The Economics and the Econometrics of Human Development and Social Mobility: Technology of Skill Formation
Part I
Introduction: Overview and Some Main Ideas

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CeMMAP Masterclass
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June 23, 2016
“The most valuable of all capital is that invested in human beings; and of that capital the most precious part is the result of the care and influence of the mother.”

—Alfred Marshall (1890)
Plan of Lectures

- **Lecture I.** Overview of the evidence and introduction to basic models of skill measurement and skill formation
- **Lecture II.** Skills: definitions and measurement issues; identification of skills from measurements. Should we trust psychologists to do their job or should we do it for them and do it better?
- **Lecture III.** Technologies of skill formation and models of skill development; identification of technologies of skill formation
- **Lecture IV.** Evidence on interventions: treatment effects and mechanisms underlying treatment effects
I. Background
Introduction

• The accident of birth plays a powerful role in explaining variability in lifetime income.

• It is estimated that 50% of the variability of the present value (discounted value) of lifetime earnings is determined by age 18.

• The remaining 50% is determined by market forces – “luck” – determined after family influence wanes.

• The modern welfare state operates to reduce both sources of inequality.

• As currently configured in most modern economies, it is far more effective at reducing market risk than family risk for disadvantaged children.
Evidence suggests that redistributing the same resources toward the early years will reduce inequality, boost productivity and promote intergenerational social mobility.

Families are under strain in many countries around the world.

Dysfunctional families account in part for a slowdown in the growth of skills in many countries.

At a time when highly skilled workers are in great demand, growth in the supply of skills has stalled or slowed down in many economies.
• Immigrant assimilation has proved to be a difficult task.
• Enclaves have formed with low rates of intergenerational mobility out of poverty.
• These and other social problems are strongly rooted in conditions children face in their formative years.
Social Mobility

- Link between inequality and social mobility featured in “Great Gatsby Curve”
Intergenerational Mobility and Inequality: The “Great Gatsby Curve”

\[ \text{IGE: } \ln Y_1 = \alpha + \beta \ln Y_0 + \varepsilon \]

Income in current generation
Income of parents

\(\beta \uparrow \) mobility \(\downarrow\)

**Source:** Corak 2011, Inequality from Generation to Generation: the United States in Comparison.

**Note:**
- Inequality is measured post-taxes and transfers.
- Gini index defined on household income.
- IGE measured by pre-tax and transfer income of individual fathers and sons.
Robustness of the Great Gatsby Curve

Link to Appendix I
Associational Evidence on Family Advantage Within Country (US) Relationship

**Figure 1:** Child Income Rank vs. Parent Income Rank by Birth Cohort

![Graph showing the relationship between child income rank and parent income rank across birth cohorts. The graph includes data points and trend lines for different cohorts: 1971-74, 1975-78, and 1979-82. The slopes are given for each cohort: 71-74 Slope = 0.299 (0.009), 75-78 Slope = 0.291 (0.007), 79-82 Slope = 0.313 (0.008).]

Source: Chetty et al. (2014).
The Mechanisms Producing the IGE and Their Quantitative Importance
Why the IGE may differ across countries and over time  
(Solon, 2004)

- Specialized version of intergenerational model.
- Budget constraint: families allocate all after-tax lifetime income to either parental consumption \((Z_0)\) or investment in the child \((l_0)\):

\[
(1 - \tau)Y_0 = Z_0 + l_0. \tag{1}
\]

- Thus, no lending, borrowing, or bequests.
- \(\tau\) is the tax rate.

Why the IGE may differ across countries and over time (Solon, 2004)

- Human capital of the child ($\theta_1$) is produced by a semi-log production function:

$$
\theta_1 = \psi \log(l_0 + G_0) + e_1 . \quad (2)
$$

- Observe $l_0$ and $G_0$ are perfect substitutes. (A property of many models in the current literature, e.g., Lee and Seshadri, 2016 and Gayle et al., 2015.)
Child endowments follow AR(1) process:

\[ e_1 = \alpha + \lambda e_0 + u_1, \]  

(3)

- \( 0 \leq \lambda \leq 1 \); \( u_1 \) is white noise.

Earnings equation:

\[ \log(Y_1) = \mu + p\theta_1. \]  

(4)

- \( p \) is the return to a unit of human capital.
• The family preferences
  \[ U_1 = (1 - \delta) \log(Z_0) + \delta \log(Y_1). \]
  \[ \delta \] measures the degree of altruism towards the child.

• Provision of governmental goods:
  \[ G_0/[(1 - \tau)Y_0] = \varphi - \gamma \log(Y_0). \]
  \[ \gamma > 0 \] ratio of government investment to after-tax income is decreasing in income.

• \[ \gamma \]: a measure of the progressivity of government spending on children.
• Maximizing the utility function with respect to parental investment and collecting terms, one arrives at

$$\log(Y_1) = \mu^* + [(1 - \gamma)\psi p] \log(Y_0) + pe_1. \quad (5)$$

• Can be used to generate the standard IGE regression.
• $e_1$ correlated with $\ln(Y_0)$ through common shock $e_0$. 
• In steady state, $\sigma_0 = \sigma_1$

$$\beta = \frac{(1 - \gamma)\psi p + \lambda}{1 + (1 - \gamma)\psi p\lambda} \uparrow \text{ as } \lambda \uparrow, \psi \uparrow, p \uparrow, \gamma \downarrow. \quad (6)$$

• Can show that out of steady state as income inequality $\uparrow$ across generations, $\beta \uparrow$. 
• Cross-section variance of log $Y$ (steady state)

$$\text{Var}(\ln Y) = \frac{[1 + (1 - \gamma)\psi p \lambda]p^2 \text{Var}(u)}{[1 - (1 - \gamma)\psi p \lambda](1 - \lambda^2)[1 - (1 - \gamma)\psi p]^2}.$$ 

• $\text{Var}(u)$ is variance in heritability of endowments.

• $\text{Var}(\ln Y)$

$$\uparrow \text{ in } \lambda, \psi, p, 1 - \gamma.$$ 

• New term not in $\beta$ is $\text{Var}(u)$. 
Becker & Tomes / Solon

Link to Appendix II
Recent Literature

- Expands on the framework to recognize:
  1. Multiple stages of childhood and adulthood
  2. Moves beyond “schooling” as investment to allow economists to address the benefits and costs of different types of investments
     a. Schooling
     b. Training
     c. Preschool and early childhood investments
  3. Recognizes the modern literature on the biology and psychology of skill formation and the literature on critical and sensitive periods in development
  4. Multiple skills (cognitive, non-cognitive, and biological skill)
  5. Child preference formation and emergence of decision making (transition from child to adult)
  6. Interactions between child and parents in shaping investment (dynamic principle-agent problems and strategic interactions)
Basic Hypotheses
Genes – one plausible explanation – genetically heterogeneous populations can produce high $\beta$ (lower social mobility).
(ii) Quality of schooling and cost of investment inputs into children.
Parenting

(iii) Parental ability and skills.
(iv) Access to credit: Inability to borrow (against own income or parents against future income for investments in children)? Featured in many economic models.
(v) Inability to insure against bad environments or poor genes? (Failure of complete markets)

(vi) Parental preferences? To duplicate themselves and their preferences across generations.

(vii) Lack of parental information about parenting and their own children.
(a) Should there be policies that attempt to lower the IGE?
(b) If so, what form should they take?
Recent research in the economics of human development and social mobility addresses these problems by focusing on skills and the technology of skill formation.

It establishes the importance of considering:

1. Multiple periods in the life cycle of childhood and adulthood and the existence of critical and sensitive periods of childhood in the formation of skills
2. Multiple skills for both parents and children that extend traditional notions about the skills required for success in life
3. Multiple forms of investment
• Some of the most exciting recent research models parent-child / mentor-child, and parent-teacher-child relationships as interactive systems, involving attachment and **scaffolding** as major determinants of child learning.

• Scaffolding is an adaptive interactive strategy that recognizes the current capacities of the child (trainee) and guides him or her to further learning without frustrating the child.

• Activities are tailored to the individual child’s ability so they are neither too hard or too easy in order to keep in the “zone of proximal development,” which is the level of difficulty at which the child can learn the most.
• The recent literature also takes a more nuanced view of child investment and accounts for parental time and lack of parental knowledge about the capacities of children and effective parenting practices.

• It creates and implements an econometric framework that unifies the study of family influence and the consequences of external interventions in child outcomes.

• There is a well-established empirical relationship between family income and child achievement.

• Many interpret this relationship as evidence of market restrictions including credit constraints.
• Although it is conceptually attractive to do so, and amenable to analysis using standard methods, much of the empirical evidence that credit constraints substantially impede child skill formation is not strong.

• **Family income** proxies many aspects of the family environment – parental education, ability, altruism, personality, and peers.

• The empirical literature suggests that unrestricted income transfers are a weak reed for promoting child skills.
Skills are Important Determinants of Inequality
**Figure 2:** Demographic changes were less important than labour market trends in explaining changes in household earnings distribution, but skills play an important role.

Percentage contributions to changes in household earnings inequality, OECD average, mid-1980s to mid-2000s.

Note: Working-age population living in a household with a working-age head. Household earnings are calculated as the sum of earnings from all household members, corrected for differences in household size with an equivalence scale (square root of household size). Percentage contributions of estimated factors were calculated with a decomposition method which relies on the imposition of specific counterfactuals such as: “What would the distribution of earnings have been in recent year if workers' attributes had remained at their early year level?”

Source: Chapter 5, Figure 5.9, OECD (2013).
Figure 3: Estimated Average Annual Percentage Change in Various Inequality Measures Accounted for by Factor Components, US 1979–2007

<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>P90/P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>0.4</td>
<td>0.82</td>
</tr>
<tr>
<td>Household Structure</td>
<td>23%</td>
<td>33%</td>
</tr>
<tr>
<td>Men's Employment</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Men's Earning Disparity</td>
<td>73%</td>
<td>50%</td>
</tr>
<tr>
<td>Women's Employment</td>
<td>-25%</td>
<td>-22%</td>
</tr>
<tr>
<td>Women's Earning Disparity</td>
<td>20%</td>
<td>29%</td>
</tr>
<tr>
<td>Assortative Mating</td>
<td>10%</td>
<td>11%</td>
</tr>
<tr>
<td>Other</td>
<td>-5%</td>
<td>-6%</td>
</tr>
</tbody>
</table>

Eight Important Facts about Skills
1. Multiple Skills

- **Multiple** skills vitally affect performance in life across a variety of dimensions.
- Will discuss, in depth, measurement of skills in Lecture II.
2. Gaps in Skills

- Gaps in skills between individuals and across socioeconomic groups open up at early ages for both cognitive and noncognitive skills.
Gaps Open Early

Trend in Mean by Age for Cognitive Score by Maternal Education

Each score standardized within observed sample. Using all observations and assuming data missing at random. Source: Brooks-Gunn et al. (2006).
• Schooling quality and school resources have relatively small effects on ability deficits and only marginally account for any divergence by age across children from different socioeconomic groups in test scores.

Consistent with many stories

(1) Genetics
(2) Credit constraints
(3) Preference formation
(4) Parental investment
(5) Parental information
(6) Sorting and peer effects
Figure 4: Early Childhood Longitudinal Study (ECLS), Reading

![Reading Score vs. Age (months) graph with data points for Low poverty and High poverty groups.]


The graph illustrates the reading score development over age (in months) for children from low and high poverty backgrounds. The data points are marked for the ages 55, 65, 75, and 85 months, showing a trend of increasing reading scores with age.
Link to Appendix III
Behaviours

Figure 5: Average Percentile Rank on Anti-Social Behavior Score, by Income Quartile
Gaps also emerge in health. These appear to be divergent with age, at least in the U.S.
3. Genes

- The early emergence of skill gaps might be interpreted as the manifestation of genetics: Smart parents earn more, achieve more, and have smarter children.

- Genes are important, but skills are not solely genetically determined.

- The role of heritability is exaggerated in many studies and in popular discussions. Nisbett et al. (2012), Tucker-Drob et al. (2009), and Turkheimer et al. (2003) show that estimated heritabilities are higher in families of higher socioeconomic status.

- Epigenetics.
Evidence on Gene Environment Interactions

- Evidence of heritability – environment interaction
4. Critical and Sensitive Periods in the Technology of Skill Formation

- There is compelling evidence for critical and sensitive periods in the development of a child.
- Different capacities are malleable at different stages of the life cycle (see Thompson and Nelson, 2001, Knudsen et al., 2006, and the body of evidence summarized in Cunha et al., 2006).
- IQ is rank stable after age 10.
- Personality skills are malleable through adolescence and into early adulthood.
See Appendix G

on Evidence of Critical and Sensitive Periods and of Dynamic Complementarities

Link
5. Family Investments

- Gaps in skills by age across different socioeconomic groups have counterparts in gaps in family investments and environments.
Children enter school with “meaningful differences” in vocabulary knowledge.

1. Emergence of the Problem
In a typical hour, the average child hears:

<table>
<thead>
<tr>
<th>Family Status</th>
<th>Actual Differences in Quantity of Words Heard</th>
<th>Actual Differences in Quality of Words Heard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>616 words</td>
<td>5 affirmatives, 11 prohibitions</td>
</tr>
<tr>
<td>Working Class</td>
<td>1,251 words</td>
<td>12 affirmatives, 7 prohibitions</td>
</tr>
<tr>
<td>Professional</td>
<td>2,153 words</td>
<td>32 affirmatives, 5 prohibitions</td>
</tr>
</tbody>
</table>

2. Cumulative Vocabulary at Age 3

<table>
<thead>
<tr>
<th>Cumulative Vocabulary at Age 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children from welfare families:</td>
</tr>
<tr>
<td>500 words</td>
</tr>
<tr>
<td>Children from working class families:</td>
</tr>
<tr>
<td>700 words</td>
</tr>
<tr>
<td>Children from professional families:</td>
</tr>
<tr>
<td>1,100 words</td>
</tr>
</tbody>
</table>
5. Family Investments (cont.)

- Disadvantaged children have compromised early environments as measured on a variety of dimensions.
- Cunha et al. (2013): the lack of parenting knowledge among disadvantaged parents.
- Parenting styles in disadvantaged families are much less supportive of learning and encouraging child exploration (see Hart and Risley, 1995; Kalil, 2013; Lareau, 2011).
See Appendix B on *Measures of Investments*
See Appendix C on Trends

[Link]
6. Resilience and Targeted Investment

- The body of evidence as a group shows that, as currently implemented, many later life remediation efforts are not effective in improving capacities and life outcomes of children from disadvantaged environments.

- As a general rule, the economic returns to these programs are smaller compared to those policies aimed at closing gaps earlier (see Cunha et al., 2006; Heckman and Kautz, 2014; Heckman et al., 1999).

- However, workplace-based adolescent intervention programs and apprenticeship programs with mentoring, surrogate parenting, and guidance show promising results.

- Will discuss in depth in Lecture IV.
7. Parent-child/Mentor-child Interactions Play Key Roles in Promoting Child Learning

- A recurrent finding from the family influence and intervention literatures is the crucial role of child-parent/child-mentor relationships that “scaffold” the child.
- Will discuss in Lectures III and IV.
8. High Returns to Early Investment

- Despite the generally low returns to interventions targeted toward the cognitive skills of disadvantaged adolescents, the empirical literature shows high economic returns for investments in young disadvantaged children.

- The evidence is explained by dynamic complementarity.
Skills, the Technology of Skill Formation, and the Essential Ingredients of a Life-Cycle Model of Human Development
Skills
• **Vector of skills at age** $t$: $\theta_t$

• Lifetime $T$.

• $\theta_t$:

\[
\theta_t = (\theta_{C,t}, \theta_{N,t}, \theta_{H,t}), \quad t = 1, \ldots, T. \tag{7}
\]

• $\theta_{C,t}$: vector of cognitive skills (e.g. IQ) at age $t$.

• $\theta_{N,t}$: vector of noncognitive skills (e.g. patience, self-control, temperament, risk aversion, discipline, and neuroticism) at age $t$.

• $\theta_{H,t}$: vector of health stocks for mental and physical health at age $t$. 
• The dimensionality of $\theta_t$ may also change with $t$.
• As people mature, they acquire new skills previously missing in their personas and sometimes shed old attributes.
• Skills determine in part
  (a) Resource constraints
  (b) Agent information sets
  (c) Expectations
Key Idea

- Core *low-dimensional* set of skills joined with incentives and constraints generates a variety of diverse outcomes.
- Both the skills and their relationship with outcomes may change with the stage of the life cycle.
• Age-specific outcome $Y_{j,t}$ for action (task) $j$ at age $t$:

$$Y_{j,t} = \psi_{j,t}(\theta_t, e_{j,t}, X_{j,t}), \quad j \in \{1, \ldots, J_t\} \quad \text{and} \quad t \in \{1, \ldots, T\}. \quad (8)$$

• $X_{j,t}$: vector of purchased inputs that affect outcomes.

• Effort $e_{j,t}$: characterized by supply function:

$$e_{j,t} = \delta_j(\theta_t, A_t, X_{j,t}, R_{j,t}^a(I_{t-1}) | \psi). \quad (9)$$

• $I_{t-1}$ is the information set.

• $R_{j,t}^a(I_{t-1})$ is the anticipated reward per unit effort in activity $j$ in period $t$.

• $A_t$ represents other determinants.

• $\psi$ represents a vector of parameters characterizing preferences.
• **Tests are just measures of performance on some tasks (i.e., some other behaviors).**
• Will develop further in Lecture II.
Incentivized boosts in achievement have not been shown to persist when the incentives are removed.
- Equation (8) suggests an important identification problem.
- Will discuss in Lecture II.
Using the empirically specified system of equations (8), and the technology of skill formation in equation (10) exposited below, one can characterize how different interventions or different family influence variables affect $\theta_t$ and hence outcomes ($Y_t$) and make comparisons across those literatures.
Define the set of possible actions for people—their *action spaces*.

This is closely related to the space of “functionings” in Sen’s “capability theory.”

A fundamental notion is that of *maximum possible flexibility* in future situations and in future selves.
Technology
• Technology of skill formation

\[ \theta_{t+1} = f^{(t)}(\theta_t, I_t, \theta_{P,t}). \]  

- \( f^{(t)} \) is assumed to be twice continuously differentiable, increasing in all arguments and concave in \( I_t \).
- As noted above, the dimension of \( \theta_t \) and \( f^{(t)} \) likely increases with the stage of the life cycle \( t \), as does the dimension of \( I_t \).
- New skills emerge along with new investment strategies.
- The technology is stage-specific, allowing for critical and sensitive periods in the formation of capabilities and the effectiveness of investment.
Figure 7: The Empirical Challenge: A Life Cycle Framework for Organizing Studies and Integrating Evidence

Capacities at $t$; $I_t$: investment at $t$;

$\theta_{t+1} = f_t(\theta_t, I_t, \theta_{P,t})$: Technology of Capability Formation

\[
\begin{align*}
\theta_{P,-1} & \rightarrow I_{-1} \rightarrow \theta_{-1} \quad \text{Prenatal} \\
\theta_{P,0} & \rightarrow I_0 \rightarrow \theta_0 \quad \text{Birth} \\
\theta_{P,1} & \rightarrow I_1 \rightarrow \theta_1 \quad \text{Early Childhood, 0–3} \\
\theta_{P,2} & \rightarrow I_2 \rightarrow \theta_2 \quad \text{Later Childhood, 3–6} \\
\theta_{P,T} & \rightarrow I_T \rightarrow \theta_T \quad \text{Adulthood and Beyond} \\
\end{align*}
\]
Crucial concept emphasized in the recent literature is complementarity between skills and investments at later stages \((t > t^*)\) of childhood:

\[
\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} > 0, \quad t > t^*.
\]

Empirical literature: consistent with the notion that investments and endowments are direct substitutes (or at least weak complements) at early ages:

\[
\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} \leq 0, \quad t < t^*, \quad \left( \text{or } \epsilon > \frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} > 0, \text{ for "small" } \epsilon \right).
\]

Complementarity increases with age:

\[
\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} \uparrow t \uparrow.
\]
Growing complementarity with the stage of the life cycle captures two key ideas.
• The first is that investments in adolescents and adults with higher levels of capacity $\theta_t$ tend to be more productive.
• This is a force for the social disequalization of investment.
Table 1: Return to one year of college for individuals at different percentiles of the math test score distribution
White males from high school and beyond

<table>
<thead>
<tr>
<th></th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return in the population</td>
<td>0.1121</td>
<td>0.1374</td>
<td>0.1606</td>
<td>0.1831</td>
<td>0.2101</td>
</tr>
<tr>
<td></td>
<td>(0.0400)</td>
<td>(0.0328)</td>
<td>(0.0357)</td>
<td>(0.0458)</td>
<td>(0.0622)</td>
</tr>
<tr>
<td>Return for those who attend college</td>
<td>0.1640</td>
<td>0.1893</td>
<td>0.2125</td>
<td>0.2350</td>
<td>0.2621</td>
</tr>
<tr>
<td></td>
<td>(0.0503)</td>
<td>(0.0582)</td>
<td>(0.0676)</td>
<td>(0.0801)</td>
<td>(0.0962)</td>
</tr>
<tr>
<td>Return for those who do not attend college</td>
<td>0.0702</td>
<td>0.0954</td>
<td>0.1187</td>
<td>0.1411</td>
<td>0.1682</td>
</tr>
<tr>
<td></td>
<td>(0.0536)</td>
<td>(0.0385)</td>
<td>(0.0298)</td>
<td>(0.0305)</td>
<td>(0.0425)</td>
</tr>
<tr>
<td>Return for those at the margin</td>
<td>0.1203</td>
<td>0.1456</td>
<td>0.1689</td>
<td>0.1913</td>
<td>0.2184</td>
</tr>
<tr>
<td></td>
<td>(0.0364)</td>
<td>(0.0300)</td>
<td>(0.0345)</td>
<td>(0.0453)</td>
<td>(0.0631)</td>
</tr>
</tbody>
</table>

Source: Carneiro and Heckman (2003).
Notes: Wages are measured in 1991 by dividing annual earnings by hours worked per week multiplied by 52. The math test score is an average of two 10th grade math test scores. There are no dropouts in the sample and the schooling variable is binary (high school-college). The gross returns to college are divided by 3.5 (this is the average difference in years of schooling between high school graduates who go to college and high school graduates who do not in a sample of white males in the similar NLSY data). To construct the numbers in the table, we proceed in two steps. First we compute the marginal treatment effect using the method of local instrumental variables as in Carneiro, Heckman, and Vytlacil (2001). The parameters in the table are different weighted averages of the marginal treatment effect. Therefore, in the second step we compute the appropriate weight for each parameter and use it to construct a weighted average of the marginal treatment effect (see also Carneiro 2002). Individuals at the margin are indifferent between attending college or not. Standard errors are in parentheses. For additional evidence see Knudsen et al. (2006) and Cunha et al. (2006).
• The second idea is that complementarity tends to increase over the life cycle as a result of investment.
• Complementarity coupled with self-productivity leads to the important concept of *dynamic complementarity* introduced in Cunha and Heckman (2007, 2009).
• \( l_t \uparrow \Rightarrow \theta_{t+1} \uparrow \)

• Because of self-productivity, \( \theta_{t+1} \uparrow \Rightarrow \theta_{t+s} \uparrow, \ s \geq 1: \)

\[
\frac{\partial^2 \theta_{t+s+1}}{\partial l_t \partial l'_{t+s}} > 0, \quad s \geq 1.
\]

• Investments in period \( t + s \) and investments in any previous period \( t \) are always complements as long as \( \theta_{t+s} \) and \( l_{t+s} \) are complements, irrespective of whether \( l_t \) and \( \theta_t \) are complements or substitutes in some earlier period \( t \).

• Dynamic complementarity is a consequence of static complementarity in later life periods.

• Because future capacities are increasing in current investments and future investments are complements with future capacities, current and future investments tend to be complements the stronger the static complementarity in future periods.
• Consider the following specification for the technology with scalar $\theta_t$ and $l_t$:

\[ \theta_{t+1} = f^{(t)}(\theta_t, l_t). \]

• Denote by $f_1^t$ and $f_2^t$ the derivatives with respect to the first and second argument, respectively,

\[
\text{sign}\left\{ \frac{\partial^2 f^{(t+s)}(\theta_{t+s}, l_{t+s})}{\partial l_{t+s} \partial l_t} \right\} = \text{sign}\{f_{21}^{(t+s)}\}
\]

independently of the sign of $f_{21}^t$, for $s \geq 1$. 
• Proof

\[
\frac{\partial^2 f(t+s)(\theta_{t+s}, l_{t+s})}{\partial l_{t+s} \partial l_t} = f_{21}^{(t+s)} \left( \prod_{j=1}^{s-1} f_1^{(t+j)} \right) f_2^{(t)} .
\]
See Appendix L on Dynamic Complementarity for the Vector Case
Figure 8: Returns to a Unit Value Invested

Programs targeted towards the earliest years

Preschool programs

Schooling

Job training

• Empirical evidence (Cunha, 2007; Cunha and Heckman, 2008; Cunha et al., 2010)
• In multiperiod models
• \[ \cdots > f_{12}^{(3)} > f_{12}^{(2)} > f_{12}^{(1)} . \]
Dynamic complementarity also suggests that limited access to parenting resources at early ages can have lasting lifetime consequences that are difficult to remediate at later ages.

Parental skills also play a disequalizing role as they enhance the productivity of investments \( \left( \frac{\partial^2 \theta_{t+1}}{\partial \theta_P, t \partial I_t'} > 0 \right) \).
• Public investments: usually thought to promote equality.
• Whether they do so depends on the patterns of substitutability with private investments and parental skills ("Matthew Effect").
• If more skilled parents are able to increase the productivity of public investments as they are estimated to do with private ones, or if public investments crowd out private investments relatively more among disadvantaged families, then common (across persons) public investments will also play a role towards disequalization. Equal provision may create disequalization.
Other Ingredients
In addition to the functions linking outcomes to skills and the technology of capability formation, a fully specified model of family influence considers *family preferences for child outcomes*. 
• This includes traditional restrictions (if any) on transfers across generations, restrictions on transfers within generations (parental lifetime liquidity constraints), and the public provision of investments in children.

• Less traditional, but central to the recent literature are other constraints on parents:
  
  (a) Information on parenting practices and parental guidance
  (b) Genes
  (c) The structure of households, including assortative matching patterns
End of Lecture 1
The IRS Databank

Results of Chetty, et al.

Conclusions

Bibliography

Questions the IGE Can and Cannot Address

The Facts and Lack of Consensus

Open Issues in the IGE Literature

<table>
<thead>
<tr>
<th>Source</th>
<th>Data</th>
<th>IGE</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solon (1992)</td>
<td>PSID</td>
<td>0.29-0.41</td>
<td>1Yr</td>
</tr>
<tr>
<td>Solon (1992)</td>
<td>PSID</td>
<td>0.41</td>
<td>5Yr</td>
</tr>
<tr>
<td>Solon (1992)</td>
<td>PSID</td>
<td>0.53</td>
<td>IV</td>
</tr>
<tr>
<td>Mazumder (2005)</td>
<td>SIPP</td>
<td>0.53</td>
<td>6Yr</td>
</tr>
<tr>
<td>Mazumder (2005)</td>
<td>SIPP</td>
<td>0.61</td>
<td>15Yr</td>
</tr>
<tr>
<td>Mazumder (2005)</td>
<td>NLSY</td>
<td>0.44</td>
<td>3Yr</td>
</tr>
<tr>
<td>Chetty, et al (2014)</td>
<td>IRS</td>
<td>0.34</td>
<td>5/2Yr</td>
</tr>
</tbody>
</table>

\(^a\) Average of log-earnings of parent only.
\(^b\) Father’s years of education as instrument.
\(^c\) Log of average earnings of child and parent.
\(^d\) Log of average earnings of parent only.
\(^e\) Log of 5 (2) year avg. of parent (child) earnings.

Source: Bradley Setzler 2015.
No Consensus: 30 IGE Estimates in 2002–2013

USA IGE Estimates since 2002 (Median: 0.40, SD: 0.12)

Source: Bradley Setzler 2015.
In Corak (2013), the Great Gatsby Curve has significant, positive slope in OLS regression.
• However, there is a methodological mistake in Corak (2013): after tax-and-transfer Gini coefficients are compared to before tax-and-transfer IGE.

• Correcting this mistake makes the Great Gatsby Curve insignificantly sloped.
Due to the anchoring scheme employed by Corak (2013), the slope of the Great Gatsby Curve is arbitrarily controlled by the partly subjective choice of USA IGE estimate.
• By replacing only three of the countries’ IGE estimates with plausible alternates from the literature, we arrive at a negatively-sloped Great Gatsby Curve.

• Conclusion: Corak’s Great Gatsby Curve is not robust. This does not disprove the hypothesis that inequality and IGE are positively related.
Corak's Great Gatsby Curve

Intergenerational Elasticity (Father to Son Earnings)

Corak's Preferred Data OLS (Slope=2.05, p-value=.008)

Source: Corak, JEP(2013).
Source: Bradley Setzler 2013.
Great Gatsby Curve using Alternate Elasticities

Intergenerational Elasticity

Gini Coefficient, After Taxes and Transfers

Alternate Data Choice
OLS (Slope=-0.02, p-value=.980)

Source: Bradley Setzler 2013.
Return to main text
Appendix II
Becker-Tomes-Solon Model:
Parental utility for generation $g$: $U_g$. $Z_g$ is parental consumption

\[
U_g = \eta(Z_g) + \delta U_{g+1}.
\]

Dynastic form of the utility function:

\[
U_g = \sum_{j=0}^{\infty} \delta^j \eta(Z_{g+j}).
\]

Parents’ Problem:
Parents allocate resources between adult consumption $Z_g$ and investment in the child $I_g$ under different market settings.
Original models in the literature discuss, but do not fully characterize

a. Endogenous altruism
b. Assortative mating
c. Fertility
d. Multiple child families and interactions between children and between parents and children
e. Learning by parents
f. Alternative structures of credit markets
g. Insurance
Intergenerational Correlations of Earnings and Education

- $Y_1$ is income in generation “1”; $Y_0$ is income in generation “0”

\[ \ln(Y_1) = \omega + \beta \log(Y_0) + L_1 \]  

- $\beta$: the intergenerational elasticity (IGE)
- $(1 - \beta)$: measure of intergenerational mobility
- Intergenerational correlation ($\rho$): an alternative to $\beta$.

$$\rho = (\sigma_0/\sigma_1)\beta.$$ (12)

- $\sigma_j$ is the standard deviation of log earnings in generation $j$.
- Factors out the cross-sectional dispersion of log earnings in the two generations.
- $\beta$ can be higher in one society than in another simply because the variance of log earnings in the child’s generation is higher in that society.
Issues in estimating the intergenerational elasticity of earnings

- All initially discussed in Becker and Tomes (1986) and refined in later work.
- \( Y \) should be a measure of permanent earnings.
- Few data sets have information that allows the calculation of lifetime earnings for both fathers and sons.
- Issues:
  - Classical measurement error
  - Alignment error (ages of father and son)
Appendix III
Figure 9: Mean Trajectories, High and Low Priority Schools (ECLS), Math

Figure 10: Average Trajectories, Grades 1–3, High and Low Poverty Schools (Sustaining Effects Study), Reading

Figure 11: Average Trajectories, Grades 1–3, High and Low Poverty Schools (Sustaining Effects Study), Math

Figure 12: Average Achievement Trajectories, Grades 8–12, (NELS 88), Science

Figure 13: Average Achievement Trajectories, Grades 8–12, (NELS 88), Math

Figure 14: Growth as a Function of Student Social Background: ECLS, Reading

Figure 15: Growth as a Function of Student Social Background: ECLS, Math

Figure 16: Growth as a Function of School Poverty for Poor Children: Sustaining Effects Data, Reading

Figure 17: Growth as a Function of School Poverty for Poor Children: Sustaining Effects Data, Math

Return to main text
Appendix M: Evidence on Gene Environment Interactions
Genes, Biological Embedding of Experience, and Gene-Environment Interactions
Figure 18: DNA methylation and histone acetylation patterns in young and old twins

Source: Fraga et al. (2005).
● Tables 2 and 3 review the main studies in the behavioral genetics literature on the heritability of capabilities.

● However, the estimates presented are highly questionable.

● The first reason of skepticism is that the standard linear additive models (ACE) used in behavioral genetics and social sciences rely on highly questionable assumptions.

● In particular, they assume that child’s genetic inheritance and parenting experience are uncorrelated.

● For this to hold, parent’s genes have to be uncorrelated with the family environment they create.

● This is internally inconsistent given that the theory postulates that genes affect behavior.
A second reason of skepticism is related to the fact that while the transmission of the genotype follows biologically determined mechanisms, the mapping of the genotype into phenotype is unclear and likely affected by the environment through epigenetic forces potentially affecting also future generations (Cole et al., 2012; Jablonka and Raz, 2009; Kuzawa and Quinn, 2009; Youngson and Whitelaw, 2008).

We conclude that while genetic influences are likely important, the ways social scientists have developed to measure them fail to provide credible estimates.

Table 2 and 3 consistently show that whenever the role of environmental effects in mediating genes expressions is considered, the estimates of heritability are highly impacted (Krueger and Johnson, 2008; Nisbett et al., 2012; Tucker-Drob et al., 2009; Turkheimer et al., 2003).
## Table 2: Heritability of Cognitive Abilities

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jencks et al. (1972)</td>
<td>Meta-analysis: 18 studies considered on IQ correlations for twins and adoptive siblings and fraternal twins</td>
<td>X</td>
<td><em>Correlations</em>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- siblings raised together: 0.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- adoptive sibs: 0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- MZ twins: 0.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- DZ twins: 0.58</td>
</tr>
<tr>
<td>Golberger (1977)</td>
<td>Meta-analysis: 7 studies considered on IQ correlations for twins and adoptive siblings</td>
<td>X</td>
<td><em>Correlations</em>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- siblings raised together: 0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- adoptive sibs: 0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- MZ twins: 0.91</td>
</tr>
<tr>
<td>Bouchard and McGue (1981)</td>
<td>Meta-analysis: 69 studies considered on IQ correlations for twins and adoptive siblings</td>
<td>X</td>
<td><em>Correlations</em>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- siblings raised together: 0.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- adoptive sibs: 0.29;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- MZ twins: 0.85</td>
</tr>
</tbody>
</table>
Table 2: Heritability of Cognitive Abilities

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
</table>
| Scarr et al. (1993) | 426 members of 93 transracial adoptive families. Analysis of IQ correlations parent-child and across siblings measured at age 7 and 17 | X                              | Correlations at age 7  
Transracial adoptees: with adoptive father 0.08, adoptive mother 0.14, adoptive midparent 0.13, birth father 0.42, birth mother 0.29, birth midparent 0.47  
Biological offspring: correlation with father 0.25, mother 0.40, midparent 0.48  
Correlations at age 17  
Transracial adoptees: with adoptive father 0.21, adoptive mother 0.21, adoptive midparent 0.27, birth father 0.28, birth mother 0.23, birth midparent 0.24  
Biological offspring: correlation with father 0.13, mother 0.45, midparent 0.40 |
# Table 2: Heritability of Cognitive Abilities

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
</table>
| Devlin et al. (1997) | Meta-analysis: 212 studies considered on IQ correlations for twins. Model comparison using Bayes factors. Allow for a role of maternal effects. | ✓                              | **Correlations:**  
- siblings raised together: 0.44  
- siblings raised apart: 0.27  
- MZ twins raised together: 0.85  
- MZ twins raised apart: 0.68  
- DZ twins raised together: 0.59  
**Variance decomposition:**  
- narrow sense heritability (additive genetic effects): 34%  
- broad-sense heritability (include non additive genetic factors): 48%  
- maternal effect (for twins): 20%  
- maternal effect (for siblings): 5%  
- common environment: 17% |
### Table 2: Heritability of Cognitive Abilities

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
</table>
| Turkheimer et al. (2003) | 319 twins pairs from the National Collaborative Perinatal Project sample. Analysis on the relationship between socio-economic status (SES) and heritability of IQ. | ✓                              | Variance decomposition:  
- genes: 0.1 for low SES, 0.8 for high SES  
- shared environment: 0.55 for low SES, 0.1 for high SES  
- non-shared environment: 0.35 for low SES, 0.1 for high SES  
- parental environments matter more for low SES families often underrepresented in samples |
| Tucker-Drob et al. (2009) | 319 pairs of twins in the National Collaborative Perinatal Project. Nonlinear factor analysis: account for the possibility that correlations in different cognitive abilities is different at different ability levels. Avoid bias in estimating the relationship of SES and heritability of cognitive abilities | ✓                              | Variance decomposition:  
- genes, 0.15 for low SES, 0.6 for high SES  
- shared environment, 0.55 for low SES, 0.25 for high SES  
- non-shared environment, 0.3 for low SES, 0.15 for high SES.  
- SES gradient in heritability (Turkheimer et al., 2003) is less steep but still present when accounting for nonlinear effects |
| Haworth et al. (2009)   | Twins of high ability (≥ 85th percentile) from samples in United States, Australia, Netherlands and United Kingdom | X                              | Variance decomposition:  
- genes 50%  
- shared environment 28%  
- non-share environment 0.22% |
## Table 2: Heritability of Cognitive Abilities

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
</table>
| Nisbett et al. (2012)        | Meta-analysis: review of recent literature on different aspects of intelligence (IQ, fluid and crystallized) and its relationships with socioeconomic status, interventions and other environmental conditions | ✓                              | IQ and SES: heritability of IQ is higher for higher SES families in the US. Less evident in Europe.  
IQ and environment: Increase from 12 to 18 points in IQ when children are adopted from working class to middle class homes.  
IQ and interventions: even if effects on IQ of interventions vanish, there are effects on educational achievements and life outcomes (limits of IQ as the only relevant characteristic) |
| Briley and Tucker-Drob (2013)| Meta-analysis: 16 articles with 11 unique samples. Total of 11,500 twin and siblings pairs reared together and with cognition measured at least twice between 6 months and 18 years old. Analysis of the changes in the role of genetic heritability over the phases of development. | X                              | IQ heritability increases over time even when controlling for cross sectional age differences. Innovative genetic influences (activation of new genes because of biological or environmental changes) are predominant until age 8 then genetic amplification (small initial genetic differences are amplified by transactional processes) dominates. Innovative influences are relevant also for the components of variance in IQ due to shared environment, but fades overtime and it is confounded with amplification from age 12. |
# Table 3: Heritability of Personality Traits

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
</table>
| Loehlin (2005) | Meta-analysis: correlations in personality measures between parents and children under different scenarios | X                              | *Biological parents raise children*: extraversion 0.14, agreeableness 0.11, conscientiousness 0.09, neuroticism 0.13, openness 0.17.  
*Adoptive parents and adopted children*: extraversion 0.03, agreeableness 0.01, conscientiousness 0.02, Neuroticism 0.05, openness 0.07.  
*Biological parents and adopted children*: extraversion 0.16, agreeableness 0.14, conscientiousness 0.11, neuroticism 0.11, openness 0.14. |
<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Krueger and Johnson (2008)</td>
<td>Twins from Minnesota Twin Family Study. 556 male twin pairs and 604 female pairs. Method: allow for parenting style (measured by regard and conflict) as a form of gene-environment interaction. Parental actions mediate the role of genetic contribution to personality.</td>
<td>✓</td>
<td>Positive emotionality (PEM): proportion of variance explained by genes (heritability) depends on level of parental regard. If low, environmental factors explain 64% of variance, genes 35%, if high, genes explain 76%, environment 23%. Conflict does not mediate genes, but environment. If low environment explains 29%, if high 50%. If parental actions are ignored (standard ACE model) genes explain 52%. Negative emotionality (NEM): low regard, genes explain 28%, high 56%. Low conflict, genes explain 0.67, high 0.31. If parental actions are ignored 40%. Shared environments explain little, but for high level of conflict 0.56%.</td>
</tr>
</tbody>
</table>
# Table 3: Heritability of Personality Traits

<table>
<thead>
<tr>
<th>Study</th>
<th>Data and Method</th>
<th>Genes-Environment Interactions</th>
<th>Findings</th>
</tr>
</thead>
</table>
| Caprara et al. (2009) | 428 Twin Pairs in the Italian Twin Register. Genetic and environmental components of self-esteem, life satisfaction and optimism. | X                               | Self-esteem: genes explain 73% of the variance  
Life satisfaction: genes explain 59% of the variance  
Optimism: genes explain 28% of the variance                                                                                           |
| Belsky et al. (2012)  | 1,116 pairs of same sex twins in the E-Risk Longitudinal Twin Study followed from birth to age 12. Analysis of borderline personality related characteristics (BRPCs) | X                               | BRPCs scale correlation in MZ twins 0.66, in dizygotic (DZ) twins is 0.29.  
Genes account for 66% of variance in BRPCs.  
Early childhood physical maltreatment and exposure to maternal negative expressed emotions correlates with BRPCs.  
Family history of psychiatric disorders increase likelihood of BRPC more in presence of harsh treatment in childhood. |
Return to main text
Appendix G: Evidence of Critical and Sensitive Periods and of Dynamic Complementarities
Figure 19: Second language learning

Source: Johnson and Newport (1989).
Table 4: Return to one year of college for individuals at different percentiles of the math test score distribution
White males from high school and beyond

<table>
<thead>
<tr>
<th></th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return in the population</td>
<td>0.1121</td>
<td>0.1374</td>
<td>0.1606</td>
<td>0.1831</td>
<td>0.2101</td>
</tr>
<tr>
<td></td>
<td>(0.0400)</td>
<td>(0.0328)</td>
<td>(0.0357)</td>
<td>(0.0458)</td>
<td>(0.0622)</td>
</tr>
<tr>
<td>Return for those who attend college</td>
<td>0.1640</td>
<td>0.1893</td>
<td>0.2125</td>
<td>0.2350</td>
<td>0.2621</td>
</tr>
<tr>
<td></td>
<td>(0.0503)</td>
<td>(0.0582)</td>
<td>(0.0676)</td>
<td>(0.0801)</td>
<td>(0.0962)</td>
</tr>
<tr>
<td>Return for those who do not attend college</td>
<td>0.0702</td>
<td>0.0954</td>
<td>0.1187</td>
<td>0.1411</td>
<td>0.1682</td>
</tr>
<tr>
<td></td>
<td>(0.0536)</td>
<td>(0.0385)</td>
<td>(0.0298)</td>
<td>(0.0305)</td>
<td>(0.0425)</td>
</tr>
<tr>
<td>Return for those at the margin</td>
<td>0.1203</td>
<td>0.1456</td>
<td>0.1689</td>
<td>0.1913</td>
<td>0.2184</td>
</tr>
<tr>
<td></td>
<td>(0.0364)</td>
<td>(0.0300)</td>
<td>(0.0345)</td>
<td>(0.0453)</td>
<td>(0.0631)</td>
</tr>
</tbody>
</table>

Source: Carneiro and Heckman (2003).
Return to main text
Appendix B: Evidence on Gaps in Family Environments and Investments in Child Care Across Socioeconomic Classes
Comparison of Ability and Personality Measures by Race
Children enter school with “meaningful differences” in vocabulary knowledge.

1. **Emergence of the Problem**

In a typical hour, the average child hears:

<table>
<thead>
<tr>
<th>Family Status</th>
<th>Actual Differences in Quantity of Words Heard</th>
<th>Actual Differences in Quality of Words Heard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>616 words</td>
<td>5 affirmatives, 11 prohibitions</td>
</tr>
<tr>
<td>Working Class</td>
<td>1,251 words</td>
<td>12 affirmatives, 7 prohibitions</td>
</tr>
<tr>
<td>Professional</td>
<td>2,153 words</td>
<td>32 affirmatives, 5 prohibitions</td>
</tr>
</tbody>
</table>

2. **Cumulative Vocabulary at Age 3**

<table>
<thead>
<tr>
<th>Cumulative Vocabulary at Age 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children from welfare families:</td>
</tr>
<tr>
<td>Children from working class families:</td>
</tr>
<tr>
<td>Children from professional families:</td>
</tr>
</tbody>
</table>
Table 5: Gaps in HOME Scores between White and Black across Ages

(A) Females

<table>
<thead>
<tr>
<th>Data</th>
<th>Age</th>
<th>Obs</th>
<th>Means</th>
<th>Differences(in s.d.)</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>White</td>
<td>Black</td>
<td></td>
</tr>
<tr>
<td>CNLSY</td>
<td>0-3</td>
<td>2587</td>
<td>102.1</td>
<td>91.2</td>
<td>0.686</td>
</tr>
<tr>
<td></td>
<td>4-7</td>
<td>3186</td>
<td>102.6</td>
<td>89.2</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>8-11</td>
<td>3054</td>
<td>103.0</td>
<td>90.5</td>
<td>0.796</td>
</tr>
<tr>
<td>CDS 1997</td>
<td>0-3</td>
<td>276</td>
<td>16.1</td>
<td>14.3</td>
<td>0.769</td>
</tr>
<tr>
<td></td>
<td>4-7</td>
<td>382</td>
<td>21.4</td>
<td>18.4</td>
<td>1.006</td>
</tr>
<tr>
<td></td>
<td>8-11</td>
<td>321</td>
<td>22.1</td>
<td>19.8</td>
<td>0.841</td>
</tr>
</tbody>
</table>

(B) Males

<table>
<thead>
<tr>
<th>Data</th>
<th>Age</th>
<th>Obs</th>
<th>Means</th>
<th>Differences(in s.d.)</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>White</td>
<td>Black</td>
<td></td>
</tr>
<tr>
<td>CNLSY</td>
<td>0-3</td>
<td>2644</td>
<td>100.9</td>
<td>90.0</td>
<td>0.677</td>
</tr>
<tr>
<td></td>
<td>4-7</td>
<td>3289</td>
<td>101.5</td>
<td>87.0</td>
<td>0.881</td>
</tr>
<tr>
<td></td>
<td>8-11</td>
<td>3118</td>
<td>101.5</td>
<td>89.4</td>
<td>0.731</td>
</tr>
<tr>
<td>CDS 1997</td>
<td>0-3</td>
<td>250</td>
<td>15.5</td>
<td>14.5</td>
<td>0.415</td>
</tr>
<tr>
<td></td>
<td>4-7</td>
<td>406</td>
<td>21.3</td>
<td>18.3</td>
<td>1.049</td>
</tr>
<tr>
<td></td>
<td>8-11</td>
<td>337</td>
<td>22.0</td>
<td>20.0</td>
<td>0.741</td>
</tr>
</tbody>
</table>

Source: Moon (2014).
Figure 20: Hispanic and Black Parental Investment in White Distribution: Unadjusted, Age 0-3

(a) Material Goods (Females)  
(b) Material Goods (Males)

Source: Moon (2014).
Figure 22: Hispanic and Black Parental Investment in White Distribution: Unadjusted, Age 0-3 Cont.

(a) Cognitive Stimulation (Females)

(b) Cognitive Stimulation (Males)

Source: Moon (2014).
Figure 24: Hispanic and Black Parental Investment in White Distribution: Unadjusted, Age 0-3 Cont.

(a) Emotional Support (Females)  
(b) Emotional Support (Males)

Source: Moon (2014).
Return to main text
Appendix C: Time Trends on Children in Single Parent Households
Time Trends on Children in Single Parent Households
Figure 26: Children Under 18 Living in Single Parent Households by Marital Status of Parent


Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of household. Children who have been married or are not living with their parents are excluded from the calculation. Separated parents are included in “Married, Spouse Absent” Category.
Figure 27: Families with Dependent Children by Family Type in the UK, 1996 and 2014


Notes: 1. Civil partnerships were introduced in the UK in December 2005. 2. Cohabiting couples include both opposite- and same-sex couples.
Figure 28: Live Births by Type of Registration, 1986, 2001, and 2013, England and Wales

Source: Live Births in England and Wales by Characteristics of Mother 1, 2013. The report was published in 2014.
Trends for Children in Single/Never Married Households by Race
Figure 29: Children in Households with Single, Never Married Parents by Race

Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation.
Trends in Children in Single/Never Married Households by Education
Figure 30: Children in Households with Single, Never Married Parents by Education - Non-Hispanic Whites
Figure 31: Children in Households with Single, Never Married Parents by Education - Non-Hispanic Blacks
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Appendix L: Dynamic Complementarity for the Vector Case
Consider the following specification for a vector-valued technology mapping a $L \times 1$ vector of parental investments $I_t$, and a $J \times 1$ vector of skills $\theta_t$, into a $J \times 1$ vector of next period capabilities $\theta_{t+1}$:

$$\theta_{t+1} = f^t(\theta_t, I_t).$$  (13)
The matrix of second-order partial derivatives of the skill vector \( \theta_{t+s+1} \) with respect to the investment vectors \( I_{t+s} \) and \( I_t \) is given by the \( J \times L^2 \) matrix:

\[
\frac{\partial^2 \theta_{t+s+1}}{\partial I_t \partial I_{t+s}} = \begin{bmatrix}
\frac{\partial^2 f_1, t(\cdot)}{\partial i_1, t+s \partial i_1, t} & \cdots & \frac{\partial^2 f_1, t(\cdot)}{\partial i_L, t+s \partial i_1, t} & \cdots & \cdots & \cdots & \frac{\partial^2 f_1, t(\cdot)}{\partial i_L, t+s \partial i_L, t} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \cdots & \vdots \\
\frac{\partial^2 f_J, t(\cdot)}{\partial i_1, t+s \partial i_1, t} & \cdots & \frac{\partial^2 f_J, t(\cdot)}{\partial i_L, t+s \partial i_1, t} & \cdots & \cdots & \cdots & \frac{\partial^2 f_J, t(\cdot)}{\partial i_L, t+s \partial i_L, t}
\end{bmatrix}
\] (14)
\[
\frac{\partial^2 f^{j,t}(\cdot)}{\partial i_{l,t+s} \partial i_{l',t}} \quad \text{for} \quad j = 1, \ldots, J \quad \text{and} \quad l, l' = 1, \ldots, L \quad (15)
\]

- Is the cross-partial derivative of the entry \( j \) of vector \( \theta_{t+s+1} \) with respect to \( i_{l,t+s} \)
- \( l^{th} \) entry of the vector of investments \( l_{t+s} \), and \( i_{l',t} \)
- \( l' \) entry of the vector \( l_t \).
The sign of each entry is determined by the sign of:

$$\frac{\partial^2 f_{j,t}(\theta_t, l_t)}{\partial i_{l,t+s} \partial \theta_{t+s}} \frac{\partial \theta_{t+s}}{\partial i'_{l,t}}$$

for  \( j = 1, \ldots J \) and  \( l, l' = 1, \ldots, L \).

A sufficient condition for the above to be positive is that each cross partial derivative \( \frac{\partial^2 f_{j,t}(\theta_t, l_t)}{\partial i_{l,t+s} \partial \theta_{j',t+s}} \) is positive for each  \( j, j' = 1, \ldots J \) and  \( l = 1 \ldots L \).

Each entry in the skill vector is increasing in each type of investment.