frac{w_2^m}{2})^2 - 2(\frac{w_1^m}{2})^2, \tag{9.38}$$

and

$$R^w = z_{02}^w - 2z_{01}^w = (\frac{w_2^w}{2})^2 - 2(\frac{w_1^w}{2})^2 = d^2 R^m < R^m. \tag{9.39}$$

The higher market return from schooling of men encourages their investment in schooling and also strengthens their incentives to marry, because schooling obtains an additional return within marriage. In contrast, the lower return to schooling for women reduces their incentives to invest and marry. These changes create excess supply of men who wish to invest and marry. Consequently, to restore equilibrium, the rates of returns that men receive within marriage must decline implying that, for any V_1, the value of V_2 that satisfies conditions (9.22) and (9.22) must decline. These shifts in the equilibrium lines are represented by the broken blue and red lines in Figure 9.4.

[15] When wages change $z_{I(i)J(j)}$ usually changes. Also, when wages differ by gender, we generally do *not* maintain symmetry in the contribution of men and women to marriage so that $z_{12} \neq z_{21}$. It is only in the special case in which the product $w_{I(i)}^m w_{J(j)}^w$ remains invariant under discrimination that the marital surplus generated by all marriages is intact. The qualitative results for shares are not affected by this simplification.

[16] In standard human capital models where the only cost of investment is forgone earnings and the only return is higher future earnings, uniform discrimination has *no* impact on investment. In this model, however, the absolute market returns are added to the returns within marriage, which together determine investment decisions (see equations (16) and (17)). Therefore, the absolute market returns to schooling matter in our model.

For moderate changes in wages, strictly positive assortative mating continues to hold. However, the equilibrium value of V_2 declines and educated men receive a *lower* share of the surplus than they do with equal wages in any marriage. That is, as market returns of men rise and more men wish to acquire education, the marriage market response is to reduce the share of educated men in *all* marriages. When the gap between R^m and R^w becomes large, the equilibrium shifts to a mixed equilibrium, where some educated men marry uneducated women. That is, because of their higher tendency to invest, some educated men must "marry down." This equilibrium is represented by the point $e\prime$ in Figure 9.4, where the broken red line representing equality in the numbers of men and women that wish to marry (condition (9.22)) intersects the green line representing the lower bound on the share that educated men obtain in the marital surplus, $z_{21} - z_{11}$. As seen, *both* V_1 and V_2 are lower in the new equilibrium so that *all* men (women), educated and uneducated, receive lower (higher) shares of the material surplus when men have stronger market incentives to invest in schooling than women.

These results regarding the shares of married men and women in the material surplus must be distinguished from the impact of the shares in the material output. If men get a higher return from schooling as singles (due to the fact that their labor-market return from schooling is higher than that of women), then their share of the material output can be higher even though they receive a lower share of the surplus. The same remark applies to our subsequent analysis as well; one can obtain sharper comparative static results on shares of the material surplus than those on shares of the material output.

9.3.3 The Impact of Household Roles

Recall that we assume that the wife alone spends time on child care. To investigate the impact of this constraint, we start again at the benchmark equilibrium and examine the impact of an increase in τ, holding the wages of men and women at their benchmark values, that is $w_1^m = w_1^w = w_1$ and $w_2^m = w_2^w = w_2$. Such an increase reduces the contribution that educated women make to marital output and raises the contribution of uneducated women. That is, z_{11} and z_{21} rise because uneducated women are more productive at home, $\gamma > w_1$, while z_{12} and z_{22} decline because educated women are less productive at home, $\gamma < w_2$. Consequently, both equilibrium lines corresponding to conditions (9.22) and (9.23) shift down so that V_2 is lower for any V_1. At the same time, the boundaries on the rate of return from schooling that men can obtain within marriage shift as $z_{21} - z_{11}$ rises and $z_{22} - z_{12}$ declines. These changes are depicted in Figure 9.5.

For moderate changes in τ, strictly positive assortative mating with equal sharing continues to hold. As long as a symmetric equilibrium is maintained, the returns to schooling that men and women receive within marriage, V_2-V_1 and U_2-U_1, are equal. Hence, men and women have the same incentives to invest. But because the material surplus (and consequently utilities within marriage) of educated men and women, $z_{22}/2$, declines with τ, while the material surplus of uneducated men and women, $z_{11}/2$, rises, both men and women will reduce their investments in schooling by the same degree.

As τ rises further, the difference in the contributions of men and women to marriage can rise to the extent that an educated man contributes to a marriage with uneducated woman more than an educated woman contributes to a marriage with an educated man.[17] That is,

$$z_{21} - z_{11} > z_{22} - z_{21}. \tag{9.40}$$

Condition (9.40) implies that the *lower* bound on the return to schooling that men receive within marriage exceeds the *upper* bound on the return to schooling that women receive within marriage. In this event, the symmetric equilibrium in Figure 9.5 is eliminated and instead there is a mixed equilibrium with some educated men marrying uneducated women (point $e\prime$ in Figure 9.5). This outcome reflects the lower incentive of educated women to enter marriage and the stronger incentive of men to invest because their return from schooling within marriage, $V_2-V_1 = z_{21}-z_{11}$, exceeds the return to schooling that women can obtain within marriage. Consequently, some educated men must "marry down" and match with uneducated women.

9.3.4 Division of Labor and Career Choice

We can further refine the family decision problem by letting the partners decide who shall take care of the children. Reinterpreting τ as a temporal choice, imagine that one of the partners must first spend τ units of time on the child and later enter the labor market and work for the remainder of the period (length $1-\tau$).

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[17] Consider the expression

$$\begin{aligned} h(w_1, w_2, \tau) \ &\equiv 2z_{21} - z_{11} - z_{22} = 2[w_2 + \tau\gamma + (1-\tau)w_1]^2 \\ &-[w_1 + \tau\gamma + (1-\tau)w_1]^2 - [w_2 + \tau\gamma + (1-\tau)w_2]^2 \end{aligned}$$

as a function of w_1 and w_2 and τ. For $w_1 = w_2 = \gamma$, $h(\gamma,\gamma,\tau) = 0$ and

$$\begin{aligned} h_1(\gamma,\gamma,\tau) &= -4\gamma\tau, \\ h_2(\gamma,\gamma,\tau) &= 4\gamma\tau. \end{aligned}$$

Therefore, for a positive τ, w_1 slightly below γ and w_2 slightly above γ, $h(w_1, w_2, \tau) > 0$. Also

$$h_3(w_1, w_2, \tau) = (w_2 - w_1)[w_2(4-2\tau) + 2\tau(2\gamma - w_1)] > 0$$

and for all $w_2 > \gamma > w_1$, $h(w_1, w_2, 0) < 0$ and $h(w_1, w_2, 1) > 0$. Hence, the larger is τ the broader will be the range in which $h(w_1, w_2, 0) > 0$.

An important idea of Becker (1991, ch. 2) is that wage differences among identical spouses can be created endogenously and *voluntarily* because of learning by doing and increasing returns. Thus, it may be *optimal* for the household for *one* of the spouses to take care of the child and for the other to enter the labor market immediately, thereby generating a higher wage in the remainder of the period. Thus, by choosing schooling ahead of marriage one can influence his\her household role within marriage.

Because we assume transferable utility between spouses, household roles will be determined efficiently by each married couple, as long as there is ability to commit to a transfer scheme, whereby the party that sacrifices outside options when he\she acts in a manner that raises the total surplus is compensated for his/her action. In particular, the partners will assign the spouse with the lower wage to take care of the child. In the previous analysis, there was no need for such a commitment because the division of the surplus was fully determined by attributes that were determined prior to marriage via competition over mates who could freely replace partners. However, if time spent on child care affects one's labor market wages subsequently, the cost of providing childcare can differ between the two spouses. Thus, implementing the efficient outcome might require some form of commitment even if (re)matching is frictionless. A simple, enforceable, prenuptial contract is one in which both partners agree to pay the equilibrium shares V_I to the husband and U_J to the wife in case of divorce. By making those shares the relevant threat points of each spouse, this contract sustains the equilibrium values V_I and U_J in marriage, which is sufficient to attain the efficient household division of labor.

If there is discrimination against women and they receive lower market wages than men, then the wife will be typically assigned to stay at home, which will erode her future market wage and reinforce the unequal division of labor. Similarly, if there are predetermined household roles such that women must take care of their child, then women will end up with lower market wages. Thus, inequality at home and the market are interrelated.[18] Models of statistical discrimination tie household roles and market wages through employers' beliefs about female participation. Typically, such models generate multiple equilibria and inefficiency (Hadfield, 1999, Lommerud and Vagstad, 2000). Here, we do not require employers' beliefs to be correct. Instead, we think of household roles and discrimination as processes that evolve slowly and can be taken as exogenous in the medium run.

[18] Related papers that emphasize the dual-feedback mechanism between the intensity of home work and labor market wages are Albanesi and Olivetti (2009) and Chichilnisky (2005).

9.3.5 Why Women May Acquire More Schooling than Men

We have examined two possible reasons why women may invest less than men in schooling. The first is that women may receive lower return from schooling investment in the market because of discrimination. The second reason is that women may receive a lower return to schooling in marriage because of the need to take care of children (due to social and cultural norms or the biological time requirements of child care).

Over time, fertility has declined and women's wages have risen in industrialized countries, a pattern being replicated in many developing countries too. This is consistent with increased investment in education by women. The fact that women are now slightly more educated than men, on average, appears surprising given the fact that women still earn substantially less than men. However, in dealing with investments in education, the crucial issue is whether the gender wage gap rises or declines with schooling, or equivalently, whether women obtain a higher rate of return from schooling. There is some evidence that this is indeed the case and that the gender wage gap declines with schooling (see Chiappori *et al*, 2009 and Dougherty, 2005).

Now consider a comparison of the following two situations. An "old" regime in which married women must spend a relatively large fraction of their time at home and a "new" regime in which, because of reductions in fertility and improved technology in home production, married women spend less time at home and work more in the market (see chapter 1 tables 8a and 8b).[19] Assume further that women suffer from statistical discrimination because employers still expect them to invest less on the job. However, this discrimination is weaker against educated women because they are expected to stay longer in the labor market than uneducated women. Finally, assume that in the old regime norms were relevant but in the new regime the roles are determined efficiently (for some evidence, see Chiappori *et al*, 2009). It is then possible that in the new regime women will invest in schooling *more* than men. The presence of discrimination *raises* the return of women relative to men because schooling serves as an instrument for women to escape discrimination. The fact that women are still tied up in home work lowers their return from schooling relative to men because women obtain lower returns from schooling within marriage. However, as women raised their labor force participation due to technological changes or break of norms, this second effect weakens and the impact of discrimination can dominate.

[19] Greenwood et al.(2005) and Fernandez (2007) discuss the impact of technological advance and change in norms on the rise in female participation. Mulligan and Rubinstein (2008) emphasize the role of higher rewards for ability (reflected in the general increase in wage inequality) in drawing married women of high ability into the labor market.

In Figure 9.6, we display the transition between the two regimes. We assume that $d_2 > d_1$ so that discrimination against women is lower at the higher level of schooling. This feature generates stronger incentives for women than men to invest in schooling. However, the fact that women must spend time working at home has the opposite effect. We then reduce the amount of time that the mother has to spend at home, τ, and raise the wage that educated women receive (so that d_2 rises), which strengthens the incentives of women to invest in schooling and to marry. Therefore, holding the marriage surplus z_{IJ} constant, an increase in V_2 relative to V_1 is required to maintain equality between the number of men who wish to invest and marry and the number of women who wish to invest and marry. This effect is represented by the upwards shifts in the broken red and blue lines in Figure 9.6.[20] The impact is assumed to be large enough to generate an equilibrium in which the two equilibrium requirements – equality of the numbers of men and women who acquire no schooling and marry (the broken blue line) and equality of the total numbers of men and women who wish to marry (the broken red line) – yield an intersection above the upper bound on the returns from schooling that men can receive within marriage. Therefore, strictly positive assortative mating cannot be sustained as an equilibrium and the outcome is a mixed equilibrium in which there are more educated women than men among the married and some educated women marry uneducated men. This new mixed equilibrium is indicated by the point e'' in Figure 9.6.

9.4 A Numerical Example

Suppose that μ and θ are uniformly and independently distributed. Although wages vary across the two regimes, we assume that in both regimes, educated women are more productive in the market and uneducated women are more productive at home. We further assume that in both regimes, men earn more than women with the same schooling level but educated women earn more than uneducated men. Finally, in both regimes, women have a higher market return from schooling. The transition from the old regime to the new regime is characterized by three features: (i) productivity at home is higher and women are required to work less at home; (ii) men and women obtain higher market returns from schooling; and (iii) couples move from a traditional mode to an efficient one in which the high-wage spouse works in the market.

All the above economic changes raise the gains from marriage and would

[20]Because the marital surplus matrix, z_{IJ}, also changes, the equilibrium curves did not shift up. In fact, for the parameters of Figure 9.6, there is a range over which the equilibrium line representing market-clearing in the marriage market shifts down. This, however, has no bearing on the equilibrium outcome.

cause higher marriage rates. To calibrate the model, we assume that the variance in the preference for marriage rises over time which, other things being the same, reduces the propensity to marry. We thus assume that in both periods μ is distributed over the interval $[-4, 4]$, while θ is distributed over the intervals $[-4, 4]$ and $[-8, 8]$ in the old and the new regimes, respectively. It is important to note that the shift in the distribution of θ has *no* impact on the equilibrium surplus shares, which are our main concern. However, it changes the proportion of individuals who invest and marry given these shares. Table 9.1 reflects these assumptions.

Table 9.1: Parameters in the old and the new regimes

Parameter	Old Regime	New Regime
Wage of uneducated men	$w_1^m = 2$	$w_1^m = 2.375$
Wage of uneducated women	$w_1^w = 1.2$	$w_1^w = 1.425$
Wage of educated men	$w_2^m = 3$	$w_2^m = 4.0$
Wage of educated women	$w_2^w = 2.4$	$w_2^w = 3.2$
Wage difference among the uneducated	$d_1 = .6$	$d_1 = .6$
Wage difference among the educated	$d_2 = .8$	$d_2 = .8$
Market return to schooling, men	$R^m = .25$	$R^m = 1.18$
Market return to schooling, women	$R^w = .72$	$R^w = 1.54$
Work requirements	$\tau = .8$	$\tau = .3$
Productivity at home	$\gamma = 2$	$\gamma = 2.5$
Distribution of tastes for schooling	$[-4, 4]$	$[-4, 4]$
Distribution of tastes for marriage	$[-4, 4]$	$[-8, 8]$
Norms	Wife at home	Efficient

The marriage market implications of these changes are summarized in Tables 9.2-9.5 below.

Table 9.2: Impact of parameter changes on marital surplus

Old regime

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 1.72$
Educ. husband	$z_{21} = 3.25$	$z_{22} = 2.76$

New Regime

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 3.90$
Educ. husband	$z_{21} = 3.75$	$z_{22} = 5.66$

A decrease in the amount of time worked at home, raises the contribution of an educated woman to the material surplus and lowers the contribution of an uneducated woman. Therefore, in the old regime with $\tau = .8$, the material surplus *declines* with the education of the wife when the husband is uneducated, while in the new regime with $\tau = .3$, it rises. This happens because educated women are more productive in the market than uneducated women but, by assumption, equally productive at home. In the old regime, if an educated wife would marry an uneducated man (which does not happen in equilibrium) she would be assigned to household work even though she has a higher wage than her husband. In the new regime, couples act efficiently, household roles are reversed and educated women *do* marry uneducated men. Note that for couples among whom both husband and wife are uneducated, the wife continues to work at home in the new regime, because she has the lower wage. The parameters are chosen in such a way that technology has no impact on the marital surplus of such couples. In the new regime, uneducated women work less time at home but their productivity at home is higher as well as the wage that they obtain from work.

Table 9.3: Impact of parameter changes on the equilibrium shares

Old regime

	Uneducated	Educated
Men	$V_1 = .76$	$V_2 = 1.68$
Women	$U_1 = 1.57$	$U_2 = 1.09$

New Regime

	Uneducated	Educated
Men	$V_1 = 1.13$	$V_2 = 2.88$
Women	$U_1 = 1.20$	$U_2 = 2.78$

Compared with the old regime, educated women receive a higher share of the marital surplus in the new regime, while uneducated women receive a lower share. These changes reflect the higher (lower) contributions to marriage of educated (uneducated) women. The marital surplus shares of both educated and uneducated men rise as a consequence of the rising productivity of their wives.

The implied returns from schooling within marriage in the old regime are

$$U_2 - U_1 = 1.09 - 1.57 = z_{22} - z_{21} = 2.76 - 3.25 = -.49 ,$$
$$V_2 - V_1 = 1.68 - .76 = z_{21} - z_{11} = 3.25 - 2.33 = .92 .$$

That is, men receive the lower bound on their return from schooling within marriage while women receive the upper bound on their return from schooling. This pattern is reversed in the new regime:

$$U_2 - U_1 = 2.78 - 1.20 = z_{12} - z_{11} = 3.90 - 2.33 = 1.58,$$
$$V_2 - V_1 = 2.88 - 1.13 = z_{22} - z_{12} = 5.66 - 3.90 = 1.75,$$

where women receive their lower bound and men receive their upper bound. Both men and women receive a higher return from schooling within marriage in the new regime, reflecting the increased efficiency although the rise for women is much sharper.

Table 9.4: Impact of parameter changes on the investment and marriage rates*

Old Regime

	Married	Unmarried	All
Educ.	.452, .335	.153, .215	.606, .550
Uned.	.211, .323	.183, .122	.394, .450
All	.662, .666	.334, .334	1

New Regime

	Married	Unmarried	All
Educ.	.577, .590	.207, .226	.784, .816
Uned.	.077, .063	.139, .121	.216, .184
All	.653, .653	.347, .347	1

∗ First and second entries in each cell refer to men and women resp.

In the old regime, more men invest in schooling than women and some educated men marry down to match with uneducated women. This pattern is reversed in the new regime and women invest in schooling more than men and some educated women marry down to join uneducated men. That is, women increase their investment in schooling more than men. Although market returns have risen for both men and women, the returns for schooling within marriage have risen substantially more for women. The basic reason for that is the release of married women from the obligation to spend most of their time at home, due to the reduction in the time requirement of child care and the change in norms that allow educated women who are married to uneducated men to enter the labor market. Uneducated men gain a higher share in the surplus in all marriages because of their new opportunity to marry educated women, while uneducated women lose part of their share in the marital surplus in all marriages because they no longer marry educated men. Notice that the proportion of educated women who remain single declines from $.215/.550 = 0.39$ to $226/.816 = 0.28$ in the new regime. In contrast, the proportion of educated men who marry remains roughly the same, $0.153/0.606 = 0.28$ and $0.207/0.784 = 0.26$ in the old and new regimes, respectively. This gender difference arises because, under the old regime, women were penalized in marriage by being forced to work at home.

We can use these examples to discuss the impact of norms. To begin with, suppose that in the old regime couples acted efficiently and, if the wife was more educated than her husband, she went to work full time and the husband engaged in child care. Comparing Tables 9.2 and 9.5, we see that the impact of such a change on the surplus matrix is only through the rise in z_{12}. Because women receive lower wages than men at all levels of schooling, the household division of labor is not affected by the norms for couples with identically educated spouses; for all such couples, the husband works in the market and the wife takes care of the child. However, the norm does affect the division of labor for couples among whom the wife has a higher education level than her husband. This is due to our assumptions that educated women have a higher wage than uneducated men in the labor market and their market wage exceeds their productivity at home. In contrast to the case in which the mother always works at home, we see in Table 9.5 that the education levels now become substitutes, namely $z_{11}+z_{22} < z_{12}+z_{21}$, implying that we can no longer assume that there will be some educated men married to educated women and some uneducated men married to uneducated women. More specifically, an educated woman contributes more to an uneducated man than she does to an educated man (that is $z_{12} - z_{11} > z_{22} - z_{21}$) so that uneducated men can bid away the educated women from educated men. Thus changes in norms can influence the patterns of assortative mating.

Table 9.5: Impact of norms on material surplus

Old regime, efficient

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 2.40$
Educ. husband	$z_{21} = 3.25$	$z_{22} = 2.76$

New Regime with norms

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 3.23$
Educ. husband	$z_{21} = 3.75$	$z_{22} = 5.66$

Consider, next, the possibility that the norms persist also in the new regime and the mother must work at home even if she is more educated than her husband. Again, the norm bites only in those marriages in which the wife is more educated than the husband. In the new regime, positive assortative mating persists independently of the norms. However, the mixing equilibrium in which some educated women marry uneducated men is replaced by strict assortative mating in which educated men marry only educated women and uneducated men marry only uneducated women. Thus, again, norms can have a qualitative impact on the type of equilibrium that emerges.

The new marriage and investment patterns are presented in the lower panel of Table 9.6. The main difference is that educated women are less likely to marry when the norms require them to work at home, where they are relatively less efficient.

Table 9.6: Impact of norms on investment and marriage rates (new regime)*

Efficient work pattern

	Married	Unmarried	All
Educ.	.577, .589	.207, .126	.784, .816
Uned.	.077, .063	.139, .121	.216, .184
All	.653, .653	.347, .347	1

Wife work pattern

	Married	Unmarried	All
Educ.	.583, .583	.207, .227	.790, .810
Uned.	.070, .070	.140, .120	.210, .190
All	.653, .653	.347, .347	1

* The first and second entry in each cell refer to men and women resp.

Consider, finally, the impact on the shares in the material surplus when norms are replaced by an efficient allocation in the new regime (see Table 9.7). The removal of social norms that the wife must work at home benefits uneducated men and harms uneducated women. This example illustrates the differences between the predictions of general equilibrium models with frictionless matching, like the one we present here, and partial equilibrium models that rely on bargaining. The latter would predict that no woman would lose from the removal of norms that forces women in general to stay at home and take care of the child, but as this example demonstrates, the market equilibrium can change and uneducated women are hurt because they can no longer marry with educated men.

Table 9.7: Impact of norms on the equilibrium shares in the new regime

Efficient pattern of work

	Uneducated	Educated
Men	$V_1 = 1.13$	$V_2 = 2.89$
Women	$U_1 = 1.20$	$U_2 = 2.78$

Wife always works at home

	Uneducated	Educated
Men	$V_1 = 1.06$	$V_2 = 2.89$
Women	$U_1 = 1.28$	$U_2 = 2.77$

9.5 Conclusions

In standard models of human capital, individuals invest in schooling with the anticipation of being employed at a higher future wage that would compensate them for the current foregone earnings. This chapter added another consideration: the anticipation of being married to a spouse with whom one can share consumption and coordinate work activities. Schooling has an added value in this context because of complementarity between agents, whereby the contribution of the agents' schooling to marital output rises with the schooling of his\her spouse. In the frictionless marriage market considered here, the matching pattern is fully predictable and supported by a unique distribution of marital gains between partners. Distribution is governed by competition because for each agent, there exists a perfect substitute that can replace him\her in marriage. There is thus no scope for bargaining and, therefore, premarital investments are efficient.

We mentioned two interrelated causes that may have diminish the incentives of women to invest in schooling in the past: lower market wages and larger amount of household work. With time, the requirement for wives to stay at home have relaxed and discrimination nay have decreased too but probably not to the same extent[21]. Although we did not fully specify the sources of discrimination against women in the market, we noted that such discrimination tends to decline with schooling, which strengthens the incentive of women to invest in schooling. This is a possible explanation for the slightly higher investment in schooling by women that we observe today. We do not view this outcome as a permanent phenomenon but rather as a part of an adjustment process, whereby women who now enter the labor market in increasing numbers, following technological changes at home

[21] Whether discrimination has declined is debated; see Mulligan and Rubinstein (2008).

and in the market that favor women, must be "armed" with additional schooling to overcome norms and beliefs that originate in the past.

We should add that there are other possible reasons for why women may invest in schooling more than men. One reason is that there are more women than men in the marriage market at the relatively young ages at which schooling is chosen, because women marry younger. Iyigun and Walsh (2007) have shown, using a similar model to the one discussed here, that in such a case women will be induced to invest more than men in competition for the scarce males. Another reason is that divorce is more harmful to women, because men are more likely to initiate divorce when the quality of match is revealed to be low. This asymmetry is due to the higher income of men and the usual custody arrangements (see Chiappori and Weiss, 2007). In such a case, women may use schooling as an insurance device that mitigates their costs from unwanted divorce.

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9.7 Appendix: Existence and Uniqueness of Equilibrium

Substitute $z_{11} - V_1$ for U_1 and $z_{22} - V_2$ for U_2 in equation (21), and define $\Psi(V_1, V_2)$ as

$$\begin{aligned}\Psi(V_1, V_2) \quad \equiv \quad & F(V_1) + \int_{V_1}^{V_2} G(R^m + V_2 - \theta) f(\theta) d\theta \\ & -F(z_{11} - V_1) - \int_{z_{11}-V_1}^{z_{22}-V_2} G(R^w + z_{22} - V_2 - \theta) f(\theta) d\theta \ .\end{aligned} \tag{A1}$$

Note, first, that

$$\Psi(0,0) = F(0) - F(z_{11}) - \int_{z_{11}}^{z_{22}} G(R^w + z_{22} - \theta) f(\theta) d\theta < 0 \tag{A2}$$

and that

$$\Psi\left(z_{11}, z_{22}\right) \equiv F\left(z_{11}\right) - F\left(0\right) + \int_{z_{11}}^{z_{22}} G(R^m + z_{22} - \theta) f(\theta) d\theta > 0 \ , \tag{A3}$$

since $z_{11} > 0$ implies that $F\left(z_{11}\right) - F\left(0\right) > 0$. By continuity, we conclude that there exists a set of couples (V_1, V_2) for which $\Psi\left(V_1, V_2\right) = 0$.

In addition, we have

$$\begin{aligned}\frac{\partial \Psi(V_1, V_2)}{\partial V_1} &= f(V_1)[1 - G(R^m + V_2 - V_1)] \\ & \quad + f(z_{11} - V_1)[1 - G(R^w + z_{22} - z_{11} - (V_2 - V_1)] > 0\end{aligned} \tag{A4}$$

and

$$\begin{aligned}\frac{\partial \Psi(V_1, V_2)}{\partial V_2} &= G(R^m) f(V_2) + G(R^w) f(z_{22} - V_2)] \\ & \quad + \int_{V_1}^{V_2} g(R^m + V_2 - \theta) f(\theta) d\theta + \int_{U_1}^{U_2} g(R^w + U_2 - \theta) f(\theta) d\theta > 0 \ .\end{aligned} \tag{A5}$$

By the implicit function theorem, $\Psi(V_1, V_2) = 0$ defines V_2 as a differentiable, decreasing function of V_1 over some open set in $\mathbb{R}$. Equivalently, the locus $\Psi(V_1, V_2) = 0$ defines a smooth, decreasing curve in the (V_1, V_2) plane.

Using (22), define $\Omega(V_1, V_2)$ as

$$\begin{aligned} \Omega(V_1, V_2) \equiv\ & F(V_1)\left[1 - G(R^m + V_2 - V_1)\right] \\ & - F(z_{11} - V_1)\left[1 - G(R^w - z_{11} + V_1 + z_{22} - V_2)\right]. \end{aligned} \tag{A6}$$

Note that Ω is continuously differentiable, increasing in V_1 and decreasing in V_2. Moreover,

$$\begin{aligned} \lim_{V_1 \to \infty} \Omega(V_1, V_2) &= 1, \\ \lim_{V_2 \to \infty} \Omega(V_1, V_2) &= -F(z_{11} - V_1) < 0. \end{aligned} \tag{A7}$$

By continuity, there exists a locus on which $\Omega(V_1, V_2) = 0$; by the implicit function theorem, it is a smooth, increasing curve in the (V_1, V_2) plane. In addition,

$$\Omega(V_1, V_2) = A(V_1, V_2 - V_1), \tag{A8}$$

where

$$A(V, X) = F(V)\left[1 - G(R^m + X)\right] - F(z_{11} - V)\left[1 - G(R^w - z_{11} + z_{22} - X)\right]. \tag{A9}$$

Since

$$\frac{\partial A(V, X)}{\partial V} = f(V)\left[1 - G(R^m + X)\right] + f(z_{11} - V)\left[1 - G(R^w - z_{11} + z_{22} - X)\right] > 0 \tag{A10}$$

and

$$\frac{\partial A(V, X)}{\partial X} = -F(V)\, g(R^m + X) - F(z_{11} - V)\, g(R^w - z_{11} + z_{22} - X) < 0, \tag{A11}$$

the equation $A(V, X) = 0$ defines X as some increasing function ϕ of V. Therefore,

$$\Omega(V_1, V_2) = A(V_1, V_2 - V_1) = 0 \tag{A12}$$

gives

$$V_2 = V_1 + \phi(V_1), \tag{A13}$$

where $\phi'(V) > 0$. Thus in the (V_1, V_2) plane, the slope of the $\Omega(V_1, V_2) = 0$ curve is always more than 1. In particular, the curve must intersect the decreasing curve $\Psi(V_1, V_2) = 0$, and this intersection (V_1^*, V_2^*) is unique.

Finally, stability requires that

$$U_1 + V_2 \geq z_{21} \qquad \text{and} \qquad U_2 + V_1 \geq z_{12} \tag{A14}$$

which implies that, at any stable match, we have

$$z_{21} - z_{11} \leq V_2 - V_1 \leq z_{22} - z_{12}, \tag{A15}$$

and

$$z_{12} - z_{11} \leq U_2 - U_1 \leq z_{22} - z_{21}. \tag{A16}$$

Three cases are thus possible:

1. If $z_{21} - z_{11} \leq V_2^* - V_1^* \leq z_{22} - z_{12}$, then (V_1^*, V_2^*) is the unique equilibrium (see figure A.1).

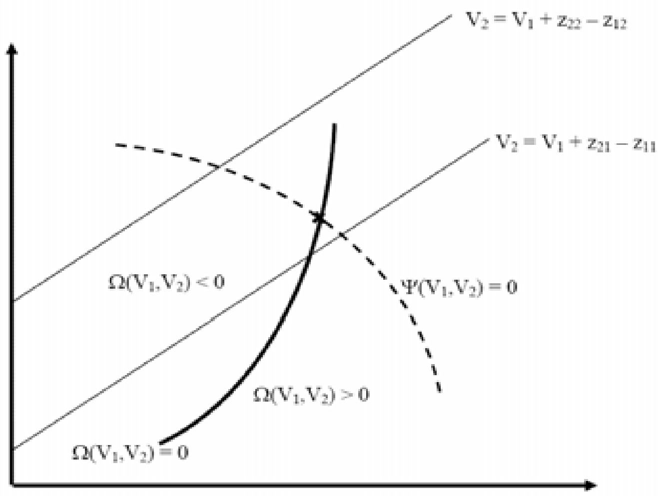

Figure A1

Indeed, it is the only equilibrium with perfectly assortative matching. Moreover, a point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{21} - z_{11} \tag{A17}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) > 0$, which contradicts the fact that the number of educated men should exceed that of educated women for such an equilibrium to exist. Similarly, a point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{22} - z_{12} \tag{A18}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) < 0$, which contradicts the fact that the number of educated women should exceed that of educated men for such an equilibrium to exist.

2. If $z_{21} - z_{11} > V_2^* - V_1^*$, then the unique equilibrium (see figure A.2) is such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{21} - z_{11}. \tag{A19}$$

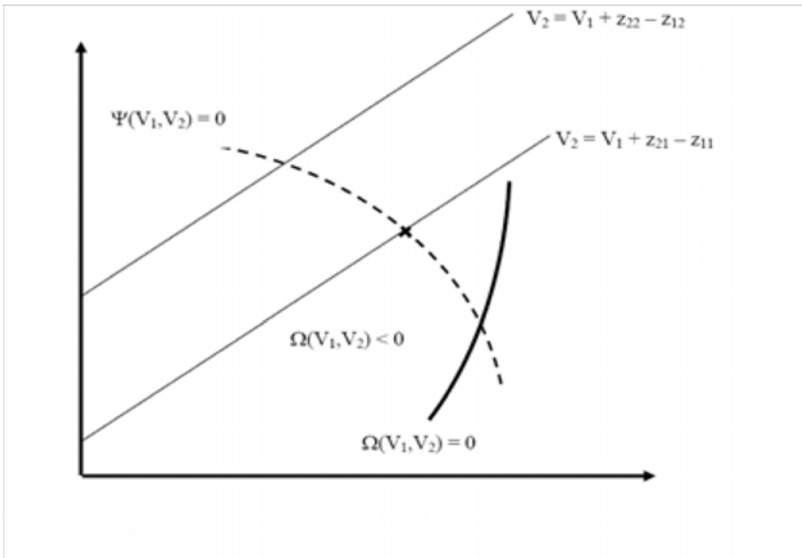

Figure A2

Indeed, a perfectly assortative matching equilibrium is not possible because the only possible candidate, (V_1^*, V_2^*), violates the condition $z_{21} - z_{11} \leq V_2^* - V_1^* \leq z_{22} - z_{12}$. A point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{22} - z_{12} \tag{A20}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) < 0$ which contradicts the fact that the number of educated women should exceed that of educated men for such an equilibrium to exist.

3. Finally, if $V_2^* - V_1^* > z_{22} - z_{12}$, then the unique equilibrium (see figure A.3) is such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{22} - z_{12}\ . \tag{A21}$$

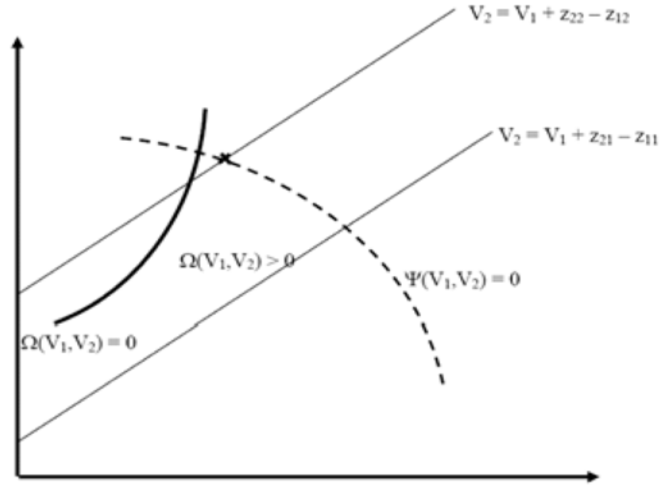

Figure A3

Indeed, a perfectly assortative matching equilibrium is not possible because the only possible candidate, (V_1^*, V_2^*), violates the condition $z_{21} - z_{11} \leq V_2^* - V_1^* \leq z_{22} - z_{12}$. A point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{21} - z_{11} \tag{A22}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) > 0$ which contradicts the fact that the number of educated men should exceed that of educated women for such an equilibrium to exist.

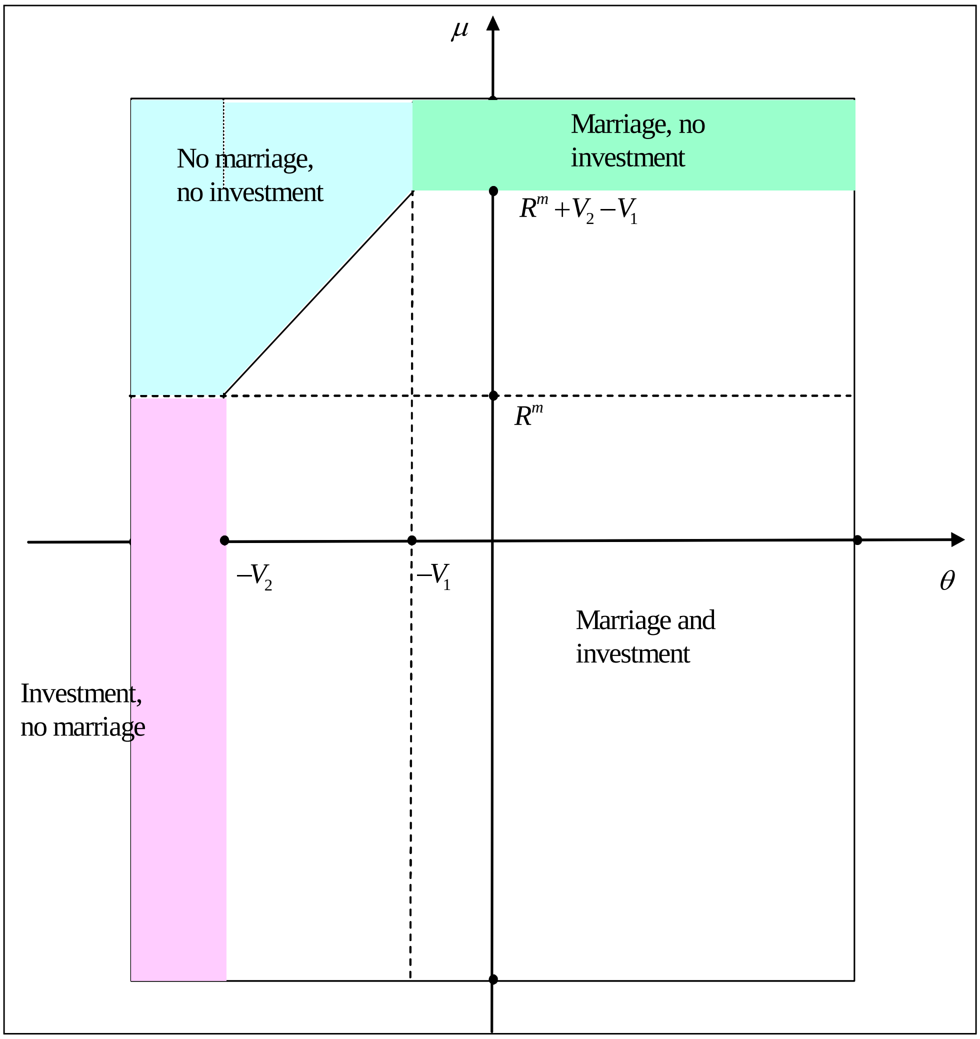

FIGURE 9.1. Regions for Marriage and Investment

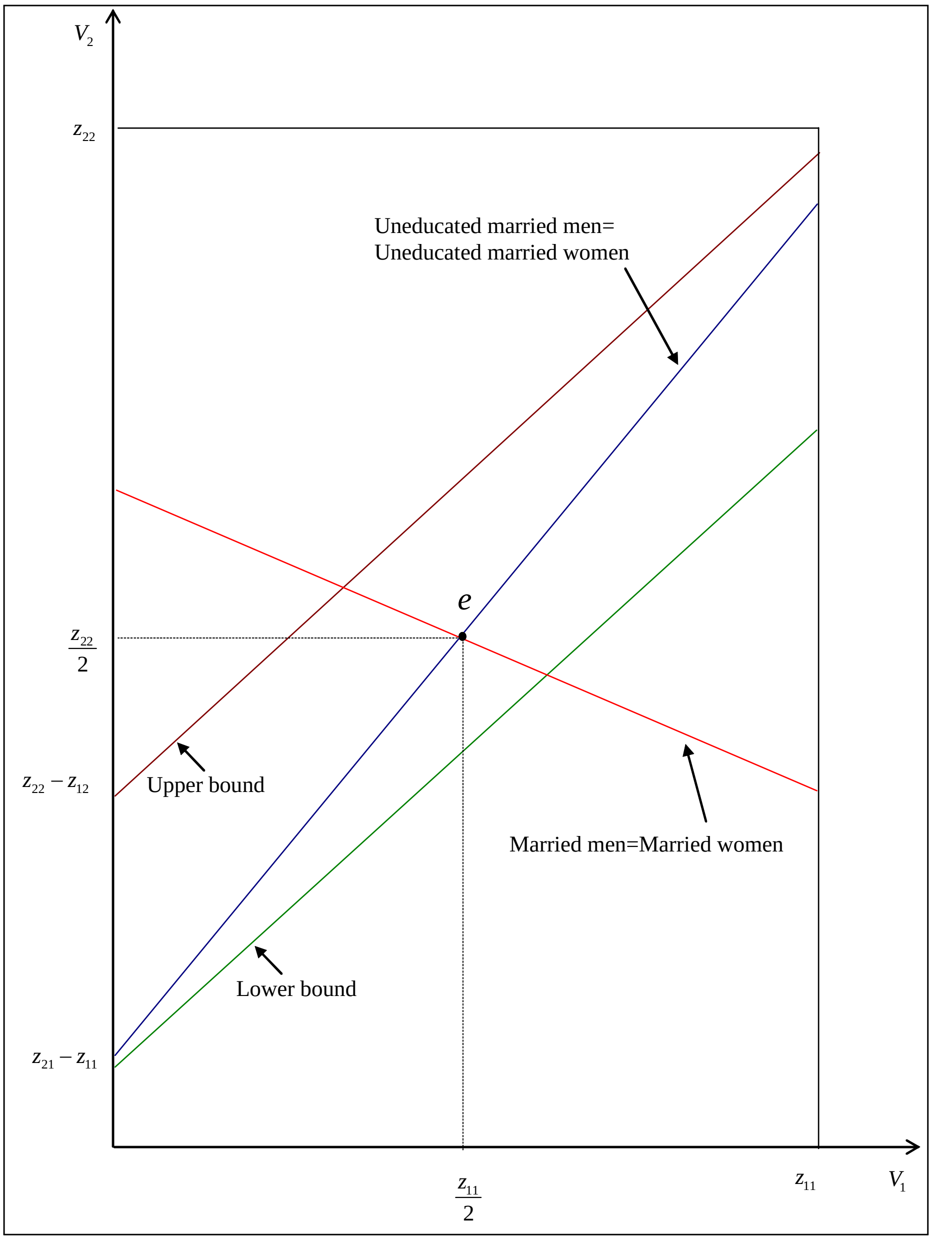

FIGURE 9.2. Equilibrium with Strictly Positive Assortative Matching

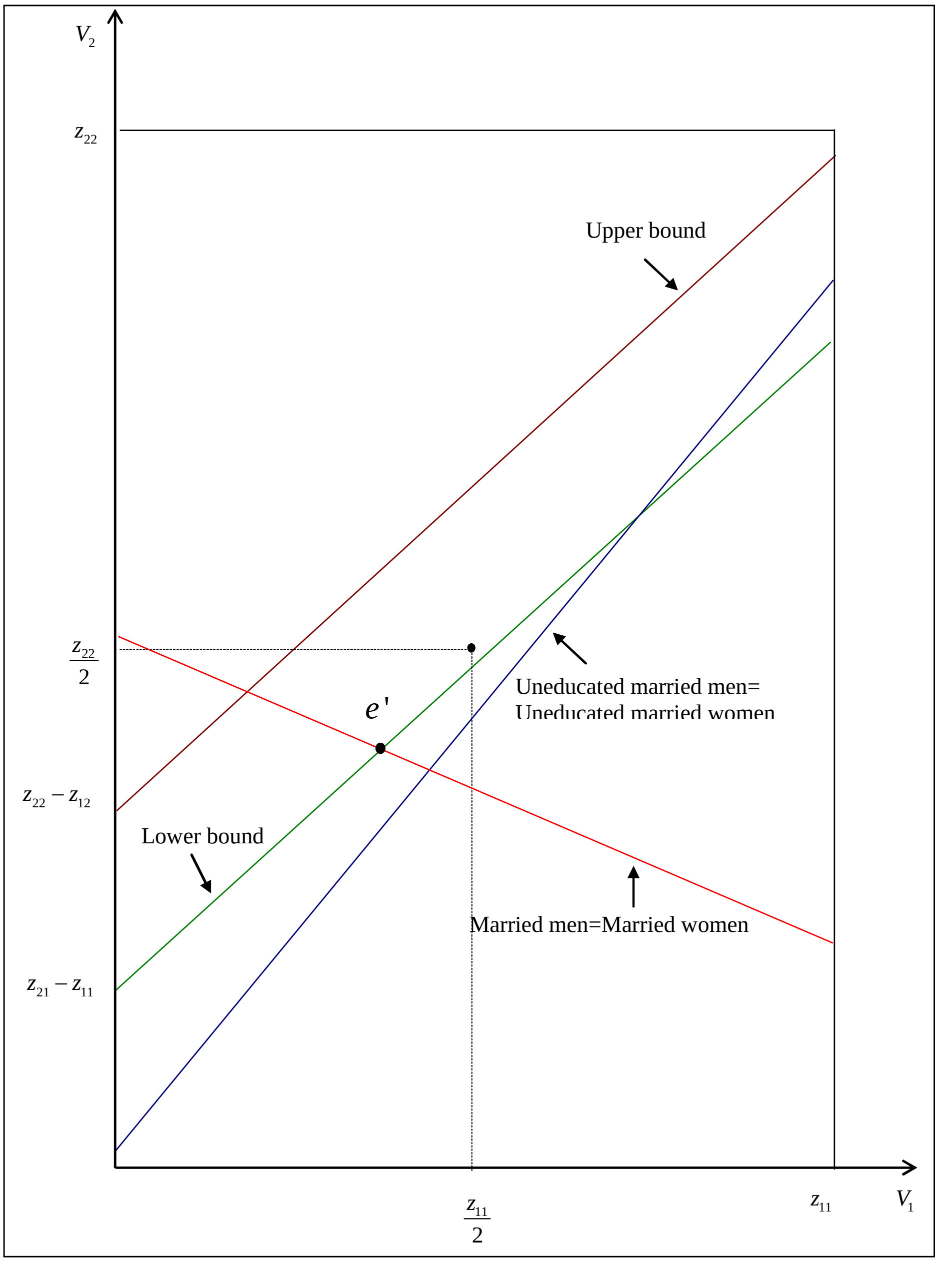

FIGURE 9.3. Mixed Equilibrium with More Educated Men than Educated Women

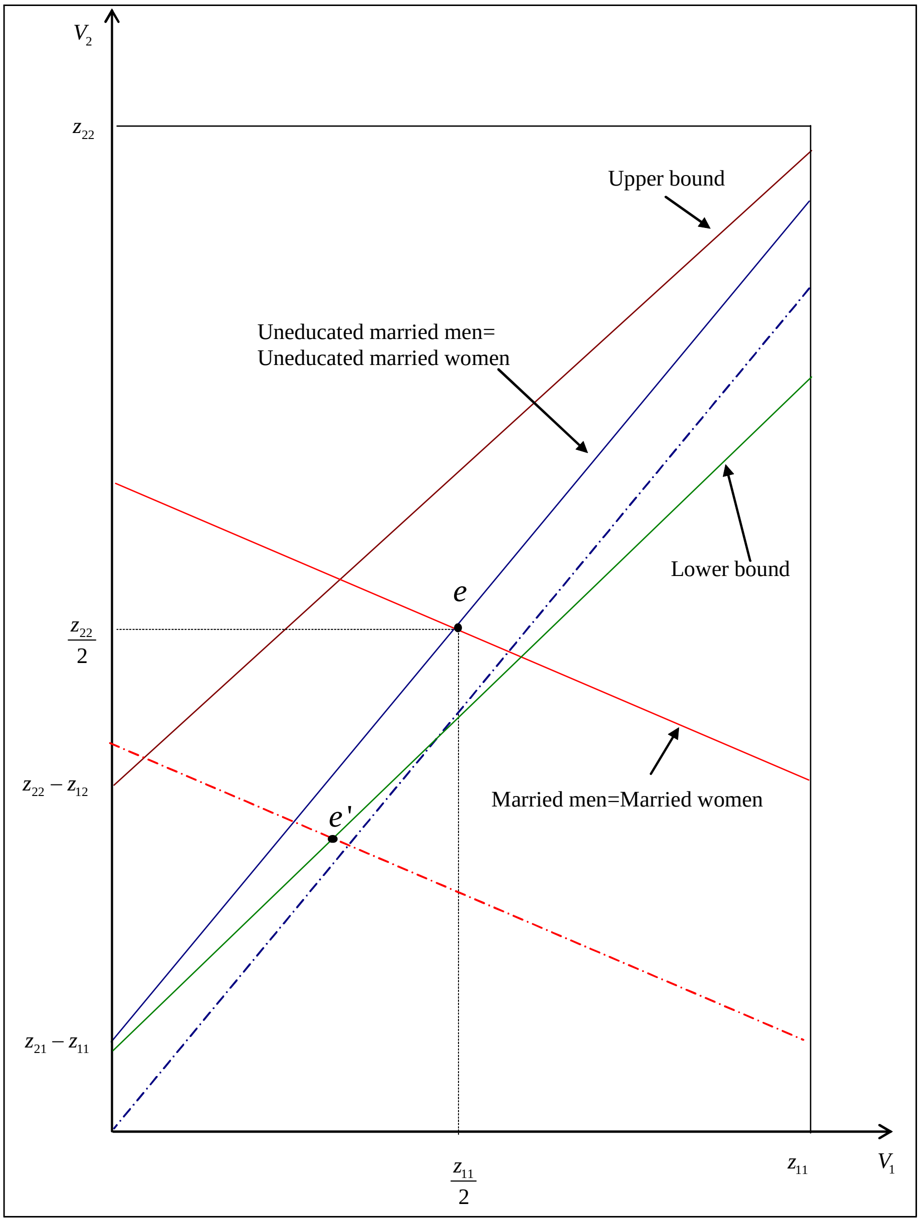

FIGURE 9.4. The Impact of an Increase in the Wage of Educated Men Combined with a Reduction in the Wage of Educated Women

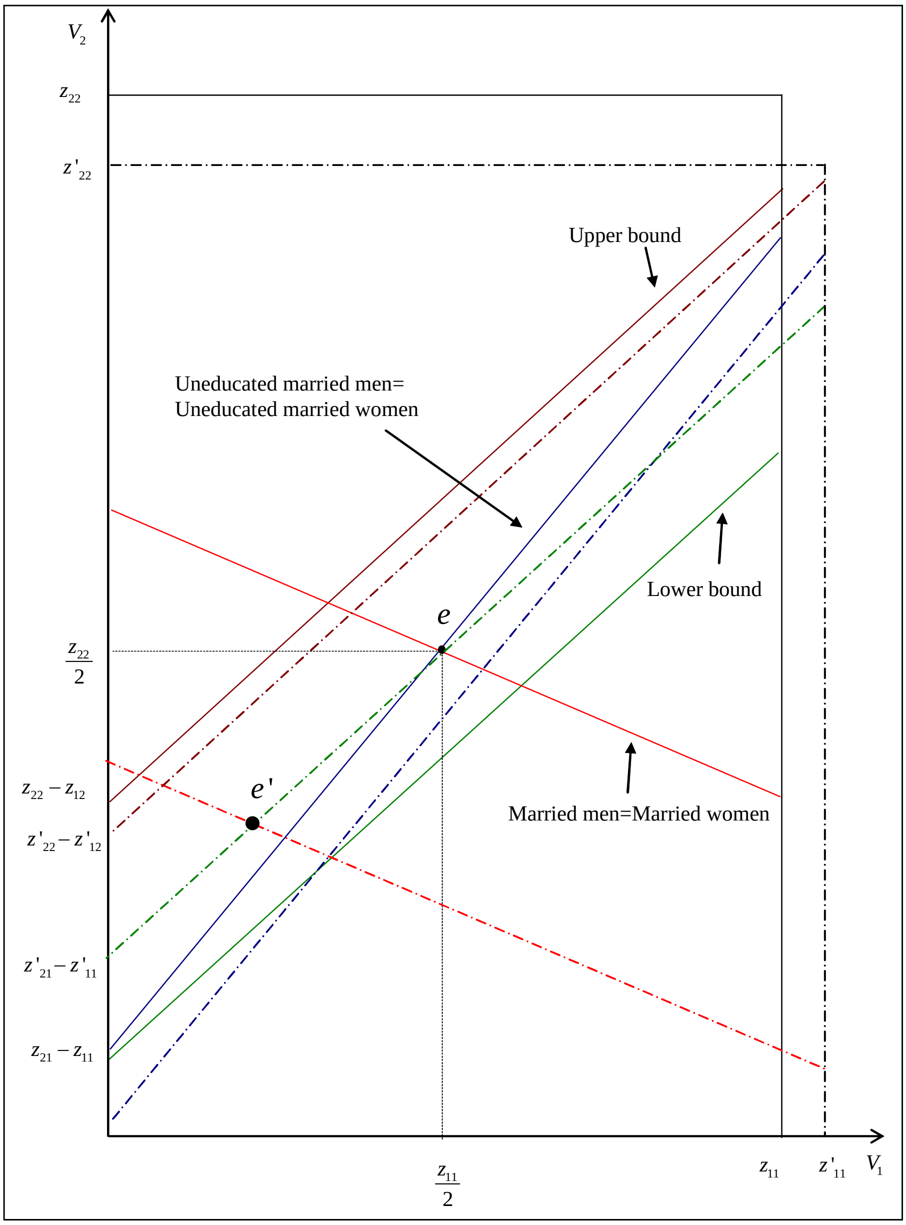

FIGURE 9.5. The Impact of an Increase in the Wife's Work at Home

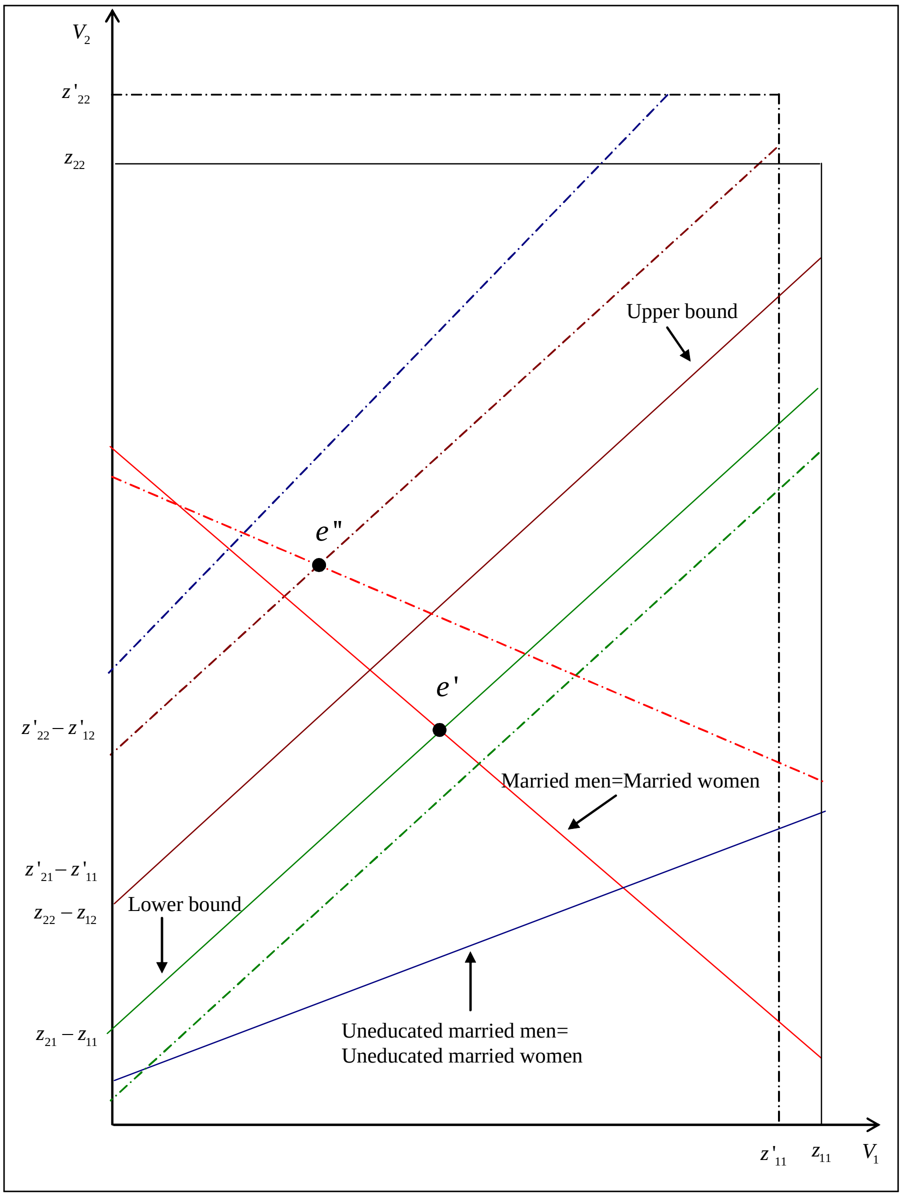

FIGURE 9.6. The Impact of a Decrease in the Wife's Work at Home Combined with an Increase in the Wage of Educated Women

10

An equilibrium model of marriage, fertility and divorce

This chapter provides a simple model of the marriage market that includes fertility, divorce and remarriage and addresses some of the basic issues associated with the higher turnover in the marriage market. For this purpose, we introduce search frictions, heterogeneity and unexpected shocks to match quality. The model is simple enough to identify the welfare implication of increasing turnover. The main result is that the prospects of remarriage generate multiple equilibria due to a positive feedback whereby a higher *aggregate* divorce rate facilitates remarriage, which, in turn, raises the incentives of each couple to divorce. Moreover, when multiple equilibria exist, an equilibrium with higher divorce and remarriage rates generates higher expected welfare for all participants in the marriage market. This is a direct outcome of the positive search externalities that are embedded in the model. The main lesson is that a high aggregate divorce rate can be beneficial because it facilitates the recovery from negative shocks to match quality, allowing couples to replace bad marriages by better ones. Related papers are Aiyagari *et al* (2000), Brien *et al* (2006) and Chiappori and Weiss (2006).

10.1 A model of the Marriage Market

Consider a society in which there is an equal number of men and women and all individuals are *ex ante* identical and live for two periods. Alone, each person consumes their own income Y. If married, the partners share consumption and each consumes $2Y$. In addition, marriage entails a non monetary return θ that both partners enjoy. This 'quality of match' is randomly distributed and different couples draw different values of θ at the time of marriage. However, the future quality of match is uncertain.

Meetings are random. At the beginning of each period, each person randomly meets a person of the opposite sex of his/her age group in a given cohort. We assume that marriage binds for at least one period. At the end of the first period divorce can occur but remarriage is possible only with unattached individuals who never married before or have divorced. In the first period, one meets an eligible partner with certainty. The probability of each individual to meet a single person of the opposite sex in their second period of life equals the proportion in the population of unattached individuals of the opposite sex, divorced or never married. This assumption is crucial for our analysis and implies an 'increasing returns meeting technology' whereby, the more singles are around, the easier it is for each single person to find a match. The logic behind this assumption is that meetings often occur at work or school and are 'wasted' if the person you meet is already married.

Marriage also provides the partners with the option to produce (exactly) two children (there is no out of wedlock birth). The production of children entails a cost to the parents in the first period, c, and a benefit which both parents enjoy in the subsequent period. The utility of a child is independent of household income but depends on the proximity to their natural parents. It equals q^* if the children live with both natural parents and to q^0 if they live with only one of the parents or in a step family; we assume $q^* > c > q^0$. Both parents treat the utility of the child as a public good and it enters additively into their preferences. Partners with children find divorce more costly, because the welfare of the children is higher if children are raised with their natural parents.

Upon meeting, the quality of match θ is revealed and the matched partners decide whether to marry or not. If they choose to marry, they can further decide whether they wish to have children. Because of the delayed benefits, the production of children is a relevant option only for partners in the first period of their life. During each period, there is a shock ε to the quality of match, which is revealed at the end of the period. Having observed the shock at the end of the first period, the partners decide whether to divorce or not. The random variables θ and ε are assumed to be independent across couples. In particular, for each remarried person the values of θ in the first and second marriage are independent. We denote the distributions of θ and ε by $G(\theta)$ and $F(\varepsilon)$ with densities $g(\theta)$ and $f(\varepsilon)$ respectively. We assume that these distributions have zero mean and are symmetric around their mean.

We assume that *all* goods in the household, consumption, match quality and children are public and both partners enjoy them equally. Hence, by assumption, men and women benefit equally from marriage or divorce. The assumptions of public goods and equal numbers of men and women generate perfect symmetry between genders that allows us to set aside, in this chapter, conflict and bargaining between the partners.

10.1.1 Individual Choices

The last stage: the remarriage decision

We first analyze the marriage, fertility and divorce decisions of individuals who take the conditions in the marriage market as given. We proceed from the last available choice, marriage at the second period and work backwards. Two unattached individuals who meet at the beginning of the second period will marry if and only if their drawn θ satisfies

$$\theta \geq -Y. \tag{10.1}$$

That is, conditioned on meeting, marriage occurs whenever the sum of monetary and non monetary gains from marriage is positive. This simple marriage rule holds because each partner gains $Y + \theta$ from the marriage and, if one of the partners has a child then, by assumption, the benefits from that child are the same whether the child lives with a single parent or in a step family. There are thus no costs associated with remarriage.

We denote the probability of remarriage conditioned on a meeting in the second period by

$$\gamma = 1 - G(-Y), \tag{10.2}$$

and the expected quality of match conditioned on marriage in the second period by

$$\beta = E(\theta/\theta \geq -Y). \tag{10.3}$$

Note that although the expected value of θ is zero, the expectation conditioned on remarriage, β, is positive, reflecting the option not to marry if the drawn θ is low.

The probability of meeting an unattached person of the opposite sex at the beginning of the second period is denoted by u. The probability that an unattached person will meet an eligible single person whom he or she will choose to marry is $p = u\gamma$. Note that men and women face the same remarriage probability p, because we assume perfect symmetry between men and women. The expected utility of an unattached person, conditioned on having children is, therefore,

$$V_{2,n} = p(2Y + \beta) + (1 - p)Y + nq^0, \tag{10.4}$$

where $n = 1$ if children are present and $n = 0$, otherwise.

The intermediate stage: the divorce decision

A married person will choose to divorce if and only if the θ drawn at the beginning of the first period and the ε drawn at the end of the first period are such that

$$2Y + \theta + \varepsilon + nq^* < V_{2,n} \ . \tag{10.5}$$

This can be rewritten as $\varepsilon + \theta < h_n$, where

$$h_n \equiv -Y + p(Y + \beta) - n(q^* - q^0) \tag{10.6}$$

is the expected *net* gain from divorce.

The probability of divorce for a married couple with initial quality of match θ is given by $F(h_n - \theta)$. This probability depends on both individual circumstances, represented by θ and n, and on market conditions, represented by $p = u\gamma$. Specifically, the probability of divorce rises with the number of singles who are eligible for remarriage, u, and is lower among couples who have children or are well matched. That is, surprises such as shocks to the quality of the match, represented here by ε, are less disruptive if the current marriage is good, the cost of separation is high or remarriage is unlikely. The influence of remarriage prospects on the decision to divorce creates a link between the aggregate divorce rate and the individual decision to divorce. If many choose to divorce then the number of singles, u, is high, which would raise the probability of remarriage, p, and the net gain from divorce, h_n, and thus the probability of divorce.

The first stage: the marriage and fertility decisions

Two unmarried individuals who meet at the beginning of the first period and observe their drawn quality of match, θ, must decide whether to marry and whether to have children upon marriage. Their expected life time utility upon marriage, conditioned on n, is given by

$$W_{1,n}(\theta) = 2Y + \theta - nc + \int_{h_n-\theta}^{\infty} (2Y + nq^* + \theta + \varepsilon) f(\varepsilon) d\varepsilon + F(h_n - \theta) V_{2,n}. \quad (10.7)$$

Differentiating $W_{1,n}(\theta)$ with respects to θ yields (details are given in the appendix):

$$\frac{\partial W_{1,n}}{\partial \theta} = 2 - F(h_n - \theta). \quad (10.8)$$

That expected utility is increasing in the quality of match is intuitively clear, because a couple with high θ can always replicate the divorce and remarriage decisions of a couple with low θ. The value of marrying without children, $W_{1,0}(\theta)$, and the value of marrying with children, $W_{1,1}(\theta)$, are continuous, increasing and convex functions of θ. A person who chooses not to marry at the beginning of the first period has expected lifetime utility given by:

$$V_1 = Y + V_{2,0}. \quad (10.9)$$

Thus, a first marriage will occur if and only if:

$$\max\left(W_{1,0}(\theta), W_{1,1}(\theta)\right) \geq V_1. \quad (10.10)$$

This maximum function inherits the properties of the individual $W_{1,n}$ functions; that is, it is continuous, increasing and convex in θ. Because the values of marriage with and without children both rise with θ, the decision whether to marry has the form of a stopping rule. That is, couples will marry if and only if $\theta \geq \theta_m$, where θ_m is determined by the condition that (10.10) holds as an equality.[1] Because the maximum is an increasing function of θ, θ_m is unique; see Figure 10.1.

The decision whether to have children can also be represented as a stopping rule, because (10.8) implies that $\frac{\partial W_{1,1}}{\partial \theta} > \frac{\partial W_{1,0}}{\partial \theta} > 0$ for all θ. That is, the quality of the first match is more important if the couple has children and are thus less likely to divorce (recall that children impede divorce, $h_1 < h_0$). Therefore, there is a unique value of θ, θ_c, that solves $W_{1,1}(\theta) = W_{1,0}(\theta)$; see Figure 10.1. Thus a very simple rule arises: those couples for whom $\theta < \theta_m$ will not marry. Those couples for whom $\theta \geq \theta_m$ will marry but they may or may not have children, depending on the costs and benefits from having children. If the cost of having children is relatively high then $\theta_c > \theta_m$ and only those married couples for whom $\theta > \theta_c$ will have children while couples for whom $\theta_c > \theta \geq \theta_m$ will choose to marry but have no children. This is the case illustrated in figure 10.1. If the cost of having children is relatively low then $\theta_c < \theta_m$ and all people that marry will have children. In terms of Figure 10.1 this is equivalent to moving $W_{1,1}(\theta)$ up until the two curves intersect at a value of θ below θ_m.

An interesting testable implication of this model is that individuals are less selective in their first marriage decision than in their remarriage decision. That is, $\theta_m \leq -Y$ (see the appendix). Conditional on θ, marriage in the first period is always more attractive because of the *option* to sample ε. There is no downside risk because one can divorce if ε is low. Such an option is not available in the second period. The option to have children makes this preference for early marriage even stronger.

Another testable result is that individuals become more selective in their first marriage decisions if more eligible singles are available for remarriage in the second period. That is, θ_m is increasing in the remarriage probability, p. This follows directly from the observation that the probability of remarriage has a stronger effect on someone who chose not to marry and is thus sure to be single in the second period than on someone who married and will be single next period with probability less than one. That is,

$$\frac{\partial W_{1,n}}{\partial p} = (Y + \beta)F(h_n - \theta) < Y + \beta = \frac{\partial V_1}{\partial p}. \tag{10.11}$$

It is also the case that the critical value for having children, θ_c, rises with the probability of remarriage, p, implying that a couple will be less inclined to have children when p is higher. This follows because childless couples are more likely to divorce and therefore the positive impact of p on couples without children is stronger, $\frac{\partial W_{1,0}}{\partial p} > \frac{\partial W_{1,1}}{\partial p}$; see the appendix.

[1] We are here implicitly assuming that the support of θ is wide enough so that some people do not marry.

Summary

We have identified two basic forces that guide marriage, divorce, and fertility choices: individual circumstances, represented here by θ and ε and market forces represented here by p. Couples who drew a good match quality upon meeting are more willing to marry and to invest in children because they expect the marriage to be more stable. High turnover in the marriage market has the opposite effect; it discourages marriage and investment in children, because of the higher risk of divorce. These two forces interact and reinforce each other. If individuals expect high turnover, they invest less in children and are therefore more likely to divorce, which raises turnover. High turnover can raise the probability of divorce even in the absence of children because partners are more willing to break a marriage when the prospects for remarriage are good.

10.1.2 Aggregation

We can now aggregate over couples with different realizations of θ and define the aggregate rate of divorce (per number of individuals in the cohort) assuming that the cost of children is large enough so that $\theta_c > \theta_m$.

$$d = \int_{\theta_m}^{\theta_c} F(h_0 - \theta)g(\theta)d\theta + \int_{\theta_c}^{\infty} F(h_1 - \theta)g(\theta)d\theta. \tag{10.12}$$

Given the value of p that individuals expect, the implied proportion of singles at the beginning of period 2 is:

$$u = U(\theta_m(p), \theta_c(p)) \equiv G(\theta_m) + d \tag{10.13}$$

and the aggregate number of remarriages (per number of individuals in the cohort) is $p = \gamma u$.

Our results on individual behavior imply that $U(.,.)$ is increasing in its two arguments. Specifically, from equations (10.12) and (10.13) and the fact that children raise the cost of divorce, $h_0 > h_1$, we obtain:

$$\begin{aligned} \frac{\partial U}{\partial \theta_m} &= (1 - F(h_0 - \theta_m))g(\theta_m) > 0, \\ \frac{\partial U}{\partial \theta_c} &= [F(h_0 - \theta_c) - F(h_1 - \theta_c)]g(\theta_c) > 0. \end{aligned} \tag{10.14}$$

Having shown that both $\theta_m(p)$ and $\theta_c(p)$ are increasing in the remarriage probability, p, we conclude that $U(\theta_m(p), \theta_c(p))$ is also increasing in p.

10.1.3 Equilibrium

Equilibrium is defined by the condition that the value of p that individuals expect is the same as the aggregate number of singles implied by the expectation. That is,

$$p = U(\theta_m(p), \theta_c(p)). \tag{10.15}$$

The function $U(.,.)$, viewed as a function of p, is a non decreasing function from $[0,1]$ to $[0,1]$. Therefore, by the Tarski fixed point theorem (see Mas-Colell *et al* (1995), section MI), there is at least one equilibrium point in the interval $[0,1]$ at which expectations are realized.

We may narrow down the range of possible equilibria, based on some *a priori* information. Because of the advantages of joint consumption and the zero mean and symmetry assumptions on $G(\theta)$ and $F(\varepsilon)$, more than half of the population will choose to marry, and those who subsequently received a sufficiently favorable shock to the quality of match will remain married even if the probability of finding a new mate is 1, implying that $p < 1$ in equilibrium. If there is not much heterogeneity in θ and the support of the shock ε is small, everyone will marry and no one will divorce so that $p = 0$ in equilibrium. However, with sufficiently large variability in θ and ε, an equilibrium p will be positive, because even in the absence of remarriage prospects, couples who draw a sufficiently low quality of match will not marry, and married couples who suffered a large negative shock will divorce, so that $U(\theta_m(0), \theta_c(0)) > 0$.

Because of the positive feedback, whereby an increase in the expected number of singles induces more people to become single, there may be multiple equilibria. Having assumed that all individuals are *ex ante* identical, we can rank the different equilibria based on their common expected value of life time utility:

$$W_1 = E\max(W_{1,1}(\theta), W_{1,0}(\theta), V_1). \tag{10.16}$$

The expectation is taken at the beginning of the first period prior to any meeting, when the quality of prospective matches is yet unknown. An equilibrium with a higher number of unattached individuals at the beginning of the second period will generally have less marriages, more divorces and fewer couples with children. Despite these apparently negative features, equilibria with higher p are in fact Pareto *superior*, because of the better option for couples who suffered a bad shock to their first marriage to recover by forming a new marriage. To see this, note that by (10.11), $\frac{\partial W_{1,n}}{\partial p}$ and $\frac{\partial V_1}{\partial p}$ are positive, implying that an increase in p causes an increase in the expected welfare of all members of society, irrespective of the value of θ that they draw. In other words, the search frictions, represented here by random meetings with members of the opposite sex, irrespective of whether or not they are already attached, imply that those who choose to divorce or remain single exert a positive externality on other members of society who find it easier to find a mate for remarriage. This externality dominates the welfare comparisons because all other factors, such as the damage to children, are internalized by the partners. The presence of children implies that married couples are more reluctant to divorce, which yields a lower equilibrium value for p. However, it is still true that all couples, including couples with children, will be better off in an equilibrium with a higher p if multiple equilibria exist.

10.2 An Example

We now introduce a simple example with multiple equilibria and discuss their properties. Assume that ε takes only two values, $-a$ and $+a$ with equal probability, while θ is distributed uniformly on $[-b, b]$. For this example, we assume that $2a > \left(q^* - q^0\right)$; that is, the variance of the match quality shock is large relative to the loss for children from divorce, so that even couples with children may divorce if the revised quality of their match is low enough.

The expected utility profile if marriage takes place, conditional on having children or not $(n = 0, 1)$, is:

$$W_{1,n}(\theta) = \begin{cases} 3Y + \theta + p(Y+\beta) + n(q^0 - c) \\ \text{if } -b \leq \theta < h_n - a \\ \frac{7Y}{2} + \frac{3\theta}{2} + \frac{p(Y+\beta)}{2} + \frac{a}{2} + n(\frac{q^0}{2} + \frac{q^*}{2} - c) \\ \text{if } h_n - a \leq \theta \leq h_n + a \\ 4Y + 2\theta + n(q^* - c) \\ \text{if } h_n + a < \theta \leq b \end{cases} \tag{10.17}$$

For a given n and conditional on marriage, couples who draw θ such that $\theta + a < h_n$ will divorce for sure at the end of the first period.[2] Couples who draw θ such that $\theta - a > h_n$ will stay married for sure (if they marry). Couples who draw θ in the intermediate range $h_n - a \leq \theta \leq h_n + a$ will divorce if the shock is negative and remain married otherwise. Using equations (10.4) and (10.9), the value of not marrying in the first period is given by:

$$V_1 = 2Y + p(Y + \beta) \tag{10.18}$$

which is independent of θ.

We now wish to identify the points θ_m and θ_c that trigger marriage and having children, respectively. For this purpose, it is useful to inspect Figure 10.2, in which we plot $W_{1,0}(\theta)$, $W_{1,1}(\theta)$ and V_1.[3] Note that the kinks in $W_{1,0}(\theta)$ always appear at higher values of θ than the kinks in $W_{1,1}(\theta)$. This happens because the expected gains from divorce are higher for couples without children, $h_0 - h_1 = q^* - q^0 > 0$. It can be seen that an

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[2] Marriage followed by certain divorce can occur if the gains from joint consumption are sufficiently large to offset the low quality of the current match $(Y + \theta > 0)$.

[3] In this figure we have $h_1 + a > h_0 - a$. This follows from the assumption that $2a > \left(q^* - q^0\right)$.

intersection of the two curves can occur only in the intervals $[h_1 - a, h_0 - a]$ or $[h_1 + a, h_0 + a]$. Moreover, it can be verified that if the costs from having children are relatively high, that is, $q^* > c > \frac{q^*+q^0}{2}$, then the only possible intersection is in the region $[h_1 + a, h_0 + a]$; see the appendix for a proof.[4] We obtain θ_c by equating $W_{1,0}(\theta)$ for the intermediate region ($h_0 - a < \theta < h_0 + a$) with $W_{1,1}(\theta)$ for $\theta > h_1 + a$. This gives:

$$\theta_c = p(Y + \beta) - Y + a - 2(q^* - c), \tag{10.19}$$

Using this expression that determines θ_c, we can now determine θ_m. Referring again to Figure 10.2, we see that $\max(W_{1,0}(\theta), W_{1,1}(\theta))$ is represented by the upper *envelope* of the $W_{1,0}(\theta)$ and $W_{1,1}(\theta)$ profiles. We thus have to consider three segments of this envelope. In the first case (with low V_1), V_1 intersects the envelope at a value of θ below $h_0 - a$, where couples would be indifferent between singlehood and a marriage without children followed by a certain divorce. In the second segment the intersection occurs at $\theta \in [h_0 - a, \theta_c]$,where couples would be indifferent between singlehood and a marriage without children followed by divorce if a negative shock occurs (this is the case illustrated in Figure 10.2). In the third case, (high V_1) the intersection is above θ_c,where couples would be indifferent between singlehood and a marriage with children that remains intact with certainty. In the appendix we show that:

$$\theta_m = \begin{cases} -Y & \text{if} \quad p(Y+\beta) > a \\ \frac{p(Y+\beta)-a}{3} - Y & \text{if} \quad a \geq p(Y+\beta) \geq 3(q^* - c) - 2a \\ p\frac{Y+\beta}{2} - \frac{(q^*-c)}{2} - Y & \text{if} \quad p(Y+\beta) < 3(q^* - c) - 2a \end{cases} . \tag{10.20}$$

Note that the assumptions $2a > \left(q^* - q^0\right)$ and $c > \frac{q^*+q^0}{2}$ ensure that interval $[3(q^* - c) - 2a, a]$ is non-empty.

From equations (10.19) and (10.20) we see that both θ_m and θ_c rise with the expected remarriage rate, p. That is, the likelihood of marrying and having children decline with p. This happens because matched partners anticipate that they are more likely to divorce if the prospect of remarriage rises. Both θ_m and θ_c decline with income, implying that the likelihood of marrying and having children rise with income. This happens in our model because of the complementarity between the incomes of the spouses that is induced by joint consumption of public goods. A dollar increase in Y raises the consumption of *each* married person by 2 dollars, while their consumption as a single will rise by only one dollar.

The proportion of singles at the beginning of the second period that is associated with a given p consists of those who did not marry in the beginning of the first period, $G(\theta_m(p))$ and the divorcees at the end of the first period among the married. These divorcees constitute of all the married

[4] The interested readers may try the case with low costs of children, see appendix.

for whom $\theta_m < h_0 - a$, half of the married for whom $h_0 - a \leq \theta_m \leq \theta_c$ and none of the married for whom $\theta_m > \theta_c$. Therefore, equation (10.13) for the proportion of singles at the beginning of period 2 can be written as

$$U(\theta_m(p), \theta_c(p)) = \begin{cases} \frac{G(h_0(p)-a)+G(\theta_c(p))}{2} \\ \text{if } p(Y+\beta) > a \\ \frac{G(\theta_m(p))+G(\theta_c(p))}{2} \\ \text{if } a \geq p(Y+\beta) \geq 3(q^* - c) - 2a \\ G(\theta_m(p)) \\ \text{if } p(Y+\beta) < 3(q^* - c) - 2a \end{cases} \tag{10.21}$$

Because in this particular example, the reservation rules for marriage and for having children are linear functions of p we obtain under the assumption that $G(.)$ is uniform that $U(\theta_m(p), \theta_c(p))$ is also a piecewise linear function of p. Consequently multiple equilibria can arise. Within the confines of our example, multiple equilibria occur only if there is not too much heterogeneity in the quality of match. We therefore choose a relatively small b and obtain Figure 10.3. As seen in this figure, there are three equilibria at $p = 0$, at $p = 0.25$ and at $p = 0.5$. Details of these three equilibria are presented in Table 10.1. In all three equilibria, everyone marries whomever they meet (this holds in both periods[5]), but the higher is the equilibrium level of p, the lower is the proportion of families that choose to have children and the higher is the proportion that divorces. At the low equilibrium, where everyone expects a remarriage rate of $p = 0$, all couples have children and no one divorces. This implies that there will be no singles in the second period, which justifies the expectations. At the equilibrium in which everyone expects a remarriage rate of $p = 0.25$, half of the couples have children and, of those who do not have children, half divorce upon the occurrence of a bad shock. This implies that at the beginning of the second period, a quarter of the population will be single, which justifies the expected remarriage rate. At the equilibrium with $p = 0.5$, no couple has children and half of them divorce upon the realization of a bad shock so, in this case too, expectations are realized. Thus, all three equilibria share the basic property that expectations are fulfilled. However, the intermediate equilibrium at $p = 0.25$ is not stable with respect to an arbitrary change in expectations. That is, if the expected remarriage rate, p, rises (declines) slightly then the aggregate number of singles $U(\theta_m(p), \theta_c(p))$ rises (declines) too.[6]

For these examples, one can easily calculate the equilibrium value of *ex ante* welfare, W_1 (see equation (10.16)). If $p = 0.5$, $W_{1,0}(\theta)$ is the highest for all θ, implying that all couples marry, have no children and divorce with

[5] In the second period, this implies that $\gamma = 1$ and $\beta = 0$.

[6] If b goes to zero and all matches are *ex ante* identical, the middle section disappears and the equilibrium function becomes a step function yielding only two stable equilibria.

probability 0.5, so that

$$W_1 = EW_{1,0}(\theta) = \frac{7}{2}Y + \frac{1}{4}Y + \frac{1}{2}a = \frac{5}{6} \tag{10.22}$$

If $p = 0$, $W_{1,1}(\theta)$ is the highest for all θ, implying that all couples marry, have children and do not divorce, so that

$$W_1 = EW_{1,1}(\theta) = 4Y + (q^* - c) = \frac{3}{6} \tag{10.23}$$

The calculation of welfare is a bit more complex if $p = 0.25$. In this case, the maximum is given by $W_0(\theta)$ if $\theta \leq 0$ and by $W_1(\theta)$ if $\theta \geq 0$. Thus,

$$W_1 = \frac{\frac{7}{2}Y + \frac{1}{4}Y + \frac{1}{2}a + \frac{3}{2}E(\theta/\theta \leq 0)}{2} + \frac{4Y + (q^* - c) + 2E(\theta/\theta \geq 0)}{2} = \frac{4}{6} \tag{10.24}$$

These calculations illustrate that ex-ante welfare rises as we move to equilibrium points with higher p, reflecting the positive externality associated with an increase in the aggregate number of singles.

10.3 Income uncertainty and ex-post heterogeneity

The simple model assumed perfect symmetry among spouses and that all individuals have the same incomes which remain fixed over time. We now allow income to change over time, which creates income heterogeneity ex-post. As before, all men and women have the same income, Y, in the first period of their life. However, with probability λ income in the second period rises to Y^h and with probability $1-\lambda$ it declines to Y^l. To maintain ex-ante symmetry, we assume that the incomes of men and women follow this same process. To simplify, we shall assume now that the quality of the match, θ, is revealed only at the end of each period. The realized value of θ at the end of the first period can trigger divorce, while the realized value of θ at the end of the second period has no behavioral consequences in our two period model. Since there are gains from marriage, and the commitment is only for one period, everyone marries in the first period. However, in this case, changes in incomes as well as changes in the quality of match can trigger divorce. We continue to assume risk neutrality and joint consumption.

The main difference from the previous model is that at the beginning of the second period there will be *two* types of potential mates, rich and poor. Let α be the expected remarriage rate and π the proportion of high income individuals among the divorcees, and let $y = \pi Y^h + (1-\pi)Y^l$ be the average income of the divorcees. Then the expected values of being unattached in the beginning of the second period for each type are

$$V^j(\alpha, \pi) = Y^j + \alpha y, \quad j = l, h. \tag{10.25}$$

This expression is obtained because type j consumes Y^j alone and expects to consume $Y^j + y$ when married and the expected value of the quality of a new match θ in the second (and last) period is zero. Clearly, a richer person has a higher expected value from being unattached.

At the end of the first period, the quality of the current match and the new income values (Y^h or Y^l) for each spouse are revealed, and each partner can choose whether to stay in the current match or divorce and seek an alternative mate. An hh couple divorces if:

$$2Y^h + \theta < Y^h + \alpha y \Rightarrow \theta < \alpha y - Y^h \tag{10.26}$$

An ll couple divorces if:

$$2Y^l + \theta < Y^l + \alpha y \Rightarrow \theta < \alpha y - Y^l \tag{10.27}$$

Note that, despite the lower value of being unattached for the two spouses, a poor couple is *more* likely to divorce, because the current marriage is less attractive.

In a mixed couple, type h will wish to divorce if

$$Y^h + Y^l + \theta < Y^h + \alpha y,$$

which is the same as condition (10.27), while type l will wish to divorce if

$$Y^h + Y^l + \theta < Y^l + \alpha y,$$

which is the same as condition (10.26). But inequality (10.26) implies inequality (10.27), which holds for a wider range of θ. Thus, the condition for marital dissolution for mixed couples is (10.27). For mixed couples there will be disagreement on the divorce decision if

$$\alpha y - Y^h \leq \theta < \alpha y - Y^l.$$

In this case, divorce is always triggered by the *high* income spouse who can do better outside the marriage.

In equilibrium, the expected remarriage rate, α, equals the divorce rate, that is,

$$\alpha = \lambda^2 G(\alpha y - Y^h) + (1 - \lambda^2) G(\alpha y - Y^l). \tag{10.28}$$

Equation 10.28 involves two endogenous variables, the expected remarriage rate α and the expected income of a divorcee, y. However, these two variables are interrelated and the equilibrium condition (10.28) can be reduced to one equation in one unknown, αy, which is the variable part of the expected gains from divorce. Then, we can deduce the separate equilibrium values of both α and y.

As a first step, note that the proportion in the population of high income divorcees of each gender is

$$\alpha\pi = \lambda[\lambda G(\alpha y - Y^h) + (1-\lambda)G(\alpha y - Y^l)] \tag{10.29}$$

Taking the difference between (10.28) and (10.29), we have

$$\alpha(1-\pi) = (1-\lambda)G(\alpha y - Y^l). \tag{10.30}$$

Using the definition of y, we have

$$1 - \pi = \frac{Y^h - y}{Y^h - Y^l}. \tag{10.31}$$

Then, substituting from (10.31) into (10.30) we get

$$\alpha = \frac{\alpha y}{Y^h} + (1-\lambda)G(\alpha y - Y^l)\frac{(Y^h - Y^l)}{Y^h}. \tag{10.32}$$

Finally, eliminating α in (10.28), we can then rewrite the equilibrium condition as an equation in αy

$$\alpha y = \lambda^2 G(\alpha y - Y^h)Y^h + (1-\lambda^2)G(\alpha y - Y^l)[\frac{\lambda Y^h}{1+\lambda} + \frac{Y^l}{1+\lambda}]. \tag{10.33}$$

To analyze this equation, we note that the expected income of a divorcee, y, is bounded between Y^l (which occurs if only low income individuals divorce, $\pi = 0$) and Y^h (which occurs if only high income individuals divorce, $\pi = 1$) and that the divorce rate α is bounded between 0 and 1. Therefore, αy is bounded between 0 and Y^h. Assuming that $G(-Y^h) > 0$, equation (10.33) has a positive solution for αy because the right hand side of (10.33) is positive at $\alpha y = 0$ and smaller than Y^h at $\alpha y = Y^h$ and $G(.)$ is continuous. However, because both sides of (10.33) are increasing in αy, this equation may have *multiple* solutions. Given an equilibrium value for αy, we can find the equilibrium divorce rate, α, from equation (10.28) and the equilibrium share of the rich among the divorcees, π, from the ratio of (10.29) to (10.28).

The comparative statics of this system are somewhat complicated, but the basic principles are quite clear. An increase in the proportion of the rich in the second period, λ, has two opposing effects on the equilibrium divorce rate. First, it raises the monetary gain from maintaining the current marriage. Second, it raises the average quality of divorcees and thus the prospects of finding a good match, which encourages divorce. The relative importance of these considerations depends on the initial proportions of the two types, the values of low and high income and the distribution of match quality. We cannot provide general results but simulations suggest that the divorce rate tends to increase with the proportion of the rich when the proportion of the rich is low in the second period. An increase in the income of the poor or the rich tends to reduce divorce. The positive income effects reflect the increasing gains from remaining married when consumption is a public good. There is no simple mapping from income risk or income inequality to the rate of divorce, but starting from equality an increase in the difference $Y^h - Y^l$ raises the divorce rate. An increase in the variability of the quality of match generally leads to a rise in the divorce rate.

The simple model outlined above generates *positive* assortative mating in the second period. This happens here because the good matches hh are less likely to break, and all types have the same remarriage probability α. So that there is a larger proportion of h among those who stay married than in the population. This can be immediately seen by noting that the term in square brackets in (10.29) is smaller than 1, so that $\alpha\pi < \lambda$. Conversely, there is a larger proportion of l among the singles than in the population, because they are more likely to divorce and are equally likely to remain single. This process of selective remarriage, via differential incentives to divorce, is quite different from the usual models (see Burdett and Coles 1999) that are built on the idea that the high type is more selective in the first marriage. In the search model, rejection of unsatisfactory mates is done when one is single, reflecting the assumption that a match is "for ever". In our model, rejection happens when married, after θ is revealed. This reflects our assumption that marriage is an "experience good". It seems that the two approaches lead to the same outcome.

10.4 Conclusion

The simple models discussed in this chapter make several important points that carry a general message for the empirical and theoretical analysis of the family. First, the marriage, fertility and divorce decisions are closely interrelated. Couples decide to marry and to have children based on the risk of divorce and the prospect of remarriage. Conversely, the fact that couples chose to marry, or have children, has implications for their subsequent divorce decisions. Second, in a marriage market, as in other search markets, individual decisions can be quite sensitive to the choices of others. In particular, if many choose to remain single, not to have children, or to divorce, this will strengthen the incentive of each couple separately to behave in a similar manner. Such markets are susceptible to sudden and large structural changes as may have happened following the introduction of the contraceptive pill in the 1970's. As we have seen, search externalities may have important policy and welfare implications. In particular, societies with high marital turnover may in fact yield better outcomes for the typical adult, because such an equilibrium allows easier recovery from bad shocks. In this chapter, we assumed that children are always worse off as a consequence of divorce. In the subsequent chapter, we shall discuss child support transfers and show that children are not necessarily harmed by divorce and, conditional on the divorce of their parents, may in fact be better off in a high divorce environment.

10.5 Appendix

10.5.1 *Properties of the expected utility, with and without children*

Using (10.7), (10.4) and (10.6):

$$W_{1,n}(\theta) = 2Y + \theta - nc + \int_{h_n-\theta}^{\infty} (2Y + nq^* + \theta + \varepsilon) f(\varepsilon) d\varepsilon + F(h_n - \theta) V_{2,n},$$

$$\begin{aligned} V_{2,n} &= p(2Y + \beta) + (1-p)Y + nq^0, \\ h_n &= -Y + p(Y+\beta) - n(q^* - q^0). \end{aligned}$$

Hence,

$$V_{2,n} = h_n + 2Y + nq^* \tag{10.34}$$

Differentiating $W_{1,n}(\theta)$ with respects to θ yields

$$1 + \int_{h_n-\theta}^{\infty} f(\varepsilon) d\varepsilon + (2Y + nq^* + \theta + h_n - \theta) f(h_n - \theta) - f(h_n - \theta) V_{2,n}, \tag{10.35}$$

where we use the fact that the derivative of an integral with respect to the lower bound equals the value of the integrand at that point. Cancelling and collecting terms, we obtain

$$\frac{\partial W_{1,n}}{\partial \theta} = 2 - F(h_n - \theta), \tag{10.36}$$

as stated in (10.8). Note that $1 \leq \frac{\partial W_{1,n}}{\partial \theta} \leq 2$ and that $\frac{\partial W_{1,n}}{\partial \theta}$ is increasing in θ. Hence, the expected values with and without children, $W_{1,1}(\theta)$ and $W_{1,0}(\theta)$ respectively, are increasing and convex functions of of θ, with slopes bounded between 1 and 2. Also, because $h_1 < h_0$, $\frac{\partial W_{1,1}}{\partial \theta} > \frac{\partial W_{1,0}}{\partial \theta}$. Finally, examining the partial impact of p, holding θ fixed we see that

$$\begin{aligned} \frac{\partial W_{1,n}(\theta)}{\partial p} &= \frac{\partial}{\partial p}[\int_{h_n-\theta}^{\infty} (2Y + nq^* + \theta + \varepsilon) f(\varepsilon) d\varepsilon \\ &\quad + F(h_n - \theta)(h_n + 2Y + nq^*)] \\ &= F(h_n - \theta)(Y + \beta), \end{aligned} \tag{10.37}$$

implying that $\frac{\partial W_{1,0}}{\partial p} > \frac{\partial W_{1,1}}{\partial p}$.

10.5.2 Properties of the trigger for having children, θ_c

The trigger for θ_c is determined by the condition $W_{1,1}(\theta_c) = W_{1,0}(\theta_c)$. If there is a solution for θ_c, it must be unique because $\frac{\partial W_{1,1}}{\partial \theta} > \frac{\partial W_{1,0}}{\partial \theta}$. Using (10.7) and (10.34), the requirement that $W_{1,1}(\theta_c) = W_{1,0}(\theta_c)$ implies

$$\begin{aligned} & -c + \int\limits_{h_1-\theta_c}^{\infty} (2Y + q^* + \theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_1 - \theta_c)(h_1 + 2Y + q^*) \\ = & \int\limits_{h_0-\theta_c}^{\infty} (2Y + \theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_0 - \theta_c)(h_0 + 2Y). \end{aligned} \tag{10.38}$$

or

$$-c + q^* + \int\limits_{h_1-\theta_c}^{\infty} (\theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_1 - \theta_c) h_1 \tag{10.39}$$

$$= \int\limits_{h_0-\theta_c}^{\infty} (\theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_0 - \theta_c) h_0.$$

By (10.6),

$$\frac{dh_0}{dp} = \frac{dh_1}{dp} = Y + \beta. \tag{10.40}$$

Differentiating both sides of (10.39) with respect to p and θ_c, we obtain

$$\begin{aligned} & (1 - F(h_1 - \theta_c)) d\theta_c + F(h_1 - \theta_c)(Y + \beta) dp \\ = & (1 - F(h_0 - \theta_c)) d\theta_c + F(h_0 - \theta_c)(Y + \beta) dp, \end{aligned} \tag{10.41}$$

implying that

$$\frac{d\theta_c}{dp} = Y + \beta.$$

10.5.3 Properties of the trigger for marriage, θ_m

By definition,

$$\max(W_{1,1}(\theta_m), W_{1,0}(\theta_m) = V_1 \tag{10.42}$$

Because $W_{1,1}(\theta_m)$ and $W_{1,0}(\theta_m)$ both increase in θ, while V_1 is independent of θ, the solution for θ_m must be unique if it exists. The solution must also satisfy $\theta_m \leq -Y$, because

$$W_{1,0}(-Y) = V_1 + \int\limits_{p(Y+\beta)}^{\infty} (-p(Y + \beta) + \varepsilon) f(\varepsilon) d\varepsilon \geq V_1. \tag{10.43}$$

There are two cases to consider.

Case 1, $W_{1,0}(\theta_m) = V_1 > W_{1,1}(\theta_m)$, which implies $\theta_c > \theta_m$. In this case

$$2Y + \theta_m + \int_{h_0-\theta_m}^{\infty} (2Y + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon + F(h_0 - \theta_m) V_{2,0} = Y + V_{2,0}, \quad (10.44)$$

or

$$Y + \theta_m + \int_{h_0-\theta_m}^{\infty} (2Y + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon = (1 - F(h_0 - \theta_m))(h_0 + 2Y). \quad (10.45)$$

Differentiating totally both sides of (10.45) yields

$$\begin{aligned} & [1 + (1 - F(h_0 - \theta_m) + f(h_0 - \theta_m)(2Y + h_0)]d\theta_m \\ & -f(h_0 - \theta_m)(h_0 + 2Y)(Y + \beta)dp \\ = \; & f(h_0 - \theta_m)((h_0 + 2Y)d\theta_m \\ & +[(1 - F(h_0 - \theta_m)) - (h_0 + 2Y)f(h_0 - \theta_m)](Y + \beta)dp \quad (10.46) \end{aligned}$$

Cancelling equal terms and rearranging, we obtain

$$\frac{\partial \theta_m}{\partial p} = (Y + \beta) \frac{1 - F(h_0 - \theta)}{2 - F(h_0 - \theta)} > 0 \text{ if } \theta_c > \theta_m. \quad (10.47)$$

Case 2, $W_{1,1}(\theta_m) = V_1 > W_{1,0}(\theta_m)$, which implies $\theta_c < \theta_m$. In this case

$$2Y + \theta_m - c + \int_{h_1-\theta_m}^{\infty} (2Y + q^* + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon + F(h_1 - \theta_m) V_{2,1} = Y + V_{2,1}, \quad (10.48)$$

or

$$Y + \theta_m - c + \int_{h_1-\theta_m}^{\infty} (2Y + q^* + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon = (1 - F(h_1 - \theta_m))(h_1 + 2Y + q^*). \quad (10.49)$$

Using the same calculations as in the previous case, we obtain

$$\frac{\partial \theta_m}{\partial p} = (Y + \beta) \frac{1 - F(h_1 - \theta)}{2 - F(h_1 - \theta)} > 0 \text{ if } \theta_c < \theta_m. \quad (10.50)$$

We conclude that

$$\frac{\partial \theta_c}{\partial p} > \frac{\partial \theta_m}{\partial p}. \quad (10.51)$$

10.5.4 Calculations for the example

Properties of θ_c in the example

We first prove that if the costs of having children are relatively high, that is if $q^* > c > \frac{q^*+q^0}{2}$, then an intersection of $W_{1,0}(\theta)$ with $W_{1,1}(\theta)$ cannot occur in the region $[h_1 - a, h_0 - a]$. The proof is by contradiction. Assume for some $\theta \in [h_1 - a, h_0 - a]$, $W_{1,0}(\theta) = W_{1,1}(\theta)$. Then this θ must satisfy

$$\begin{aligned} & \frac{7}{2}Y + \frac{3}{2}\theta + \frac{1}{2}p(Y+\beta) + \frac{1}{2}a + (\frac{1}{2}q^0 + \frac{1}{2}q^* - c) \\ = \; & 3Y + \theta + p(Y+\beta). \end{aligned} \qquad (10.52)$$

Solving for θ and denoting the solution by θ_c , we have

$$\theta_c = p(Y+\beta) - Y - a + 2c - (q^* + q^0). \qquad (10.53)$$

Recalling equation (10.6) for $n = 0$:

$$h_0 = -Y + p(Y+\beta),$$

we obtain, using $c > \frac{q^*+q^0}{2}$,

$$\theta_c = h_0 - a + 2c - (q^* + q^0) > h_0 - a. \qquad (10.54)$$

Properties of θ_m in the example

Proof of (10.20). Consulting Figure 10.2 and allowing V_1 to move up or down, we see that we have to consider three cases for equation 10.42

First, low values of V_1 give an intersection with $W_{1,0}(\theta)$ below $\theta = h_0 - a$. Equating V_1 with $W_{1,0}(\theta)$ this gives:

$$\theta_m = -Y \qquad (10.55)$$

This requires that:

$$\begin{aligned} -Y \; = \; & \theta_m \leq h_0 - a = -Y + p(Y+\beta) - a \\ \Rightarrow \; & p(Y+\beta) \geq a \end{aligned} \qquad (10.56)$$

For intermediate values of $\theta \in [h_0 - a, \theta_c]$ we equate V_1 with $W_{1,0}(\theta)$ evaluated in the intermediate region of equation (10.17). This gives:

$$\theta_m = \frac{1}{3}(p(Y+\beta) - a) - Y. \qquad (10.57)$$

Since we have $\theta_m \leq \theta_c$ this value and (10.19) requires that:

$$p(Y+\beta) \geq 3(q^* - c) - 2a \qquad (10.58)$$

Finally we can consider high values of θ, such that $\theta \geq \theta_c$. Equality for equation (10.10) requires equating V_1 with $W_{1,1}(\theta)$ evaluated for $\theta \geq \theta_c$(that is, the third region of equation (10.17)). This gives

$$\theta_m = \frac{1}{2}p(Y+\beta) - \frac{1}{2}(q^* - c) - Y \tag{10.59}$$

This case requires $\theta_m > \theta_c$ which gives:

$$p(Y+\beta) < 3(q^* - c) - 2a \tag{10.60}$$

Properties of the proportion of singles, $U(\theta_m(p), \theta_c(p))$, in the example (proof of 10.21)

The proportion of singles at the beginning of the second period consists of those who did not marry in the beginning of the first period, $G(\theta_m(p))$, and of the divorcees at the end of the first period among the married. The proportion of divorcees depends on the location of θ_m. If V_1 is low and intersects $W_{1,0}(\theta)$ below $h_0 - a$, then all of the married for whom $\theta_m < \theta < h_0 - a$ divorce for sure, and all of the married for whom $h_0 - a < \theta < \theta_c$ divorce upon a bad shock, that is with a probability of $\frac{1}{2}$, while those married with children for whom $\theta > \theta_c$ do not divorce. Therefore,

$$\begin{aligned} U(\theta_m(p), \theta_c(p)) &= G(\theta_m(p)) + (G(h_0(p) - a) - G(\theta_m(p))) \\ &\quad + \frac{1}{2}(G(\theta_c(p)) - G(h_0(p) - a)) \\ &= \frac{1}{2}[G(\theta_c(p)) + G(h_0(p) - a)]. \end{aligned} \tag{10.61}$$

For intermediate values of V_1, the intersection with $W_{1,0}(\theta)$ is in the range $[h_0 - a, \theta_c]$, where the married with children for whom $\theta_m < \theta < \theta_c$ divorce upon the occurrence of a bad shock. In this case,

$$\begin{aligned} U(\theta_m(p), \theta_c(p)) &= G(\theta_m(p)) + \frac{1}{2}(G(\theta_c(p)) - G(\theta_m(p))) \\ &= \frac{1}{2}(G(\theta_c(p)) + G(\theta_m(p))). \end{aligned} \tag{10.62}$$

Finally, for high values of V_1, the intersection is with $W_{1,1}(\theta)$ above θ_c, where all married people have children, and no one divorces. In this case,

$$U(\theta_m(p), \theta_c(p)) = G(\theta_m(p)). \tag{10.63}$$

10.5.5 *Low costs of raising children*

For completeness, we discuss briefly the case with low costs of raising children, $\frac{q^*+q^0}{2} > c > q^0$. In this case, the intersection is at $\theta \in [h_1 - a, h_0 - a]$.

Therefore, we equate $W_{1,0}(\theta)$ evaluated in the first region of equation (10.17) with $W_{1,1}(\theta)$ evaluated in the intermediate region of equation (10.17), implying

$$\theta_c = p(Y+\beta) - Y - a + 2c - q^* - q^0, \tag{10.64}$$

and

$$\theta_m = \begin{cases} -Y \\ \text{if } p(Y+\beta) > a + q^0 + q^* - 2c \\ \frac{p(Y+\beta)-a+2c-q^*-q^0}{3} - Y \\ \text{if } a + q^0 + q^* - 2c \geq p(Y+\beta) \geq c + q^* - 2q^0 - 2a \\ \frac{p(Y+\beta)}{2} - \frac{q^*-c}{2} - Y \\ \text{if } p(Y+\beta) < c + q^* - 2q^0 - 2a \end{cases}. \tag{10.65}$$

Note that the assumptions $2a > \left(q^* - q^0\right)$ and $c < \frac{q^*+q^0}{2}$ ensure that interval
$\left[c + q^* - 2q^0 - 2a, a + q^0 + q^* - 2c\right]$ is non-empty.
The aggregate number of singles associated with a given p is

$$U(\theta_m(p), \theta_c(p)) = \begin{cases} \frac{G(h_1(p)+a)+G(\theta_c(p))}{2} \\ \text{if } p(Y+\beta) > a + q^0 + q^* - 2c \\ \frac{(G(h_1(p)+a)+G(\theta_m(p)))}{2} \\ \text{if } a + q^0 + q^* - 2c \geq p(Y+\beta) \geq c + q^* - 2q^0 - 2a \\ G(\theta_m(p)) \\ \text{if } p(Y+\beta) < c + q^* - 2q^0 - 2a \end{cases}. \tag{10.66}$$

10.6 References

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[4] Chiappori, Pierre-Andre and Yoram Weiss, 'Divorce, Remarriage and Welfare: A general Equilibrium Approach', **Journal of the European Economic Association**, 4 (2006), 415-426.

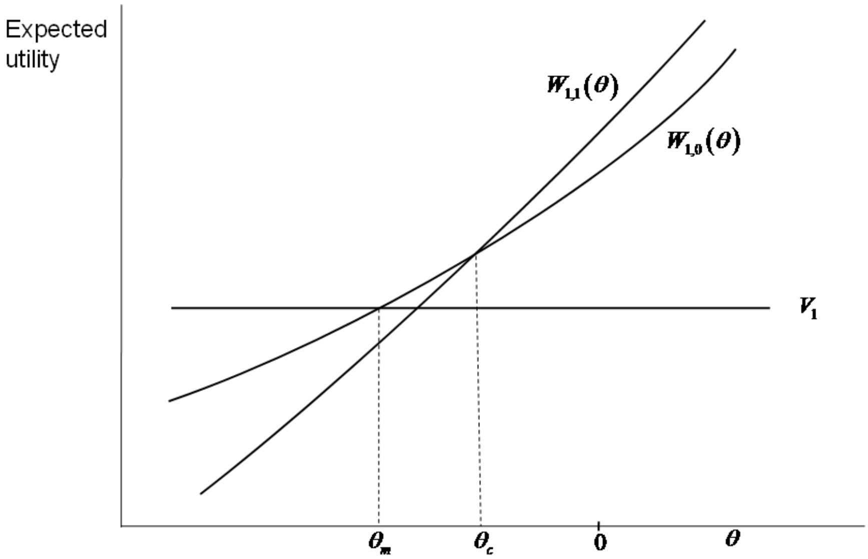

FIGURE 10.1. Expected utility profiles.

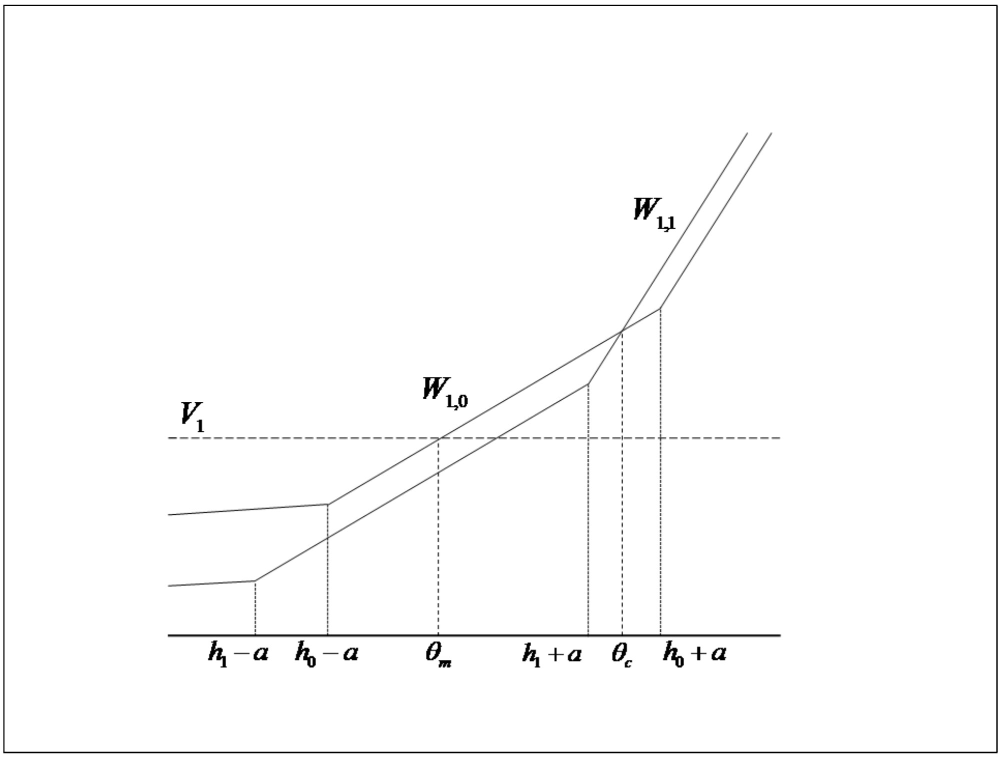

FIGURE 10.2. Expected utility profiles for example

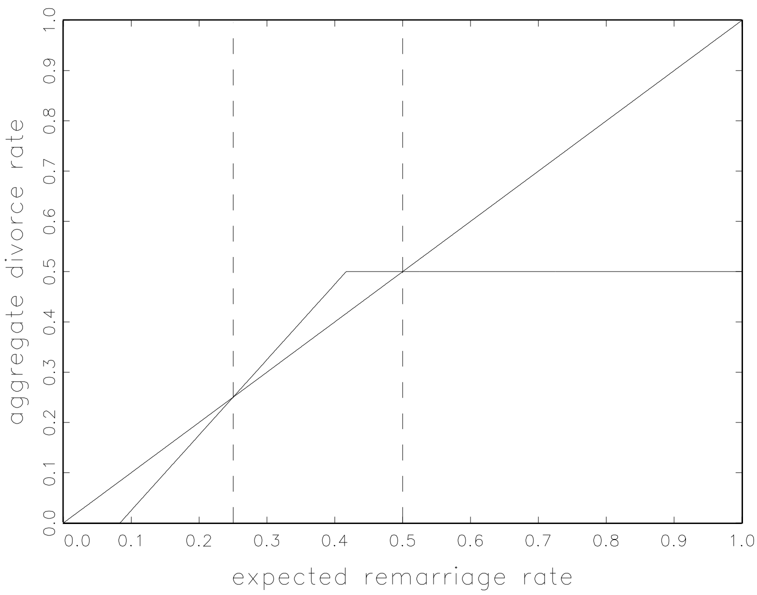

FIGURE 10.3. Equilibrium Divorce Rates

Table 10.1: Example with Multiple Equilibria

	$p = 0.0$	$p = 0.25$	$p = 0.5$
Critical value for marriage, θ_m	-1.31	-1.22	-1.14
Critical value for children, θ_c	$-.25$	.00	.25
Percent married	100	100	100
Percent with children	100	50	0
Percent divorced with children	0	0	0
Percent divorced without children	–	50	50
Percent single	0	25	50
Lifetime utility	4.083	4.094	4.208

Parameter values:
Income, $Y = 1$
Range for the match quality, $\theta \in [-\frac{1}{6}, \frac{1}{6}]$
Size of shock to match quality, $a = \pm\frac{11}{12}$
Utility of children in intact family, $q^* = 1$
Utility of children following divorce, $q^0 = 0$
Cost of raising children, $c = \frac{11}{12}$
Probability of remarriage, $\gamma = 1$
Expected quality of match conditioned on remarriage,
$\beta = E(\theta/\theta \geq -Y) = 0$

11

Marriage, Divorce, Children

11.1 Introduction

The purpose of this chapter is to examine in more detail the role of children in marriage and divorce.[1] In particular, we wish to discuss the determination of expenditures on children and their welfare under various living arrangements, with and without the intervention of the courts. There is a growing concern that the higher turnover in the marriage market causes more children to live with single mothers or step parents. In the US, year 2005, 68 percent of children less than 18 years old lived with two parents (including step parents), 23 percent lived only with their mother, 5 percent lived only with their father and the rest lived in households with neither parent present. This may be harmful to the children.[2] Part of the problem is that, following separation, fathers are less willing to transfer resources to the custodial mothers (that is, their ex-wives). A major objective of our analysis is to explain how transfers between separated parents are determined and how they vary with marriage market conditions.

Separation may entail an inefficient level of expenditures on children for several reasons: 1) If the parents remarry, the presence of a new spouse who cares less about step children reduces the incentives to spend on children from previous marriages. 2) If the parents remain single then, in addition to the loss of the gains from joint consumption, the custodial parent may determine child expenditures without regard to the interest of their ex-spouse. 3) Parents that live apart from their children can contribute less time and goods to their children and may derive less satisfaction from them. These

[1] This chapter extends the results reported in Chiappori and Weiss (2007) to include both time and money as inputs to the child welfare. See also Weiss and Willis (1985, 1993), Del-Boca (2003), and Case et al (2003).

[2] There is substantial evidence that children of divorced parents do not perform as well as comparable children in intact families. See Argys et al.(1998), Lamb et al.(1999), Hetherington and Stanley-Hagan (1999), Gruber (2004) and Stafford and Yeung (2005). Such empirical evidence should be interpreted with some care, for two reasons. First, dysfunctional families are more likely to generate both divorce *and* poor child performance. Bj́orklund and Sundstrom (2006) argue that inferior performances of divorced children can largely be attributed to selection effects. Second, even if divorce causes poor performance at the individual level, the impact of the *aggregate* divorce rate on the welfare of children is a different issue. As shown by Piketty (2003) the increase in the divorce rate in France has *reduced* the gap in school performance between children of divorced parents and children from intact families.

problems are amplified if the partners differ in income and cannot share custody to overcome the indivisibility of children. The custodial parent is usually the mother who has some comparative advantage in caring for children but has lower income. The father has often limited access to the child and low incentive to provide for him. The outcome is that the level of child expenditures following separation is generally below the level that would be attained in an intact family, reducing the welfare of the children and possibly the welfare of their parents.

An important consequence of having children is that they create ex-post wage differences between men and women. The basic reason for such differences is biological in nature. The mother is the one who gives birth as she is more capable of taking care of the child at least initially. As noted by Becker (1993) this initial difference may have large economic consequences. When the mother takes care of the child, her future earning capacity erodes. Then, because of the reduced earning capacity of the mother and her inherent advantage in child care, a pattern of specialization arises, whereby the father works more in the market and the mother works more at home; see, also, Chichilnisky (2005) and Albanesi and Olivetti (2009). This pattern is most pronounced if the couple remains married and can coordinate activities. Following separation, however, the allocation of time may change, and a custodial mother may spend less time on her child if she remarries, because a foster father cares less about the child than a natural father.

The ex-post asymmetry between parents can have strong implications for the divorce decision and the incentive to produce children. Because men maintain or increase their earning capacity during marriage, they have higher expected gains from divorce. Under divorce at will, they will initiate the divorce, at some situations in which the mother would like to maintain the marriage. If transfers within marriage are limited due to a large component of public consumption, separations will be inefficient, implying that the gains from having children are smaller to the mother than to the father. Because the production of children requires both parents, the mother may avoid birth in some situations in which the husband would like to have a child. The consequence is then an inefficient production of children.

To overcome these problems, the partners have an incentive to sign *binding* contracts that will determine some transfers between the spouses. The purpose of the transfers is to induce an efficient level of child expenditures following divorce and to guarantee efficient separation and child production by restoring the symmetry between the parents. It is generally not possible to obtain such a first best outcome, because of some important limitations on transfers. First, transfers within marriage can only partially compensate for *common* factors that affect both partners, such as the failure of the marriage. If the partners separate then transfers can compensate for differences in the gains and costs from divorce, but these transfers are limited too. In particular, it is not possible to condition the transfer on the allocation within a household which is usually not observed by a third party.

Legal intervention is required to enforce binding contracts. In practice, enforcement of alimony and child support contracts is imperfect. This is not simply a matter of lack of resources or determination on the part of the legal authorities. There is a basic conflict between private needs and social needs that results from the externalities that prevail in the marriage market. One issue is that parents and child interests may conflict, even if parents care about their children. For instance, a mother may choose to remarry even if the child under her custody is harmed, because she gains more than the child from the presence of a new spouse. Another issue is the impact of the divorce and fertility decisions of a given couple on the prospects for remarriage and the gains from remarriage of others. In marriage markets with frictions, competition does not force a couple to internalize the impact on potential mates, because meetings are to a large extent random and rents prevail. Therefore, a contract that a couple is willing to sign is not necessarily optimal from a social point of view. A related issue is that contracts that couples are willing to sign may at the time of marriage, before the quality of match is observed, may be inefficient ex-post after divorce has occurred and the impact of the contract on the divorce and fertility decisions is not relevant any more. In this case, the partners may wish to renegotiate, thereby creating a lower level of welfare for both of them from an ex-ante point of view.

The benefits from having children depend on the contracts that the parents employ to regulate these decisions and on the prospects of remarriage that are determined in the marriage market. Consequently, the incentives to produce children depend not only on the risk of divorce, triggered by changing circumstances in a specific household, such as falling out of love, but also on the general situation in the marriage market. The larger is the proportion of couples that divorce, the better are the remarriage prospects. In the absence of children, or with children but adequate transfers, this would increase the probability of divorce. However, with children, remarriage may have a negative effect on the child because the new husband of the custodial mother may be less interested in its welfare. We may refer to this problem as the "Cinderella effect" (see Case *et al.*, 1999). This effect reduces the incentive of the non custodial father to support the child, because part of the transfer is "eaten" by the new husband. In addition, non custodial parents who are committed to their custodial ex-spouse are less attractive as potential mates for remarriage. Thus, the larger is the proportion of such individuals among the divorcees, the less likely it is that a particular couple will divorce, and the more likely it is that each couple will have children. In this chapter, we use a simple model to illustrate the interactions among these considerations in a general equilibrium framework and highlight the potential consequences for parents and children.

11.2 The Model

We consider here a given cohort with equal number of men and women. Individuals live for two periods and can be married or single in each of these periods. A household consists of one or two adults and possibly one child. We treat fertility as a choice variable and each couple decides on whether or not it should have a child in the first period. We assume that childless men and women are identical and both earn the same wage w_f. However, if a couple has a child then, because the mother is the one who gives birth, her second period wage drops to a lower level, w_m.

11.2.1 The technology and preferences

The household pools the incomes of its members and allocates it to buy an adult good a and a child good c. Each parent has one unit of time which can be allocated between market work and child care. Let h_m and h_f denote the time spent by mother and father in market work, respectively. Then, the amount of time they spend at home is $t_j = 1 - h_j$, where $0 \leq h_j \leq 1$ for $j = m, f$.

The household production function is

$$q = \alpha a + t + g(c) \tag{11.1}$$

where

$$t = \beta t_f + \gamma t_m. \tag{11.2}$$

The output q is interpreted as the child's utility or 'quality'. The parameter α describes the marginal effect of the adult good, a, on the child's quality, the parameters β and γ represent the productivities of the father and mother, respectively, in household work and t is total time spent on the child, measured in efficiency units. The function $g(c)$ is assumed to be increasing and concave, with $g(0) = 0$. The linearity in t is assumed to allow corner solutions whereby family members *specialize* either in household work or market work. To determine the pattern of specialization under different household structures, we assume

$$\begin{aligned} \gamma &> w_m(1+\alpha) \\ \beta &< w_f(1+\alpha) \end{aligned} \tag{11.3}$$

where w_m is the wage of the mother and w_f is the wage of the father. That is, the mother is more productive at home, while the father is more productive in the market. This may hold either because the mother has an absolute advantage in home production $\gamma > \beta$ or that she has an absolute disadvantage in market work, $w_m < w_f$, because of the erosion in her wage due to her withdrawal from the labor force during child birth.

The adult good a is shared by all members of a household. The marginal utility of each adult from the adult good is set to 1, while the marginal utility of the child is set to the constant α that is smaller than 1. In contrast, the child good, c, is consumed only by the child. However, indirectly, child consumption matters to the parents of the child, who care about its welfare. The utility of a child is defined to be identical to its quality, q, and the utility of each parent is defined as the sum $a + q$. Thus, both parents care about their joint child, wherever the child lives. In this sense, child quality is a collective good for the natural parents.[3]

Taken together, these assumptions impose a quasi-linear structure that implies that the time and specific goods spent on the child do not depend on the household's total income if a positive amount of the adult good is consumed. In that case, any additional income is spent only on the adult good. Income effects on the child are present, however, if the adult good is not consumed.

Married couples also enjoy a match specific "love" factor which we denote by θ. This factor is random and not known at the time of marriage. The quality of match θ is revealed at the end of each period.[4] We assume that θ is independent across couples and is distributed with some known distribution $F(\theta)$ that is symmetric and has a mean of zero.[5] Individuals, who marry at the beginning of the first period, observe θ at the end of the first period and can then decide whether or not to break the marriage and look for a new match. If marriage continues it will have the same θ. If a new marriage is formed its θ will be a random draw from $F(\theta)$.

A negative shock to θ can cause dissolution of the marriage. Following divorce, the parents may remain single or remarry, so that the child may live in a household that consists of one or two adults. Household structure affects both the technology and the household decision making. We assume that a parent can spend time on a child only if they live in the same household, but may spend money on the child even if they live apart. If both parents live together with their child in an intact family, all household goods are public and there is no conflict as to how much should be spent on the child. However, if the family breaks the parents may have conflicting interests, because the costs of caring for the child good will not be the same when they do not share the adult good. In addition, if the custodial parent

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[3] A parent that lives apart from the child may enjoy it to a lesser degree, and we may set the parent's utility to $a + \delta q$ if the parent and child live in a separate households. The parameter δ may be interpreted as a discount factor that captures the idea that "far from sight is far from heart".

[4] In contrast to chapter 10, we simplify here by eliminating the premarital signal of the quality of the match.

[5] The zero mean assumption implies that in the second period agents marry only "for money". In the first period, however, the average married couple enjoys a positive non monetary gain, because the option of divorce eliminates some of the downward risk. The model can be easily generalized to the case with $\bar{\theta} > 0$.

remarries, then child quality is influenced by the foster parent who may care less about the child than his natural parents.

11.2.2 The legal framework

We consider a modern society in which individuals can marry or divorce at will. However, the partners can sign binding contracts, enforced by law, that specify the custody arrangement and child support payments following divorce or remarriage. Such contracts may be signed at the time of divorce or at the time of marriage. An interim contract signed at the time of divorce takes the presence of children and the separation as given, and it's main objective is to influence the expenditures on children under the different household structures that may arise if each parent remarries or remains single. An ex-ante contract, signed at the time of marriage, aims to influence the fertility and separation decisions as well. We discuss here simple and familiar contracts in which the mother obtains custody and the father commits, at the time of divorce, to pay the mother a fixed amount that is not contingent on whether one or both of the parents remarry. Such binding contracts are in fact enforced by law.

Except for the enforcement of arrangement that the partners may reach, the law may also intervene by setting standards within which the partners can operate. Custody is most often given to the mother on the ground that she can take better care of the child, while the father obtains visitation rights. Unless stated otherwise, we shall assume that, following divorce the mother is the sole custodian.[6] Often, custody assignment is associated with some amount of child support that is mandated by law. The guiding principle is that the custody assignment and the mandated payments should minimize the harm to the child. Such legal constraints may affect the agreements that partners would reach when bargaining in the "shadow of the Law" (see Mnookin and Kornhauser, 1977).

11.2.3 The meeting's technology

As in chapter 10, we assume that, each period, a person meets a *random* draw from the population of the opposite sex in the same age group. If this person is already married then such a meeting is "wasted" and no new marriage is formed. This feature creates "increasing returns" in meetings (see Diamond and Maskin, 1979), whereby it is more likely to meet a single

[6] See Cancian and Meyer (1998). However, the share of joint physical custody has increased over time. Halla (2009) examines the impact of state differences in this trend and concludes that the option of joint custody has raised the incentives of men to marry, with little impact on divorce.

person if there are more singles around.[7]

Only if two singles meet, they can form a new household. We denote by p the proportion of singles (divorcees) that one meets in the second period of life. Because the expected quality of match, θ, which is revealed with a lag, equals zero and the material gains from marriage are positive, every meeting of two divorcees results in a marriage, so that p is also the probability of remarriage. A new element in this chapter is that, in the second period, one can meet individuals that differ in their attractiveness as partners for remarriage, because of the presence of children, lack of income or commitments to previous spouses. However, the consequences of having a child may differ for men and women. For women it implies a lower wage. For men it may imply commitments to the ex-wife. In either case, a person with a child is a less attractive match.

11.3 Household structure and child care

We begin our analysis with the allocation of resources by couples with one child. This allocation depends on whether the partners live with their child or are separated.

11.3.1 Intact family

If both parents live with their child in an intact family, the utility of husband and wife is the same for all allocations and the two spouses will agree to maximize their common utility subject to the household budget constraint

$$a + c = w_m h_m + w_f h_f. \tag{11.4}$$

Because the mother is assumed to have the comparative advantage in home production, the father spends all his discretionary time in the market, $h_f = 1$, while the mother will spend all her discretionary time at home. To verify the optimality of this outcome, note that, due to assumption (11.3), an increase the father's work in the market raises the utility of *both* parents by $w_f(1 + \alpha) - \beta$, while an increase in the mother's work at home raises their common utility by $\gamma - w_m(1 + \alpha)$.

[7] Lauman *et al.* (1994, Table 6.1) report that about half of the marriages arise from meeting in school, work, and private party and only 12 percent originate in specialized channels such as social clubs or bars. The establishment of more focused channels, where singles meet only singles, is costly and they will be created only if the "size of the market" is large enough. Also, as noted by Mortensen (1988), the search intensity of the unattached decrease with the proportion of attached people in the population. The reason is that attached individuals are less likely to respond to an offer, which lowers the return for search (see Chapter 7).

Given this specialization pattern, the amount spent on the child is determined by equalizing the marginal utilities from a and c, that is,

$$1 + \alpha = g'(c). \tag{11.5}$$

We denote the unique solution to (11.5) by c^* and assume that $c^* < w_f$ so that a positive amount is spent on the adult good a.

The utilities of the three family members in an intact family are

$$\begin{aligned} u_c &= g(c^*) + \gamma + \alpha(w_f - c^*) \\ u_m &= u_f = g(c^*) + \gamma + (1+\alpha)(w_f - c^*). \end{aligned} \tag{11.6}$$

We shall denote the above common utility of father and mother in an intact family (with a child) by u^*.

11.3.2 Separation, custody and voluntary transfers

If the parents separate, one of the parents receives custody over the child. Only the custodial parent can contribute household time to the child, but both parents can participate in the child expenditures. The non custodial parent continues to care about the child and may wish to transfer resources to it voluntarily. Transfers can be earmarked in the form of tuition and health care, for instance, or fungible in which case the custodian treats it as regular income that can be allocated according to the custodian's preferences. Generally, an ear-marked transfer is preferred by the non custodial parent, because part of the fungible transfer is 'taxed' by the custodial parent (and also the new spouse if remarriage occurs) and does not reach the child. Realistically, the father can rarely transfer money directly to the child, especially when he is young. We shall, therefore, discuss here only fungible transfers. The transfers are determined at the time of separation, prior to meeting a new partner and are binding when the new marital status of the parents is realized. Such voluntary commitments are enforced by law and typical examples are child support and alimony agreements. We shall discuss separately non contingent contracts, such as child support in which the payment usually does not depend on the marital status of the parents, and contingent contracts, such as alimony in which the payment may stop if the ex-spouse remarries.

In practice, custody is often given to the mother, based on the idea that she can or willing to take better care of the child. In terms of our model, this is rationalized by the assumed comparative advantage that the mother has in housework. However, it is possible that the child is better of with the father because of the higher adult consumption that he provides. The custody choice is, therefore, related to the transfers that occur between the divorced parents. If transfers are sufficiently high, it is possible to restore at least partially the efficient division of labor that is attained under marriage, so that the child and consequently the parents are less harmed by the divorce. For the time being, we shall assume that the mother receives custody and will address this issue again after we derive the equilibrium level of transfers.

Single custodial mother

If the custodial mother remains single, she will choose the amount of time that she spends at work h_m, her adult consumption a_m and the amount of child goods c so as to maximize her own utility subject to her budget constraint, taking as given the amount that the ex-husband transfers to her, s. Her utility is then defined as the solution to the program

$$\begin{aligned} u_m(s) &= \max_{a_m,c,h_m} \{(1+\alpha)a_m + \gamma(1-h_m) + g(c)\} \\ &\quad \text{subject to} \\ a_m + c &= w_m h_m + s, \\ 0 &\leq h_m \leq 1. \end{aligned} \tag{11.7}$$

The mother's choices as a function of s are summarized in Figure 11.1 below. Because of the quasi-linear structure of the problem, the solution has three distinctly different regions. For low levels of s, the mother withdraws some time from the child and works in the market part time. She then spends all her disposable income on child goods. The optimum conditions in this region are

$$\begin{aligned} w_m g'(c) &= \gamma, \\ c &= w_m h_m + s. \end{aligned} \tag{11.8}$$

Thus, the mother spends a *fixed* amount of money, $\hat{c}$ on child goods and works in the market the minimal amount of time required to achieve this target. As s rises, the mother reduces her market work until it reaches zero, spending more time on child care.

For high levels of s, the mother does not work in the market and allocates her disposable income between the child and adult goods. The optimum conditions in this region are

$$\begin{aligned} g'(c) &= 1+\alpha, \\ c + a_m &= s. \end{aligned} \tag{11.9}$$

That is, the mother will spend a *fixed* amount of money, c^*, on the child and adjust her adult consumption according to the level of s.

For intermediate values of s, satisfying

$$\frac{\gamma}{w_m} > g'(s) > 1+\alpha, \tag{11.10}$$

the mother will not work and will not consume adult goods, so that all her income and free time are devoted to the child.

This pattern of behavior reflects our assumption that the mother has comparative advantage in child care $\gamma > w_m(1+\alpha)$ which is seen to imply that, for the mother, the child comes first and she spends resources on herself only when she is sufficiently wealthy. The utility of the child is then

$$q(s) = \begin{cases} g(\hat{c}) + \gamma(1 - \frac{\hat{c}-s}{w_m}) & if \quad s \leq \hat{c}, \\ g(s) + \gamma & if \quad \hat{c} < s < c^*, \\ g(c^*) + \gamma + \alpha(s - c^*) & if \quad s \geq c^*, \end{cases} \tag{11.11}$$

and the utility of the mother is

$$u_m(s) = \begin{cases} g(\hat{c}) + \gamma(1 - \frac{\hat{c}-s}{w_m}) & if \quad s \leq \hat{c}, \\ g(s) + \gamma & if \quad \hat{c} < s < c^*, \\ g(c^*) + \gamma + (1+\alpha)(s - c^*) & if \quad s \geq c^*. \end{cases} \tag{11.12}$$

Remarried custodial mother

If the custodial mother remarries, she may spend less time and money on the child, because her *new* husband receives little or no benefits from such spending. This is in contrast to the case of an intact family where, by assumption, *both* parents benefit equally from the time and money spent on the child. To sharpen our results, we assume that the new husband derives no utility at all from the step child and depends only on the adult good, a, that the remarried couple purchases.[8] The mother, however, cares about both the adult good, a, and the child good, c. All this means that the child good is a private good for the wife in the new household. Because of the potential conflict of interests, bargaining is required to determine the amount spent on the adult good in the new household.[9] Assuming that the bargaining outcome is efficient, it must be on the Pareto frontier within the new household. Let $y_h = w_f - s'$ denote the income that the new husband brings into marriage, net of his obligations to his ex-wife, $s\prime$.[10] Then, the Pareto utility frontier is given by

$$u_m = \begin{cases} (1+\alpha)a + \gamma + g(y_h + s - a) \\ \text{if } y_h < a < y_h + s - \hat{c} \\ (1+\alpha)a + \frac{\gamma}{w_m}(w_m + y_h + s - a - \hat{c}) + g(\hat{c}) \\ \text{if } y_h + s - \hat{c} \leq a \leq y_h + w_m + s - \hat{c} \\ (1+\alpha)a + g(y_h + w_m + s - a) \\ \text{if } y_h + w_m + s - \hat{c} < a < y_h + w_m + s \end{cases} \tag{11.13}$$

[8] The new husband's utility also depends on the utility of his child from the previous marriage, which is taken as given in the bargaining of the remarried couple.

[9] Akashi-Ronquest (2009) reports lower child investments following remarriage (compared with intact families) and that an increase in the hourly wage of a biological mother significantly improves her child investment when her husband is a stepfather of the child, while there is no such effect for mothers living with the biological father of the child. The author interprets these findings as bargaining on child quality in step families.

[10] Since we shall examine only symmetrical equilibria, there is no loss of generality in assuming that all other fathers make the same payments.

and described in Figure 11.2, which is drawn for the case in which the transfer from the father, s, exceeds the efficient level of expenditure on the child good, c^*. For levels of a close to y_h, the mother spends all her time on the child and the implied expenditure on the child good c exceeds c^*. In this case, *both* spouses want to reduce c and increase a, although the child may be hurt from such a substitution. As a is raised sufficiently so that c reaches c^* the newly formed couple enters the region of *conflict.* Any further increase in a and the associated decrease in c, benefits the new husband but reduces the mother's (and the child's) utility. Initially, the mother continues to spends all her time on the child but when a reaches $y_h + s - \hat{c}$, she starts to work part time and continues to do so until a reaches $y_h + w_m + s - \hat{c}$. In this segment, the Pareto frontier is *linear* because the child good is held at a fixed level, $c = \hat{c}$, and any increase in a is achieved by an increase in h_m which raises the father's utility by $w_m dh_m$ and reduces the mother's utility by $((1 + \alpha)w_m - \gamma)dh_m$. At high levels of a, exceeding $y_h + w_m + s - \hat{c}$, the mother works full time in the market and as a rises, the amount of child good is reduced until it reaches zero and the new husband obtains all the household resources, $y_h + w_m + s$.

To proceed with the analysis, one must determine how the conflict between the spouses is resolved and which particular point on the Pareto frontier is selected. For simplicity, we assume that the new husband obtains *all* the surplus from remarriage, so that the point on the Pareto frontier is selected so as to make the mother *indifferent* between remarriage and remaining single. This allows us to illustrate the general equilibrium issues in a relatively simple manner. The reader may interpret the model as a worst case scenario from the point of view of the mother and child.[11] The efficient level of adult consumption is then defined as the solution of the following maximization program

$$\begin{aligned} a(s, y_h) &= \max_{a,h_m,c} a \\ &\quad \text{subject to} \\ a + c &= w_m h_m + y_h + s, \\ (1+\alpha)a + \gamma(1 - h_m) + g(c) &\geq u_m(s), \\ 0 &\leq h_m \leq 1. \end{aligned} \tag{11.14}$$

For remarriage to take place, it must be the case that the solution of this

[11] We could use instead a symmetric Nash-Bargaining solution to determine the bargaining outcome (see Chiappori and Weiss, 2007). The Nash axioms imply that the bargaining outcome must maximize the product of the gains from remarriage, relative to remaining single, of the two partners. This model yields similar qualitative results, because the mother is assumed to have lower income and therefore her options outside marriage are worse than those of men. The magnitudes of the welfare loss of the child and mother would, of course, be smaller if the mother gets a larger share of the gains from marriage.

program satisfies $a(s, y_h) \geq y_h$, otherwise the new husband would be better off as single.

The first order conditions for the efficient allocation within the new household are

$$\begin{aligned} -1 - \lambda(g'(c) - (1+\alpha)) &= 0, \\ w_m - \lambda((1+\alpha)w_m - \gamma) &= 0 \text{ if } 0 < h_m < 1, \\ w_m - \lambda((1+\alpha)w_m - \gamma) &\leq 0 \text{ if } h_m = 0, \\ w_m - \lambda((1+\alpha)w_m - \gamma) &\geq 0 \text{ if } h_m = 1, \end{aligned} \tag{11.15}$$

where λ is a Lagrange multiplier such that $\frac{1}{\lambda}$ equals the slope of the Pareto frontier in the new household (see equation (11.13) and Figure 11.2)). In an interior solution with $0 < h_m < 1$ we have

$$\lambda = \frac{w_m}{(1+\alpha)w_m - \gamma} < 0, \tag{11.16}$$

because a marginal increase raises the utility of the husband by w_m and reduces the utility of the wife by $(1+\alpha)w_m - \gamma$. As in the case of a single mother household, $w_m g'(c) = \gamma$ and $c = \hat{c}$ as long as the mother works part time in the market. If h_m is at the boundaries of 0 or 1, the level of child expenditures c is determined by the requirement that the utility of the mother is equated to her reservation utility $u_m(s)$ and

$$\lambda = \frac{1}{(1+\alpha) - g'(c)}. \tag{11.17}$$

In this section, we shall consider only equilibria with moderate levels of transfers such that the mother spends some time in the labor market when she is remarried. This allows us to exploit the linear Pareto frontier that arises in this case, which substantially simplifies the calculations. Such interior solution requires that the net income of the new husband is sufficiently high to motivate the mother to work part time but not so large as to cause her to work full time.

If the remarried mother has no surplus then

$$a(s, y_h) + q(s, y_h) = u_m(s). \tag{11.18}$$

When the mother remarries, she obtains more of the adult good that she shares with her new husband but the child's utility $q(s, y_h)$ is lower. This implies that the mother's remarriage has a negative impact on the father, but he can mitigate this effect by transferring money to the mother, that is by increasing s. Specifically,

$$\begin{aligned}
\frac{\partial a(s,y_h)}{\partial s} &= \lambda(u_m'(s)-\frac{\gamma}{w_m}) \geq 0, \\
\frac{\partial a(s,y_h)}{\partial y_h} &= 1-\lambda(1+\alpha) > 1, \\
\frac{\partial q(s,y_h)}{\partial s} &= u_m'(s)-\frac{\partial a(s,y_h)}{\partial s} > 0, \\
\frac{\partial q(s,y_h)}{\partial y_h} &= -\frac{\partial a(s,y_h)}{\partial y_h} \leq 0.
\end{aligned} \tag{11.19}$$

An increase in the transfer s raises the utility that the mother would receive as single and improves her bargaining position in the newly formed household. Consequently, the remarried mother works less and spends more time with the child, which raises the utility of the child. [12] However, an increase in s also has the unintended effect of raising the new husband's utility, who "eats" part of the transfer. An increase in the net income of the new husband raises his gain from marriage $a(s,y_h)-y_h$ because the mother spends less time with the child and more time in the market. The mother is willing to do such a sacrifice of child quality because she is compensated by a higher level of adult consumption, jointly with the new husband.

The result that remarried mothers work more in the market may seem counterfactual.[13] We emphasize that market work is just one way of transferring resources from the child to the new husband and the crucial assumption is the availability of a linear transfer in some non-trivial range. For instance, the mother may spend less time with the child and more time with the new husband in joint leisure activities. As long as such substitutions are available at a fixed rate of exchange, the results are the same as if the remarried mother would spend time working in the market.

11.4 Transfers, the interim perspective

Following separation, the parents can be in four different states, depending on the new marital status of their ex spouses:

1. Both parents are single, which happens with probability $(1-p)^2$.

[12] Note that $\frac{\partial a(s,y_h)}{\partial s}=0$ if $s=\hat{c}$, $\frac{\partial a(s,y_h)}{\partial s}=1$ if $s=c^*$ and $1>\frac{\partial a(s,y_h)}{\partial s}>0$ for $\hat{c}<s<c^*$.

[13] As seen in Chapter 1, Figure 1.13 , the raw data suggests that divorced women work more than married women. However Seitz (1999) shows that correcting for selection and unobserved attributes, there is no significant difference in labor supply of divorced and married women, while remarried women work significantly more than married women, as our model suggests.

2. The father remains single while the mother is remarried, which happens with probability $p(1-p)$.

3. The mother remains single but the father is remarried, which happens with probability $(1-p)p$.

4. Both parents remarry, which happens with probability p^2.

Note that, by assumption, the probability of remarriage is the same for the husband and wife, and that meetings and subsequent remarriages are independent across parents.

Anticipating these contingencies, the father may be willing to commit to transfer money to the custodial mother with the intention to influence the welfare of the child, of whom he continues to care.[14] Each father makes his choice of s separately, taking the choice of others, $s\prime$ as given. These payments are made at the time of divorce, before the marital status of the ex-spouses is known. We, therefore, must use expectations in determining the optimal level of the transfer. The expected utility of the father is, therefore,

$$\begin{aligned} V_f &= (1-p)^2[w_f - s + q(s)] + (1-p)p[w_f - s + q(s, w_f - s')] \\ &\quad +p(1-p)[a(s', w_f - s) + q(s)] \\ &\quad +p^2[a(s', w_f - s) + q(s, y - s')] \end{aligned} \tag{11.20}$$

and

$$\frac{\partial V_f}{\partial s} = (1-p)[q'(s) - 1] + p[\frac{\partial q}{\partial s} - \frac{\partial a}{\partial y_h}] \tag{11.21}$$

We first note that the father will never choose voluntarily transfer s that exceeds c^* because, in this case, the single mother would spend the marginal dollar on the adult good. The father then receives a marginal benefit of α from the transfer if the mother remarries single and $1 + a - \frac{\gamma}{\gamma-(1+\alpha)w_m}$ if she remarries. But his expected cost in terms of the adult good is higher, because a transfer of a dollar costs the father 1 dollar if he remains single and $\frac{\partial a}{\partial y_h} = \frac{\gamma}{\gamma-(1+\alpha)w_m}$ if he remarries (see equations (11.11) and (11.19)).

Under our maintained assumption that the remarried mother works part time, equation (11.21) can be rewritten as

$$\frac{\partial V_f}{\partial s} = \begin{cases} (1-p)(\frac{\gamma}{w_m} - 1) + p(\frac{\gamma}{w_m} - \frac{\partial a}{\partial y_h}) & \text{if} \quad 0 \le s < \hat{c}, \\ (1-p)(g'(s) - 1) + p(g'(s) - 2\frac{\partial a}{\partial y_h}) & \text{if} \quad \hat{c} \le s \le c^*, \end{cases} \tag{11.22}$$

where

$$\frac{\partial a}{\partial y_h} = 1 - \lambda(1+\alpha) = \frac{\gamma}{\gamma - (1+\alpha)w_m}$$

[14] Another possible motive is that the father maintains an altruistic motive towards his ex-wife. In this chapter, however, we ignore this added altruistic link and confine our attention only to the case in which parents care about their children.

The two branches in (11.22) reflect changes in the mother's behavior as a function of the transfer s that she would receive if she would have remained single. In the region $0 < s < \hat{c}$, the single mother would work part time, so that $c = \hat{c}$ and $q'(s) = \frac{\gamma}{w_m}$. The independence $\frac{\partial V_f}{\partial s}$ from s in this region implies that either the father will contribute nothing or he will voluntarily commit on a transfer of at least $\hat{c}$. Which of these two possibilities applies depends on the basic parameters of the model and the probability of remarriage. The father is certainly willing to transfer resources to his ex-wife if he and the mother remain single with high probability, because then the marginal benefit in terms of child quality, $\frac{\gamma}{w_m}$, exceeds the marginal costs in terms of the forgone consumption of the father, which is 1. The father will be more reluctant to contribute if p is large, because then the cost for him is larger, $\frac{\partial a}{\partial y_h}$. The father will contribute at all p if $\gamma > (2 + \alpha)w_m$ which means that the mother is highly effective in caring for the child. If this requirement is not satisfied, there will be some critical p below which the father will give at least the amount $\hat{c}$, but above which he will give nothing. In the region $\hat{c} < s < c^*$, the mother uses the transfer to increase child consumption and because of the concavity of $g(s)$, the marginal value of the transfer $\frac{\partial V_f}{\partial s}$ declines with s. Hence, an interior solution can exist in this region.

We can now characterize the father's incentives to support the mother.

Proposition 11.1 *Let $s^*(p)$ be the* optimal *level of voluntary commitment that the father is willing to make at the time of divorce. Then, $s^*(p) \le c^*$, declines in the probability of remarriage, p, and is independent of the income of the new husband whom the mother may remarry. For a sufficiently high comparative advantage of the mother in child care, that is, $\gamma > (2+\alpha)w_m$, the optimal transfer, $s^*(p)$, exceeds $\hat{c}$. The transfer is then set to $s^*(p) = c^*$ if $p < p_1$, where p_1 satisfies*

$$1 + \alpha = (1 - p_1) + 2p_1 \frac{\partial a}{\partial y_h}. \tag{11.23}$$

Otherwise, if $p \ge p_1$, $s^*(p)$ is determined by the unique solution to

$$g'(s) = (1 - p) + 2p \frac{\partial a}{\partial y_h}. \tag{11.24}$$

The optimal transfer declines with the remarriage probability for two reasons: the marginal impact of a transfer on child quality is larger (or equal) when the mother is single, and the cost of giving are higher if the father remarries.

The independence of the transfer from the new husband's income implies that although the expected utility of each husband depends on the transfers by others, the *marginal* impact of s is not and, therefore, s is independent of $s\prime$. This feature is reflected in the fact that

$$\frac{\partial a}{\partial y_h} = \frac{\gamma}{\gamma - (1 + \alpha)w_m}$$

is a constant. However, if the mother does not work when she is remarried, or works full time, the utility frontier for a remarried couple is no longer linear and the marginal impact of the transfer to the mother will depend on the net income of her new husband y_h.

11.4.1 Partial equilibrium

Suppose that all couples have children. Then, at given probability of remarriage, p, the equilibrium outcome is that all fathers will transfer the *same* amount $s^*(p)$ to the mother if the marriage dissolves. That is, given that other fathers choose $s\prime = s^*(p)$, each father independently chooses $s = s^*(p)$. This equilibrium requirement is trivially satisfied here, because the optimal choice of each father is (locally) independent of the choices of others. We refer to the equilibrium as partial because, as we shall see shortly, the remarriage and fertility rates must also be set at equilibrium levels.

In this partial equilibrium, the mother works part time when she is remarried but not as single. The reason for this difference is that she must compensate her new husband for the option of sharing the adult good.

The amount of time that the mother spends in market work is

$$h_m(p) = \frac{(1+\alpha)(w_f - \hat{c}) + g(\hat{c}) - g(s^*(p))}{\gamma - (1+\alpha)w_m} \tag{11.25}$$

and a sufficient condition for an interior solution $0 < h_m(p) < 1$ for all p is that:

$$\gamma - (1+\alpha)w_m > (1+\alpha)(w_f - \hat{c}) > g(c^*) - g(\hat{c}). \tag{11.26}$$

Basically, the mother should have a sufficiently high comparative advantage in child care to motivate her to spend some time at home and the net income of the new husband should not be so high that the mother is driven completely into the market, contrary to her comparative advantage.[15]

We see that when p rises and *all* husbands reduce their contribution, the remarried mothers increase their hours of work and thus reduce the amount of time spent with the child. The implied adult consumption in the remarried household

$$a(p) = w_m h_m(p) + w_f - \hat{c} \tag{11.27}$$

rises in p but the child's utility if the mother remarries

$$q(p) = \alpha a(p) + \gamma(1 - h_m(p)) + g(\hat{c}) \tag{11.28}$$

[15] The sufficient condition (11.26) is much stronger than we need because, as we shall show shortly, the equilibrium remarriage rate is bounded by $\frac{1}{2}$.

declines in p, because the mother's time is more important for the child than the added adult good.

The expected utilities of the three family members, evaluated at the time of divorce, are

$$\begin{aligned} V_m(p) &= g(s^*(p)) + \gamma, \\ V_c(p) &= g(s^*(p)) + \gamma - pa(p), \\ V_f(p) &= g(s^*(p)) + \gamma + (1-p)(w_f - s^*(p)). \end{aligned} \tag{11.29}$$

Compared with an intact family with $\theta = 0$, all three family members are worse off if the marriage breaks. The child received less child goods because the transfer from the father $s^*(p)$ is lower than c^* (except at low probability of remarriage $p < p_1$) and also less time if the mother remarries. Both the mother and the father suffer from the reduction in child quality. In addition, there is a loss of resources resulting from the inability to share consumption goods when the parents remain single. This cost is born mainly by the mother. The assumption that the mother receives no surplus implies that she pays for the adult good in terms of the child's quality, so that her utility is unaffected by remarriage but that of the child is reduced by $a(p)$. The father, on the other hand, gets the benefits from sharing $a(p)$ with the new wife and, in addition, he consumes the adult good when he is single. In fact, he consumes as single more of the adult good than he would under marriage. The outcome of this asymmetry is that the father's expected utility following separation is higher than the mother's.

The expected utility of all family members in the aftermath of divorce declines with the probability of remarriage, p. This is a surprising result, given that remarriage is voluntary. It can be traced to the fact that a higher remarriage rate does not only make it easier to remarry, which is *individually* welfare enhancing, but also affects behavior in a way that may be harmful to others. Thus, although the mother fully internalizes that the child is worse off upon remarriage, this does not stop her from remarrying if she is compensated by higher adult consumption. Nor does she take into account the negative impact of her remarriage on her ex-husband. The father's incentives to transfer money to the custodial mother decline as the probability of remarriage rises, because he anticipates that part of it will be spent on adult goods that are not as useful to the child, mainly because of a presence of a third party in the form of the new husband. As a result of this reluctance to contribute, mothers are worse off even if they remain single. Finally, each father is worse off mainly because the child is worse off when the mother remarries and he cannot fully remedy that by the use of transfers, due to the principal-agent issues that we described. This loss of control is sufficiently costly to offset the gains that the father receives when he remarries and obtains all the surplus.

11.5 Divorce

Having observed the realized quality of the current match, each spouse may consider whether or not to continue the marriage. A parent will agree to continue the marriage, if given the observed θ the utility in marriage exceeds his/her expected gains from divorce. Under divorce at will, the marriage breaks if

$$u^* + \theta < \max(V_m, V_f) - b \tag{11.30}$$

where u^* is the common utility of the husband and wife if the marriage would continue (not incorporating the quality of the match) and b is a fixed cost associated with divorce. The fixed costs reflect the emotional, legal and relocation costs associated with the change in marital status that affects the child and parents. We assume that these costs are higher for couples with children and are shared equally by the two spouses.

The particular value of θ that triggers divorce is given by

$$\theta^*(p) = \max(V_m(p), V_f(p)) - u^* - b. \tag{11.31}$$

The critical value θ^* is seen to equal the expected gains from divorce, relative to remaining married, evaluated at $\theta = 0$, which is the mean value of θ in the population. In other words, the couples that divorce are those with a realized quality of match that is below the unconditional expectation of the gains from divorce, *before* θ is observed. These expected gains are negative, because an intact marriage with $\theta = 0$ is better for all parties. The probability that a couple will divorce is then

$$\Pr\{\theta \leq \theta^*\} = F(\theta^*) \tag{11.32}$$

where $F(.)$ is the cumulative distribution of θ.

Our previous analysis implies that

Proposition 11.2 *If all couples have a child, then in a partial equilibrium where all fathers choose the optimal transfer $s^*(p)$, the divorce decision at any expected remarriage rate, p, is determined by the father. The critical value of θ that triggers divorce $\theta^*(p)$ is negative and declines in the probability of remarriage, p.*

Because of the ex-post asymmetry between the partners, separation may be inefficient. The father, who has strictly higher expected gain from divorce than the mother, will initiate the divorce at some θ such that the mother wants the marriage to continue and inflict on her a loss of match quality.[16]

[16] We note, however, that if $\delta < 1$ so that the non custodial father suffers from the distance from his child, the father's gains from divorce decline and may be lower than the mother's.

Couples without children

If the parents do not have a child, the "material" utility (not including the love component θ) of *each* parent in an intact family is $u^* = 2w_f$, reflecting the assumptions of income pooling and joint consumption of the adult public good. However, a parent that remains single consumes only w_f. If *all* couples do not have a child, the symmetry between the parents is reestablished and both expect upon separation to receive

$$V_m(p) = V_f(p) = (1-p)w_f + p2w_f. \tag{11.33}$$

An important difference from the case with children is that the expected utility of the two parents, as evaluated at the time of divorce, *rises* with the probability of remarriage. This is simply an outcome of the option to share consumption upon remarriage, without any negative impact of divorce on child quality.

The critical value of θ that triggers divorce is now given by

$$\theta^*(p) = -(1-p)w_f \tag{11.34}$$

which rises with p. That is, the higher is the probability of remarriage the more likely it is that a particular couple will divorce. This result is in sharp contrast to that in Proposition 11.2, illustrating the marked difference that children might have on the divorce decisions.

11.6 Fertility

So far, we took the number of children as given and assumed that all couples have children. We now examine the decision to have children.

We view children as an investment good that the parents produce at some cost during the first period of marriage, before the quality of match is revealed. To simplify, we assume that only one child can be produced. The costs of having a child are the forgone earnings of the mother associated with child birth and child rearing. We assume that the mother cannot work in the first period if she gives birth, so that w_f is lost in the first period. Also, because of the mother's withdrawal from the labor force, her second period wage erodes from w_f to w_m. The benefits from the child that accrue in the second period depend on the probabilities of divorce and remarriage and on the parents' ability to care for the child in the aftermath of divorce. To avoid trivial solutions, we assume that children may be a bad or good investment, depending on the circumstances. In particular, a couple that obtains the average draw $\theta = 0$ and chooses not to divorce gains from having had children. This is equivalent to saying that children are desired if divorce is not an option. However, when divorce is an option, children may be a liability if they lock the parents into bad matches.

An important feature of the analysis is that the decisions of each couple whether to divorce or to have a child depend not only on the circumstance of the couple, for example, if it suffered a negative shock, but also on the decisions of other couples to have children and to divorce, as well as on the contracts that they sign. These decisions by other couples influence the prospects of remarriage and the quality of potential mates. To simplify our analysis, we focus here on the case in which, *in equilibrium,* all couples have children or all couples do not have children. Therefore, we only need to consider the benefits of a particular couple from having a child, conditioned on whether or not *all* other couples have children.

Given the choices of others, the expected life time utility of a parent j in a particular couple is

$$W_{j,n}(p) = u_n^0 + \int_{\theta_n^*(p)}^{\infty} (u_n^* + \theta) f(\theta) d\theta + F(\theta_n^*(p))(V_{j,n}(p) - b_n) \qquad (11.35)$$

where, $j = f$ for the (potential) father and, $j = m$ for the (potential) mother, and n is a choice variable that equals to 1 if the couple has children and 0 otherwise. The term u_n^0 in equation (11.35) represents the utility of the two partners in the first period, which is $2w_f$ if the couple has no children and only w_f if a child is born, because of the mother's withdrawal from the labor force during child birth. The term u_n^* represents the parents utility if marriage continues, which is $2w_f$ if the couple has no children and u^* if there is a child. The fixed costs of separation b_n are assumed to be larger when the couple has children.[17]

The expected life time utility is higher for the partner with the higher gains from divorce who determines the divorce decision. In fact, the expected life time utility can be rewritten as

$$W_{j,n}(p) = \begin{cases} u_n^0 + u_n^* + \int_{-\infty}^{\theta_n^*(p)} (\theta_n^*(p) - \theta) f(\theta) d\theta & if \quad V_{j,n}(p) \geq V_{i,n}(p) \\ u_n^0 + u_n^* + \int_{-\infty}^{\theta_n^*(p)} (\theta_n^*(p) - \theta) f(\theta) d\theta & \\ \quad -F(\theta_n^*(p))(V_{i,n}(p) - V_{j,n}(p)) & if \quad V_{j,n}(p) < V_{i,n}(p) \end{cases} \qquad (11.36)$$

where the term $u_n^0 + u_n^*$ is the value of the marriage if it never breaks and the term $\int_{-\infty}^{\theta_n^*(p)} (\theta_n^*(p) - \theta) f(\theta) d\theta$ is the *option value* of breaking the marriage if it turns sour because of a bad draw of θ. The option to sample from the distribution of θ is a motivation for marriage that exists even

[17] It would be more realistic to allow discounting of future utilities in (11.35). However, this does not add any new conceptual issues and to economize on notation we set the discount factor to unity.

if marriage provides no other benefits. However, this option is available only to the person with the higher gains from divorce, who determines the divorce. When the marriage breaks, an event that happens with probability $F(\theta_n^*(p))$, the spouse who does not initiate the divorce and is left behind suffers a capital loss given by $V_{i,n}(p) - V_{j,n}(p)$. The value of the option for the spouse who determines the divorce, increases in the gains from divorce, $F(\theta_n^*(p)$, and also with the variability in the quality of match, because then the ability to avoid negative shocks becomes more valuable.

We define the benefit of spouse j from having a child as

$$B_j(p) = W_{j,1}(p) - W_{j,0}(p). \tag{11.37}$$

Because the production of children must involve both partners, a couple will have a child if and only if *both* partners agree to have a child. That is if,

$$B(p) = Min\{B_m(p), B_f(p)\} \geq 0. \tag{11.38}$$

Because the father determines the divorce decision, his life time utility must also be higher, $W_{f,1}(p) \geq W_{m,1}(p)$. Now imagine that a particular couple departs from the general pattern and chooses not to have a child. If the husband remains single, he will get his income w_f and if he remarries he will get $w_f + w_m h_w$, because his new wife is a custodial mother who spends some of her time on the child. The wife also gets w_f if she remains single, because she has no child, and has the same wage as her husband. However if she remarries she will get $w_f + w_f - s^*(p)$. For a sufficiently large gap between w_f and w_m, a non custodial father brings more income into the marriage than a custodial mother who generally works only part time.[18] Therefore, if all other couples have a child, the wife in a couple that chooses not to have a child expects to gain from divorce *more* than her husband, $V_{f,0}(p) < V_{m,0}(p)$. In this case, she will determine the divorce decision, and consequently, her life time utility is higher, $W_{f,1}(p) < W_{m,1}(p)$. It follows that the mother has lower benefits from having the child, $B_m(p) < B_f(p)$.

Proposition 11.3 *The wife determines whether or not the couple have children.*

When the value of having a child is strictly lower for the wife, there may be some p such that she will prefer not to have a child when the husband would like to have a child. In such a case, the father may be willing to sign a binding ex-ante contract which would transfer money to the mother upon separation if this would induce her to have a child. We shall return to this issue in the concluding section.[19]

[18] Recalling that $s^*(p) \leq c^*$, a sufficient condition is that $w_f > w_m + c^*$.

[19] We note again that if $\delta < 1$, so that the non custodial father suffers from the distance from his child, the father's benefit from having a child could be smaller than the mother's.

11.7 Equilibrium

Equilibrium requires consistency among the choices of the participants in the marriage market and realization of their expectations. The first consistency requirement is that the aggregate divorce rate coincides with the expected remarriage rate. Assuming independence of the marital shocks across couples and a large population, the proportion of couples that will choose to divorce is the same as the probability that a particular couple divorces. The decision of each couple to divorce depends on the expected remarriage rate, p. Assuming that a person can remarry only with a divorcee and that meetings are random, we require that, in equilibrium, the realized aggregate divorce rate must equal the expected remarriage rate of all agents. That is,

$$p = F(\theta^*(p)). \tag{11.39}$$

Because the gains from divorce for a couple with $\theta = 0$ are negative, the threshold $\theta^*(p)$ is negative and it then follows from our assumptions on $F(p)$ that any solution of (11.39) must be such that $p < \frac{1}{2}$.

When fertility is endogenous, we have the additional requirement that the expected gain from divorce must reflect the optimal fertility choices of the participants in the marriage market. Thus, in an equilibrium without children we must have that

$$p = F(\theta_0^*(p)), \tag{11.40}$$

and $B(p) < 0$. That is, the expected gains from divorce are calculated based on the assumption that all singles are childless, and given these expectations no couple wishes to have a child. Similarly, in an equilibrium in which every couple has a child we must have

$$p = F(\theta_1^*(p)), \tag{11.41}$$

and $B(p) > 0$.

The third requirement from equilibrium is that the participants contracting choices must be optimal, given by $s^*(p)$. These equilibrium requirements implicitly assume symmetric equilibria in which all agents behave in the same manner. Such equilibria are a natural choice given that all agents are initially identical, but other equilibrium may exist. In a more general analysis, one can incorporate also mixed equilibria such that some couples choose to have a child, some choose to remain childless and all couples are *indifferent* between having and not having a child. However, because such equilibria tend to be unstable, we are less interested in them and will not introduce the additional notation that is required to characterize them.

For any given legal environment, it is convenient to rewrite the equilibrium condition in the form

$$F^{-1}(p) = \theta^*(p). \tag{11.42}$$

This formulation separates the properties of the distribution of the unanticipated shocks from the properties of the trigger $\theta^*(p)$ that summarized the impact of the expected remarriage rate on the expected gains from divorce. Because $F^{-1}(p)$ rises in p, while $\theta^*(p)$ declines (rises) in p when a child is (not) present, there may be two equilibrium points: a high divorce (remarriage) without children and a low divorce (remarriage) with children.

Numerical Example 11.1

We now present a numerical example that illustrates some of the results. We adopt here a slightly more flexible formulation, allowing the father to suffer a utility loss when he lives separately from the child, $\delta < 1$, and allowing discounting of future utilities, $\Delta < 1$. With these modifications (11.20) and (11.35) become:

$$\begin{aligned} V_f \quad = \quad & (1-p)^2[w_f - s + \delta q(s)] + (1-p)p[w_f - s + \delta q(s, w_f - s')] \\ & + p(1-p)[a(s', w_f - s) + \delta q(s)] + p^2[a(s', w_f - s) + \delta q(s, y - s')] \end{aligned}$$

and

$$W_{j,ch}(p) = u^0_{ch} + \Delta\{ \int_{\theta^*_{ch}(p)}^{\infty} (u^*_{ch} + \theta) f(\theta) d\theta + F(\theta^*_{ch}(p))(V_{j,ch}(p) - b_{ch})\}$$

respectively, where $0 < \delta$, $\Delta \leq 1$. In the figures, we set $\delta = 0.75$ and $\Delta = 0.625$.

Figure 11.3 shows the optimal transfers that the father promises to his ex-wife at the time of divorce as a function of the prospective remarriage rate, p, and the implied consequences for the child when the mother remarries and remains single.[20] The optimal transfer from the father to his ex-wife, $s^*(p)$, declines with p because the marginal impact of the transfer on the child is lower when the mother remarries. As a consequence, the utility of the child when the mother remains single, $\gamma +\ g(s^*(p))$, declines too. The child's utility when the mother remarries, $\alpha a(p) + \gamma(1 - h_m(p)) + g(\hat{c})$, declines because a lower transfer implies that the mother works more and spends less time on the child; but the reduction in the mother's caring time has a stronger effect than the child's gains from the higher consumption of the adult good. The child's utility when the mother is single exceeds the child's utility when the mother remarries, because the mother is "paying" for her gain of adult good by reducing the utility of the child. Therefore, the

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[20] The graphs are drawn for the case in which the mother's productivity at home is $\gamma = 1.75$, the mother's wage is $w_m = 0.5$ and the father's wage is $w_f = 1$. We specify $g(c) = g_1(1 - (1 + c)^{-g_2})$ where $g_1 = g_2 = \frac{9}{4}$ and $\alpha = \frac{1}{4}$. Thus c^* satisfies $g_1 g_2 (1 + c)^{-g_2 - 1} = 1 + \alpha$, implying $c^* = 0.5378$ and $g(c^*) = 1.3956$. $\hat{c}$ satisfies $g^2(1 + c)^{-g_2 - 1} = \frac{\gamma}{w_f}$ implying $\hat{c} = 0.1203$.

child' loss from remarriage equals $a(p)$, which rises with p, as the bargaining position of the mother worsens when the father transfers less.

As a consequence of the decline in the optimal transfer, the expected utilities at the time of divorce of the child, mother and father all decline (see Figure 11.4). Assuming a moderate loss for the father when he lives apart from the child, $\delta = 0.75$, the expected utility of the father is higher than that of the mother through most of the relevant range of p. Consequently, the father determines the divorce decision if $0.05 < p < 0.5$, while the mother determines the divorce decision if $p < 0.05$.[21]

In Figure 11.5, we plot the maximum of the husband's and wife's expected gains (losses) from divorce, including the fixed cost of divorce, for couples with and without children. These gains rise for couples without children because remarriage enhances joint consumption and decline for couples with children, because remarriage implies lower spending on the child that dominate the gains from joint consumption. The intersections of these curves with the inverse probability function at $p = 0.214$ and $p = 0.334$ represent potential equilibria, where the realized divorce rate equals the expected remarriage rate. [22] A higher potential equilibrium point arises when all couples do not have children because, by assumption, such couples have lower fixed cost of separation and, in addition, they do not suffer from the reduced welfare of the child when the marriage breaks. To make sure that the two intersections in Figure 11.5 satisfy all the requirements for equilibrium, we must further verify that, at the higher intersection with $p = 0.334$, no couple without a child wants to deviate and have a child when all the others do not have a child, while in the low intersection with $p = 0.214$, no couple with a child wants to deviate and have no child when all others have a child.

Figure 11.6 shows the incentives of the husband and wife to deviate and have no child when all other couples have a child and their child support is set at the optimal level $s^*(p)$. The expected life time utilities of the husband and wife when all couples have children decrease with the probability of remarriage, with the mother's life time utility being slightly lower than the father's (except for very low p, $p < 0.05$), reflecting the father's higher expected gain from divorce. In contrast, the life time utilities that the parents obtain upon deviating to not having a child *rise* with the probability of remarriage because of the gain from joint consumption. With this structure, a deviation would occur only at a sufficiently high probability

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[21] The difference between the father's and mother's expected gains from divorce is

$$(g(s^*(p)) + \gamma)(\delta - 1) + (1 - p)(w_f - s^*(p)),$$

which is always positive if $\delta = 1$, but can be negative for $\delta < 1$.

[22] The inverse probability is drawn for the case in which the match quality, θ, is uniformly distributed over $[-d, d]$ so that $p = prob\{\theta \leq x\} = \frac{d+x}{2d}$ and $x = d(2p - 1)$. In the figures, we set $d = 2$.

of remarriage. Because both partners are required to produce a child, it is sufficient for a deviation to occur that *one* of the two parents refuses to have a child. We see that the wife wants to deviate only if the remarriage rate exceeds 0.28, while the husband wants to deviate only if the remarriage rate exceeds 0.36. Thus, the intersection in Figure 11.5 at $p = 0.214$ is an equilibrium with children. By a similar argument, it can be seen that the intersection in Figure 11.5 at $p = 0.334$ is an equilibrium without children, because neither the husband nor the wife wish to have a child if all others do not have a child (see Figure 11.7).[23]

In Table 11.1, we provide some comparative static results. The first panel shows the benchmark parameters. The second panel shows the impact of changes in the variance of the quality of match, holding the mean constant. The inverse probability is drawn for the case in which the match quality, θ, is uniformly distributed over $[-d, d]$, so that such an increase is represented by an increase in d. The higher is d, the more likely it is that the realized match will be sufficiently low to trigger divorce. Therefore, the equilibrium divorce rate rises with d. At a low d, $d = 1.5$, the only equilibrium is the one with children and for a high d, $d = 2.5$, the only equilibrium is without children. For intermediate values of d ($d = 2.0$ and $d = 2.2$) there are two equilibria for each value of d. It is then possible that a small exogenous change that is, a rise in d from 2 to 2.5 will cause a large change in the divorce rate, shifting the equilibrium from a divorce (remarriage) rate of $p = 0.214$, (with children) to 0.375 (without children), with a noticeable rise in the utility of both parents. This change illustrates a *social multiplier* effect where the higher willingness of each couple to divorce, as a consequence of the exogenous shock (that is, the rise in d), increases the aggregate divorce rate, which further increases the incentives to divorce. The rise in the life time utility of the parents with d illustrates our observation that marriage has an option value, because bad outcomes to the quality of the match can be avoided through divorce. However, the child, who is a passive agent that cannot directly influence the divorce decision, suffers from the dissolution of the marriage.

The third panel of Table 11.1 illustrates the impact of an increase in the fixed costs of divorce in the presence of children, b_1.[24] An increase in b_1 reduces the divorce rate of couples with children and thereby reduces the expected life time utility of the parents who cannot so easily recover from bad matches. The child, of course, gains from such a change, because he is better off in an intact family and, by assumption, does not suffer from a bad quality of match. Although such a change does not directly affect

[23] If all other couples do not have a child, a husband would like to deviate and have a child only if $p < 0.19$, while a wife would like to have a child only if $p < 0.24$. In calculating the deviation, we take into account that when the couple will have the child the father will commit to pay child support according to $s^*(p)$.

[24] We assume no fixed cost of separation in the absence of children; that is, $b_0 = 0$.

the outcomes if all couples do not have children, it still may influence the equilibrium outcome through the impact on the incentives to have children. Thus if b_1 is reduced from 0.25 to 0.05 then the equilibrium without children disappears and the only equilibrium is with children.

The last panel of Table 11.1 illustrates the impact of changes in the utility of the father, as δ rises and he suffers less from living apart from the child. Such an increase in proximity raises the utility of the father directly, but it also raises his willingness to transfer money to the custodial mother and, consequently, the child and mother gain too. Notice that for $\delta = 1$, the father would like to have a child but the mother prefers not to have a child if all other couples do not have a child. This conflict could, in principle, be resolved by *ex ante* contracting at the time of marriage.

11.8 Further issues

In this concluding section we discuss some departures from the standard contract that we analyzed and examine their implications.

11.8.1 The custody assignment

If the father has the custody, he will spend only the minimal amount of time on child care because, under our assumption that $w_f(1+\alpha) > \beta$, any hour spent on child care could be better used in the market. He will spend on the child good c^* and spend the remainder of his income on the adult good. This is true whether or not he receives transfers from the mother. Therefore, the mother has no incentive to transfer to the custodial father, as any additional dollar is spent on the adult good and $\alpha < 1$. If all couples choose father custody then in a remarried couple, the new wife will work in the market, because she cannot spend time on her child who lives in a different household. The expected utilities of the family members are then

$$\begin{aligned} V_m(p) &= \alpha(w_f - c^*) + g(c^*) + p\alpha w_m + w_m + p(w_f - c^*), \\ V_c(p) &= \alpha(w_f - c^*) + g(c^*) + p\alpha w_m, \\ V_f(p) &= (1+\alpha)(w_f - c^*) + g(c^*) + p(1+\alpha)w_m. \end{aligned} \tag{11.43}$$

We see that under father's custody, the child receives less time but consumes more of the adult and child goods. Thus, the justification for the prevalence of mother custody must rest on the assumption that, in the case of children, time is more important then money, that is γ is large relative to $\alpha(w_f - c^*)$. For a small remarriage probability, the condition $\gamma > \alpha(w_f - c^*)$ is sufficient to ensure that mother custody is better for the child. In this case, the father also prefers that the child will be with the mother, because for a small p his expense on child support, $s^*(p)$, is about the same as he would spend himself on the child, c^*, and the potential gain from the mother contribution of time exceeds the gains that the father has from sharing adult consumption with the child. However, if $\gamma < \alpha(w_f - c^*) + w_m$ then, for a small p, the mother would prefer that the father will have the custody, because this would free her to earn some extra money in the labor market. In this case, the child is a "hot potato" that each parent prefers that the *other* will take care of it. This reflects, of course, the potential for free riding that exists in the provision of public goods. Thus, γ must exceed $\alpha(w_f - c^*) + w_m$ for the two parties to agree on mother custody.[25]

An increase in the probability of remarriage p decreases the welfare of the child under the mother's custody but raises it under the father's custody. This difference is caused by the shift of the custodial mother towards market work when she remarries. The custodial father works at the same intensity whether he is married or not and the child gains from the added adult consumption when the father remarries. Therefore, for a large probability of remarriage father's custody becomes more attractive and a larger gap between γ and $\alpha(w_f - c^*)$ is required to justify mother's custody under the voluntary commitments discussed so far. A possible resolution is to mandate (and enforce) some *minimal* child support transfer from the non custodial father to the custodial mother.

11.8.2 Mandated and contingent contracts

The courts often consider the "accustomed standard of living" of the parties as a standard for divorce settlements. Because living alone is more costly then living together and there is always a risk of remaining single, it is *impossible* to restore the same standard of living for *all* parties. The problem is exacerbated by the principal-agent issues emphasized here that imply a level of transfers that is insufficient to restore efficiency. We now discuss some alternative contracting options that may restore efficiency.

The law also singles out children as worthy of special consideration in divorce settlements. This concern is justified because, as we have seen, even if parents are altruistic and internalize the welfare of the child, the child as a passive party can be hurt by the divorce. It is then natural to apply the accustomed standard of living principle only to the child, as a constraint on the parents' contracting choices. For instance, the law may mandate a level of child support $s = c^*$. As we have shown, such a transfer would indeed induce a single mother to choose the efficient level of child care, spending all her time on child care. This, however, is not true for

[25] For alternative models of custody assignment see Atteneder and Halla (2007) and Rasul (2006).

a remarried mother, who still may be forced to work part time to comply with the interest of her new husband. As long as the courts cannot interfere with within-household allocations (that are hard to verify) and the father cannot transfer directly to the child, because his money transfer is fungible and can be consumed by the mother and her new husband, it is hard to expect that the child interests will be maintained simply by mandating a money transfer. There is, however, one notable exception. If the mother has sufficient bargaining power to take *all* the gains from marriage, then she would solve the mirror image of problem (11.13) and maximize her utility subject to the constraint that the new husband is just indifferent between remaining single and remarriage. Formally, this problem is the same as problem (11.7) that the mother solves as single and setting $s = c^*$ would indeed induce her to maintain the efficient outcome when she remarries.[26]

This brief discussion illustrates that in search markets with rents that are subject to bargaining, it is important to specify the relative bargaining power of the parties that determines the share of the surplus that each party gets. In the Nash bargaining model this is determined by considerations such as impatience and risk aversion that, of course, need not be equal across genders. More broadly, social norms such as egalitarianism and sex roles may also affect the bargaining outcome.

Another, and potentially more fruitful, direction is to enlarge the set of contracts that the courts are willing to enforce. In principle, child support payments should depend on the marital status of both parents, because the costs and benefits of post divorce transfers depend on these states.[27] In practice, child support is not contingent on marital status but there are other payments such as alimony that are often contingent on the marital status of the mother. Because we assume that all transfers are fungible, the name attached to these payments does not really matter, but it does matter how flexible they are and to what contingences they respond.

Now imagine that a father can pay different amounts to the custodial mother depending upon whether or not she is remarried. Suppose further that the father is forced by law to pay a fixed amount of child support $s = c^*$ but can augment it by an additional payment σ that he pays the custodial mother only if she is single. Then, the efficient allocation within the remarried household is determined by

$$\max_{h_m, c} E(a|\sigma, \sigma') = w_m h_m + w_f - c - (1-p)\sigma' \tag{11.44}$$

[26] Aiyagari et al. (2000) also discuss mandatory child support payments in a general equilibrium framework. They show that an increase in such payments raises welfare of parents and children.

[27] In theory, the transfers should depend on the marital status of *all* agents that participate in the marriage market. But this, of course, is highly impractical.

subject to the constraints:

$$(1+\alpha)[w_m h_m + w_f - c - (1-p)\sigma'] + \gamma(1-h_m) + g(c) \geq u_m(c^* + \sigma)$$

and $0 \leq h_m \leq 1$. The chosen values of h_m and c depend on the transfer that the father promises the mother if she remains single, σ, and the *expected value* of the new husband's gross income $w_f - (1-p)\sigma'$. Only the expectation matters because the remarried partners are risk neutral with respect to a and because the mother's work time, h_m, and the expenditures on the child good, c, are determined before the marital status of the ex-wife of the new husband is known.

In contrast to a non contingent transfer, a transfer given to the mother *only* when she is single does not change the utility frontier of the remarried couple and therefore must *reduce* the expected utility of the new husband. This implies that with contingent payments, the father is able to attain a larger impact on the child's utility and is willing to contribute more to the custodial mother. In fact, by setting $\sigma = w_f - c^* - (1-p)\sigma'$ the father can eliminate *all* the gains from marriage of the new husband and restore efficiency. We then obtain the following characterization (see Appendix).

Proposition 11.4 *If all couples have children, then the commitment equilibrium for a given remarriage probability p, is such that: For $p < p_0$, the only symmetric equilibrium is one in which all fathers pay only the mandatory payment $s = c^*$ and $\sigma = 0$. For $p > p_1$, the only symmetric equilibrium is one in which all fathers voluntarily commits to pay their ex-wife $\sigma = \frac{w_f - c^*}{2-p}$ if she remains single. For $p_1 \geq p \geq p_0$, both types of equilibrium can arise. The equilibrium $\sigma = \frac{w_f - c^*}{2-p}$ is efficient and*

$$\begin{aligned} V_c(p) &= \gamma + g(c^*) + \alpha\frac{w_f - c^*}{2-p}, \\ V_m(p) &= V_f(p) = \gamma + g(c^*) + (1+\alpha)\frac{w_f - c^*}{2-p}. \end{aligned} \tag{11.45}$$

The pattern described in the proposition suggests reinforcement; one is willing to commit to his wife if others do, but not if they do not. As is well known, such positive feed backs can yield multiple equilibria. We also see that higher probability of remarriage is conducive to equilibria in which fathers are willing to commit on a payment that is conditioned on the event that the mother remains single, because such promises are carried out less often and are more likely to yield benefits.

When efficiency is restored, the child suffers only from the reduced adult consumption that is caused by the risk of remaining single. If the mother is sure to remarry, that is, $p = 1$, then the child is as well off as in an intact family. That is, the father was practically replaced by the new husband with no harm to the child. This favorable outcome was achieved by eliminating the marital surplus completely and effectively eliminating the power of the new husband from extracting any rents which may harm the child upon remarriage. Importantly, the contingent transfer restores ex-post symmetry between the parents, which implies that divorce will also be efficient. Finally, because of the efficiency in child care, all family members benefit from a higher probability of remarriage.

These results are in sharp contrast to the case of non contingent transfers and raise the question why contingent contracts are not more prevalent. The basic problem with such contracts is that they are not attractive when the probability of remarriage is low, because then the father is very likely to bear the costs, when the mother remains single, and correspondingly unlikely to reap the benefits when she remarries. As we shall show in a subsequent section, this problem can be mitigated if the courts would also enforce contracts that are signed at the time of marriage. Before we turn to that case, however, let us illustrate the impact of contingent contracts with a numerical example.

Numerical Example 11.2

We now present an example with contingent contracts. The parameters are the same as in numerical example 11.1, except that we now set $\delta = 1$. This change is made to increase the motivation of the father to support the child and the motivation of the couple to have children, so that an equilibrium with voluntary transfers and children can be supported.

Figure 11.8 presents three potential equilibrium points for the divorce and remarriage rates associated with the following alternatives:

1. All couples have children and the father pays the mother a *fixed* payment c^* and, in addition, a *contingent* payment $\sigma = \frac{w_f - c^*}{2-p}$ that the mother receives only if she remains single.

2. All couples have children and the father pays the mother only the *fixed* payment c^*.

3. All couples have no children and no transfers are made upon divorce.

Case 3 is identical to the one discussed in the previous section. Case 2 is a modification of the case discussed in the previous section; the child support payment is still unconditional but fathers are forced to pay more than they would pay voluntarily. However, because part of this payment is eaten by the new husband of the remarried mother, the child and consequently the father are still harmed by remarriage, which is the same qualitative result that we had before. The main departure is case 1, where the contingent payment given to the mother raises her bargaining power to the extent that the child is not harmed from remarriage and, consequently, the gains from divorce of both parents rise with the prospects of remarriage. For the chosen parameters, this case yields an equilibrium divorce rate $p = 0.356$[28]

Figures 11.9 and 11.10 describe the impacts of deviations from the transfer patterns when all parents have children. Figure 11.9 shows that if all parents commit on the transfer $\sigma = \frac{w_f - c^*}{2-p}$ that restores efficiency then for $p < 0.142$, each father, taken separately, would be better off by unilaterally deviating to $\sigma = 0$. Thus, a symmetric equilibrium in which everyone commits to $\sigma = \frac{w_f - c^*}{2-p}$ cannot exist in this range. Figure 11.10 shows that if all parents pay only the compulsory payment c^* and $\sigma = 0$ then, for $p > 0.302$, each father taken separately would be better off if he unilaterally commits to the mother to pay her *all* his disposable income, $w_f - c^*$, if she remains single. Thus an equilibrium where no one wishes to commit cannot exist in this range. Proposition 11.4 states that if $p_1 > p > p_0$ there may be *two* partial equilibria, one in which every father commits on a positive σ and one in which no father commits. However, for the chosen parameters, the equilibrium divorce rate associated with having children is above $p_1 = 0.302$, implying that the equilibrium in which every father commits at $p = 0.356$ is the only potential equilibrium when all parents have children. Table 11.2 shows that the equilibrium at $p = 0.356$ is indeed a full equilibrium in the sense that, with the implied child support transfers, all couples prefer to have children. For the assumed parameters, this is the only equilibrium because, if all couples have no children, there is an incentive to deviate to a situation with a child and full commitment $\sigma = w_f - c^*$. Thus, the full equilibrium is *unique.*

Comparing the results in Table 11.2 to the last row in Table 11.1, we can see the impact of different legal regimes when all parameters of the model are the same. Suppose that all couples have children. Then, if transfers are not contingent and determined optimally, the child's expected utility is 2.613. If fathers are forced to pay the mother a transfer of $s = c^*$, the child's expected utility rises to 2.811 and if contingent transfers are also enforced, the child's expected utility is 3.216, which is only slightly less than the child's utility in an intact family, 3.260. As we move across these alternatives, the transfer from each father to his ex-wife rises when the marriage breaks and, consequently, the expected life time utility of each mother rises. The surprising result, however, is that the father is also better off and his expected utility levels are 3.456, 3.475 and 3.485, respectively. The result that a *compulsory* increase in child support above the individually optimal level, $s^*(p)$ raises the father's expected utility reflects a positive *contract externality*, whereby the commitment made by each father to his ex-wife benefits other fathers when they remarry. The second increase, associated

[28] An interesting point is that if the efficiency of child expenditures is restored by appropriate transfers then the gains from divorce with children can exceed the gains from divorce without children, despite the higher fixed costs of divorce associated with children. The reason is that couples without children have higher joint consumption (in terms of the adult good), which can make divorce more costly for them.

with raising the contingent payment, σ, from 0 to $\frac{w_f - c^*}{2-p}$ benefits each father separately because of the rise in the expected utility of the child. The rise in the remarriage prospects as p rises from 0.344 to 0.356 raises the incentives of all fathers to contribute to their ex-wives. In this respect, a higher *aggregate* divorce rate can serve as a coordination device that can benefit children and raises the incentives to have children.

11.8.3 Transfers, the ex-ante perspective

At the interim stage, when the fertility has already been determined, the purpose of the contract is to induce the custodial mother to spend all her time on the child if she remarries. A contract that is signed at the time of marriage can also influence the divorce and the father would be willing to commit for a broader range of p. Thus, in contrast to the ex-post contract, where the husband gains from the commitment to pay the single mother only if she remarries, the *ex ante* contract can benefit the father even if the mother remains single. Of course, the husband is willing to pay only if the mother would not have the child in the absence of contract. It is easy to find parameters of the model such that the mother would prefer to have a child even without a contract, in which case there is no role for voluntary ex-ante commitments by the husband. However, we shall focus here on the case in which, in the absence of binding contracts, the mother does not wish to have the child but if binding contracts are enforced then the mother may prefer to have a child, depending on the expected remarriage rate and the decision of others to have children.

Suppose that all couples have children and sign an ex-ante contract, at the time of marriage, that promises the mother $\sigma = \frac{w_f - c^*}{2-p}$ if the mother remains single. Then, the gains from divorce are the same for the two partners and separations are efficient for all p and, therefore, the expected utility at the time of marriage is the same for the husband and wife. Since under such contract both partners want the child at the same values of p, the production of children is efficient too. *Both* partners would agree to sign such a contract at the time of marriage, if their expected utility is higher than it would be in the absence of contract and no children. Finally, the ex-ante contract is renegotiation proof, because it coincides with the interim contract.

It is puzzling why ex-ante contracts that are signed at the time of marriage are not prevalent among all couples with children. The implementation of such contracts in earlier times suggests that the enforcement of ex-ante contracts is not the issue. Rather, in modern societies with free marriage, based in part on mutual attraction, the general sense is that emotional commitments are more important than legal agreements and thinking of contingencies and writing them down may "kill love". However, prenuptial contracts are often signed, at the time of marriage, by couples in which one (or both) of the partners bring into the marriage substantial property, which is more common on second marriages.

11.9 Conclusion

As the last two chapters illustrate, marriage markets with search frictions, in which the meeting technology displays increasing returns, may have multiple equilibria, because of the various search and contracting externalities. In chapter 10, we did not allow any contracting and, as a consequence, obtained the result that equilibria with higher turnover, that is, higher divorce and remarriage rates, provides all participants with a higher welfare. The reason is that an increase in the *aggregate* divorce rate, raises the prospects of remarriage, which makes it easier to replace bad marriages by better ones. In chapter 11, we allowed parents to transfer resources in the aftermath of divorce, based on the insight that, in the presence of children, marriage dissolution does not eliminate all ties between the partners because both parents continue to care about their child, which motivates post divorce transfers.[29] However, the impact of transfers on the marriage market and the welfare of children is quite complex, because the willingness of each parent to transfer to his\her ex-spouse depends on the transfers that potential mates for remarriages expect from their ex-spouses. This contract externality can operate in different ways, depending on the type of contracts that are enforced by law. If only unconditional transfers are enforced, higher divorce and remarriage rates reduce the incentive to transfer money to the custodial mother, because a dollar transferred to her is less likely to reach the child than if she remarries. The consequence is that children may be worse off in high divorce equilibria. The outcome is completely reversed if the contracts environment is enriched and contingent contracts are also enforced. If the non custodial father promises the mother a payment that is contingent on her remaining single, then her bargaining position vis-a-vis her new husband is improved and the welfare of the child can be protected. Fathers have a stronger incentive to make such commitments when the remarriage rate is high, because then the payments to the custodial mothers are made relatively rarely, while the non custodial fathers are rewarded for their commitments more often. The outcome, in this case, is that equilibria with higher *aggregate* divorce (and remarriage) can be welfare enhancing. In particular, children who would suffer from the break of the marriage of their parents if it would happen in isolation, can gain from being in environment in which a higher proportion of marriages dissolve.

[29] This is in sharp contrast to employment relationships that end in separation, in which case the ex-partners are no longer tied with each other.

11.10 Appendix: Contingent contracts

The purpose of this appendix is to prove Proposition 11.4. We assume throughout a mandatory child support payment of c^* so that $s = c^* + \sigma$ and $\sigma \geq 0$. Hence, by (11.12), $u'_m(s) = 1 + \alpha$ and $q\prime(s) = \alpha$.

11.10.1 The choice of contract

The expected utility of the father is now

$$\begin{aligned} V_f \quad = \quad & (1-p)^2[w_f - (c^* + \sigma) + q(c^* + \sigma)] + (1-p)p[w_f - c^* + E(q|\sigma, \sigma')], \\ & +p(1-p)[E(a|\sigma', \sigma) + q(c^* + \sigma)], \\ & +p^2[E(a|\sigma', \sigma) + E(q|\sigma, \sigma')]. \end{aligned} \tag{11.46}$$

From (11.44) we have, that in any household

$$\begin{aligned} \frac{\partial E(a|\sigma, \sigma')}{\partial \sigma} &= \lambda u'_m(c^* + \sigma) = \lambda(1+\alpha) < 0, \\ \frac{\partial E(q|\sigma, \sigma')}{\partial \sigma} &= u'_m(c^* + \sigma) - \frac{\partial E(a|\sigma, \sigma')}{\partial \sigma} = (1+\alpha)(1-\lambda) > 0, \\ \frac{\partial E(a|\sigma, \sigma')}{\partial \sigma'} &= -(1-p)(1-\lambda(1+\alpha)) < 0, \\ \frac{\partial E(q|\sigma, \sigma')}{\partial \sigma'} &= -\frac{\partial E(a|\sigma, \sigma')}{\partial \sigma} > 0, \end{aligned} \tag{11.47}$$

where λ is the Lagrange multiplier for the participation constraint of the wife. Therefore,

$$\begin{aligned} \frac{\partial V_f}{\partial \sigma} \quad = \quad & (1-p)^2[-1+\alpha] + (1-p)p[(1+\alpha)(1-\lambda_m)] + \\ & p(1-p)[-(1-p)(1-\lambda_f(1+\alpha)) + \alpha)] + p^2[-(1-p)(1-\lambda_f(1+\alpha)) \\ & +(1+\alpha)(1-\lambda_m)], \end{aligned} \tag{11.48}$$

where λ_m and λ_f denote the Lagrange multipliers if the mother or father remarry, respectively.

An interior solution for σ exists if $\frac{\partial V_f}{\partial \sigma} = 0$ and

$$\frac{\partial^2 V_f}{\partial \sigma^2} = (1+\alpha)p[-\frac{\partial \lambda_m}{\partial \sigma} + (1-p)\frac{\partial \lambda_f}{\partial \sigma}] < 0. \tag{11.49}$$

From the first order conditions to (11.13), if both the mother and the new wife of the father work part time then $\lambda_m = \lambda_f = \frac{w_m}{(1+\alpha)w_m - \gamma}$. Otherwise, $\lambda_j = \frac{1}{1+\alpha-g'(c)}$ for $j = m, f$. When the mother does not work, any increase in σ increases the consumption of the child and decreases λ_m. Similarly, when the father remarries and his wife does not work any increase in his commitment to his ex-wife raises the consumption of the step child and decreases λ_f. Hence, $\frac{\partial \lambda_m}{\partial \sigma} \leq 0$ and $\frac{\partial \lambda_f}{\partial \sigma} \leq 0$. But in a symmetric equilibrium all couples make the same choices and $\frac{\partial \lambda_m}{\partial \sigma} = \frac{\partial \lambda_f}{\partial \sigma}$, which would imply that $\frac{\partial^2 V_f}{\partial \sigma^2} \geq 0$. Therefore, there is *no* interior symmetric equilibrium, and the only symmetric equilibria are such that all couples must be at one of the boundaries, $\sigma = 0$ or $\sigma = w_f - c^* - (1-p)\sigma'$. Notice that the upper boundary is not determined by the budget constraint but by the requirement that the mother is just indifferent between remarriage and remaining single.

Suppose that all other couples set $\sigma' = \frac{w_f - c^*}{2-p}$. Then by setting

$$\sigma = w_f - c^* - (1-p)\sigma' = \frac{w_f - c^*}{2-p} \tag{11.50}$$

the father can guarantee that the mother is just indifferent between marriage and remaining single. This is seen by noting that the mother participation constraint becomes

$$(1+\alpha)[w_m h_m + \frac{w_f - c^*}{2-p} + c^* - c] + \gamma(1-h_m) + g(c) \geq \gamma + g(c^*) + (1+\alpha)\frac{w_f - c^*}{2-p} \tag{11.51}$$

But

$$\begin{aligned} & \underset{c,h_m}{Max}\{(1+\alpha)[w_m h_m + c^* - c] + \gamma(1-h_m) + g(c)\} \\ \leq \; & \underset{c,h_m}{Max}\{\gamma h_m + (1+\alpha)(c^* - c) + \gamma(1-h_m) + g(c)\} \\ = \; & \gamma + g(c^*). \end{aligned} \tag{11.52}$$

Therefore, (11.51) must hold as equality.

It remains to show that $\sigma = \frac{w_f - c^*}{2-p}$ is indeed an optimal choice, given that others maintain $\sigma' = \frac{w_f - c^*}{2-p}$. For a marginally lower σ we have that $\frac{\partial V_f}{\partial \sigma}$ approaches ∞ because $\lambda_m = \frac{1}{1+\alpha - g'(c)}$ approaches $-\infty$ as c approaches c^*. For marginally higher σ we have that $\frac{\partial V_f}{\partial \sigma}$ approaches $-\infty$, because when the mother chooses not to marry the father suffers a discrete loss since he pays the mother σ with certainty. Thus, $\sigma = \frac{w_f - c^*}{2-p}$ is a *local* maximum. However, it need not be a global maximum. In particular, for a small p, it is always the case that $\frac{\partial V_f}{\partial \sigma} < 0$, because the father bears the costs with high probability and the benefits with a low probability, so that $\sigma = 0$ is also a *local* maximum. However, in contrast to the selection of $\sigma = \frac{w_f - c^*}{2-p}$, the selection of $\sigma = 0$ is a local maximum *only* if p is small. The difference arises because at low levels of σ, $c < c^*$ so that λ_m and λ_f are finite and $\frac{\partial V_f}{\partial \sigma}$ must change sign from negative to positive as p rises from zero to one.

We have noted in the text that if remarried mothers work part time then $\frac{\partial V_f}{\partial \sigma}$ is independent of σ'. However, if the mother does not work or works full time so that $\lambda_m = \frac{1}{1+\alpha - g'(c)}$, then an increase in σ' will raise c,

λ_m becomes more negative and $\frac{\partial V_f}{\partial \sigma}$ rises. That is, σ and σ' are *strategic complements*.

The characterization in the text follows from the following observations: For any fixed σ', the global maximum is at $\sigma = 0$ if p is sufficiently small, say less than p_0, and at $\sigma = \frac{w_f - c^*}{2-p}$ if p is sufficiently large, say larger than p_1. Because of complementarity, one is more inclined to give if others do, and therefore p_1 must exceed p_0.

11.11 References

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Table 11.1: Impact of Change in Parameter on the Equilibrium Divorce (Remarriage) Rate and Life Time Utilities of Family Members

Part 1: Benchmark Parameters

Husband's wage	$w_f = 1$
Wife's wage	$w_m = .5$
Mother's productivity in child care	$\gamma = 1.75$
Father's productivity in child care	$\beta = 0$
Child's marginal utility from the adult good	$\alpha = .25$
Distribution of shocks	$\theta\tilde{}U[-d, d],\ d = 2$
Utility from child expenditures	$g(c) = 2.25 * (1 - \frac{1}{(1+c)^{2.25}})$
Child expenditure levels	$c^* = 0.5378,\ \hat{c} = 0.1203,\ g(c^*) = 1.4$
Proximity factor of father's utility from child	$\delta = .75$
Discount factor	$\Delta = .625$
Fixed cost of separation without a child	$b_0 = 0$
Fixed cost of separation with a child	$b_1 = .35$
Father's second period utility	$\gamma + g(c^*) + (1 + \alpha)(w_f - c^*) = 3.72$
Mother's second period utility	$\gamma + g(c^*) + (1 + \alpha)(w_f - c^*) = 3.72$
Child's second period utility	$\gamma + g(c^*) + \alpha(w_f - c^*) = 3.26$

Part 2: Change in the Variability of Shock, d

	Equilibrium with children					Equilibrium without children			
	Divorce rate:	Utilities:	Husband	Wife	Child	Divorce rate:	Utilities:	Husband	Wife
$d = 1.5$	0.139	Life Time	3.345	3.340	2.784	0.250	Life Time	3.309	3.309
		Deviation	3.287	3.286			Deviation	3.329	3.333
		Second period	2.989	2.932			Second period	1.250	1.250
$d = 2$	0.214	Life Time	3.384	3.373	2.606	0.334	Life Time	3.389	3.389
		Deviation	3.351	3.352			Deviation	3.356	3.358
		Second period	2.925	2.844			Second period	1.334	1.334
$d = 2.2$	0.235	Life Time	3.403	3.390	2.556	0.353	Life Time	3.421	3.421
		Deviation	3.379	3.381			Deviation	3.373	3.374
		Second period	2.905	2.819			Second period	1.353	1.353
$d = 2.5$	0.262	Life Time	3.434	3.419	2.492	0.375	Life Time	3.470	3.470
		Deviation	3.423	3.426			Deviation	3.401	3.401
		Second period	2.879	2.788			Second period	1.375	1.375

Part 3: Change in the Fixed Cost of Separation with a Child, b1

	Equilibrium with children					Equilibrium without children			
	Divorce rate:	Utilities:	Husband	Wife	Child	Divorce rate:	Utilities:	Husband	Wife
$b_1 = 0.05$	0.274	Life Time	3.420	3.405	2.463	0.334	Life Time	3.389	3.389
		Deviation	3.357	3.360			Deviation	3.392	3.395
		Second period	2.866	2.775			Second period	1.334	1.334
$b_1 = 0.25$	0.234	Life Time	3.395	3.383	2.559	0.334	Life Time	3.389	3.389
		Deviation	3.353	3.354			Deviation	3.366	3.369
		Second period	2.906	2.820			Second period	1.334	1.334
$b_1 = 0.35$	0.214	Life Time	3.384	3.373	2.606	0.334	Life Time	3.389	3.389
		Deviation	3.351	3.352			Deviation	3.356	3.358
		Second period	2.925	2.844			Second period	1.334	1.334
$b_1 = 0.45$	0.193	Life Time	3.374	3.364	2.656	0.334	Life Time	3.389	3.389
		Deviation	3.349	3.349			Deviation	3.347	3.349
		Second period	2.944	2.868			Second period	1.334	1.334

Part 4: Change in the Proximity Factor, δ

	Equilibrium with children					Equilibrium without children			
	Divorce rate:	Utilities:	Husband	Wife	Child	Divorce rate:	Utilities:	Husband	Wife
$\delta = 0.5$	0.126	Life Time	3.347	3.349	2.454	0.334	Life Time	3.389	3.389
		Deviation	3.340	3.344			Deviation	3.324	3.331
		Second period	2.576	2.607			Second period	1.334	1.334
$\delta = 0.75$	0.214	Life Time	3.384	3.373	2.606	0.334	Life Time	3.389	3.389
		Deviation	3.351	3.352			Deviation	3.356	3.358
		Second period	2.925	2.844			Second period	1.334	1.334
$\delta = 0.85$	0.255	Life Time	3.408	3.377	2.620	0.334	Life Time	3.389	3.389
		Deviation	3.357	3.355			Deviation	3.385	3.369
		Second period	3.092	2.896			Second period	1.334	1.334
$\delta = 1$	0.321	Life Time	3.456	3.375	2.613	0.334	Life Time	3.389	3.389
		Deviation	3.367	3.361			Deviation	3.448	3.375
		Second period	3.357	2.953			Second period	1.334	1.334

Table 11.2: The Incentives to Deviate at Alternative Potential Equilibria

Equilibrium with children, $\sigma = \frac{w_f - c^*}{2-p}$

Divorce rate:	Life Time Utilities:	Husband	Wife	child
0.356	With a child, $\sigma = \frac{w_h - c^*}{2-p}$	3.485	3.485	3.216
	With a child, $\sigma = 0$	3.422	3.417	
	No child	3.369	3.343	

Equilibrium with children, $\sigma = 0$

Divorce rate:	Life Time Utilities:	Husband	Wife	child
0.344	With a child, $\sigma = 0$	3.475	3.410	2.811
	With a child, $\sigma = w_h - c^*$	3.477	3.540	
	No child	3.378	3.354	

Equilibrium without children

Divorce rate:	Life Time Utilities:	Husband	Wife
0.334	No child	3.389	3.389
	With a child, $\sigma = w_h - c^*$	3.453	3.540
	With a child, $\sigma = 0$	3.452	3.414

Parameters are the same as in the benchmark of Table 11.1, except that $\delta = 1$, instead of $\delta = .75$.

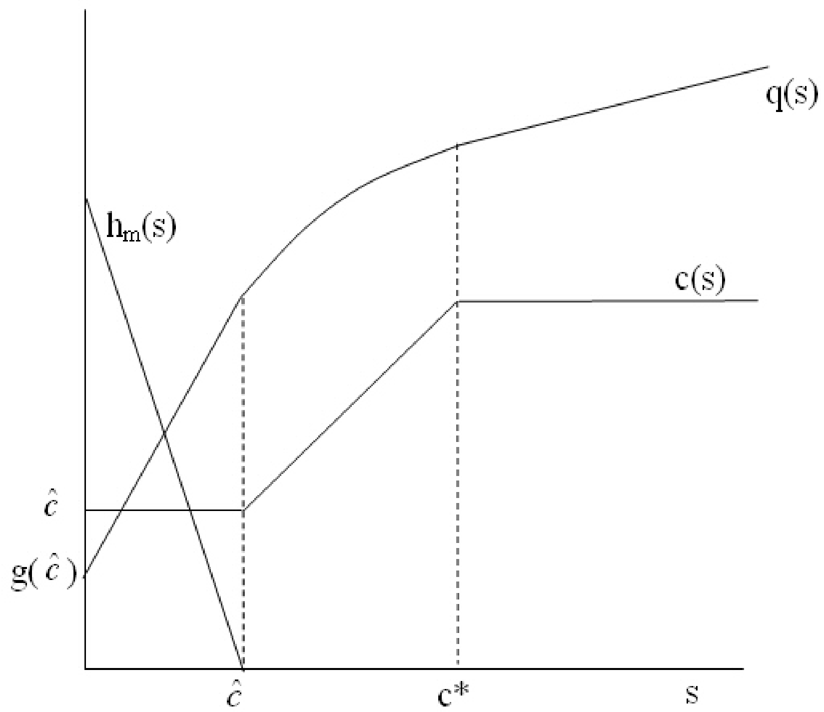

FIGURE 11.1. Effect of Transfer on Mother's Work, Child's Consumption and Child's Utility

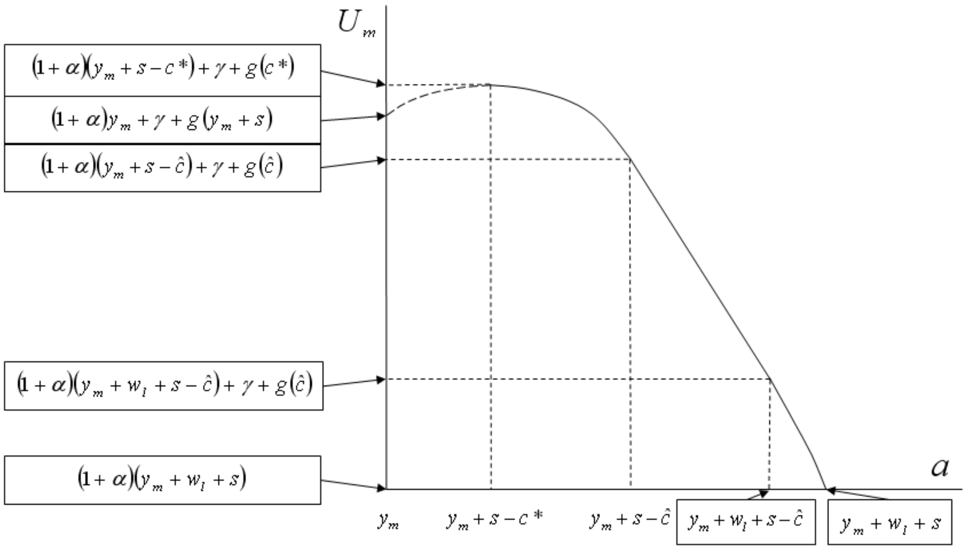

FIGURE 11.2. The Pareto Utility Frontier

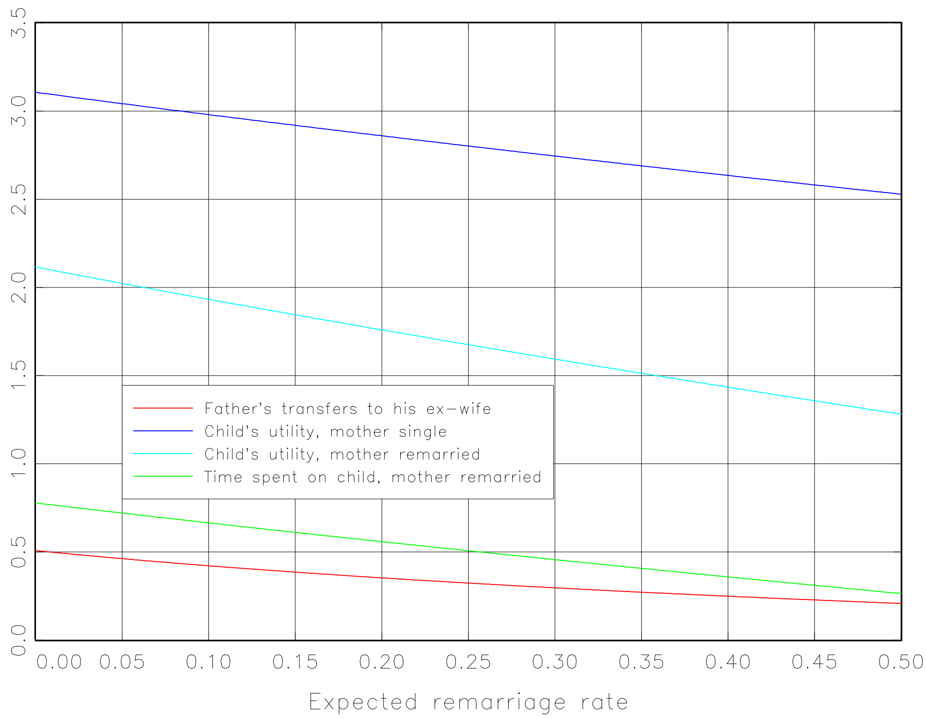

FIGURE 11.3. Optimal Transfers and Child's Utility if the Mother Remmaries or Remains Single

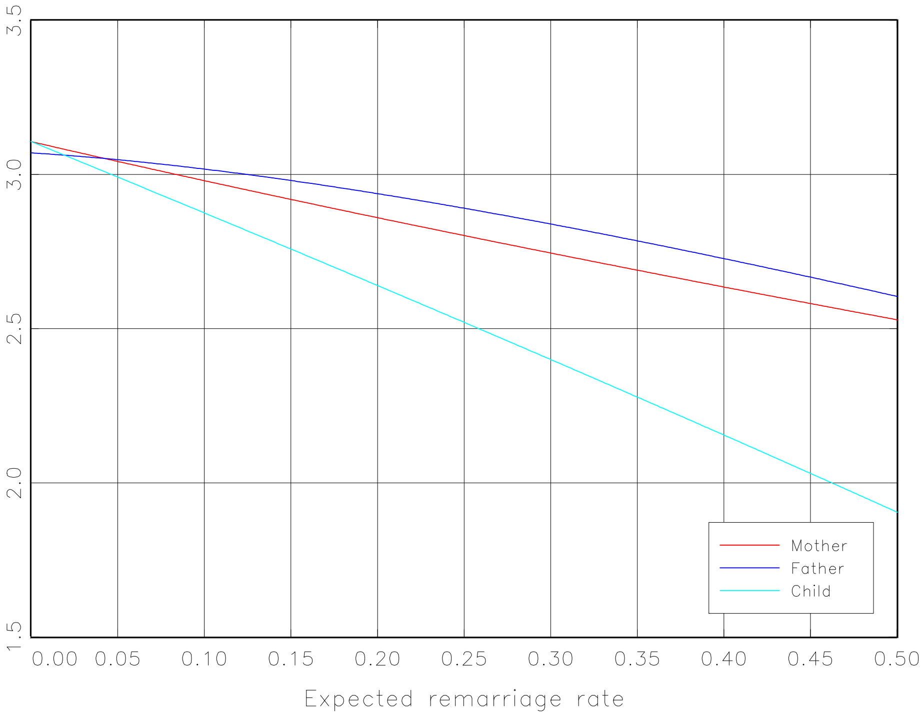

FIGURE 11.4. Expected Utilities of Father, Mother and Child at the Time of Divorce

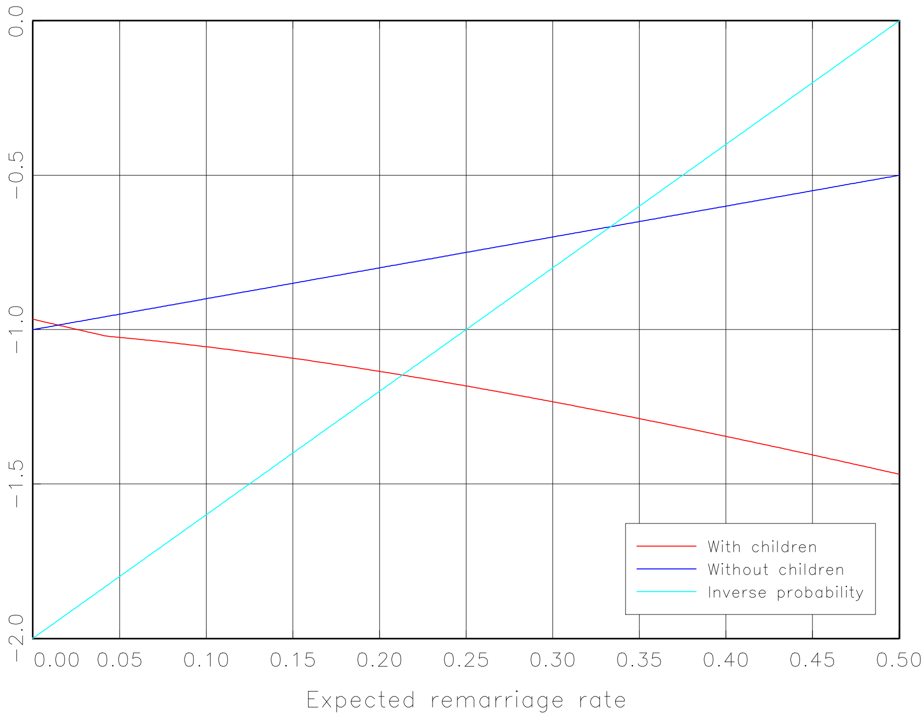

FIGURE 11.5. Potential Marriage Market Equilibria, with and without Children

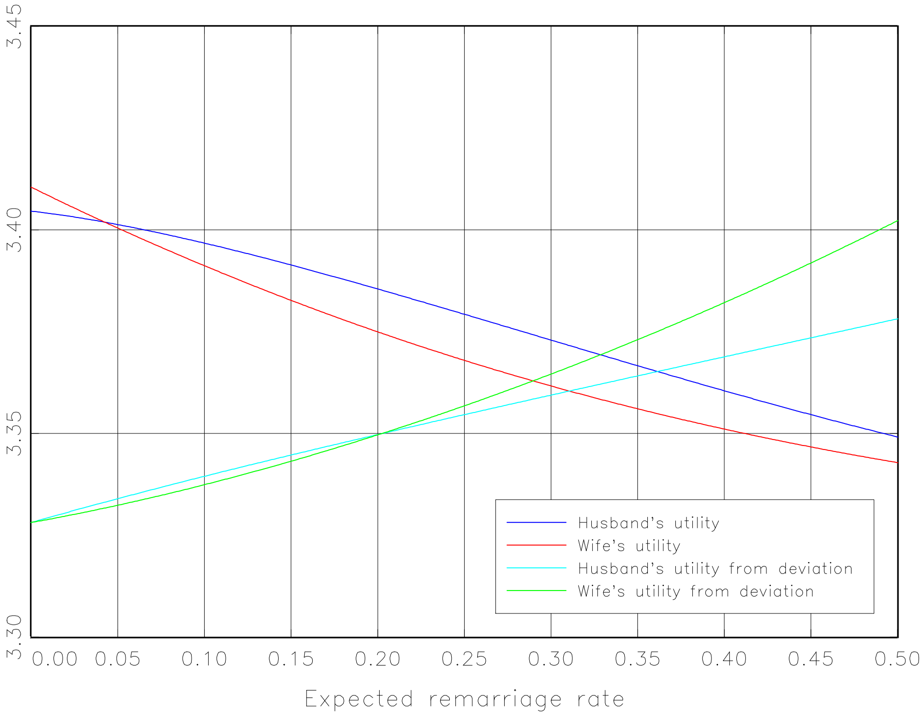

FIGURE 11.6. The Impact of Deviation to not having a Child on the Expected Life Time Utility when all other Couples have a Child

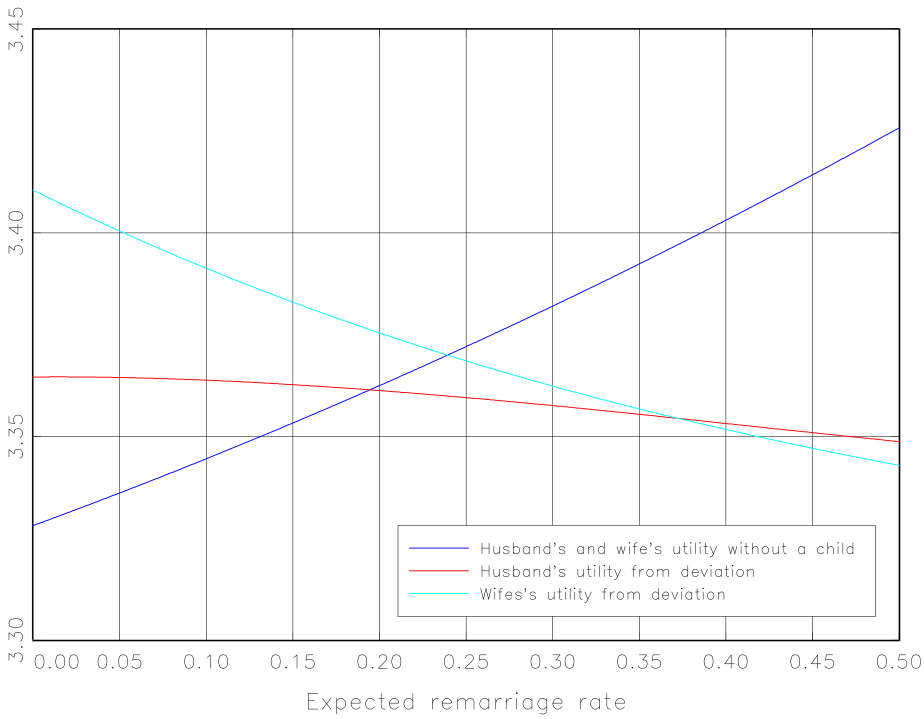

FIGURE 11.7. The Impact of Deviation to Having a Child on the Expected Life Time Utility when all other Couples Have no Child

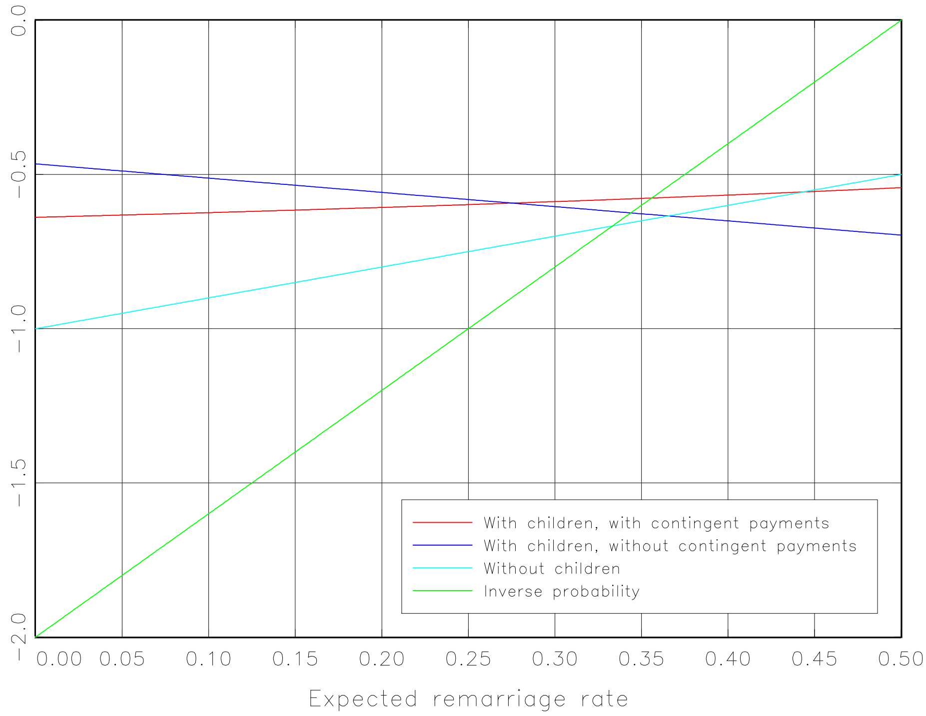

FIGURE 11.8. with Children under Different Payment Schemes and without Children

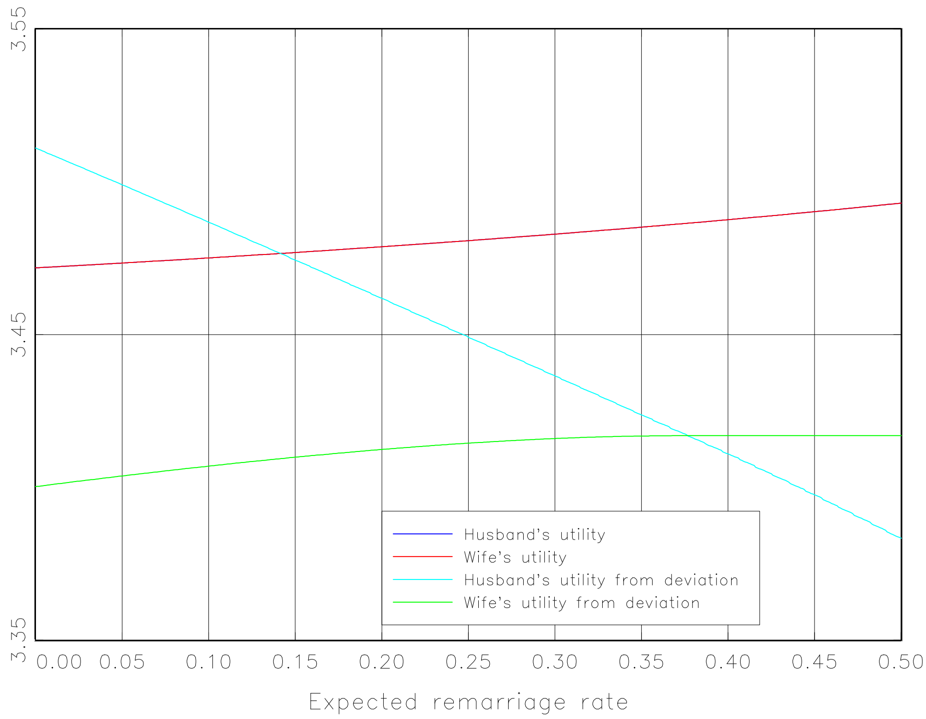

FIGURE 11.9. The Impact of Deviation to a Fixed Payment when all other Fathers give a Contingent Payment

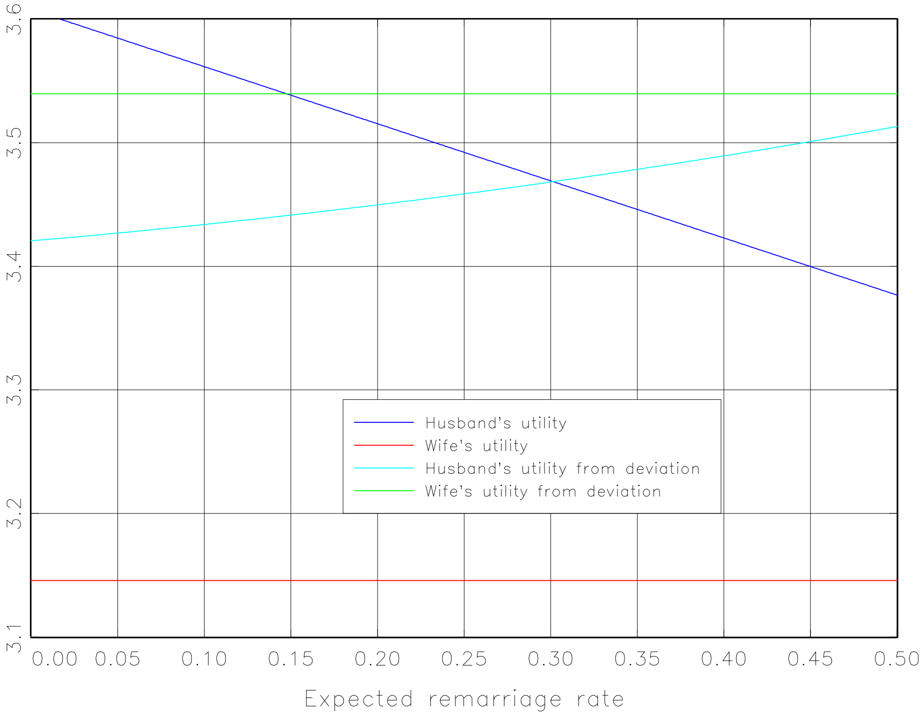

FIGURE 11.10. The Impact of Deviation to a Contingent Payment when all other Fathers give no Contingent Payment