Household Sharing and Commitment
Evidence from Panel Data on Individual Expenditures and Time Use

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Introduction

There is substantial evidence looking between households that relative allocations within households are related to relative bargaining positions (relative market productivity, marriage market opportunities etc).

- There is also evidence that changes to bargaining position affects relative allocations during marriage.

Mazzocco (2007) rejects full commitment, but cannot test the alternative commitment technology.

We provide evidence on the dynamics of intra-household allocations using a unique panel data set on individual private consumption expenditure allocations, public expenditures, and individual allocations of time to market production, home production and leisure.
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- We provide evidence on the dynamics of intra-household allocations using a unique panel data set on individual private consumption expenditure allocations, public expenditures, and individual allocations of time to market production, home production and leisure.
Related Literature (Incomplete)

- **Theory:**
  - Becker (1965); Chiappori (1988, 1992); Apps & Rees (1988)

- **Extension and evidence:**
  - Thomas (1990); Browning, Bourguignon, Chiappori & Lechene (1994); Lundberg, Pollak & Wales (1997); Blundell, Chiappori & Meghir (2005); Lise & Seitz (2011); Cherchye, De Rock & Vermeulen (2012); Attanasio & Lechene (2013)

- **Dynamics:**
  - Mazzocco (2007); Mazzocco, Ruiz & Yamaguchi (2007); Voena (2013)
A Dynamic Model of Household Decision Making

Households maximize the discounted weighted sum of individual utility:

$$\max_{\{c_jt, m_jt, \ell_jt, h_jt, g_t\}_{j=A,B,t=0,...,T}} \mathbb{E}_0 \sum_{t=0}^{T} \delta^t \left( \mu_t u^A(c_{At}, \ell_{At}, q_t; x_{1At}, x_{2At}) 
+ (1 - \mu_t) u^B(c_{Bt}, \ell_{Bt}, q_t; x_{1Bt}, x_{2Bt}) \right)$$

subject to

- home production: $q_t = q(g_t, h_{At}, h_{Bt}; x_{3At}, x_{3Bt})$
- time constraint: $\ell_jt + h_jt + m_jt = 1, \quad j = A, B$
- budget constraint:

$$c_{At} + c_{Bt} + g_t + w_{At} (\ell_{At} + h_{At}) + w_{Bt} (\ell_{Bt} + h_{Bt})$$

$$= w_{At} + w_{Bt} + (1 + r_t) a_t - a_{t+1} \equiv y_t,$$

- feasibility: $c_{jt}, g_{jt}, \ell_{jt}, h_{jt}, m_{jt} \geq 0$
- expectations about the stochastic process for wages
- some process describing $\mu_t$
The Pareto Weight and Distribution Factors

It is useful to distinguish between information known (or forecastable) at the time of marriage, and new information revealed during marriage.

- Let $z_0$ be the set of distribution factors that are known or forecastable at the time of marriage

$$z_0 \equiv \{\mathbb{E}_0 z_t\}_{t=0}^T$$

- Let $z_{1t}$ be the set of innovations realized at time $t$ during marriage

$$z_{1t} \equiv z_{1t} - \mathbb{E}_0 z_t$$
The Pareto Weight and Commitment

Statements about the ability of households to commit are equivalent to statements about what enters the Pareto weight
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- Full commitment (ex ante efficient allocations)

\[ \mu_t = \mu(z_0) \quad \forall t \]
The Pareto Weight and Commitment

Statements about the ability of households to commit are equivalent to statements about what enters the Pareto weight

- Full commitment (ex ante efficient allocations)
  \[ \mu_t = \mu(z_0) \quad \forall t \]

- Lack of commitment (allocations are efficient within period, but less insurance than ex ante efficient) i.e., continuous bargaining
  \[ \mu_t = \mu(z_0, z_{1t}) \]
The Pareto Weight and Commitment

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- Full commitment (ex ante efficient allocations)

  \[ \mu_t = \mu(z_0) \quad \forall t \]

- Lack of commitment ( allocations are efficient within period, but less insurance than ex ante efficient) i.e., continuous bargaining

  \[ \mu_t = \mu(z_0, z_{1t}) \]

- An interesting special case is when renegotiation only occurs if the participation constraints binds for one of the spouses

  \[ \mu_t = \begin{cases} 
    \mu_{t-1} & \text{if } \forall j \quad V_{jt}^{\text{married}} \geq V_{jt}^{\text{single}} \\ 
    \mu(z_0, z_{1t}) & \text{if } \exists j \quad V_{jt}^{\text{married}} < V_{jt}^{\text{single}} 
  \end{cases} \]

- Rich data on demographics, education, wages and labor supply
- Key: JPSC has a consumption expenditure module and a time use module.
  - Cohort 1 comprises 1,500 women aged 24 to 34 in 1993
  - Cohort 2 comprises 500 women aged 24 to 27 in 1997
  - Cohort 3 comprises 836 women aged 24 to 29 in 2003
Household Consumption Expenditures by Category

Please write down your household expenditure in September this year. (Including not only cash purchases, but also purchases with the credit loan(s), or those charged to your bank/post office account. (If there was no expenditure corresponding to the items below, put “0” for each answer.)

Foods (including eating-out/food-dispensing); House rent, land rent and home repairs (excluding housing loans); Utilities (light, fuel, water and sewerage); Furniture and housekeeping equipments (include bedclothing); Clothing and shoes; Healthcare (including nutritious drinks, health foods); Transportation (including the purchase of an automobile, fuel, or commuter pass); Communication (Postal fees, telephone, the Internet, etc.); Education (school fees, private tutoring schools for entrance exams or supplementary lessons, textbooks, reference books, etc.); Culture and entertainment (including lessons except for those for entrance exams or supplementary tutoring, or durable goods for culture and entertainment); Social expenses; Pocket money for you, your husband, your child(ren) and allowance for your child(ren); Allowance or pocket money for your and your husband’s parent(s); Other expenses.
Allocation of Total Expenditures to Individuals

Importantly for our purposes, the JPSC also asks for the breakdown of total household expenditures into the following five categories:

1. Expenses for all of your family
2. Expenses for you
3. Expenses for your husband
4. Expenses for your child(ren)
5. Expenses for the other(s)

We treat categories (1), (4) and (5) as expenditures on household public goods $g$, category (2) as private consumption of the wife $c_A$, and category (3) as the private consumption of the husband $c_B$. 
Allocation of Time

How many hours do you or does your husband spend in total per workday and day off (if you don’t work, answer about your husband’s day off.) for each of 6 activities listed below? (Enter the time in hour and decade of minutes.) If you or your husband has two or more activities in the same period of time, choose the most important of them.

1. For attending school or workplace
2. For work
3. For schoolwork (studies)
4. For housekeeping and child care
5. For hobby, leisure, social intercourse, etc
6. For other activities such as sleeping, meals, taking a bath, etc.

We treat activities (1), (2), and (3) as market hours $m_j$, activity (4) as home hours $h_j$, and activities (5) and (6) as leisure hours $l_j$. 
## Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Wife</th>
<th></th>
<th>Husband</th>
<th></th>
<th>Household</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td><strong>Expenditure</strong> (percent of household total)</td>
<td>34,305</td>
<td>59,199</td>
<td>77,834</td>
<td>65,618</td>
<td>420,657</td>
<td>23,541</td>
</tr>
<tr>
<td><strong>Time use, hours per week (share of own time)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market work</td>
<td>30.1h</td>
<td>20.3</td>
<td>60.9h</td>
<td>14.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- including commuting</td>
<td>(17.9%)</td>
<td></td>
<td>(36.3%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home production</td>
<td>44.0h</td>
<td>24.6</td>
<td>7.4h</td>
<td>9.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- including child care</td>
<td>(26.2%)</td>
<td></td>
<td>(4.4%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure</td>
<td>93.8h</td>
<td>19.7</td>
<td>99.6h</td>
<td>15.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- including sleep</td>
<td>(55.9%)</td>
<td></td>
<td>(59.3%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Observables</strong></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>35.2</td>
<td>5.4</td>
<td>37.8</td>
<td>6.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education (years)</td>
<td>13.1</td>
<td>1.5</td>
<td>13.4</td>
<td>2.2</td>
<td></td>
<td></td>
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<tr>
<td>Wage</td>
<td>889</td>
<td>565</td>
<td>1638</td>
<td>582</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children aged 0–6</td>
<td></td>
<td></td>
<td>0.68</td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children aged 7–17</td>
<td></td>
<td></td>
<td>0.95</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Wife’s Wage Share: $w_A/(w_A + w_B)$

Cross-section
Wife’s Private Consumption Share: $c_A/(c_A + c_B)$
Wife’s Leisure Share: $\ell_A/(\ell_A + \ell_B)$
Public Expenditure Share: \( \frac{g}{(c_A + c_B + g)} \)
Wife’s Market Hours Share: \( \frac{m_A}{m_A + m_B} \)
Wife’s Home Hours Share: $h_A/(h_A + h_B)$
We derive 13 estimating equations (nonlinear GMM) from the first-order conditions (intra- and inter-temporal).

\[ u_t^A = \frac{\zeta_t}{1-\sigma} \left( \alpha_1 t c_{At}^\phi + \alpha_2 t \ell_{At}^\phi + (1 - \alpha_1 t - \alpha_2 t) q_t^\phi \right)^{\frac{1-\sigma}{\phi}} \]

\[ u_t^B = \frac{\xi_t}{1-\varsigma} \left( \beta_1 t c_{Bt}^\phi + \beta_2 t \ell_{Bt}^\phi + (1 - \beta_1 t - \beta_2 t) q_t^\phi \right)^{\frac{1-\varsigma}{\phi}} \]

\[ \alpha_{kt} = \frac{\exp(\alpha'_k x_{1At})}{1+\exp(\alpha'_1 x_{1At})+\exp(\alpha'_2 x_{1At})}, \quad \beta_{kt} = \frac{\exp(\beta'_k x_{1Bt})}{1+\exp(\beta'_1 x_{1Bt})+\exp(\beta'_2 x_{1Bt})} \]

\[ \zeta_t = \exp(\zeta' x_{2At}), \quad \xi_t = \exp(\xi' x_{2Bt}) \]

\[ q_t = (\pi_t h_{At}^\gamma + (1 - \pi_t) h_{Bt}^\gamma)^\frac{\rho}{\gamma} g_t^{1-\rho} \]

\[ \pi_t = \frac{\exp(\pi' x_{3t})}{1+\exp(\pi' x_{3t})} \]

\[ \log w_{ijt} = \omega_{0i}^j + \omega_1^j \log w_{ij,t-1} + \varepsilon_{ijt}, \quad \mathbb{E} [\varepsilon_{ijt}] = 0 \]

\[ \mu_t = \frac{\exp(\mu'_0 z_0 + \mu'_1 z_{1t})}{1+\exp(\mu'_0 z_0 + \mu'_1 z_{1t})} \]
Estimation: Observable Shifters

- Pareto weight: $z_0$
  - relative market productivity known at the time of marriage
  - household resources known at the time of marriage
  - sex ratio; parents’ income; father’s occupational prestige

- Pareto weight: $z_{1t}$
  - deviations in relative wage from time zero predictions
  - deviations in household resources from time zero predictions

- Intra-temporal preferences: $x_1$
  - age and education

- Inter-temporal preferences: $x_2$
  - household size

- Efficiency of home hours: $x_3$
  - number of children aged 0–6

- Productivity at marriage: $\ln w_{ij0}$
  - education, experience, parent’s education, place of birth
Results: Home Production and Preferences

- Home production:
  - Husband’s and wife’s home hours are mildly complementary.
  - Husbands’ and wives’ home hours are equally efficient.

- Preferences:
  - Preferences are quite similar between husbands and wives.
    - The wife’s weight on the public good increases with a rise in the number of children.
  - Private consumption, leisure and public goods are complements.

The implication is that most of the differences in allocations between husbands and wives result from relative wages and the Pareto weight.
<table>
<thead>
<tr>
<th>Pareto weight</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu / (1 - \mu)$ (at sample mean)</td>
<td>0.614</td>
<td>0.648</td>
<td>0.615</td>
<td>0.614</td>
<td>0.614</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.082)</td>
<td>(0.073)</td>
<td>(0.074)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>$\mu_01: \omega_0A - \omega_0B$</td>
<td>0.626</td>
<td>0.627</td>
<td>0.624</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.077)</td>
<td>(0.074)</td>
<td>(0.075)</td>
<td>(0.073)</td>
</tr>
<tr>
<td>$\mu_02: \nu_0$</td>
<td>-0.025</td>
<td>-0.025</td>
<td>-0.024</td>
<td>-0.025</td>
<td>-0.042</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.030)</td>
<td>(0.028)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$\mu_03: \log (ed_A / ed_B)$</td>
<td>-0.213</td>
<td>-0.215</td>
<td>-0.228</td>
<td>-0.214</td>
<td>-0.199</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.124)</td>
<td>(0.124)</td>
<td>(0.125)</td>
<td>(0.137)</td>
</tr>
<tr>
<td>$\mu_04: \log (xp_A / xp_B)$</td>
<td>-0.061</td>
<td>-0.064</td>
<td>-0.056</td>
<td>-0.061</td>
<td>-0.056</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.026)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>$\mu_05: \frac{1}{2} \log (sr_A sr_B)$</td>
<td>1.167</td>
<td>0.025</td>
<td>0.024</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(1.062)</td>
<td>(0.033)</td>
<td>(0.012)</td>
<td>(0.073)</td>
<td>(0.073)</td>
</tr>
<tr>
<td>$\mu_06: \log (y_A^P / y_B^P)$</td>
<td>0.260</td>
<td>0.266</td>
<td>0.259</td>
<td>0.262</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.065)</td>
<td>(0.064)</td>
<td>(0.064)</td>
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</tr>
<tr>
<td>$\mu_07: \log (y_A^P + y_B^P)$</td>
<td>0.017</td>
<td>0.016</td>
<td>0.018</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.017)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\mu_08: \log (occ_A^P / occ_B^P)$</td>
<td>-0.024</td>
<td>-0.024</td>
<td>-0.024</td>
<td>-0.024</td>
<td>-0.024</td>
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<tr>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.012)</td>
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<tr>
<td>$\mu_11: z_{1t}$</td>
<td>0.260</td>
<td>0.266</td>
<td>0.259</td>
<td>0.262</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.065)</td>
<td>(0.064)</td>
<td>(0.064)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>$\mu_12: z_{2t}$</td>
<td>0.017</td>
<td>0.016</td>
<td>0.018</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.017)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\mu_13: z_{1t} \times 1{q_1 &lt; z_{1t} &lt; q_3}$</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(0.234)</td>
<td>(0.234)</td>
<td>(0.234)</td>
<td>(0.234)</td>
<td>(0.234)</td>
</tr>
<tr>
<td>$\mu_14: z_{1t} \times 1{z_{1t} \leq q_1 \cup z_{1t} \geq q_3}$</td>
<td>0.254</td>
<td>0.254</td>
<td>0.254</td>
<td>0.254</td>
<td>0.254</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.064)</td>
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<td>(0.064)</td>
<td>(0.064)</td>
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</tbody>
</table>
Decomposing $\mu_t / (1 - \mu_t) = \mu_0(z_0) \times \mu_1(z_{1t})$

- There is substantial heterogeneity at the time of marriage related to relative earnings potential.
- Revisions to the weight during marriage are rare, and small relative to initial dispersion.
Robustness

As a robustness exercise, we estimate the Pareto weight using only the following first-order conditions, without imposing parametric assumptions on preferences (we impose Pareto weight and marginal utility are positive):

\[
\frac{\mu_t}{1 - \mu_t} \frac{\partial u^A / \partial c_{At}}{\partial u^B / \partial c_{Bt}} = \frac{w_{At}}{w_{Bt}},
\]

\[
\frac{\mu_t}{1 - \mu_t} \frac{\partial u^A / \partial l_{At}}{\partial u^B / \partial l_{Bt}} = 1,
\]

<table>
<thead>
<tr>
<th>Pareto weight</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu / (1 - \mu)$ (at sample mean)</td>
<td>0.509</td>
<td>(0.179)</td>
</tr>
<tr>
<td>$\mu_{01}: \omega_{0A} - \omega_{0B}$</td>
<td>0.823</td>
<td>(0.363)</td>
</tr>
<tr>
<td>$\mu_{02}: \nu_0$</td>
<td>-0.011</td>
<td>(0.193)</td>
</tr>
<tr>
<td>$\mu_{03}: \log (ed_A / ed_B)$</td>
<td>-0.253</td>
<td>(1.130)</td>
</tr>
<tr>
<td>$\mu_{04}: \log (xp_{A0} / xp_{B0})$</td>
<td>0.014</td>
<td>(0.121)</td>
</tr>
<tr>
<td>$\mu_{11}: z_{1t}$</td>
<td>0.520</td>
<td>(0.206)</td>
</tr>
<tr>
<td>$\mu_{12}: z_{2t}$</td>
<td>-0.000</td>
<td>(0.100)</td>
</tr>
</tbody>
</table>
The optimal mix of \( \{h_A, h_B, g\} \) depends on relative wages, but does not depend on the Pareto weight (except indirectly at the corner for market hours).

The estimated share of total expenditure devoted to public goods is only mildly sensitive to the wage share.
Summary

1. Relative wages affect relative allocations in the cross section.

2. Innovations to relative wages lead to changes in relative allocations during marriage.
   - The effect of innovations is much smaller than the effect of differences between households.
   - Within marriage, husbands and wives are insured against small innovations to wages, but re-bargain in the face of large innovations.

3. The household public good is largely unaffected by changes in the Pareto weight.
Home production

\[
\left( \frac{\pi_t}{1 - \pi_t} \right) \left( \frac{h_{At}}{h_{Bt}} \right)^{\gamma - 1} = \frac{w_{At}}{w_{Bt}},
\]

(1)

\[
\pi_t \left( \frac{\rho}{1 - \rho} \right) \left( \frac{h_{At}^{\gamma - 1}}{H_t} \right) g_t = w_{At},
\]

(2)

\[
(1 - \pi_t) \left( \frac{\rho}{1 - \rho} \right) \left( \frac{h_{Bt}^{\gamma - 1}}{H_t} \right) g_t = w_{Bt},
\]

(3)

\[
H_t = \pi_t h_{At}^\gamma + (1 - \pi_t) h_{Bt}^\gamma.
\]
Intra-household

\[
\frac{\alpha_{1t}}{\alpha_{2t}} \left( \frac{c_{At}}{\ell_{At}} \right)^{\phi - 1} = \frac{1}{w_{At}}, \quad (4)
\]

\[
\frac{\beta_{1t}}{\beta_{2t}} \left( \frac{c_{Bt}}{\ell_{Bt}} \right)^{\varphi - 1} = \frac{1}{w_{Bt}}, \quad (5)
\]

\[
\frac{\mu_{t} \zeta_{t} A_{t}^{\frac{1-\sigma-\phi}{\phi}} \alpha_{2t} \ell_{At}^{\phi-1}}{\xi_{t} B_{t}^{\frac{1-\sigma-\varphi}{\varphi}} \beta_{2t} \ell_{Bt}^{\varphi-1}} = \frac{w_{At}}{w_{Bt}}, \quad (6)
\]

\[
\frac{\mu_{t} \zeta_{t} A_{t}^{\frac{1-\sigma-\phi}{\phi}} \alpha_{1t} c_{At}^{\phi-1}}{\xi_{t} B_{t}^{\frac{1-\sigma-\varphi}{\varphi}} \beta_{1t} c_{Bt}^{\varphi-1}} = 1, \quad (7)
\]

\[
A_{t} = \alpha_{1t} c_{At}^{\phi} + \alpha_{2t} \ell_{At}^{\phi} + (1 - \alpha_{1t} - \alpha_{2t}) q_{t}^{\phi} \quad \text{and}
\]

\[
B_{t} = \beta_{1t} c_{Bt}^{\varphi} + \beta_{2t} \ell_{Bt}^{\varphi} + (1 - \beta_{1t} - \beta_{2t}) q_{t}^{\varphi}.
\]
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\[
\mu_t \zeta_t A_t^{\frac{1-\sigma-\phi}{\phi}} \alpha_{2t} \ell_{A_t}^{\phi-1} = \pi_t \rho h_{A_t}^{\gamma-1} H_t^{\frac{\rho-\gamma}{\gamma}} g_t^{1-\rho} D_t, \quad (8)
\]

\[
\mu_t \zeta_t A_t^{\frac{1-\sigma-\phi}{\phi}} \alpha_{1t} c_{A_t}^{\phi-1} = (1 - \rho) g_t^{\frac{1}{\gamma}} H_t^{\frac{1}{\rho}} D_t, \quad (9)
\]

\[
\xi_t B_t^{\frac{1-\varsigma-\phi}{\phi}} \beta_{2t} \ell_{B_t}^{\phi-1} = (1 - \pi_t) \rho h_{B_t}^{\gamma-1} H_t^{\frac{\rho-\gamma}{\gamma}} g_t^{1-\rho} D_t, \quad (10)
\]

\[
\xi_t B_t^{\frac{1-\varsigma-\phi}{\phi}} \beta_{1t} c_{B_t}^{\phi-1} = (1 - \rho) g_t^{\frac{1}{\gamma}} H_t^{\frac{1}{\rho}} D_t, \quad (11)
\]

\[
D_t = \mu_t \zeta_t A_t^{\frac{1-\sigma-\phi}{\phi}} (1 - \alpha_{1t} - \alpha_{2t}) q_t^{\phi-1} + \xi_t B_t^{\frac{1-\varsigma-\phi}{\phi}} (1 - \beta_{1t} - \beta_{2t}) q_t^{\phi-1}.
\]

**Euler Equations**

\[
\mathbb{E}_{t-1} \left[ \frac{\mu_t \zeta_t}{\mu_{t-1} \zeta_{t-1}} \frac{\alpha_{1t}}{\alpha_{1,t-1}} \left( \frac{A_t}{A_{t-1}} \right)^{\frac{1-\sigma-\phi}{\phi}} \left( \frac{c_{At}}{c_{A,t-1}} \right)^{\phi-1} R_t \delta \right] = 1 \quad (12)
\]

\[
\mathbb{E}_{t-1} \left[ \frac{\xi_t \beta_{1t}}{\xi_{t-1} \beta_{1,t-1}} \left( \frac{B_t}{B_{t-1}} \right)^{\frac{1-\varsigma-\phi}{\phi}} \left( \frac{c_{Bt}}{c_{B,t-1}} \right)^{\phi-1} R_t \delta \right] = 1. \quad (13)
\]
Caring Preferences

With caring preferences the household is maximizing

$$U_0^H = \mathbb{E}_0 \sum_{t=0}^{T} \delta^t \left( \tilde{\mu}_t u^A(c_{At}, \ell_{At}, q_t, u^B_t; x_{At}) + u^B(c_{Bt}, \ell_{Bt}, q_t, u^A_{t}; x_{Bt}) \right), \quad (14)$$

Suppose that caring takes the following special form

$$u^j_t = u^j_t(c_{jt}, \ell_{jt}, q_t; x_{jt}) + \tau_j u^k_t(c_{kt}, \ell_{kt}, q_t; x_{kt}), \quad j \neq k \in \{A, B\},$$

where $\tau_j \in (0, 1)$. Then we can rewrite equation (14) as

$$U_0^H = \mathbb{E}_0 \sum_{t=0}^{T} \delta^t \left( \mu_t u^A(c_{At}, \ell_{At}, q_t; x_{At}) + u^B(c_{Bt}, \ell_{Bt}, q_t; x_{Bt}) \right),$$

where

$$\mu_t = \frac{\tilde{\mu}_t + \tau_B}{1 + \tau_A \tilde{\mu}_t}.$$

If $\tau_A = \tau_B = 1$ the effective Pareto weight is equal to one, independent of the size of $\tilde{\mu}_t$, and distribution factors do not enter the allocation problem.
Robust Estimating Equations

\[ Z_0 \mu_0 + Z_1 t \mu_1 + X_{At} \Gamma_A - X_{Bt} \Gamma_B - \log \left( \frac{w_{At}}{w_{Bt}} \right) = 0 \]

\[ Z_0 \mu_0 + Z_1 t \mu_1 + X_{At} \Lambda_A - X_{Bt} \Lambda_B = 0 \]

where \( X_{At} \) contains \( \{c_{At}, \ell_{At}, h_{At}, h_{Bt}, g_t, x_{At}\} \), plus all the squares, and \( X_{Bt} \) contains \( \{c_{Bt}, \ell_{Bt}, h_{At}, h_{Bt}, g_t, x_{Bt}\} \), plus all the squares not already included in \( X_{At} \).