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Estimating Private Provision of Public Goods with Heterogenous Participants: A Structural Analysis*

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Abstract

This paper estimates a structural model of private provision of public goods to provide some new empirical evidence on individuals' strategic contributing behaviors. In the model, individuals' contributing behaviors are allowed to be heterogenous and time-varying. We show that all the main components of the model including the number of different contributing strategies, functional form for each strategy, and how individuals adjust their strategies are identified from the revealed contribution choices of individuals. Further, the structural model is estimated using the data collected in a threshold public good experiment. The empirical results suggest that subjects in our experiment employ three contributing strategies, and they strategically respond to provision history by adjusting their preceding behavior. In addition, the response is heterogenous and dependent on subjects' contributing strategies.

JEL Classification: H41, C14

Keywords: Public Goods, Private Provision, Measurement Errors, Unobserved Heterogeneity, Nonparametric Identification and Estimation, Experimental Data.

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1 Introduction

Private provision of public goods is important for governments or organizations to seek private support to cover costs of projects partially or entirely. Prominent examples include the newly emerged crowd-funding industry, annual fundraising of nonprofit organizations such as Wikipedia and National Public Radio (NPR).[1] Because of the prevalence of provision of public goods in our society, understanding individuals' behaviors in private provision of public goods is an important economic question in its own right. Moreover, inference of individuals' behavior in contributing to public goods could shed lights on some policy related issues such as setting appropriate mechanism for the provision.

A large body of literature has been devoted to the study of private provision of public goods with a focus on individual behavior. It has been documented that individuals do not always reveal their true values toward the public good, (e.g., see Andreoni (1988), Weimann (1994) and Olson (1965)) and that they exhibit strategic and heterogenous contributing behaviors (Oliveira et al. (2014) and Fischbacher and Gächter (2010)). However, existing studies of individuals' behavior mainly rely on behavioral assumptions on their beliefs or preferences, and little rigorous structural and empirical work has been undertaken on private provision of public goods by modeling individuals' strategic behaviors and interactions explicitly. To fill the gap, we propose a structural model of private provision of public goods, which allows individuals' contributing behaviors to be heterogenous and evolve over time. The model primitives including the number of different contributing strategies, functional form for each strategy, and the transition probabilities among all possible strategies are shown to be identifiable and estimable from the revealed contribution choices of individuals. We apply our method to the data we collected in a threshold public good experiment and provide some new evidence on private provision of public goods. The empirical results suggest that subjects employ three contributing strategies and they strategically respond to provision history by adjusting their preceding contributing behaviors. Furthermore, the response is heterogenous and dependent on subjects' contributing strategies.

This paper focuses on threshold public good games (Bergstrom et al., 1986; Cadsby and Maynes, 1999; Croson and Marks, 2000), where the public good is provided only if the aggregated contributions reach or surpass the predetermined cost (or the provision point); otherwise contributions will be returned to individuals.[2] We collect data of individual contributions from

[1]The Crowd-funding Industry Report's data indicating the overall crowd-funding industry has raised \$2.7 billion in 2012, across more than 1 million individual campaigns globally. In 2013 the industry is projected to grow to \$5.1 billion. Wikipedia organizes an annual fundraising campaign to support its operations, which usually lasts from mid-November to mid-January. The total money raised increases from \$94,000 in 2005 to \$25 million in 2012.

[2]Bagnoli and McKee (1991) find that provision point mechanism together with money back guarantee (MBG) can potentially induce Pareto efficient outcome in a single unit provision environment.

a threshold public good experiment. Subjects in a group with fixed membership make contributions toward a public good with predetermined cost across 10 periods, with their induced values being randomly drawn from a uniform distribution. Subjects observe the outcome of the game, i.e., whether the public good is provided, and not other group members' contributions after each period. A reduced-form analysis demonstrates that subjects contribute using heterogenous strategies. Furthermore, they also adjust their strategies mainly based on the outcome as well as their own strategies in the preceding period.

To further qualify the reduced-form findings and understand the interaction among individuals within a group, we propose a structural model describing individuals' behaviors in public good provision and estimate the model using our experimental data. Our model allows the individuals to employ heterogenous contributing strategies (we label all individuals employing the same strategy as a "type") . In line with Fischbacher and Gächter (2010), which focus on "linear" public goods,[3] the heterogeneity in our model is originated in their beliefs about other contributors' behaviors as well as their own preference. Both the beliefs and preferences may change over time, hence subjects may adjust their contributing strategies based on the provision history, and our model also allows such adjustments. Without specifying the number of different contributing strategies, functional form of the strategies, and how individuals change their strategies *ex ante*, we indicate that all these objectives can be directly recovered from individuals' contributions. The main requirement of our approach is that each individual participates in three public good provision games (makes three contributions); however their induced values are not required to be known for our analysis. The three observations for each individual enable us to apply the recently developed results in nonclassical measurement errors, namely from that of Hu (2008), to identify and estimate the structural model we propose. The underlying link between our model and a model with nonclassical measurement errors is that the unobserved type of an individual is treated as the latent variable, and her contributions are the corresponding measurements. The main idea of our identification is that the contributions are used as instrumental variables for the unobserved type of individuals.

We employ a two-stage procedure for estimation. First, we back out the number of type as well as the contributing strategy for each type by a fully nonparametric approach. Second, we use maximum likelihood estimation to estimate the transition probability among different types based on provision history using multi-periods data. A Monte Carlo experiment demonstrates that our proposed method performs very well for samples with a similar size as that of our

[3]In a standard linear public good game, subjects are asked to allocate their tokens between a private fund that benefits only the individual investor and a group fund that generates profits for everyone. The private fund yields a higher rate of return than the public fund for the private investor, but the public fund provides the group with a higher total return. The marginal return for the group fund is normally set such that the social optimum occurs when individuals give everything to the group fund, while the individuals' optimum occurs when one keeps all tokens in their private fund.

experimental data. The results of estimation, using our experimental data, suggest that subjects are of three types with different contributing strategies: types 1 and 3 contribute the least and the most, respectively and type 2 is in the middle. We estimate the proportions of three types to be 36.2%, 37.4%, and 26.4% for type 1, 2, and 3, respectively. The contributing strategies of all three types are highly nonlinear, i.e., subjects with different values contribute very different proportions of their values. Type 1 contributes comparatively lesser than type 2 and 3, whereas the strategies of the latter two types are similar. Heterogenous behaviors in "linear" public good games have been documented in Fischbacher et al. (2001) and Fischbacher and Gächter (2010). These studies on linear public goods have indicated the existence of a substantial portion of free-rider. However, in our experiment with provision point mechanism, we find that the least generous type (type 1) still contributes a significant proportion of their induced values rather than employ a complete free-riding strategy.

We estimate the transition among types by matrices with each element being a probability of type k $(k = 1, 2, 3)$ in the current period conditional on one's type j $(j = 1, 2, 3)$ and the outcome of provision in the preceding period. Whether the provision is successful in the preceding period affects all three types' transition significantly: subjects maintain their proceeding contributing strategies with a probability greater than 70% in response to a successful provision. By contrast, both type 1 and 2 would adjust to higher types with a substantial probability to respond to an unsucessful outcome. Moreover, type 2 is more "sensitive" to the pervious outcome than the other two types. Considering the possibility that subjects' adjustment of types is variant over time, we estimate the model separately for the first five periods and last five periods of data and compare the results with that estimated from all the 10 periods of data. The main findings are from the first to the last five periods (1) the difference between types 2 and 3 diminishes; (2) subjects are less reluctant to adjust their contributing strategies. A possible interpretation is that subjects do learn to corporate over time.

The main contribution of our paper is to provide some new findings on individuals' contributing strategies and how they are adjusted. A fast-growing experimental literature on private provision of public goods focuses on investigating heterogenous behaviors of individuals, e.g., Oliveira et al. (2014) and Fischbacher and Gächter (2010). However, to the best of our knowledge, ours is the first paper that explicitly estimates the individuals' heterogenous strategies, and the transition probabilities between any two strategies in private provision of public goods without imposing behavioral assumptions on individuals' beliefs or preferences. These estimates in our paper constitute systematic evidence on individuals' heterogeneity and their strategic response to others' behaviors as well as their own.

The novelty of our paper is that the analysis of individuals' heterogenous behaviors is grounded on revealed contribution choices instead of relying on prior behavioral assumptions on individuals' beliefs or preferences. Moreover, individuals' induced values are not needed for our approach however, only the distribution of the values is necessary. Existing studies of individuals' heterogenous behavior (e.g., Fischbacher et al. (2001), Fischbacher and Gächter (2010), and **?**) mainly rely on experimental controls to classify different contributing strategies, which is either not applicable or costly to data contexts in fields. By contrast, our approach does not specify the number of strategies and what they are *ex ante* but identifies these objectives from the revealed contributions by exploring the structural connection between individuals' strategies and their multiple contributions. Therefore, our approach can be applied to more general data contexts. To our knowledge, this is the first paper that provides empirical evidences on threshold public goods using a structural model.

Another main contribution of our paper is that contributing strategies can be estimated without imposing a functional form or solving equilibria explicitly. This is a great advantage since the existing studies on the threshold public good provision lack detailed analyses on individuals' contributing strategies, partially due to the difficulty of deriving an analytical solution. There are several attempts to characterize the Bayesian-Nash equilibrium for two-player's threshold public good provision game (e.g., Alboth et al. (2001), Barbieri and Malueg (2008), and Laussel and Palfrey (2003)). However, once the group size grows to three or more, an analytical solution is almost impossible without much more stringent assumptions. The possible non-equilibrium and heterogenous contributing strategies of individuals might be rationalized by various behavioral models, e.g., level-k thinking (Crawford and Iriberri, 2007) or cognitive hierarchy (Camerer et al., 2004). Therefore, our paper sheds some lights on the analysis of non-equilibrium behavior without imposing too many structural restrictions. Furthermore, our paper contributes to the public good learning literature (Clemens and Riechmann, 2002; Healy, 2006). Our learning results show individuals will adjust their contributions based on the history of outcome and their own strategies and such learning adjustments are contingent on unobserved individual types.

The methodology of this paper is related to some recent studies of unobserved heterogeneity in environments of strategic interactions using results of measurement errors (e.g. Hu (2008) and Hu and Schennach (2008)). For example, Li et al. (2000), Krasnokutskaya (2011) and Hu et al. (2013a) consider auction models unobserved heterogeneity. Hu et al. (2013b) use bandit experiments to nonparametrically estimate the learning rule using auxiliary measurements of beliefs. Xiao (2013) considers multiple equilibria in static and dynamic games. The connection between the unobserved heterogeneity and observables in these studies is similar to our paper. Nevertheless, to the best of our knowledge this paper is the first study of private provision of public goods with rigorous identification and estimation in a framework of measurement errors.

This paper is organized as follow. Section 2 provides an overview of the experiment and the data, and further presents some reduced-form evidences for subjects' heterogenous contributing strategies and learning. Section 3 proposes a structural model of threshold public goods with heterogenous subjects and shows the model is nonparametrically identifiable and estimable. Section

4 conducts Monte Carlo experiments to illustrate our method. Section 5 presents the estimated results for our experimental data. Section 6 concludes. Proofs, tables, figures and experiment instructions are collected in the Appendix.

2 The Experiment and Data

We conducted six experiment sessions in the CANR (College of Agriculture and Natural Resources) Lab, University of Connecticut (UConn). Subjects were recruited primarily through UConn Daily Digest where we advertised requesting volunteer participation in economic experiments. Our subject pool consist mostly undergraduates and a few graduate students from various academic majors who have indicated a willingness to participate in economic experiments. We checked the participants names and email addresses, before confirming their attendance, to ensure each subject participated only once in this sequence of experiments. We conduct experiments through networked computer terminals using z-Tree (Fischbacher (2007)). Inter-participant communications during the experiment were prohibited and subjects could not observe each others choices. Experiment instructions were read aloud and the group size was kept constant at five. Each experiment session consist two groups and the group memberships are kept the same during the 10 decision periods, i.e., individuals know that they will play with the same people during the 10-periods experiment.

Our experiment uses provision point mechanism where each individual is asked to contribute according to the induced value. Furthermore, each individual is asked to enter their subjective probability, from 0 to 1, to indicate how likely they think their group will provide the public goods. After each decision period, we randomly re-assigned the induced value. At the beginning of each decision period, individuals were told their induced values, which simulate the valuations for the public goods. Induced values followed a uniform distribution on the interval $[7.95, 20.05)$ and are rounded to one decimal place. Subjects know the value distribution and their own induced values, however, not the induced value of the others. The unit cost, c, is public information. After each decision period, subjects will be informed the provision result and their own profits but not others' profit in the last period. We set the provision cost for one unit equal to 60% of the expected induced value for an individual times the number of all individuals in a session; thus, the cost is $60\% * 14 * 5 = 42$. A total of 60 subjects participated in the treatment, producing 600 individual level observations. Individuals receive an average earning of about \$20. Actual earnings vary across individuals and sessions.

Table 1 presents simple summary statistics of the data. In each column (period), the variable "Provided" is a binary variable indicating the outcome of the public good game: provided=1 if provided and 0 otherwise.; "Contr./Value" is defined as the ratio of contribution over value. The table demonstrates that the proportion of groups that successfully provide the public good varies a lot across period with the minimum 0.333 and the maximum 0.833 even though the change of value and contribution is relatively small. The ratio of contribution to value ranges from 0.569 to 0.661, which implies variation of subjects' contributing behavior.

[Table 1 is about here.]

2.1 A preliminary analysis of data

We provide a preliminary analysis of the data to show that subjects are heterogenous in their contributing strategies, which might be varying across periods too. Let b_i and v_i be subject i's contribution and value respectively, and $w_{t-s}, s = 1, 2, ...$ be a dummy variable of outcome, which equals to 1 if the public good is provided by the group in period $t-s$ and zero otherwise. Denote $p_i \in [0, 1]$ subjective i's subject belief on the probability that the good would be provided. To investigate an aggregated pattern of the data, we run a linear regression of contributions on values, periods, beliefs as well as the interactions terms as follows:

$$b_i = \alpha_0 + \alpha_1 v_i + \alpha_2 t + \alpha_3 p_i + \alpha_4 v_i * t + \alpha_5 v_i * p_i + \alpha_6 t * p_i + u_i. \quad (1)$$

The results are presented in Table 2. Predictably, subjects' values are important for their contributions and on average a subject contributes approximately 50%-60% of her value. The effects of beliefs on contributions are positive and significant, which implies that one tends to corporate if she believes other group members would corporate. The evidence is consistent with the empirical findings in Fischbacher and Gächter (2010) for linear public goods provision games. We find that the period is insignificant and the magnitude is small. This result indicates that the overall contributing behavior does not change over time.

As subjects observe the outcomes in previous periods in the game, dependence between their contributing behavior in two consecutive periods may exist. For this purpose, we further analyze the data by period. Let b_{it}, v_{it} and p_{it} be subject i's contribution, induced value and belief in period t, respectively. Figure 1 illustrates the relationship between subjects' value v_{it} and contribution b_{it} conditional on the outcome w_{t-1} in the preceding period. The blue and red markers are for $w_{t-1} = 1$ and $w_{t-1} = 0$, respectively. There are two important observations regarding this figure: First, for a given outcome the relationship between value and contribution varies across periods. For example, the blue markers are concentrated in period 5, however, they are scattered in period 9, implying that contributing behavior in those two periods are distinctive. Second, the relationship between value and contribution is different across outcomes; e.g., in period 2 those subjects who had a successful outcome in period 1 contribute relatively less: for a given value, contributions indicated by the blue markers are smaller than the red ones. However, such a pattern is less obvious for period 3. A possible interpretation is that subjects respond to the preceding period's different outcomes differently and such response could also be distinctive across subjects and/or periods.

[Figure 1 is about here.]

To further explore the observations in Figure 1 quantitatively, we consider a linear regression model by period:

$$b_{it} = \beta_0 + \beta_1 v_{it} + \beta_2 w_{t-1} + \beta_3 w_{t-2} + \beta_4 w_{t-1} * r_{i,t-1} + \beta_5 w_{t-2} * r_{i,t-2} + \epsilon_{it}, t = 1, 2, \cdots, 10, \qquad (2)$$

where the ratio of contribution to value $r_{i,t} \equiv b_{i,t}/v_{i,t}$ is an approximation of the contributing strategy of subject i at period t. In this regression equation, β_1 describes how subjects' value affects their contribution after controlling for other factors. The coefficient β_2 and β_3 captures how subjects respond to the different outcomes of period $t-1$ and $t-2$ respectively , and β_4 and β_5 summarizes how subjects with different contributing strategies respond the previous periods' outcome distinctively.

Table 3 provides the regression results for 10 periods. Being consistent with the results of regression (1), induced value is significant in determining subjects' contribution across all periods (β_1). Specifically, on average subjects contribute 35%-70% of their value. The effect of the preceding period's outcome (β_2) is negative and significant for most of the periods. The results indicate that subjects respond to a successful provision in the last period by decreasing their contribution in the current period. In addition, such response varies for different contributing strategies and this is shown by the estimate of coefficient β_4. The total effects of last period's outcome on the current period's contribution can be calculated as $\hat{\beta}_2 + \hat{\beta}_4 \times r_{i,t-1}$, for example, in period 2, $\hat{\beta}_2 + \hat{\beta}_4 \times r_{i,t-1} = -4.208 + 2.919 \times 0.609 = -2.430$. Thus if the good is provided in period 1, on average subjects would contribute -2.430 less than if the good is not provided, which is 31.2% of the average contribution in period 2. The estimates $\hat{\beta}_3$ and $\hat{\beta}_5$ are insignificant in almost all the periods and the results demonstrate that subjects learn mainly from the most recent history. In summary, the results for the coefficients of w_{t-1} and $w_{t-1} \times r_{i,t-1}$ demonstrate that subjects adjust their contributing behavior based on the outcome of the preceding period (it is called "learning rule" hereafter) and the adjustment is heterogenous. To further qualify subjects' contributing behavior and their learning rule, we present a structural model of private provision of public goods and indicate that the model is nonparametrically identified and estimated in the next section.

[Table 3 is about here.]

3 A Structural Model with Heterogenous Subjects

In this section, we propose a structural model of public goods provision with heterogenous subjects to rationalize the findings in Section 2. The main components of the model are shown to be nonparametrically identifiable and estimable under mild conditions.

3.1 The model

A group of $I \geq 2$ risk-neutral subjects contribute to a public good across $T \geq 3$ periods. The private values of subjects $v_{it} \in [\underline{v}, \overline{v}]$, $i = 1, 2, ..., I; t = 1, 2, \cdots T$ are i.i.d. draws across i and t from a cumulative distribution function $G(\cdot)$ with density $g(\cdot)$.[4] At period t, subject i makes a contribution $b_{it} \in [0, v_{it}]$ and the public good is provided only if the total contribution of all the I subjects exceeds the cost (threshold) $c > 0$, i.e., $\sum_i b_{it} \geq c$ where c is a known constant over periods. We maintain that $\overline{v} < c$ such that it is impossible for an individual subject to provide the good. Subject i obtains a payoff $v_{it} - b_{it}$ if the public good is provided and zero otherwise. The common knowledge among subjects at the beginning of period t includes the value distribution $G(\cdot)$, group size I, cost c and the outcome of previous periods $w_{-t} \equiv \{w_1, w_2, \cdots, w_{t-1}\}$, which are binary variables with $w_s = 1$ indicating a successful provision in period s, and $w_s = 0$ otherwise. In summary, subject i solves the following maximization problem in period t:

$$\max_{b_{it}} (v_{it} - b_{it}) \Pr\left(\sum_{j=1}^{I} b_{jt} \geq c \middle| \mathcal{I}_{-it} \right), \tag{3}$$

where $\mathcal{I}_{-it} \equiv \{(w_s, b_{is}, v_{is}), \ s = 1, 2, ..., t-1\}$ is a set of information available for subject i prior to period t. The probability in (3) summarizes both a subject's belief about others' behaviors and her own preference, as described in Fischbacher and Gächter (2010). To model the heterogenous contributing behaviors of the subjects, we follow the previous findings in the literature and assume that the probability may be different across subjects. Given a subject's value v_{it}, each possible probability implies a corresponding b_{it} as the optimal solution to problem (3). We use "type" to indicate a certain probability and the resulting contributing strategy. Without loss of generality, let all the subjects be one of the K ($K \leq I$) (discrete) private types with each type is corresponding to a specific contributing strategy. Subject i's type is denoted as $\tau_i \in \{1, 2, ..., K\}$, then her contributing strategy (we only consider those strategies monotone in value) is a mapping from her private value and type to her contribution, i.e.,

$$s_i(\cdot, \cdot) : [\underline{v}, \overline{v}] \times \{1, 2, ..., K\} \to [0, \overline{v}].$$

For ease of notation, we rewrite $s_i(v_i, \tau_i = k)$ as $s_k(v_i)$. This strategy also depends on the group size I, value distribution $G(\cdot)$, and the threshold c; however, we suppress the argument I, G and

[4] In the simulation, we demonstrate that our method still works well if there is modest correlation of values.

c in $s_k(\cdot)$ to simplify the notation. We assume that the number of types K does not vary across time and each subject's type is private information. Each subject potentially adjusts her type over time based on the outcome of the game in previous periods. Instead of providing a model to rationalize why subjects change their contributing strategies, we summarize such adjustments by a transition matrix of type, $\Pr(\tau_{t'}|\tau_t, w_{-t})$. Given a vector of outcome history w_{-t}, $\Pr(\tau_{t'}|\tau_t, w_{-t})$ is a $K \times K$ matrix with its (i, j)-th element being the probability for a type j in period t that changes to type i in period t'.

Note that we did not explicitly model subjects' interaction and subjects' contributing strategies do not necessarily constitute a Nash equilibrium in our model. The type τ can be understood as a "reduced-form" description of subjects' belief, preference and behavior. For the possible non-equilibrium behavior, the heterogenous contributing strategies of subjects might be rationalized by various behavioral models, e.g., level-k thinking (Crawford and Iriberri, 2007) or cognitive hierarchy (Camerer et al., 2004). It will be interesting to investigate which behavioral model best describes subjects' behavior in private provision of public goods. However, we will leave that for future research.

The data report three values and the corresponding contributions for each subject and the outcomes for each period, then the joint distribution of b_1, b_2, b_3 can be directly identified. Our goal of identification is to uniquely determine the number of type K, the proportion and bidding strategy for each type, and the transition matrix $\Pr(\tau_{t'}|\tau_t, w_{-t})$. Let $F(\cdot)$ be the distribution of subjects' contributions and $F(\cdot|\tau = k), k = 1, 2, \cdots, K$ be the distribution for subjects of type k. Then the model provides a finite mixture of distributions for all the types:

$$F(b_1, b_2, b_3) = \sum_{k=1}^{K} F(b_1, b_2, b_3|\tau = k)p_k, \tag{4}$$

where p_k is the proportion of type k. To explore the dependence of model primitives on the relationship above, we consider a similar equation for one period,

$$F(b) = \sum_{k=1}^{K} F(b|\tau = k)p_k = \sum_{k=1}^{K} G\big(s^{-1}_{\tau=k}(b)\big)p_k, \tag{5}$$

where the second equation holds because $F(b|\tau = k) = \Pr(B \leq b|\tau = k) = \Pr(s_{\tau=k}(V) \leq b) = \Pr(V \leq s^{-1}_{\tau=k}(b)) = G\big(s^{-1}_{\tau=k}(b)\big)$. We have two observations from (5) regarding identification of the model. First, cross-sectional observations of subjects' values and contributions are insufficient for identification. The cross-sectional data allow us to recover a relationship between values and contributions, which is the combined contributing strategies for all the types. Without prior information about the number of types as well as proportion and functional form of the strategy for each type, it is impossible to back out the contributing strategy for each type. Second, we have a short panel with multiple observations for each subject and the subject's identity; however, identification still requires a novel method. With a panel data of values and contributions being observed, a possible approach is to apply the method in Athey and Haile (2002) to recover a subject's contributing strategy using her multiple values and contributions under the assumption that her type is invariant. Such an approach requires a lot of observations for each subject, i.e., a long panel, which is unlikely to be satisfied in both experimental and field data.

We apply the recent development in the literature of measurement error, namely Hu (2008) to identify the model based upon (4) and (5). It is worth noting that our methodology of identification only requires researchers to observe three contributions for each subject, and the distribution of induced values but not individuals' values. This allows us to accommodate more flexible data structures, e.g., in many field data individuals' values are unknown but researchers may have prior information about the distribution of values.

3.2 Identification

We consider the case where M groups of subjects sequentially participate in T games of provision for the public good. The cost of the public good or the threshold is fixed for all the game. Similarly, the group size and the group members remain the same. As subjects' contributing strategies may depend on group size and cost, maintaining them fixed allows us to control for their effects when we conduct our analysis. Suppose we observe an i.i.d. sample $\{b_{it}^m, w_t^m\}, i = 1, 2, \cdots, I; m = 1, 2, \cdots, M; t = 1, 2, \cdots, T$, where i, m and t indicate individual, group, and time period, respectively,[5] and we use $N \equiv M \cdot I$ to denote the sample size or the total number of individuals. We assume individuals' values are unknown to researchers but the distribution is known. For ease of notation, we suppress the superscript m and subscript i. As will be shown, three periods of data ($T = 3$) are sufficient for identification, hence the sample is denoted as $\{b_1, w_1; b_2, w_2; b_3; w_3\}$. As discussed previously, subject i's type in period t is denoted as $\tau_{it} \in \{1, 2, ..., K\}$, where K is unknown, and the type may evolve across periods. The objectives of interest are: (1) number of type, (2) contributing strategy for each of the type, (3) the proportion of each type in the first period and (4) the transition matrix of type across period, or the "learning rules".

We start our identification strategy from a joint distribution of subjects' contributions and the provision outcome, b_1, b_2 and b_3 and w_2. By the law of total probability, we have

$$\begin{aligned} f_{b_3,w_2,b_2,b_1} &= \sum_{\tau_3} \sum_{\tau_2} f_{b_3,\tau_3,w_2,b_2,\tau_2,b_1} \\ &= \sum_{\tau_3} \sum_{\tau_2} f_{b_3|\tau_3,w_2,b_2,\tau_2,b_1} f_{\tau_3|w_2,b_2,\tau_2,b_1} f_{w_2|b_2,\tau_2,b_1} f_{b_2|\tau_2,b_1} f_{\tau_2,b_1}, \end{aligned} \tag{6}$$

where f_{R_1,R_2} and $f_{R_1|R_2}$ denote the joint and conditional densities R_1 and R_2 respectively. For simplicity of exposition, we still use the notation of $f(\cdot)$ when R_1 and/or R_2 are discrete whenever

[5]In our experiment $M = 12, I = 5, T = 10$.

there is no ambiguity. Let $\Omega_{-t} \equiv \{(w_s, b_s, \tau_s) \text{ for } s = 1, 2, ..., t-1\}$ be a set of information available for subjects prior to period t, where τ_s contains all the subjects' types from period 1 to $t-1$. Our first assumption specifies the dependence of subjects' contributions on the information set Ω_{-t}.

Assumption 1. *A subject's contribution in each period is only determined by her induced value and her current type, i.e.,* $b_{it} = s_{it}(v_{it}, \tau_{it})$*, which is* $s_k(v_{it})$ *for* $\tau_{it} = k$*,* $k \in \{1, 2, \cdots, K\}$*.*

This assumption excludes the dependence of the current contribution on the preceding information Ω_{-t}. It states all the information available to a subject is absorbed into her current type. That is, a subject sufficiently utilizes the history of outcomes, her contributions and strategies to determine the strategy at the current period, which implies that the type is a "sufficient statistic" of the information set Ω_{-t}. This assumption simplifies the conditional density $f_{b_3|\tau_3,w_2,b_2,\tau_2,b_1}$ as $f_{b_3|\tau_3}$ and $f_{b_2|\tau_2,b_1}$ as $f_{b_2|\tau_2}$. Accordingly, (6) can be rewritten as

$$f_{b_3,w_2,b_2,b_1} = \sum_{\tau_3} \sum_{\tau_2} f_{b_3|\tau_3} f_{\tau_3|w_2,b_2,\tau_2,b_1} f_{w_2|b_2,\tau_2,b_1} f_{b_2|\tau_2} f_{\tau_2,b_1}. \tag{7}$$

In the abovementioned equation, $f_{w_2|b_2,\tau_2,b_1}$ is the probability that the public good is provided successfully in period 2 for $w_2 = 1$. Recall that $w_2 = 1$ only if the summation of all the contributions in this period exceeds the cost, hence the probability $f_{w_2|b_2,\tau_2,b_1}$ is independent of any additional information if b_2 is given, i.e., $f_{w_2|b_2,\tau_2,b_1} = f_{w_2|b_2}$. The conditional probability $f_{\tau_3|w_2,b_2,\tau_2,b_1}$ captures the transition process of subjects' type from period $t = 2$ to $t = 3$. Similar to Assumption 1, we impose some restrictions on how subjects' type evolves.

Assumption 2. *The contributing strategy in the next period for a subject only depends on the outcome of provision and her contributing strategy in the current period.*

Under this assumption, the transition of types $\Pr(\tau_{t+1}|w_t, b_t, \tau_t, \Omega_{-t})$ can be simplified to $\Pr(\tau_{t+1}|w_t, \tau_t)$. The restriction imposed by this assumption is twofold: first, the history Ω_{-t}, especially outcomes before period t play no role in subjects' learning rule given the current period's information. We do not rule out the possibility that subjects consider the information Ω_{-t}, however, it's irrelevant under Assumption 1 since the current type τ_t absorbs the history Ω_{-t}. This leaves us with the transition probability being $\Pr(\tau_{t+1}|w_t, b_t, \tau_t)$. This part of assumption is also supported by the reduced-form evidence in the proceeding section, where w_{t-2} has little impact on b_{it} after controlling w_{t-1}. Second, a subject's contribution in the preceding period has no impact on her strategy for this period given the previous outcome and her previous strategy. This restriction is a natural consequence of the independence of subjects' values across periods: since values are independent, a subject can only learn from the provision outcome and her type in the last period. Intuitively, the contribution b_t contains no additional information other than τ_t for subjects with independent values across period. Nevertheless, it is worth noting that the independence of type τ_{t+1} and the information set Ω_{-t} is an assumption of first-order Markov process, which is widely used in the literature, and it can be relaxed when more periods of data are available for each subject.

Under Assumption 2 we further simplify (7) as

$$f_{b_3,b_1|w_2,b_2} f_{b_2} = \sum_{\tau_2} f_{b_3|w_2,\tau_2} f_{b_2|\tau_2} f_{\tau_2,b_1}. \tag{8}$$

Integrating out b_2 on both sides of the equation above, we obtain

$$\int f_{b_3,b_1|w_2,b_2}(\cdot,\cdot|\cdot,u) f_{b_2}(u) du = \sum_{\tau_2} f_{b_3|w_2,\tau_2} f_{\tau_2,b_1}. \tag{9}$$

The two equations above provide a structural link between directly observed objectives on the L.H.S. and unknowns on the R.H.S. Following Hu (2008), we adopt a matrix form of equations (8) and (9) for the purpose of identification. Specifically, we discretize the contributions b_1 and b_3, which are both continuous variables, as L values and denote the discretized contributions as d_1 and d_3, respectively.[6]

For a given outcome $w_2 \in \{0,1\}$, and discretized contributions d_1 and d_3, we define the following matrices:

$$\begin{aligned}
A_{ij} &\equiv \Pr(d_3 = i, d_1 = j|w_2, b_2) f_{b_2}, \\
E_{ij} &\equiv \int \Pr(d_3 = i, d_1 = j|w_2, b_2) f_{b_2} db_2, \\
(B_{d_3|w_2,\tau_2})_{i,k} &\equiv [\Pr(d_3 = i|w_2, \tau_2 = k)]_{ik}, \\
(C_{\tau_2,d_1})_{k,j} &\equiv [\Pr(\tau_2 = k, d_1 = j)]_{kj}, \\
D_{b_2|\tau_2} &\equiv \text{diag}[f(b_2|\tau_2 = 1) \;\; f(b_2|\tau_2 = 2) \;\; \cdots \;\; f(b_2|\tau_2 = K)].
\end{aligned} \tag{10}$$

All the matrices are pointwise in b_2, where A and E are of dimension $L \times L$, B, C and D are of dimension $L \times K$, $K \times L$ and $K \times K$, respectively, where the number of types K is still unknown. Similar to their continuous counter-parts, the matrices defined above describe the distributions of observed and unobserved variables. For example, the (i,k)-th element in $B_{d_3|w_2=0,\tau_2}$ is the

[6] The discrete contribution d_t is determined by the following method of discretization.

$$d_t = \begin{cases} 1 & \text{if } b_t \in [\underline{b}, b_t(1)], \\ 2 & \text{if } b_t \in (b_t(1), b_t(2)], \\ \dots & \\ L & \text{if } b_t \in (b_t(L-1), \bar{b}], \end{cases}$$

where the support of contribution, $[\underline{b}, \bar{b}]$ is divided into L segments by the $L-1$ cutoff points $b(1), b(2), ..., b(L-1)$, $\underline{b} < b(1) < b(2) < ... < b(L-1) < \bar{b}$, and $d_t \in \{1, 2, ..., L\} (L \geq 2)$ is the discretized contribution. Both d_1 and d_3 take values from $\{1, 2, ..., L\}$, however, the cutoff points for discretizing b_1 and b_3 can be different. Then $\Pr(d_t = l) \equiv \int_{b_t(l-1)}^{b_t(l)} f_{b_t}(u) du$.

probability that the discretized contributions of the third period for those subjects who are of type k is in the i-th segment given the second period's outcome is "not provided". The k-th element of $D_{b_2|\tau_2}$ is the density $f_{b_2|\tau_2}$ for the type $\tau_2 = k$ evaluated at b_2.

The matrices defined above allow us to express (8) and (9) in a matrix form as follows:

$$\begin{aligned} A &\equiv B_{d_3|w_2,\tau_2} D_{b_2|\tau_2} C_{\tau_2,d_1}, \\ E &\equiv B_{d_3|w_2,\tau_2} C_{\tau_2,d_1}. \end{aligned} \tag{11}$$

For a given value of w_2, the matrix $E = B_{d_3|w_2,\tau_2} C_{\tau_2,d_1}$ describes the joint distribution of two discretized contributions d_1 and d_3. As argued in An (2010), the rank of this matrix can be used to identify the number of types under two conditions: first, the support of τ_t does not change along with t; second, contribution distribution of any type is not a linear combination of those for other types. We employ this insight here and make the following assumption.

Assumption 3. *The inverse contributing functions $s_k^{-1}(\cdot)$ for $k = 1, 2, \cdots, K$ are linearly independent. Formally, there does not exist some $c_k \in \mathbb{R}, k = 1, 2, \cdots, K$ not all zero such that $\sum_{k=1}^K c_k s_k^{-1}(b) = 0$ for all $b \in [0, \bar{v}]$.*

The restrictions imposed by this assumption on the inverse contributing strategies $s_k^{-1}(\cdot)$ can be described as a nonzero Wronskian if $s_k^{-1}(\cdot)$ has $(K-1)$-th continuous derivatives.[7] Recall that the distribution of contributions for type k, $F(b|\tau = k)$ is equal to $G\big(s_k^{-1}(b)\big)$, which can be further simplified as $s_k^{-1}(b)/(\bar{v} - \underline{v})$ because the induced values are uniformly distributed in our experiment. Thus Assumption 3 implies that the distributions of contributions for different types are linearly independent. We require the linear independence holds regardless the conditioning on the outcome. The unconditional linear independence implies that the row rank of C_{τ_2,d_1} is equal to K, the number of types. Similarly, the linear independence conditional on the outcome w_2 guarantees that the column rank of $B_{d_3|w_2,\tau_2}$ for any $w_2 \in \{0, 1\}$ is also K. The essential restriction of this assumption is that there are enough variations of contributing strategies across type. Recall that the values of subjects who are of different types are i.i.d. It is unlikely that two different mappings from values to contributions (contributing strategies) lead to linearly dependent distributions of contributions. Similar assumptions of full rank have been widely imposed to identify structural models in econometrics. For example, in Newey and Powell (2003) and Chernozhukov et al. (2007) the full rank condition is essential for the identification of nonparametric instrumental variable models.

Lemma 1. *Under Assumptions 1-3, the number of types $K = rank(E)$.*

[7]See e.g., chapter 2 in Shilov (2013) for details.

The assumption of invertibility implies $E^{-1} = C^{-1}_{\tau_2,d_1} B^{-1}_{d_3|w_2,\tau_2}$. Combining (11) with the relationship above, we obtain

$$A \times E^{-1} = B_{d_3|w_2,\tau_2} D_{d_2|\tau_2} B^{-1}_{d_3|w_2,\tau_2}, \tag{12}$$

where $D_{b_2|\tau_2}$ and $B_{b_3|w_2,\tau_2}$ are matrices of eigenvalues and eigenvectors, respectively for the observed matrix $A \times E^{-1}$. Especially, each of the diagonal element of $D_{b_2|\tau_2=k}, k \in \{1, 2, ..., K\}$ is the density of contributions for subjects of type k evaluated at b_2. Employing the strategies of identification proposed in Hu (2008), if the matrix decomposition in (12) is unique, then both $B_{d_3|w_2,\tau_2}$ and $D_{b_2|\tau_2}$ are identified since the L.H.S of the equation can be recovered from data.

To achieve the uniqueness of the decomposition, it is necessary to normalize the eigenvector matrix $B_{d_3|w_2,\tau_2}$ and make the eigenvector unique for each given eigenvalue. Considering that for a given outcome $w_2 \in \{0, 1\}$, each element in the eigenvector matrix $B_{d_3|w_2,\tau_2}$ is a conditional probability, hence each column of the matrix sums up to one, i.e., $\sum_{d_3} B_{d_3|w_2,\tau_2} = 1$. Then a plausible method of normalization is to divide each column by the corresponding column sum. To achieve the uniqueness of eigenvector for each eigenvalue, it is necessary for the eigenvalues to be distinctive, which is guaranteed by the following lemma.

Lemma 2. *If subjects' values are uniformly distributed, then the distributions of contributions for any two different types of subjects are distinct, i.e., for any two different types $k, j \in \{1, 2, \cdots, K\}$, the density $f_{b|\tau}(b|\tau = k)$ is different from $f_{b|\tau}(b|\tau = j)$.*[8]

Proof. See Appendix. ∎

The result in lemma 2 is testable from (12) because once we obtain all the eigenvalues for each contribution b_2, it is straightforward to verify whether the result is violated, i.e., whether there exist at least two types whose distributions of contributions are always the same for any b_2.

In our practice, as in most of the experiments, the distribution of subjects' values is known to the researcher. Combining this distribution with the identified conditional density $f(b_2|\tau_2)$ allows us to recover the contributing strategies for $\tau_2 = 1, 2, \cdots, K$, i.e.,

$$s_k^{-1}(b) = (\overline{v} - \underline{v}) F_{B|\tau}(b|\tau = k) + \underline{v}, \ k = 1, 2, \cdots, K.$$

Assumption 4. *The inverse contributing strategies $s_k^{-1}(b), k = 1, 2, \cdots, K$ can be strictly ordered at either a known quantile of $b \in [0, \bar{v}]$ or the mean.*

This assumption states that at some known quantiles, the contributing strategies of K types can be strictly ordered. For example, let $b_{0.5}$ be the median of the contribution b, then a possible

[8]To express it rigorously: for any two different types $k, j \in \{1, 2, \cdots, K\}$, the set $\{b : f_{b|\tau}(b|\tau = k) \neq f_{b|\tau}(b|\tau = j)\}$ has nonzero Lebesgue measure.

condition to order $s_k^{-1}(\cdot)$ is $s_1^{-1}(b_{0.5}) > s_2^{-1}(b_{0.5}) > \cdots > s_K^{-1}(b_{0.5})$, which implies subjects of type 1 would have the largest value to contribute $b_{0.5}$ and type K have the smallest value, i.e., type 1 is the least generous type. The restriction of this assumption is flexible and in the following identification we assume that the average contributions for different types can be ordered. Recall that $s_k^{-1}(b)/(\bar{v} - \underline{v}) = F(b|\tau = k)$, Assumption 4 implies that we can distinguish different types according to their average contribution. Without loss of generality, we always label types in an ascending order according to expected contribution, i.e., on average type 1 contributes the least while type K contributes most generously. The approach to label the types is consistent with the findings in the literature of public good. For example, in Fischbacher and Gächter (2010) the three types free riders, learners and contributors are classified according to how much they contribute. By imposing assumption 4, the ordering of eigenvalues (eigenvectors) is fixed and the eigenvector matrix $B_{d_3|w_2,\tau_2}$ is uniquely determined from the eigenvalue-eigenvector decomposition of the observed matrix $A \times E^{-1}$. The ordering of eigenvalues may be achieved by imposing alternative restrictions. More generally, the distribution of contributions for different types $f_{b|\tau}$ can be ordered if there exists a functional $\varpi(\cdot)$ such that $\varpi(f(b|\tau))$ is strictly increasing or decreasing in τ.

For each period, the observed distribution of contributions is a weighted average of distributions for all the possible types, i.e.,

$$f(b) = \sum_{\tau} f(b|\tau) \Pr(\tau). \tag{13}$$

This relationship allows us to identify the proportion of each type $\Pr(\tau_2)$ in period 2 once the distribution for each type $f(b_2|\tau_2)$ is identified from the eigenvalue-eigenvector decomposition. In summary, all the important components of the model are identified from (12) and the results are summered as follows.

Proposition 1. *Under Assumptions 1-4, the distribution of contributions in period 3 conditioning on the outcome and type in the last period* $\Pr(d_3|w_2,\tau_2)$*, the distribution of contributions* ($f_{b_2|\tau_2}$) *and the proportion for each type* ($\Pr(\tau_2)$) *in period 2 are uniquely determined by the joint distribution of outcome in period 2 and contributions in three periods,* f_{b_3,w_2,b_2,b_1}*. Furthermore, if the distribution of values is known, the contributing strategy of each type* $s_k(\cdot)$ *is also identified.*

Based on the results of identification in proposition 1, we show next that the two learning rules $\Pr(\tau_3|w_2,\tau_2)$ and $\Pr(\tau_2|w_1,\tau_1)$ are identified, too. First of all, the identified distribution of period 3 conditional on the outcome and type in period 2, $\Pr(d_3|w_2,\tau_2)$ is associated with the learning rule $\Pr(\tau_3|w_2,\tau_2)$ as

$$\begin{aligned}\Pr(d_3|w_2,\tau_2) &= \sum_{\tau_3} \Pr(d_3|\tau_3,w_2,\tau_2)\Pr(\tau_3|w_2,\tau_2) \\ &= \sum_{\tau_3} \Pr(d_3|\tau_3)\Pr(\tau_3|w_2,\tau_2). \end{aligned} \tag{14}$$

It is necessary to utilize an important implication of our model: the distribution of subjects' contributions for a certain type is invariant across periods, i.e., $f_{b_3|\tau_3} = f_{b_2|\tau_2} = f_{b_1|\tau_1}$. This conclusion is due to fact that the provision game is homogeneous and subjects' values are i.i.d. in each period, therefore, the distribution of contributions must remain the same for each type in different periods. Using this property, $\Pr(d_3|\tau_3)$ can be obtained from the identified conditional density $f_{b_3|\tau_3} = f_{b_2|\tau_2}$, and the learning rule $\Pr(\tau_3|w_2, \tau_2)$ is identified from (14). We exemplify the procedure by assuming subjects are of two types, and correspondingly the discretized contribution d_3 takes two values. Then the abovementioned equation can be expressed in a matrix form:

$$
\begin{bmatrix} \Pr(d_3 = 1|w_2, \tau_2 = 1) & \Pr(d_3 = 1|w_2, \tau_2 = 2) \\ \Pr(d_3 = 2|w_2, \tau_2 = 1) & \Pr(d_3 = 2|w_2, \tau_2 = 2) \\ \end{bmatrix} =
$$

$$
\begin{bmatrix} \Pr(d_3 = 1|\tau_3 = 1) & \Pr(d_3 = 1|\tau_3 = 2) \\ \Pr(d_3 = 2|\tau_3 = 1) & \Pr(d_3 = 2|\tau_3 = 2) \\ \end{bmatrix} \times \begin{bmatrix} \Pr(\tau_3 = 1|w_2, \tau_2 = 1) & \Pr(\tau_3 = 1|w_2, \tau_2 = 2) \\ \Pr(\tau_3 = 2|w_2, \tau_2 = 1) & \Pr(\tau_3 = 2|w_2, \tau_2 = 2) \\ \end{bmatrix}, \tag{15}
$$

where $w_2 \in \{0, 1\}$. This is a linear system and the learning rule $\Pr(\tau_3|w_2, \tau_2)$ can be uniquely solved from it only if the first matrix on the R.H.S. is full rank, which is guaranteed under Assumption 3. A similar argument can be applied to identify the learning rule of subjects from the first to the second period $\Pr(\tau_2|w_1, \tau_1)$. Alternatively, we might identify the learning rule as follows. Considering the observed joint density of contribution b_2, b_1 and the outcome w_1, f_{b_2,w_1,b_1}, we employ the law of total probability to obtain

$$
\begin{aligned}
f_{b_2,w_1,b_1} &= \sum_{\tau_2}\sum_{\tau_1} f_{b_2,\tau_2,w_1,b_1,\tau_1} \\
&= \sum_{\tau_2}\sum_{\tau_1} f_{b_2|\tau_2,w_1,b_1,\tau_1} f_{\tau_2|w_1,b_1,\tau_1} f_{w_1|b_1,\tau_1} f_{b_1|\tau_1} f_{\tau_1} \\
&= \sum_{\tau_2}\sum_{\tau_1} f_{b_2|\tau_2} \Pr(\tau_2|w_1, \tau_1) f_{w_1|b_1} f_{b_1|\tau_1} f_{\tau_1}, \qquad (16)
\end{aligned}
$$

where the first two equalities hold without any assumption and the third equality is due to Assumptions 1 and 2. In the equation above, the L.H.S. as well as $f_{w_1|b_1}$ are directly observed from the data. The distribution for each type $f_{b_2|\tau_2} = f_{b_1|\tau_1}$ and f_{τ_1} are identified using Proposition 1.

Proposition 2. *Under Assumptions 1-4, the learning rules regarding how subjects adjust their contributing strategies $f_{\tau_2|w_1,\tau_1}$ and $f_{\tau_3|w_2,\tau_2}$ are uniquely determined by the joint distribution of outcomes and contributions in three periods, f_{b_3,w_2,b_2,w_1,b_1}.*

The results of identification in Propositions 1 and 2 are constructive and they suggest a convenient multi-step procedure for estimation. We discuss the procedure briefly and leave the technical details of estimation in Appendix B. The first step of estimation is to determine the number of types by testing the rank of the matrix E. Next, by the eigenvalue-eigenvector decomposition in (12), we obtain the eigenvector matrix as well as the conditional distribution of contributions for each type in the second period, where the L.H.S. of (12) is estimated nonparametrically by kernel estimation. Consequently, the corresponding probability of each type can be estimated from (13). Lastly, based on (16) the learning rules are estimated by maximum likelihood estimation (MLE) since the learning rule only contains K^2 parameters, where K is the number of types.

4 Monte Carlo Experiments

In this section, we present some Monte Carlo evidence to demonstrate the performance of estimator. We consider a game of public good provision similar to the experimental setting in Section 2. The game is played by groups with size $m = 5$ for three periods ($T = 3$). Values V_{it} are drawn from a standard uniform distribution and independent across individuals and over the three periods. The cost of the public good is set to be $c = 0.6 \times E[V_{it}] \times m = 1.5$. Individuals are of three types with their contributing strategies respectively being as follows:[9]

$$s_1(v) = \sqrt{v+1} - 1, \;\; s_2(v) = \frac{2v}{3}, \;\; s_3(v) = \Phi^{-1}\big((\Phi(1) - \Phi(0))v + \Phi(0)\big), \tag{17}$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard norm distribution. Notice that all the three strategies are strictly increasing in value on the support $[0, 1]$.

Starting from period $t = 1$, we randomly draw N values from a standard uniform distribution $U[0, 1]$, then assign one of the three types to the N individuals according to the probability $\Pr(\tau_1 = 1) = 0.4, \Pr(\tau_1 = 2) = 0.3$ and $\Pr(\tau_1 = 3) = 0.3$. After we simulate the contributions for all the individuals based on their values and the contributing strategies in (17), the indicator of outcome w_1 is generated as $w_1 = 1(\sum_{m=1}^{5} b_{1m} \geq c = 1.5)$. Conditioning on w_1 and individuals' type τ_1 in period $t = 1$, we simulate their type τ_2 in period $t = 2$ according to the following transition matrix of types:

$$f(\tau'|\tau, w = 1) = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.6 & 0.4 \\ 0.3 & 0.1 & 0.4 \end{bmatrix}, f(\tau'|\tau, w = 0) = \begin{bmatrix} 0.8 & 0.1 & 0.2 \\ 0.1 & 0.7 & 0.6 \\ 0.1 & 0.2 & 0.2 \end{bmatrix},$$

where τ' indicates the type in the next period. For simplicity, it is assumed that the two transition matrices are invariant across periods. For example, if an individual was type 1 in a

[9] For simplicity we assume away the dependence of contributing strategies on group size. Nevertheless, the estimation still relies on group size because the winning indicator w is determined by contributions and the cost c, which is a linear function of group size m.

certain period, and her group successfully provides the public good, then she will be type 1, 2, and 3 with probabilities $0.5, 0.3$ and 0.2, respectively in the next period. By applying this procedure repeatedly, we simulate a sample of contributions and outcomes $\{b_{i1}, w_1, b_{i2}, w_2, b_{i3}\}, i = 1, 2, \cdots, N$ for 1000 replications.

We first estimate the number of types through the rank of E defined previously. Instead of conducting a rigorous test of the rank (e.g., Robin and Smith (2000)), we provide some statistics of the condition number and determinant for the matrix E under the hypotheses of different number of types.[10] The condition number is a measure of how close a matrix is singular: a matrix with large condition number is nearly singular, whereas a matrix with condition number close to 1 is far from being singular. In the simulation, we discretize $b_{it}, t = 1, 3$ into 2-6 segments, and compute the condition number and the determinant of the matrix E for each segment. Tables 4 and 5 present the results for $w = 0, N = 500$ and $w = 1, N = 1000$, respectively and the results for $w = 1, N = 500$ and $w = 0, N = 1000$ are similar, hence omitted for abbrevity.[11] As the results show, both the condition number and the determinant jump between 3 and 4 at different quantiles. For instance, the median of condition number for $N = 500$ jumps more than three-fold from 53-167.45 and a similar pattern is also observed for the case with $N = 1000$. The pattern of determinants is consistent with the condition number, and this offers some statistical confirmation for the rank of E, i.e., the number of types being three.

[Tables 4 and 5 are about here]

The estimates of contributing strategies for three types, together with the corresponding $[10\%, 90\%]$ point-wise confidence intervals are illustrated in Figures 2. The estimates perform well for modest sample-size datasets of $N = 500, 1000$. For the three types, the estimated contributing strategies track the actual ones very closely. Notice that for types 1 and 2 the estimated contribution is larger than the true one when the value is close to its upper bound. This is because the estimate of contribution distribution is less accurate when contribution is close to the upper bound due to the sparse observations.

[Figures 2 is about here]

[10]Condition number of a matrix A is defined as $||A|| \cdot ||A||^{-1}$, where $|| \cdot ||$ is a matrix norm. We adopt the Euclidean norm, i.e., $||A||_2$, which is defined as the largest eigenvalue of the matrix $A'A$.

[11]The presented results are obtained by discretizing the subjects' contributions equally on the support. A different approach of discretization might change the reported numbers but the pattern that both condition number and determinant jump from 3 to 4 does not change with discretizations.

We provide the estimation of the initial type probabilities, i.e., at period $t = 1$ in Table 6. The probabilities are accurately estimated for both sample size. Tables 7 and 8 present the estimated transition matrices of types. Notice that the estimate of $f(\tau'|\tau, w = 1)$ performs very well while that of $f(\tau'|\tau, w = 0)$ is a bit noisy, and this is because the observations for $w = 0$ is smaller than $w = 1$ due to the setting of our transition matrix of type. Nevertheless, both estimated matrices are accurate enough to capture the transition pattern of the type.

[Tables 6, 7 and 8 are about here]

In summary, the Monte Carlo evidence illustrates that our procedure of estimation performs well for modest-sized samples.

Robustness check. In the data generating process of the simulation as well as in our experiment, it is assumed that individuals' induced values are independent across periods. Such an assumption might be violated for some field data and values may be correlated. As a robustness check, we allow values to be correlated across three periods and then estimate the model by the proposed method assuming independence of the values.[12] Figure 3 and Table 9 present the estimated contributing strategies and the transition matrices, respectively for sample size 500 when the values of two consecutive periods are correlated with a coefficient 0.2. The estimate of strategies are very close to that in Figure 2. Similarly, the probabilities of transition in Table 9 also closely track the corresponding elements in the true transition matrices. A comparison of the estimated results in Tables 7 and 9 illustrates that the correlation of values negatively affect the accuracy of estimates. For example, the probability $\Pr(\tau_2 = 1|w = 1, \tau_1 = 1)$ is estimated to be 0.494 in Table 7, which is close to the true value 0.50, while the estimate is 0.590 in Table 9. Nevertheless, with the modest correlation of values, our proposed method based on independence of values still performs well in estimating the model.

[12]To generate the uniformly distributed values with Pearson correlation, we first generate normally distributed draws with Spearman correlations then apply the uniform transformation to those random draws. Please see Embrechts et al. (2003) for details.

5 Empirical Results

In this section we apply our methodology to the experimental data described in Section 2 and provide some new evidence on the private provision of public goods. As discussed in Section 3.2, three periods of data are sufficient for our analysis. Thus each of the three consecutive periods of the 10 periods in our sample can be employed for empirical analysis.[13] To maximize the number of observations and explore the possibility that subjects may change their learning rules across periods, we consider the following three approaches to aggregate the observations: (a) pool all the 10 periods' data. By applying this approach, we are estimating an average learning rule $\Pr(\tau'|w,\tau)$, which is treated invariant between any two periods. (b) use only the first five periods' data and (c) use the last five periods' data, where we assume that subjects' learning behavior is invariant in the first and the last five periods, respectively. The results of (a) are baseline and (b) and (c) are used for robustness check.

The first set of results are condition numbers and determinants of matrix E_{ij}, which are used to determine the number of types. Tables 10 presents the results when we pool the 10 periods' data together. The first and last five periods' of data both lead to very similar results and are hence omitted. The top panel of the table is conditional on the outcome that the public good is not provided, $w_2 = 0$ and the bottom panel is for the outcome $w_2 = 1$. Both panels reveal a clear pattern that the condition number and determinant jump when the number of discretization changes from 3 to 4, and this identifies the number of types to be 3.

[Table 10 is about here]

Next, the procedure of identification using matrix decomposition enables us to recover the conditional density $f_{b_2|\tau_2}$. It is used to obtain the contributing strategies for three types and the initial type probability through f_{b_1} and $f_{b_2|\tau_2} = f_{b_1|\tau_1}$.[14] The estimate of probability and contributing strategy for each type are provided in Table 11 and Figure 4, respectively. We label the three types such that type 3 contributes the most, whereas type 1 the least. The results in Table 11 indicate that proportion of each of the three types is significantly positive. The first two rows are both estimates for the first period; however using different sample of data they reveal a similar pattern: the proportion of type 3 is the smallest (about 20%), whereas type 2 is the largest (about 36%). By contrast, the type probability of period 5 (using the data of the last five periods) displays a different pattern: the proportion of type 1 is the smallest (about 20%)

[13] We use $\{1,2,3\},\{2,3,4\},\cdots\{8,9,10\}$. More generally, any sample of three periods t, $t+s$ and $t+2s$ ($s \geq 1$) can be used for estimation. The corresponding learning rule will be for s periods.

[14] We focus on the type probability in the first period because the probability in the next period is just the product of learning rule and the initial type probability.

whereas the other two types both have a proportion approximately 40%. The results imply that subjects tend to contribute less at the beginning (being types 1 and 2), probably because of less corporation. As they continue to participate in the game, more subjects learn to cooperate and contribute more.

[Table 11 and Figure 4 are about here.]

The three subplots in Figure 4 illustrate the contribution as a function of the value for subjects of three types. The contributing strategies for three types can be classified into two groups: type 1 and type 2-3, where the difference between type 2 and 3 is relative small in the cases of overall and last five periods. A formal test indicates that the contributing strategies of types 2 and 3 are not significantly different at the 5% significance level for those two cases, whereas type 1 significantly differs from both types 2 and 3.[15] A different pattern displays for the first five periods, where all the three types are tested to be significantly different at the 5% significance level. An implication of these results is that learning of subjects may lead to "convergence" of some contributing strategies. Nevertheless, some subjects may not conform to others even after 10 periods' learning. Particularly, subjects of type 1 contribute significantly lesser than the other two types in all three cases. When their values are near the lower bound 8, type 1 behave as free-riders and contribute nothing; whereas types 2 and 3 contribute up to 4. However, as the value increases type 1 may contribute a significant proportion of the value, e.g., for a value 14 the contribution is approximately 6, which is 43% of the value. Furthermore, it is worth noting that the difference between type 1 and types 2& 3 is larger when the value is small and the discrepancy diminishes as the value increases. Especially, when value is greater than 18, which is near the upper bound (the upper bound of value is 20), the three types are very close to each other. A comparison between type 2 and 3 implies that in the last five periods subjects of type 1 contribute more generously than the first five periods when their value is greater than 16. These result reinforce our early findings that subjects tend to behave more generous as they spend more time with other group members, and this may reflect subjects' "learn to corporate" process.

[Figure 4 is about here]

The last set of results are on the learning rule of different types and they are presented in Tables 12-14, where the two subtables (a) and (b) of each table are transition matrices of types conditional on two outcome. For every matrix, each column contains the probabilities that a

[15] According to Lemma 2, testing the difference between two bidding strategies is equivalent to that of two distributions of contributions. Therefore, we conduct Kolmogorov-Smirnov tests on the distributions of contributions for any of the two types.

certain type is being adjusted to three types in the next period, hence the column sum is one. For example, in Table 12 (a) the second column implies that if the public good is provided successfully, then the subjects of type 2 in the current period would continue to be type 2 with probability 70.1% and adjust to be type 1 (contributes less) with probability 29.9% in the next period. However, they never transit to type 3 and contribute more.

Across all the three tables, subtables (a) are diagonally dominant, whereas (b) are not. Such a difference reveals that subjects' contributing strategies are negatively affected by the outcome, i.e., they will maintain their strategies or change to a less generous one if the good is provided successfully but contribute more generously or at least the same for a not successful outcome. In Table 12 (a) almost all the type 1, 70.1% of types 2 and 75.6% of type 3 would keep their own type in the next period conditioning on a successful outcome. Moreover, the remaining proportion of types 2 and 3 would change to type 1 and contribute less generously in the next period. By contrast, if the good is not provided in the current period, types 1 and 2 are more likely to be higher types and contribute more in the next period.[16] Table 14 (b) provides a typical illustration: more than half of type 1 and all of type 2 adjust to types 2 and 3, respectively conditional on a unsuccessful outcome.

Another important observation is that the estimates in Tables 13 and 14 indicate different patterns of subjects' learning. Subjects in the first five periods are more reluctant to adjust their contributing strategies than in the last five periods. For instance, conditioning on an unsuccessful provision, with probability 60.5% of type 1 moves to type 2 in the last five periods. By contrast, this probability is only 34.6% for the first five periods. In response to a successful provision, almost all the subjects of type 1 remain as type 1 in the first five periods, whereas this probability is 62.2% in the last five periods. The difference between the first and the last five periods suggest that the learning can also be dynamic, which is out of the primary focus of our paper but provides an interesting venue for future research.

[Tables 12-14 are about here.]

6 Conclusion

We study the identification and estimation of a structural model for private provision of public goods with heterogenous participants. The main motivation of the model is the need

[16] This is not true for type 1 in Table 12 (b). Nevertheless, a significant proportion of type 1 (25.4%) changes to type 2.

to explain individuals' heterogenous contributing behavior and possible adjustments of their strategies based on provision history (learning). The heterogeneity and the learning of individuals have been documented in previous experimental studies and also confirmed by our reduced-form analysis on our experimental data of threshold public good games. Our structural model allows for individuals to employ heterogenous contributing strategies, which can be adjusted upon observing the outcome of the provision. A prominent advantage of our approach over the existing studies is that from the revealed contributions of individuals, we are able to recover the number of different strategies, function form of each strategy and the transition probability among the strategies without imposing any parametric assumptions on these objectives.

The structural estimates of our experimental data suggest that subjects can be classified into three types who employ three different contributing strategies to make contributions. A subject of type 1 contributes a much smaller share of her value than types 2 and 3 while the contributing strategies of the latter two types are similar. The estimates of learning indicate that subjects tend to keep their strategies in response to a successful provision in the last period. By contrast, they become more generous in contributing if the good is not provided in the last period. Nevertheless, the three types display different patterns of learning: type 1 makes relative smaller adjustment to their strategy than the other two types. By dividing the data into two time intervals, i.e., the first and last five periods, we find that subjects in the first five periods are more reluctant to adjust their contributing strategies than in the last five periods.

There are a few directions for future research. First, our methodology for threshold public good might be applied to another large category of experiments: linear public good provision, where the contributing strategy is a mapping from endowment to the ratio of contributions made to public good over their own account. Furthermore, we allow individuals change their learning behavior across 10 periods but leave out their forward-looking behavior. It will be interesting to incorporate dynamics into our model and explore deeper regarding individuals learning behaviors.

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Appendix

A Estimation

In this section, we propose a procedure to estimate the objectives that are identified nonparametrically in Section 3 . The procedure follows directly from the argument of identification, and a similar approach is also applied in An et al. (2010). We estimate all the objectives in multiple steps.

Step one: Estimation of the conditional distribution $f(b_2|\tau_2)$. Recall our identification is mainly based on equation (12), which holds for all b_2. To improve the performance of our estimator, we take integral of this equation with respect to b_2 and use the aggregated version for estimation:

$$\int_{b_2} b_2 A \times E^{-1} db_2 = B_{b_3|w_2,\tau_2} D_{Eb_2|\tau_2} B^{-1}_{b_3|w_2,\tau_2} Eb_2, \tag{A.1}$$

where $D_{Eb_2|\tau_2} \equiv \int_{b_2} b_2 D_{b_2|\tau_2} db_2$. The L.H.S. of the equation above is estimable from data, then both $B_{b_3|w_2,\tau_2}$ and $D_{Eb_2|\tau_2}$ can be estimated by the eigenvalue-eigenvector decomposition described in (12). The details can be found in An et al. (2010) and An (2010), and thus omitted here. Let $\widehat{B}_{b_3|w_2,\tau_2}$ be the estimated eigenvector matrix, we estimate the conditional density $f(b_2|\tau_2)$ from the joint density $f(b_2, \tau_2)$ and the probability distribution $\Pr(\tau_2)$. First we consider the relationship

$$f(b_2, \tau_2) = \sum_{w_2 \in \{0,1\}} f(b_2, \tau_2|w_2) \Pr(w_2),$$

where $\Pr(w_2)$ can be directly recovered from data and the joint distribution of b_2, τ_2 conditional on the outcome w_2, $f(b_2, \tau_2|w_2)$ is determined by the following equation:

$$f(b_2, d_3|w_2) \quad = \quad \sum_{\tau_2} f(b_2, d_3, \tau_2|w_2) = \sum_{\tau_2} f(d_3|w_2, b_2, \tau_2) f(b_2, \tau_2|w_2) = \sum_{\tau_2} f(d_3|w_2, \tau_2) f(b_2, \tau_2|w_2).$$

The L.H.S. of the equation above is estimable from data, and $f(d_3|w_2, b_2, \tau_2)$ is obtained from the eigenvalue-eigenvector decomposition. Thus, we get an estimator of $f(b_2, d_3|w_2)$. We exemplify the estimation for $w_2 = 0$:

$$f(b_2, d_3|0) = B_{d_3|0,\tau_2} f(b_2, \tau_2|0) \Rightarrow \widehat{f}(b_2, \tau_2|0) = \widehat{B}^{-1}_{d_3|0,\tau_2} \widehat{f}(b_2, d_3|0),$$

where $\widehat{B}_{d_3|0,\tau_2}$ is invertible by construction, and $\widehat{f}(b_2, d_3|0)$ is a kernel estimator defined as:

$$\widehat{f}(b_2, d_3 = j|0) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{b_2 - b_{2i}}{h}\right) \mathbf{1}(b_{3i} = j).$$

Consequently we have the estimator of the joint distribution (b_2, τ_2),

$$\widehat{f}(b_2, \tau_2) = \widehat{f}(b_2, \tau_2|0)\widehat{\Pr}(w_2 = 0) + \widehat{f}(b_2, \tau_2|1)\widehat{\Pr}(w_2 = 1) \tag{A.2}$$

Similarly, the type distribution $\Pr(\tau_2)$ can be estimated from

$$\Pr(\tau_2) = \sum_{w_2 \in \{0,1\}} \Pr(\tau_2|w_2) \Pr(w_2),$$

where $\Pr(\tau_2|w_2)$ is associated with estimable $\Pr(d_3|w_2)$ and estimated $\Pr(d_3|\tau_2, w_2)$.

$$\Pr(d_3|w_2) = \sum_{\tau_2} \Pr(d_3, \tau_2|w_2) = \sum_{\tau_2} \frac{\Pr(d_3|\tau_2, w_2) \Pr(\tau_2, w_2)}{\Pr(w_2)} = \sum_{\tau_2} \Pr(d_3|\tau_2, w_2) \Pr(\tau_2|w_2).$$

We again illustrate our estimator for $w_2 = 0$. Let $\overrightarrow{\Pr}(d_3|0)$ denote a column vector with three elements $[\Pr(d_3 = 1|w_2 = 0) \Pr(d_3 = 2|w_2 = 0) \Pr(d_3 = 3|w_2 = 0)]^T$, and $\overrightarrow{\Pr}(\tau_2|0)$ is similarly defined. Then the last equation can be rewritten as

$$\overrightarrow{\Pr}(d_3|0) = B_{d_3|0,\tau_2} \overrightarrow{\Pr}(\tau_2|0),$$

which implies an estimator $\widehat{\overrightarrow{\Pr}}(\tau_2|0) = \widehat{B}^{-1}_{d_3|0,\tau_2} \widehat{\overrightarrow{\Pr}}(d_3|0)$. Then the type probabilities are estimated as

$$\widehat{\overrightarrow{\Pr}}(\tau_2) = \widehat{\overrightarrow{\Pr}}(\tau_2|0)\widehat{\Pr}(w_2 = 0) + \widehat{\overrightarrow{\Pr}}(\tau_2|1)\widehat{\Pr}(w_2 = 1).$$

Step two: Estimation of contributing strategies. In our paper, as in most of the experiments, the distribution of subjects' values is known to the researcher. Combining this distribution with the estimated conditional density $\widehat{f}(b_2|\tau_2)$ allows us to recover the contributing strategies for $\tau_2 = 1, 2, 3$. Let $F_{B|\tau}$ denote the conditional cdf of the observed contributions for a type τ, then lemma 2 states that

$$F_{B|\tau}(b|\tau = k) = \frac{s_k^{-1}(b) - \underline{v}}{\overline{v} - \underline{v}}, \ k = 1, 2, 3,$$

The relationship above implies that

$$s_k^{-1}(b) = (\overline{v} - \underline{v}) F_{B|\tau}(b|\tau = k) + \underline{v}, \ k = 1, 2, 3.$$

Then, our estimate of $s_k^{-1}(b)$ is

$$\widehat{s_k^{-1}}(b) = (\overline{v} - \underline{v}) \widehat{F}_{B|\tau}(b|\tau = k) + \underline{v}, \ k = 1, 2, 3. \tag{A.3}$$

Step three: Estimation of transition matrices of types. The learning rule $f_{\tau_2|w_1,\tau_1}$ is estimated from (16), which is repeated as follows.

$$f_{b_2,w_1,b_1} = \sum_{\tau_2} \sum_{\tau_1} f_{b_2|\tau_2} f_{\tau_2|w_1,\tau_1} f_{w_1|b_1} f_{b_1|\tau_1} f_{\tau_1}.$$

Based on the equation above, we maximize the likelihood function of the L.H.S. to estimate the learning rule on the R.H.S. Specifically, suppose we fix $w_1 = 1$ then the log likelihood function is expressed as:

$$\begin{aligned}
\log \ \mathcal{L} &= \sum_{i=1}^{N} \log \sum_{\tau_2=1}^{3} \left(f_{b_{2i}|\tau_2} \sum_{\tau_1=1}^{3} \Pr(\tau_2|w_1 = 1, \tau_1) f_{w=1|b_{1i}} f_{b_{1i}|\tau_1} \Pr(\tau_1) \right) \\
&= \sum_{i=1}^{N} \log \sum_{\tau_2=1}^{3} \left(f_{b_{2i}|\tau_2} \frac{f_{w=1,b_{1i}}}{f_{b_{1i}}} \sum_{\tau_1=1}^{3} \Pr(\tau_2|w_1 = 1, \tau_1) f_{b_{1i}|\tau_1} \Pr(\tau_1) \right) \\
&= \sum_{i=1}^{N} \log \sum_{\tau_2=1}^{3} \left(f_{b_{2i}|\tau_2} \frac{f_{b_{1i}|w_1=1} \Pr(w_1 = 1)}{f_{b_{1i}}} \sum_{\tau_1=1}^{3} \Pr(\tau_2|w_1 = 1, \tau_1) f_{b_{1i}|\tau_1} \Pr(\tau_1) \right).
\end{aligned} \tag{A.4}$$

Recall that the unknown transition matrix $\Pr(\tau_2|w_1 = 1, \tau_1)$ contains six independent parameters (denoted by θ). Given the estimated results in the proceeding steps, MLE of $\Pr(\tau_2|w_1 = 1, \tau_1; \theta)$ is

$$\widehat{\Pr}(\tau_2|w_1 = 1, \tau_1; \theta) \equiv \max_{\theta \in [0,1]^6} \log \mathcal{M}, \tag{A.5}$$

where $\log \mathcal{M}$ is the log-likelihood function $\log \mathcal{L}$ with all the terms but the transition matrix being replaced by their corresponding estimates. Especially, we employ the relationship $f_{b_1|\tau_1} = f_{b_2|\tau_2}$ and $f(b_1) = \sum_{\tau_1} f_{b_1|\tau_1} \Pr(\tau_1)$ in estimating $f_{b_1|\tau_1}$ and $\Pr(\tau_1)$, respectively.

Properties of the estimators can be proved by standard methods and we refer interested reader to An (2010) and An et al. (2010) for details.

B Tables and Figures

Table 1: Summary statistics (by period)

	Period									
Variable	1	2	3	4	5	6	7	8	9	10
Sample size	60	60	60	60	60	60	60	60	60	60
Provided	0.583	0.333	0.333	0.333	0.417	0.25	0.75	0.833	0.5	0.5
	(0.497)	(0.475)	(0.475)	(0.475)	(0.497)	(0.437)	(0.437)	(0.376)	(0.504)	(0.504)
Value	14.863	13.233	14.373	13.038	14.152	13.567	13.745	13.837	14.952	13.958
	(3.280)	(3.606)	(3.365)	(3.186)	(3.311)	(3.350)	(3.793)	(3.525)	(3.195)	(3.391)
Contribution	8.821	7.775	8.178	7.997	8.323	8.323	8.941	8.992	8.605	7.937
	(4.068)	(4.311)	(3.621)	(3.945)	(2.626)	(3.115)	(3.776)	(3.373)	(3.470)	(3.339)
Subj. Prob.	0.662	0.555	0.594	0.534	0.484	0.513	0.488	0.517	0.521	0.519
	(0.240)	(0.291)	(0.281)	(0.300)	(0.300)	(0.326)	(0.314)	(0.332)	(0.292)	(0.337)
Contr./Value	0.609	0.586	0.569	0.615	0.598	0.615	0.646	0.661	0.583	0.580
	(0.326)	(0.299)	(0.232)	(0.263)	(0.169)	(0.202)	(0.188)	(0.255)	(0.213)	(0.212)

Table 2: Reduced-form analysis: overall regression

	(1)	(2)	(3)	(4)
Constant	1.441***	1.332**	0.499	-1.495
	(0.542)	(0.593)	(0.603)	(1.553)
Value	0.497***	0.497***	0.462***	0.600***
	(0.0376)	(0.0377)	(0.0375)	(0.110)
Period		0.0205	0.0483	0.202
		(0.0450)	(0.0444)	(0.200)
Belief			2.185***	4.716**
			(0.426)	(1.880)
Value * Period				-0.00953
				(0.0132)
Value * Belief				-0.172
				(0.125)
Period * Belief				-0.0321
				(0.151)
N	600	600	600	600
R^2	0.226	0.226	0.259	0.262
adj. R^2	0.225	0.224	0.255	0.254

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3: Reduced-form analysis of subjects' contributing behavior: by period

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Value	0.304*	0.598***	0.523***	0.465***	0.394***	0.479***	0.651***	0.358***	0.436***	0.329***
	(0.156)	(0.127)	(0.101)	(0.150)	(0.0983)	(0.0954)	(0.0790)	(0.100)	(0.133)	(0.112)
Belief	3.622*	5.551***	1.086	2.187	1.072	1.212	2.248**	2.547**	0.808	3.510***
	(2.137)	(1.520)	(1.235)	(1.674)	(1.057)	(1.048)	(0.988)	(1.000)	(1.479)	(1.127)
Outcome (w_{t-1})		-4.208***	-5.366***	-3.156	-0.815	-6.845***	-10.94***	-6.824***	-4.308**	-4.682
		(1.327)	(1.497)	(2.951)	(1.625)	(1.736)	(2.393)	(1.989)	(1.842)	(2.853)
$\frac{b_{i,t-1}}{v_{i,t-1}} * w_{t-1}$		2.919*	5.486***	3.361	0.252	9.558***	15.90***	10.30***	4.135**	8.159**
		(1.562)	(1.799)	(4.162)	(1.986)	(2.524)	(3.231)	(2.734)	(2.045)	(4.045)
Outcome (w_{t-2})			-4.379***	-4.278**	-1.754	-2.627	-0.968	1.950	-1.123	-0.224
			(1.064)	(2.037)	(2.021)	(1.586)	(1.769)	(3.345)	(2.121)	(1.640)
$\frac{b_{i,t-2}}{v_{i,t-2}} * w_{t-2}$			4.369***	3.257	2.367	4.479**	1.526	-2.814	0.417	1.490
			(1.157)	(2.452)	(2.844)	(1.855)	(2.579)	(4.673)	(2.813)	(1.779)
Constant	1.911	-1.860	1.398	1.717	2.489	1.303	-1.128	2.580	3.550	0.462
	(2.544)	(2.064)	(1.622)	(1.984)	(1.510)	(1.303)	(1.257)	(1.652)	(2.279)	(1.943)
N	60	60	60	60	60	60	60	60	60	60
R^2	0.124	0.446	0.567	0.315	0.310	0.555	0.689	0.515	0.251	0.348
adj. R^2	0.093	0.406	0.518	0.237	0.232	0.505	0.653	0.461	0.167	0.274

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Identification of number of types, $w = 0, N = 500$

Discretize Level	2	3	4	5	6
Condition Number					
Mean	190.03	126.14	792.29	2124.35	1164.67
25 percentile	18.40	26.86	101.93	93.31	138.51
Median	32.03	53.00	167.45	199.17	247.45
75 percentile	63.20	93.47	425.37	490.10	590.13
Determinant					
Mean	4.83E-03	-2.75E-02	2.16E-04	1.96E-07	-3.49E-05
25 percentile	-4.66E-04	-3.01E-05	-6.54E-08	-8.15E-11	-9.75E-13
Median	1.08E-03	-1.02E-06	-2.99E-10	0.00E+00	-7.24E-18
75 percentile	5.05E-03	4.48E-06	8.59E-09	1.92E-10	2.14E-13

Table 5: Identification of number of types, $w = 1, N = 1000$

Discretize Level	2	3	4	5	6
Condition Number					
Mean	115.89	145.99	1307.84	774.56	849.63
25 percentile	20.54	32.48	104.16	135.55	142.82
Median	35.87	60.07	196.40	220.76	253.25
75 percentile	77.65	116.02	440.30	461.40	476.87
Determinant					
Mean	1.50E-04	9.20E-05	4.24E-07	5.91E-09	5.54E-13
25 percentile	-2.72E-04	-4.61E-06	-8.23E-09	-4.79E-11	-6.84E-14
Median	8.53E-04	2.41E-07	1.60E-10	-8.34E-14	0.00E+00
75 percentile	2.95E-03	1.09E-05	1.12E-08	1.52E-11	6.16E-14

Table 6: Estimate of type probability

	Type 1	Type 2	Type 3
True value	0.40	0.30	0.30
$N = 500$	0.400***	0.301***	0.299***
	(0.047)	(0.059)	(0.045)
$N = 1000$	0.401***	0.297***	0.301***
	(0.032)	(0.041)	(0.031)

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Estimated transition matrix of type, $N = 500$

(a) $\Pr(\tau_2 | w = 1, \tau_1)$

	Type1	Type2	Type3
Type1	0.494***	0.308*	0.206
	(0.170)	(0.240)	(0.170)
Type2	0.226	0.543**	0.423**
	(0.193)	(0.285)	(0.225)
Type3	0.279***	0.150	0.370**
	(0.143)	(0.167)	(0.172)

(b) $\Pr(\tau_2 | w = 0, \tau_1)$

	Type1	Type2	Type3
Type1	0.735***	0.218	0.213
	(0.192)	(0.279)	(0.288)
Type2	0.160	0.555*	0.618*
	(0.188)	(0.358)	(0.401)
Type3	0.105	0.227	0.169
	(0.110)	(0.234)	(0.250)

Table 8: Estimated transition matrix of type, $N = 1000$

(a) $\Pr(\tau_2 | w = 1, \tau_1)$

	Type1	Type2	Type3
Type1	0.504***	0.317*	0.176*
	(0.125)	(0.203)	(0.136)
Type2	0.206	0.556***	0.441***
	(0.146)	(0.230)	(0.179)
Type3	0.291***	0.127	0.382***
	(0.102)	(0.115)	(0.130)

(b) $\Pr(\tau_2 | w = 0, \tau_1)$

	Type1	Type2	Type3
Type1	0.766***	0.192	0.168
	(0.153)	(0.236)	(0.239)
Type2	0.140	0.608**	0.615**
	(0.138)	(0.320)	(0.362)
Type3	0.099	0.204	0.215
	(0.084)	(0.190)	(0.244)

Table 9: Estimated transition matrix of type: correlated values ($N = 500$)

(a) $\Pr(\tau_2 | w = 1, \tau_1)$

	Type1	Type2	Type3
Type1	0.590***	0.258	0.104
	(0.152)	(0.216)	(0.134)
Type2	0.174	0.607**	0.408**
	(0.167)	(0.275)	(0.214)
Type3	0.236**	0.135	0.488***
	(0.135)	(0.175)	(0.193)

(b) $\Pr(\tau_2 | w = 0, \tau_1)$

	Type1	Type2	Type3
Type1	0.843***	0.207	0.196
	(0.157)	(0.295)	(0.314)
Type2	0.087	0.593**	0.607*
	(0.144)	(0.359)	(0.394)
Type3	0.069	0.2	0.197
	(0.092)	(0.248)	(0.293)

Table 10: Estimation of number of types

Discretize Level	2	3	4	5	6
		$w = 0$			
Condition Number					
Original Sample	11.87	12.09	355.40	64.15	64.08
Mean	14.07	18.35	6.34E+14	1.40E+16	1.33E+16
25 percentile	8.54	10.57	1.37E+02	93.85	198.28
Median	11.48	13.25	340	194.06	399.48
75 percentile	16.04	18.28	1083.6	737.08	848.88
Determinant					
Original Sample	0.03	6.70E-04	5.04E-07	1.22E-08	9.71E-11
Mean	0.02	5.29E-04	4.93E-07	5.74E-09	7.44E-12
25 percentile	0.01	3.07E-04	-2.10E-07	5.85E-10	-4.24E-12
Median	0.02	5.02E-04	1.10E-07	2.60E-09	2.68E-12
75 percentile	0.03	5.03E-04	1.03E-06	8.62E-09	1.42E-11
		$w = 1$			
Condition Number					
Original Sample	18.34	16.43	137.96	90.78	128.50
Mean	17.56	27.81	9.86E+14	1.90E+16	1.66E+16
25 percentile	8.42	9.73	125.50	110.79	176.20
Median	10.92	13.67	306.80	238.21	418.00
75 percentile	16.95	23.38	1104.30	786.92	1219.10
Determinant					
Original Sample	0.03	4.35E-04	5.04E-07	1.22E-08	9.71E-11
Mean	0.02	5.54E-04	5.52E-07	4.91E-09	1.06E-11
25 percentile	0.01	3.06E-04	-1.66E-07	0.00E+00	-3.91E-11
Median	0.02	5.03E-04	1.24E-07	2.54E-09	2.01E-12
75 percentile	0.03	7.33E-04	1.20E-06	6.83E-09	2.13E-11

Table 11: Estimate of type probability in the first period

	Type 1	Type 2	Type 3
pooled data	0.362***	0.374***	0.264***
	(0.115)	(0.071)	(0.080)
the first 5 periods	0.346***	0.493***	0.161***
	(0.053)	(0.071)	(0.087)
the last 5 periods	0.233***	0.381***	0.386***
	(0.040)	(0.064)	(0.066)

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 12: Learning rule: pooled data

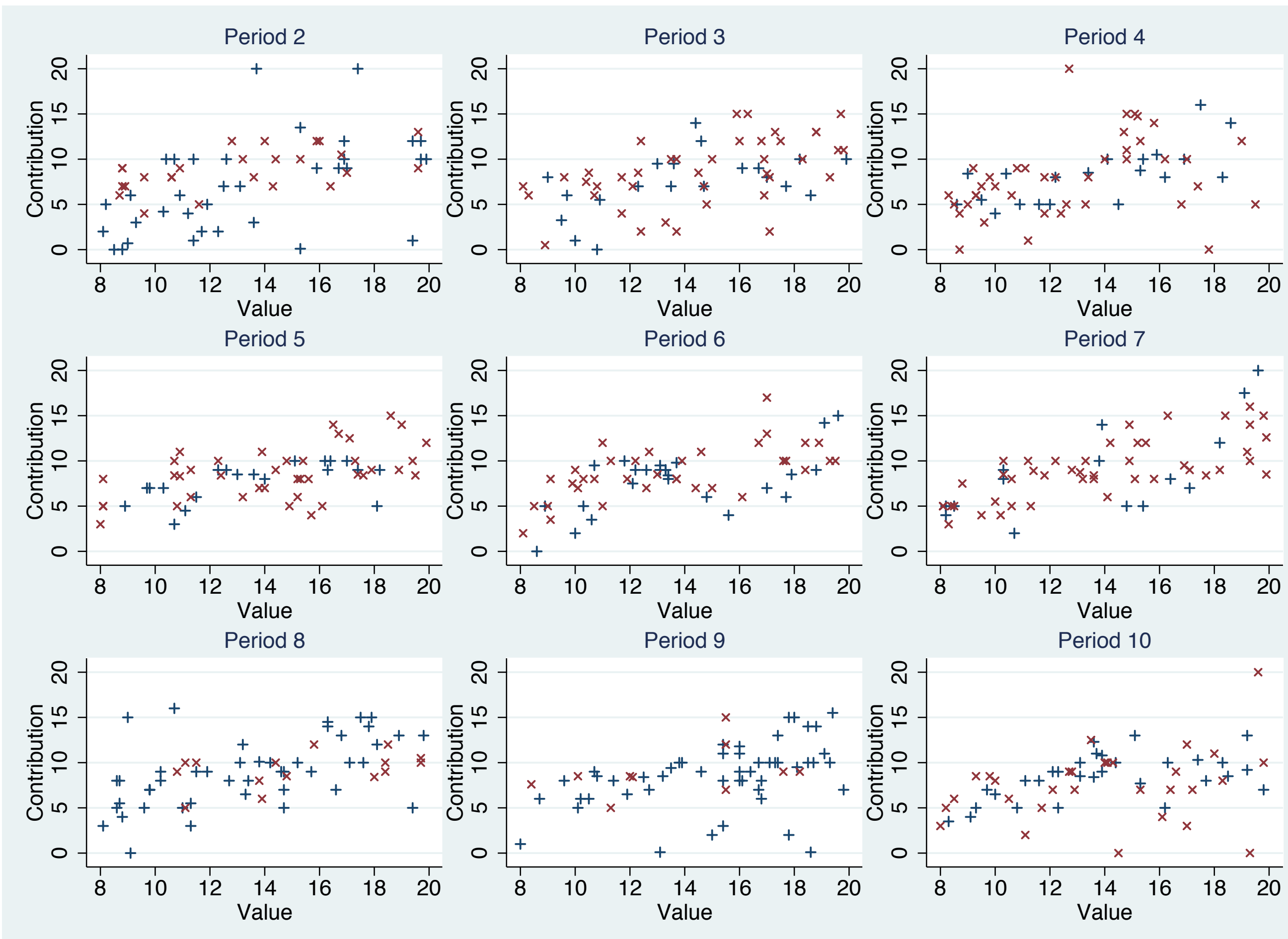
(a) $\Pr(\tau_2 | w = 1, \tau_1)$

	Type1	Type2	Type3
Type1	1.000***	0.259	0.240
	(0.103)	(0.281)	(0.160)
Type2	0.000	0.741***	0.000
	(0.073)	(0.273)	(0.219)
Type3	0.000	0.000	0.760***
	(0.044)	(0.279)	(0.174)

(b) $\Pr(\tau_2 | w = 0, \tau_1)$

	Type1	Type2	Type3
Type1	0.762***	0.000	0.000
	(0.127)	(0.073)	(0.096)
Type2	0.238**	0.000	0.173
	(0.124)	(0.449)	(0.266)
Type3	0.000	1.000**	0.827***
	(0.150)	(0.449)	(0.292)

Figure 1: Value-Contribution Conditioning on Outcome



× – – conditional on "not provided"

+ – – conditional on "provided"

Figure 2: Estimate of contributing strategies

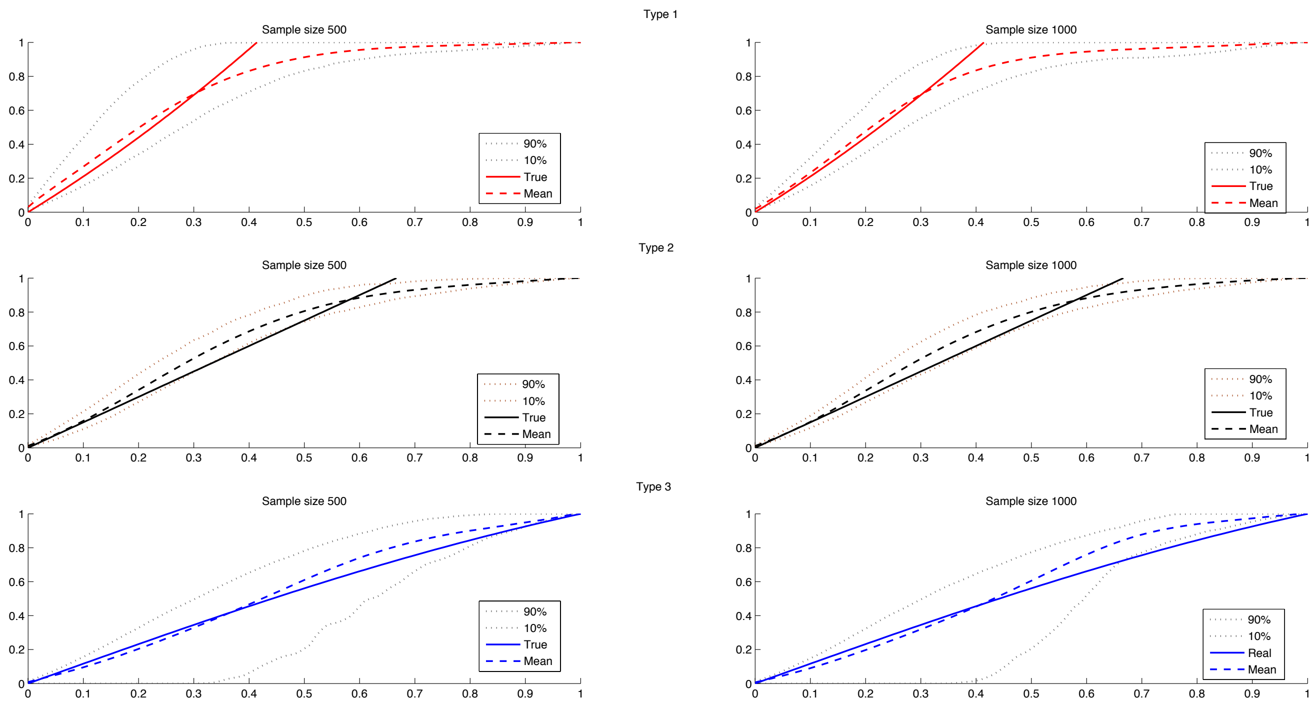


Figure 3: Estimated contributing strategies with correlated values, $N = 500$

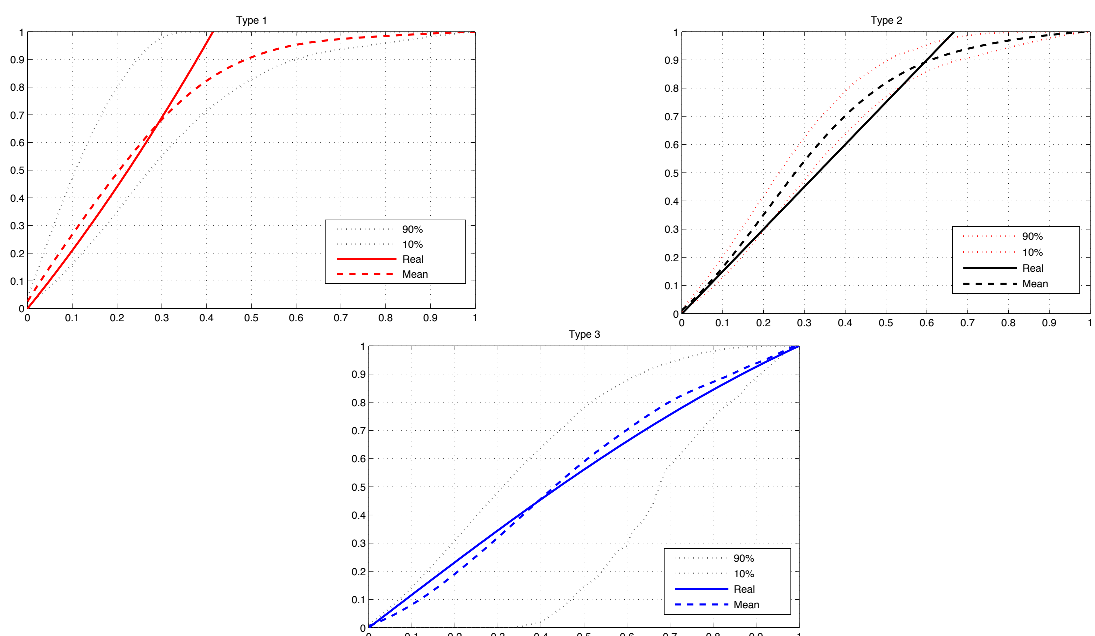


Table 13: Learning rule: first 5 periods

(a) $\Pr(\tau_2|w=1,\tau_1)$

	Type1	Type2	Type3
Type1	1.000***	0.353	0.000
	(0.180)	(0.238)	(0.320)
Type2	0.000	0.647**	0.223
	(0.073)	(0.273)	(0.219)
Type3	0.000	0.000	0.777***
	(0.162)	(0.173)	(0.320)

(b) $\Pr(\tau_2|w=0,\tau_1)$

	Type1	Type2	Type3
Type1	0.654***	0.000	0.000
	(0.187)	(0.185)	(0.006)
Type2	0.346*	0.520	0.000
	(0.189)	(0.385)	(0.258)
Type3	0.000	0.480	1.000***
	(0.116)	(0.395)	(0.260)

Table 14: Learning rule: last 5 periods

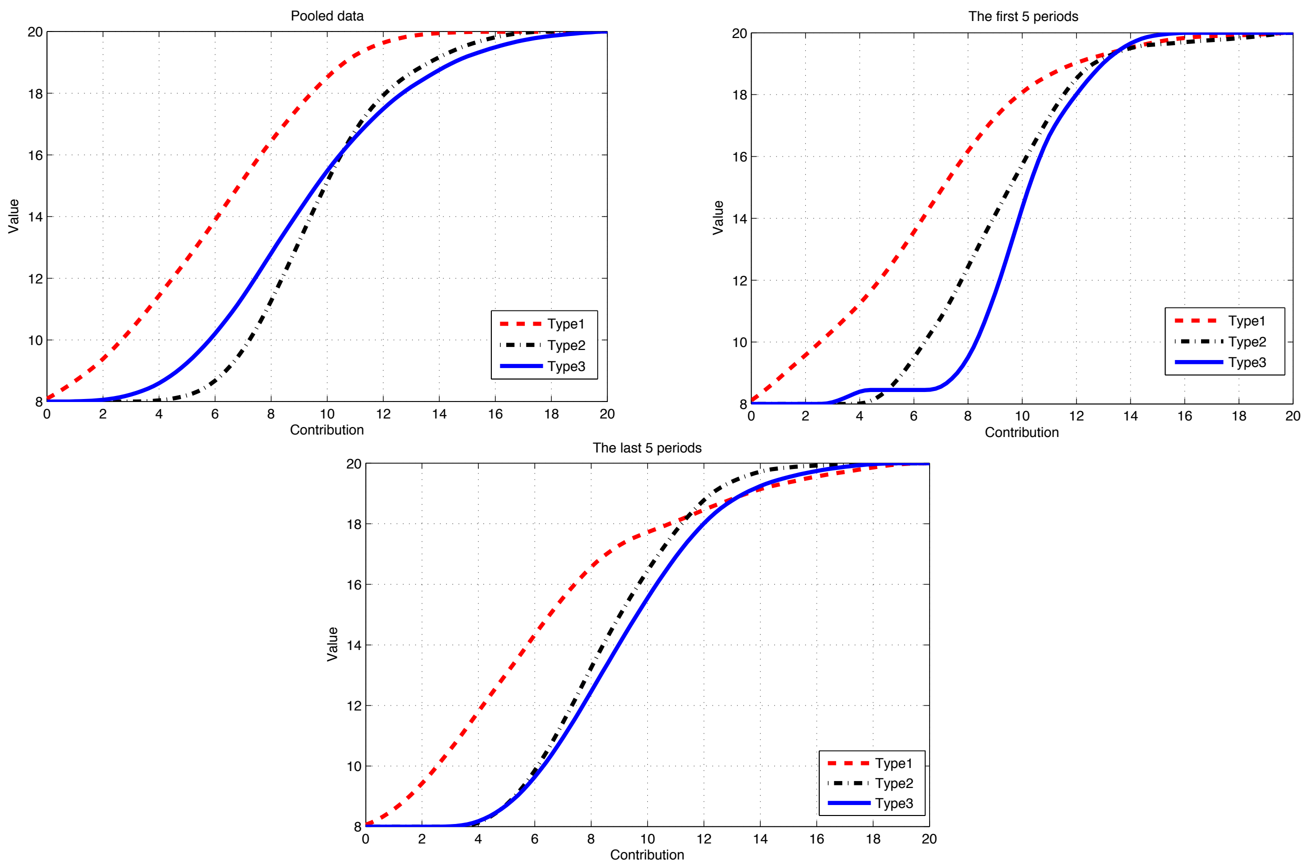
(a) $\Pr(\tau_2|w=1,\tau_1)$

	Type1	Type2	Type3
Type1	0.622***	0.199*	0.000
	(0.218)	(0.154)	(0.059)
Type2	0.378**	0.801***	0.172
	(0.219)	(0.209)	(0.162)
Type3	0.000	0.000	0.828***
	(0.009)	(0.120)	(0.160)

(b) $\Pr(\tau_2|w=0,\tau_1)$

	Type1	Type2	Type3
Type1	0.397***	0.214*	0.000
	(0.153)	(0.165)	(0.074)
Type2	0.603***	0.212	0.110
	(0.212)	(0.278)	(0.221)
Type3	0.000	0.574**	0.890***
	(0.184)	(0.309)	(0.242)

Figure 4: Heterogenous contributing strategies



C. Experiment Instruction

In this experiment, you will be divided into different groups where each group can provide one unit of public good. If the sum of contributions from your group exceeds the cost, the public good is provided, and your profit is your value minus your contribution; otherwise your profit is zero. Your value is randomly drawn from 8 to 20; that is, someone may have a value as low as 8, and someone may have a value as high as 20, while for the most of the time, your value is between 8 and 20. Your value will vary across periods.

Your goal is to maximize your profit. *In order to make better decisions, you may need to guess how much other people would contribute in your group.* In each period, you need to enter 1) your guess on how likely your group will provide the public good (subjective probability, between 0 and 1); 2) your contribution to the public good.

What you need to do?
Once the program is activated, please enter your guess on how likely your group will provide the public good and then make an offer to the public good.

How is your profit calculated?

- Your profit= Your benefit - Your cost.

- Your benefit= your value, if the public good if provided;
  Your benefit= 0, if the public good if not provided.

Suppose that your value is $10, if the public good is provided, you benefit equals your value, which is $10; if the public good is not provided, you benefit is 0.

- Your cost= your offer, if the public good if provided;
  Your cost= 0, if the public good if not provided.

Suppose that you make an offer of $5, if the public good is provided, you cost is $5; if the public good is not provided, you cost is 0.

- Under this situation, your profit=$10-$5=$5 if the public good is provided; your profit=$0 if the public good is not provided.

All the numbers used in examples serve only illustrative purpose; please do not try to use these examples to guess what would actually happen in the experiment.

How to decide if the public good can be provided?

- We will compare the total offer of your group with the cost of the public good. If the group's total offer is higher or equal to the cost for the public good, we will provide the public good, otherwise not.

Quiz (4 mins):

1. If your offer on the public good is $10, you value $20, what's if your profit if the public good is provided /not provided?

2. If the total offers of your group is $50, the cost of the public good is $40, is the public good provided? What if the cost of the public good is $60?

Instructions At-A-Glance

- You will be asked to decide how much money to offer towards the cost of the public good.
- The administrator will use the offers of everyone in your group to determine if we can provide the public good.
- If you offer more, in exchange for incurring some of the costs, you may get a higher profit by increasing the probability of the public good being provided.
- If you offer less, you may decrease the probability of the public good being provided; however, you may get a higher profit since you pay less if the public good is provided.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Now please make your decisions!