Investment in Schooling and the Marriage Market*

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Abstract

We present a model with pre-marital schooling investment, endogenous marital matching and spousal specialization in homework and market production. Investment in schooling raises wages and generates two kinds of returns in our framework: a labor-market return and a marriage-market return because education can affect the intra-marital share of the surplus one can extract from marriage. When the returns to education are gender neutral, men and women educate in equal proportions and there is pure positive assortative matching in the marriage markets. But if the market returns are not gender neutral, then there may be mixing in equilibrium where some educated individuals marry uneducated spouses and those who educate less because extract a relatively larger share of the marital surplus. Conditional on the choice of schooling, couples’ career decisions affect the size of their marital surplus, but the existence of large and frictionless marriage markets can still produce efficient household specialization where the higher-wage spouse specializes in market production and the lower-wage spouse engages in homework. Even when cultural and social norms dictate that wives devote relatively more time to homework, women can acquire more schooling than men if a gender wage gap exists but it narrows with the level of education. In contrast, men can lower their educational attainment levels even if their education premium rises.

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1 Introduction

One of the salient trends in recent decades is the increased investment in education by women and the closing of the gap in schooling between men and women. In several developed countries, women now have more schooling than men. Beyond the important impact of schooling on the earnings capacity of women and their economic independence, such trends influence the gains from marriage and marriage patterns. Couples sort according to schooling and, therefore, the educational gap between the spouses has declined too. At the same time we observe a decline in marriage and higher divorce rates, suggesting that the closing of the educational gaps had a larger influence on those who remain single than on those who marry. Women today receive lower wages and spend more time at home than men, although these gaps have narrowed over the years. Hence, one could think that women should invest less in schooling, which appears to be less useful for them both at home and in the market.

Goldin et al. (2006) discuss the rise in investments in schooling of men and women and show that starting with the 1970 birth cohort, women have attained higher college graduation rates than men in the United States. Of the 17 OECD countries with sufficient data, they document that tertiary school enrollment rates of women were below those of men in 13 countries in the mid-1980s, but by 2002 women’s college enrollment rates exceeded those of men in 15 countries. Goldin (1997) provides a vivid and detailed description of the changes in the career and schooling choices of women in the last century. She compares female college graduates of several birth cohorts. Women who graduated from college during the early part of the twentieth century (1900 -1920) had sacrificed family to pursue a career; 50 percent of them had no children by age 35-44 and 30 percent of them never married. Women of later cohorts were better able to mix family life with a career, but they altered the timing of their career and family-life choices. Those who graduated from college prior 1965 gradually raised their marriage and fertility rates and they typically had children before entering the labor market. In contrast, women who graduated from college after 1965 had lower marriage and fertility rates and they tended to start to work before having children.

These demographic changes were also associated with changes in the patterns of who marries whom. The degree of positive assortative matching by schooling has not changed much over time. However, marriage patterns responded to the increased number of educated women. In the past, 35 percent of college women had a more educated spouse and 24 percent had a less educated husband; these proportion changed to 18 percent and 35 percent respectively in recent years. That is, more women marry down and fewer women marry up now than in the 1970s. In contrast,
men with a college degree are now more likely to marry up (matching with women with advanced degrees) and less likely to marry down (pairing up with women with some college or high school degrees).

Unlike other attributes such as race and ethnic background, schooling is an acquired trait and within some limits subject to choice. Presumably, agents who invest in schooling take into account the potential gains of education both in the labor market and within marriage. However, the gains from schooling within marriage strongly depend on the decisions of others to acquire schooling. Because much of schooling happens before marriage, partners cannot coordinate their investments. Rather, men and women make their choices separately, based on the anticipation of marrying a “suitable” educated spouse with whom schooling investments are expected to generate a higher return.

The purpose of this paper is to provide a simple general equilibrium framework for the joint determination of pre-marital schooling and choices couples make during their marriage. An important feature of the model is that the investment choices of both men and women are established simultaneously. In our model, the returns to pre-marital investments can be decomposed into two parts: First, higher education raises one's wage rate and increases the payoff from time on the job (the labor-market return). Second, it can improve the intra-marital share of the surplus one can extract from marriage (the marriage-market return).¹

The basic ingredients of our model are as follows. We consider a frictionless marriage market in which, conditional on the predetermined spousal schooling levels, the assignments are stable. That is, there are no men or women (married or single) who wish to form a new union and there are no men or women who are married but wish to be single. We then assume transferable utility between the spouses to characterize the stable assignment. We further assume that men and women can be divided into schooling classes (high and low) and the interactions between married spouses depend only on their education classes. In particular, although men and women have idiosyncratic preferences for marriage and investment in schooling, they all have the same ranking over spouses of the opposite sex which depend only on their schooling. Thus, every educated man (woman) and every uneducated man (woman) has a perfect substitute. The absence of rents allows us to pin down the shares of the marital surplus of men and women in each schooling class. These shares, together with the known returns as singles, are sufficient to determine the investments in schooling of men and women.

¹Educational attainment could influence intra-marital spousal allocations directly (due to the fact that education raises household income) or indirectly (to the extent that educational attainment influences prospects for marriage and the determination of spousal roles within marriage).
When the market return to education and household roles are gender neutral, men and women acquire education in equal proportions and, under the assumption that the schooling of the two spouses complement each other, a strictly positive assortative matching arises in the marriage markets. That is, educated men marry only educated women and uneducated men marry only uneducated women. Regardless of whether they are educated or not, married couples in such an equilibrium share their marital surplus equally. But if the returns are not gender neutral, then the shares within marriage adjust and the gender with higher market return receives a lower return within marriage. For a sufficiently large gap in the market returns, a mixed equilibrium arises where some educated individuals of the gender that has a higher overall return to schooling marry “down” with uneducated spouses. In such an asymmetric equilibrium, the gender-education class that is in short supply obtains the upper bound on the return from schooling in marriage, which is the marginal contribution of an educated man (woman) to an educated spouse.

We use this simple model to explain why women may overtake men in schooling despite their lower market wage rate and higher amount of housework compared with men. We hypothesize that the increase in the levels of schooling investment by women to and above the levels of men is a consequence of the higher return that women receive for schooling, reflecting lower labor market “discrimination” at higher levels of schooling.\(^2\) The essence of the argument we make is that education can serve as a means to escape discrimination.\(^3\) Therefore, although women today still receive lower wages and spend more at time in the household than men, women may acquire more schooling than men because of their higher returns to schooling in the labor market. In the past, the higher market return was washed out by the lower returns for schooling that women received within marriage. Now, however, women work less at home and market returns are more important. Because of their reduced household chores, educated women are now also more likely to be married and have children.

While ours is not the only framework in which women can invest more in education than men, it provides a general equilibrium framework for the joint determination of schooling and marriage patterns of men and women. In fact, the investment patterns of men have been very different than those of women and equally puzzling.

\(^2\)Mincer and Polachek (1974) and Weiss and Gronau (1981) provided explanations for the main patterns of the gender wage gap even in the absence of any discrimination based on lower investments on the job resulting from expected interruptions in participation. At that time, women also acquired less schooling. The current reversal in the schooling gender gap poses a challenge to this approach.

\(^3\)Discrimination here simply means that conditioned on their level of schooling, women expect lower wages than men during their work careers. This outcome can result from a variety of causes including self-selection of women into part-time jobs with lower wages and weaker incentives for women to acquire, or for employers to provide, on the job training.
While the returns to schooling have risen steadily for them too since the late-1970s, men’s college graduation rates have peaked for the cohort born in the mid-1940s (i.e., around the mid-1960s). And, after falling for the cohorts that followed, men’s college graduation rates have plateaued for the most recent cohorts at levels slightly below their peak. In terms of our model, a possible reason for this phenomenon is the reduction in men’s return for schooling within marriage. In particular, our model suggests that as women became more educated than men and some of them started to marry down to match with uneducated men, the latter began to obtain a higher share of the marital surplus in all marriages. Consequently, men’s returns from schooling within marriage have declined. Thus, incorporating spousal matching into a model of schooling investment can explain the divergent patterns of educational attainment and propensity for marriage between men and women, by generating equilibrium adjustments in the spousal returns to education within marriage.

2 Background

We begin with a brief description of the main facts that we wish to address. Figure 1 describes the time trends in levels of school completion for men and women, aged 30 to 40, in the United States. As seen, the proportions of women with some college education, college completion and advanced degrees (M.A., Ph.D.) have increased much faster than the corresponding proportions for men. By 2003, women had overtaken men in all of these three categories. Goldin et al. (2006) present trends for college graduation by gender and show that, starting with the 1970 birth cohort, women have attained higher college graduation rates than men.

Figure 2 presents the time trends in the hourly wage differentials by schooling for men and women in the United States (for those who work at least 20 hours a week and adjusted for potential work experience). Compared with high school, women receive a higher increase in wages than men when they acquire college or advanced degrees. These gender differences in the returns to schooling tend to widen with the level of schooling and narrow with time. They are positive and statistically significant for most years.

[Figures 1 and 2 about here.]

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4 See Goldin et al. (2006) and Goldin (1997) for more details.
5 The sample includes whites aged 25 to 60. Observations with hourly wages of less than 1 or more than 400 dollars were excluded. The presented coefficients are from year-by-year separate regressions for men and women with log-hourly wages as the dependent variable and highest degree attained, experience and experience-squared as explanatory variables.
As is well known, gender wage differences are confounded by a variety of selection processes: selection into marriage, selection into schooling and selection into work. To disentangle all these effects is beyond the scope of this work (see Mulligan and Rubinstein, 2005). However, if we restrict the sample to “full time, full year” workers (who reported at least 35 hours a week and 51 weeks of work last year), the female advantage in the returns to schooling in the CPS is significant only for advanced degrees between the years 1968 and 1999, but is usually insignificant for other degrees. This suggests that additional schooling favors women mainly in terms access (or commitment) to full-time jobs.6

Couples typically sort positively by the level of schooling. That is, men with higher levels of schooling tend to marry women with higher (but not necessarily the same) levels of schooling. This process is driven not only by the mutual gain from marriage, but also by the availability of partners with different levels of schooling in the population and the likelihood of meeting potential mates in school or the workplace (see Lewis and Oppenheimer 2000). While the proportion of couples in which the husband and wife have the same schooling has remained stable at about 50 percent, 30 percent of the couples in the earlier cohorts had husbands who were more educated whereas 30 percent of the couples in recent cohorts had wives with higher levels of educational attainment (see Figure 3).

Figure 4.a shows the distribution of spousal education levels by husbands and wives with different level of schooling for young couples aged 30 to 40 in the period 1970 – 1979. Figure 4.b displays the same distribution for the period 1996 – 2005. At low schooling levels, each gender mainly marries with individuals of the opposite sex with similar education levels during both time periods. However, at higher levels of schooling, the two time periods display very different marriage patterns. For instance, among the most educated segments, we see that a woman with an advanced degree had a 64 percent chance of marrying a man with an advanced degree in the 1970s, while this likelihood had declined to 46 percent in the late 1990s and early 2000s. That is, because of the increase in the proportion of highly-educated women, some had to marry down and match with less-educated men, mainly college graduates.

6 A familiar conceptual issue is which variables should be held fixed when one considers the impact of schooling. It seems that for the analysis of schooling investment, variables such as hours of work and job characteristics, and perhaps even experience, should be allowed to vary. Mulligan and Rubinstein (2005) show that, in the CPS, the gender wage gap declines with schooling if one compares men and women who work full time without controlling for experience. Dougherty (2005) and O’Neill and O’Neill (2006) show, using NLSY data, that the gender differences in the impact of schooling are eliminated when detailed employment and occupational characteristics are added. Gronau (1998) shows, using PSID data, that education strongly affects access to on-the-job-training opportunities, but the difference between men and women in this regard is not significant.
On the other hand, highly-educated men benefited from the increase in the number of educated women: the probability that a man with an advanced degree married a woman with an advanced degree rose from 19 percent in the 1970s to 37 percent between 1996 – 2005, although it was still more likely for a highly-educated man to marry down than for a highly-educated woman to do so.

For intermediate education categories, we see a large increase in the proportion of marriages where both the husband and wife have some college education, reflecting the sharp increase in the number of women with some college training. In addition, husbands with some college education have replaced wives with a high school degree with wives who have a college degree, while wives with some college education replaced men who have a college and higher degrees with men with high school degrees. Specifically, the proportion of men with some college education who were married to women with high school degrees has declined from 53 percent in the period 1970 – 79 to 25 percent in 1996 – 2005, while the proportion of men with some college education married to women with a college degree has risen from 10 percent to 20 percent over the same time interval. In contrast, the proportion of women with some college education married to men with high school degree has risen from 23 percent in 1970 – 79 to 29 percent in 1996 – 2005, while the proportion married to men with a college degree has declined from 26 percent to 18 percent.

[Figures 3 and 4 about here.]

3 The Basic Model

We begin with a benchmark model in which men and women are completely symmetric in their preferences and opportunities. However, by investing in schooling, agents can influence their marriage prospects and labor market opportunities. Competition over mates determines the assignment (i.e., who marries whom) and the shares in the marital surplus of men and women with different levels of schooling, depending on the aggregate number of women and men that acquire schooling. In turn, these shares together with the known market wages guide the individual decisions to invest in schooling and to marry. We investigate the rational-expectations equilibrium that arises under such circumstances.

3.1 Assumptions

There are two equally large populations of men and women to be matched. Individuals live for two periods. Each person can choose whether to acquire schooling or not and
whether and whom to marry. Investment takes place in the first period of life and marriage in the second period. Investment in schooling is lumpy and takes one period so that a person who invests in schooling works only in the second period, while a person who does not invest works in both periods. To simplify, we assume no credit markets. All individuals with the same schooling and of the same gender earn the same wage rate, but wages may differ by gender. We denote the wage of educated men by $w_2^m$ and the wage of uneducated men by $w_1^m$, where $w_2^m > w_1^m$. The wage of educated women is denoted by $w_2^w$ and that of uneducated women by $w_1^w$, where $w_2^w > w_1^w$. Market wages are taken as exogenous and we do not attempt to analyze here the feedbacks from the marriage market and investments in schooling to the labor market. We shall discuss, however, different wage structures.

We denote a particular man by $i$ and a particular woman by $j$. We represent the schooling level (class) of man $i$ by $I(i)$ where $I(i) = 1$ if $i$ is uneducated and $I(i) = 2$ if he is educated. Similarly, we denote the class of woman $j$ by $J(j)$ where $J(j) = 1$ if $j$ is uneducated and $J(j) = 2$ if she is educated. The surplus generated by a marriage of man $i$ and woman $j$ is

$$s_{ij} = z_{I(i)J(j)} + \theta_i + \theta_j,$$  

(1)

where $\theta_i$ and $\theta_j$ represent the non-economic gains of man $i$ and woman $j$ from their marriage and $z_{I(i)J(j)}$ is the "material" surplus that their marriage generates. Married partners can divide their material surplus and their utilities are linear in their shares (transferable utility).

We assume that the schooling levels of married partners complement each other so that

$$z_{11} + z_{22} > z_{12} + z_{21}. \quad (2)$$

Except for special cases associated with the presence of children, we assume that the surplus rises with the schooling of both partners. When men and women are viewed symmetrically, we also have $z_{12} = z_{21}$.

The per-period material utilities of man $i$ and woman $j$ as singles are denoted by $z_{I(i)0}$ and $z_{0J(j)}$, which are assumed to increase in $I(i)$ and $J(j)$. Thus, a more educated person has a higher utility as a single. Men and women who acquire no schooling and never marry have life time utilities of $2z_{10}$ and $2z_{01}$, respectively. A person that invests in schooling must give up the first period utility and, if he/she remains single, the life time utilities are $z_{20}$ for men and $z_{02}$ for women. Thus, the (absolute) return from schooling for never married men and women are $R^m = z_{20} - 2z_{10}$ and $R^w = z_{02} - 2z_{01}$.

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7 Allowing borrowing and lending raises issues such as whether or not one can borrow based on the income of the future spouse and enter marriage in debt (see Browning et al., in progress, ch. 7).
respectively.\(^8\) The return to schooling of never married individuals depends only on their own market wages and we shall refer to it as the labor-market return. However, investment in schooling raises the probability of marriage and those who marry have an additional return from schooling investment in the form of increased share in the material surplus, which we shall refer to as the marriage-market return to schooling. In addition to the returns in the market or marriage, investment in schooling is associated with idiosyncratic costs (benefits) denoted by \(\mu_i\) for men and \(\mu_j\) for women.

The idiosyncratic preference parameters are assumed to be independent of each other and across individuals. We denote the distributions of \(\theta\) and \(\mu\) by \(F(\theta)\) and \(G(\mu)\) and assume that these distributions are symmetric around their zero means. This specification is rather restrictive because one might expect some correlations between the taste parameters and the observable attributes. For instance, individuals that have a low cost of schooling may also have a high earning capacity and individuals may derive different benefits from marriage depending on the observed quality of their spouses. One may also expect a correlation between the emotional valuations of the marriage by the two spouses. Thus, the model is very basic and intended mainly as an illustration of the possible feedbacks between the marriage market and investment in schooling.

### 3.2 The Marriage Market

Any stable assignment of men to women must maximize the aggregate surplus over all possible assignments (Shapley and Shubik, 1972). The dual of this linear programming problem posits the existence of non-negative shadow prices associated with the constraints of the primal that each person can be either single or married to one spouse. We denote the shadow price of woman \(j\) by \(u_j\) and the shadow price of man \(i\) by \(v_i\). The complementarity slackness conditions require that

\[
z_{I(i)J(j)} + \theta_i + \theta_j \leq v_i + u_j, \tag{3}\]

with equality if \(i\) and \(j\) are married and inequality otherwise.

Condition (3) is equivalent to

\[
v_i = \max \{ \max_{j} [z_{I(i)J(j)} + \theta_i + \theta_j - u_j], 0]\}
\]

\[
u_j = \max \{ \max_{i} [z_{I(i)J(j)} + \theta_i + \theta_j - v_i], 0]\}, \tag{4}\]

\(^8\)Because we assume away the credit market, the rate of return from schooling investment depends on consumption decisions and is in utility terms.
which means that the assignment problem can be decentralized. That is, given the shadow prices \( u_j \) and \( v_i \), each agent marries a spouse that yields the highest “profit.” Alternatively, we can view the shadow prices \( u_j \) and \( v_i \) as the reservation utility levels that woman \( j \) and man \( i \) require to participate in any marriage.

Our specification imposes a restrictive but convenient structure in which the interactions between agents depend on their group affiliation only, i.e., their level of schooling. Therefore, the endogenously-determined shadow prices of married man \( i \) in \( I(i) \) and married woman \( j \) in \( J(j) \) can be written in the form,

\[
v_i = V_{I(i)} + \theta_i \quad \text{and} \quad u_j = U_{J(j)} + \theta_j, \tag{5}
\]

where

\[
V_i = \max_J [z_{IJ} - U_j] \quad \text{and} \quad U_j = \max_I [z_{IJ} - V_i] \tag{6}
\]

are the shares that the partners receive from the material surplus of the marriage (not accounting for the idiosyncratic effects \( \theta_i \) and \( \theta_j \)). All agents of a given type receive the same share of the material surplus \( z_{IJ} \) no matter whom they marry, because all the agents on the other side rank them in the same manner. Any man (woman) of a given type who asks for a higher share than the “going rate” cannot obtain it because he (she) can be replaced by an equivalent alternative.

Although we assume equal numbers of men and women in total, it is possible that the equilibrium numbers of educated men and women will differ. We shall assume throughout that there are some uneducated men who marry uneducated women and some educated men who marry educated women. This means that the equilibrium shares must satisfy

\[
U_2 + V_2 = z_{22}, \tag{7}
\]
\[
U_1 + V_1 = z_{11}. \tag{8}
\]

We can then classify the possible matching patterns as follows. Under strict positive assortative mating, educated men marry only educated women and uneducated men marry only uneducated women. Then,

\[
U_1 + V_2 \geq z_{21}, \tag{9}
\]
\[
U_2 + V_1 \geq z_{12}. \tag{10}
\]

If there are more educated men than women among the married, some educated men will marry uneducated women and condition (9) also will hold as an equality. If there are more educated women than men among the married, equation (10) will
hold as an equality. It is impossible that all four conditions will hold as equalities because this would imply

$$z_{22} + z_{11} = z_{12} + z_{21},$$  \hspace{1cm} (11)

which violates assumption (2) that the education levels of the spouses are comple-
ments. Thus, either educated men marry uneducated women or educated women marry uneducated men but not both.

When types mix and there are more educated men than educated women among
the married, conditions (7) to (9) imply

$$U_2 - U_1 = z_{22} - z_{21},$$

$$V_2 - V_1 = z_{21} - z_{11}.$$  \hspace{1cm} (12)

If there are more educated women then men among the married then conditions (7),
(8) and (10) imply

$$V_2 - V_1 = z_{22} - z_{12},$$

$$U_2 - U_1 = z_{12} - z_{11}.$$  \hspace{1cm} (13)

One may interpret the differences $U_2 - U_1$ and $V_2 - V_1$ as the *return to schooling in marriage* for women and men, respectively. The quantity $z_{22} - z_{21}$ which reflects
the contribution of an educated woman to the material surplus of a marriage with
an educated man provides an *upper bound* on the return that a woman can obtain
through marriage, while her contribution to a marriage with an uneducated man,
$z_{12} - z_{11}$, provides a *lower bound*. When there are more educated women than men,
analogous bounds apply to men. When types mix in the marriage market equilibrium,
we see that the side that is in short supply receives the marginal contribution to
a marriage with an educated spouse, while the side in excess supply receives the
marginal contribution to a marriage with an uneducated spouse.

One issue of concern is whether the “material shares” defined above are non-
negative. In practice, if the only means to transfer utility within couples is via the
transfer of consumption goods, which are bounded from below at zero, then the non-
negativity constraints on consumption bind, and utility is no longer transferable. To
avoid such outcomes, we provide in the Appendix in Section 6 the sufficient conditions
which ensure that, in equilibrium, $V_I$ and $U_J$ are all strictly positive. And in the
analysis hereafter, we shall provide examples in which the material shares are strictly
positive in equilibrium.\(^9\)

\(^9\)In a more general analysis, one may consider additional means to transfer utility within marriage,
3.3 Investment Decisions

We assume rational expectations so that, in equilibrium, individuals know $V_I$ and $U_J$, which are sufficient statistics for investment decisions. Given these shares and knowledge of their own idiosyncratic preferences for marriage, $\theta$, and costs of schooling, $\mu$, agents know for sure whether or not they will marry in the second period, conditional on their choice of schooling in the first period.

Man $i$ chooses to invest in schooling if

$$z_{20} - \mu_i + \max(V_2 + \theta_i, 0) > 2z_{10} + \max(V_1 + \theta_i, 0).$$

(14)

Similarly, woman $j$ chooses to invest in schooling if

$$z_{02} - \mu_j + \max(U_2 + \theta_j, 0) > 2z_{01} + \max(U_1 + \theta_j, 0).$$

(15)

Figure 5 describes the choices made by different men. Men for whom $\theta < -V_2$ do not marry and invest in schooling if and only if $\mu < R^m \equiv z_{20} - 2z_{10}$. Men for whom $\theta > -V_1$ always marry and they invest in schooling if and only if $\mu < R^m + V_2 - V_1$. Finally, men for whom $-V_2 < \theta < -V_1$ marry if they acquire education and do not marry if they do not invest in schooling. These individuals will acquire education if $\mu < R^m + V_2 + \theta$. In this range, there are two motivations for schooling: to raise future earning capacity and to enhance marriage. We shall assume that the variability in $\theta$ and $\mu$ is large enough to ensure that all these regions are non empty in an equilibrium with positive $V_I$ and $U_J$. In particular, we assume that, irrespective of marital status, there are some men and women who prefer not to invest in schooling and some men and women who prefer to invest in schooling. That is, $\mu_{\max} > \max[R^m + z_{22} - z_{12}, R^m + z_{22} - z_{21}]$ and $\mu_{\min} < \min[R^m, R^w]$. We shall also assume that $\theta_{\min} < -z_{22}$ so that, irrespective of the education decision, there are some individuals who wish not to marry. Note, finally, that because the support of $F(.)$ extends to the positive range, there are always some educated men and women who marry and some uneducated men and women who marry.

[Figure 5 about here.]

The proportion of men who invest in schooling is

$$G(R^m)F(-V_2) + [1 - F(-V_1)]G(R^m + V_2 - V_1) + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta,$$

(16)

such as “signs of endearment”. In this case, the marital shares are no longer material only and can be negative or positive.
the proportion of men who marry is

\[ [1 - F(-V_1)] + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta, \]  

(17)

and the proportion of men who invest and marry is

\[ [1 - F(-V_1)]G(R^m + V_2 - V_1) + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta. \]  

(18)

The higher are the returns from schooling in the labor market, \( R^m \), and in marriage, \( V_2 - V_1 \), the higher is proportion of men who acquire schooling. A common increase in the levels \( V_2 \) and \( V_1 \) also raises investment because it makes marriage more attractive and schooling obtains an extra return within marriage. For the same reason, an increase in the market return \( R^m \) raises the proportion of men that marry. Analogous expressions hold for women.

### 3.4 Equilibrium

In the marriage market equilibrium, the numbers of men and women who marry must be the same. Using equation (17) and applying symmetry, we can write this condition as

\[ F(V_1) + \int_{V_1}^{V_2} G(R^m + V_2 - \theta)f(\theta)d\theta = F(U_1) + \int_{U_1}^{U_2} G(R^w + U_2 - \theta)f(\theta)d\theta. \]  

(19)

Under strictly positive assortative mating, the numbers of men and women in each education group are equal. Given that we impose condition (19), it is necessary and sufficient to require that the numbers of men and women who marry but do not invest in schooling are the same. Using condition (18) and symmetry, we can derive this condition as

\[ F(V_1)G(-R^m + V_1 - V_2) = F(U_1)G(-R^w + U_1 - U_2). \]  

(20)

Together with conditions (7) and (8), conditions (19) and (20) yield a system of four equations in four unknowns that are, in principle, solvable.

If there is some mixing of types, equation (20) is replaced by an inequality and the shares are determined by the boundary conditions on the returns to schooling.
within marriage for either men or women, whichever is applicable. If there are more educated men than women among the married,

\[ F(V_1)G(-R^m + V_1 - V_2) < F(U_1)G(-R^w + U_1 - U_2) \]  

(20a)

and educated women receive their maximal return from marriage while men receive their minimal return so that condition (12) holds. Conversely, if there are more educated women than men among the married we have

\[ F(V_1)G(-R^m + V_1 - V_2) > F(U_1)G(-R^w + U_1 - U_2) \]  

(20b)

and educated men receive their maximal return from marriage while educated women receive their minimal return so that condition (13) holds. Together with conditions (7) and (8), we have four equations in four unknowns that are again, in principle, solvable.\(^\text{10}\)

The two types of solutions are described in Figures 6 and 7, where we depict the equilibrium conditions in terms of \( V_1 \) and \( V_2 \) after we eliminate \( U_1 \) and \( U_2 \) using (7) and (8). The two positively-sloped and parallel green lines in these figures describe the boundaries on the returns to schooling of men within marriage. The negatively sloped red line describes the combinations of \( V_1 \) and \( V_2 \) that maintain equality in the numbers of men and women who wish to marry. The positively-sloped blue line describes the combinations of \( V_1 \) and \( V_2 \) that maintain equality in the numbers of men and women that acquire no schooling and marry. The slopes of these lines are determined by the following considerations: An increase in \( V_1 \) (and a reduction in \( U_1 \)), keeping \( V_2 \) and \( U_2 \) constant, induces more men and fewer women to prefer marriage.

An increase in \( V_2 \) holding \( V_1 \) has a similar effect. Thus, \( V_1 \) and \( V_2 \) are substitutes in terms of their impact on the incentives of men to marry and \( U_1 \) and \( U_2 \) are substitutes in terms of their impact on the incentives of women to marry. Therefore, equality in the number of men and women who wish to marry can be maintained only if \( V_2 \) declines when \( V_1 \) rises.\(^\text{11}\) At the same time, an increase in \( V_1 \) (and a reduction in \( U_1 \)), keeping \( V_2 \) and \( U_2 \) constant, increases the number of men that would not invest and marry and reduces the number of women who wish to acquire no schooling and

\(^{10}\)Note the system of equations consisting of (7), (8) and (12) and the system consisting of (7), (8) and (13) impose only three independent requirements.

\(^{11}\)Differentiating (19),

\[
0 = \left\{ f(V_1)[1 - G(R^m + V_2 - V_1)] + f(z_{11} - V_1)[1 - G(R^w + z_{22} - z_{11} - (V_2 - V_1))] \right\} dV_1 \\
+ \left\{ (G(R^m)f(V_2) + G(R^w)f(z_{22} - V_2)) \right\} dV_2 \\
+ \int_{V_1}^{U_2} g(R^m + V_2 - \theta)f(\theta)d\theta + \int_{\hat{U}_1}^{U_2} g(R^w + U_2 - \theta)f(\theta)d\theta \right\} dV_2
\]
marry. Therefore, equality in the numbers of uneducated men and women who wish to marry can be maintained only if $V_2$ rises when $V_1$ rises so that the rates of return to education within marriage are restored.\textsuperscript{12}

As long as the model is completely symmetric, that is $R^m = R^w$ and $z_{12} = z_{21}$, the equilibrium is characterized by equal sharing: $V_2 = U_2 = z_{22}/2$ and $U_1 = V_1 = z_{11}/2$. With these shares, men and women have identical investment incentives. Hence, the number of educated (uneducated) men equals the number of educated (uneducated) women, both among the singles and the married. Such a solution is described by point $e$ in Figure 6, where the lines satisfying conditions (19) and (20) intersect. There is a unique symmetric equilibrium. However, with asymmetry, when either $R^m \neq R^w$ or $z_{12} \neq z_{21}$, there may be a mixed equilibrium where the line representing condition (19) intersects either the lower or upper bound on $V_2 - V_1$ so that condition (20) holds as an inequality. Such a case is illustrated by the point $e\prime$ in Figure 7. In this equilibrium, educated men obtain the lower bound on their return to education within marriage, $z_{21} - z_{11}$. The equilibrium point $e\prime$ is on the lower bound and above the blue line satisfying condition (20), indicating excess supply of educated men.

[Figures 6 and 7 about here.]

3.4.1 Efficiency

An important issue is whether premarital investments in education are efficient. The concern arises when ex-post bargaining within marriage determines the division of the gains between the two partners. Because each person bears the full cost of his/her investment prior to marriage and receives only part of the gains, there is a potential for under investment. This is known as the “hold-up problem.” In contrast, models that allow endogenous assignments or intra-marital spousal allocations can generate over-investment in schooling as individuals seek to increase their attractiveness to potential

implying that

$$\frac{dV_2}{dV_1} < 0.$$  

\textsuperscript{12}The slope line satisfying condition (20) must exceed 1 because

$$f(V_1)G(R^m - (V_1 - V_2)) + f(z_{11} - V_1)G(R^m - (z_{22} - z_{11})) + (V_1 - V_2)\frac{dV_1}{dV_1}$$  

$$= F(V_1)g(R^m - (V_1 - V_2)) + F(z_{11} - V_1)g(R^m - (z_{22} - z_{11})) + (V_1 - V_2)\frac{dV_2}{dV_1}$$

and therefore

$$\frac{d(V_2 - V_1)}{dV_1} = \frac{dV_2}{dV_1} - 1 > 0.$$
mates or they recognize that their intra-marital allocations depend on their outside options (which are in turn influenced by their educational attainment). Nonetheless, due to the fact that our marriage markets operate frictionlessly and they are large, we can demonstrate that individuals’ pre-marital investments are efficient.

Consider, first, a “mismatched” couple \((i, j)\) in a mixed equilibrium, such that the wife is not educated and the husband is. Recall that the partners selected their schooling levels prior to being matched and without any coordination. Could the mismatched partners have both been better off if woman \(j\) had acquired schooling? If woman \(j\) had gotten educated, the partners together would have gained \(z_{22} - z_{21}\) in terms of marital output but the cost of schooling for woman \(j\) would have been her forgone earnings, \(2z_{01} - z_{02}\), plus her idiosyncratic non-monetary cost, \(\mu_j\). By assumption, woman \(j\) married and chose not to invest. Hence, by (15),

\[
z_{02} - \mu_j + U_2 < 2z_{01} + U_1. \tag{21}
\]

Also, because woman \(j\) anticipated correctly that there is an excess supply of educated men, by (12), we have

\[
U_2 - U_1 = z_{22} - z_{21}. \tag{22}
\]

Therefore,

\[
z_{22} - z_{21} < 2z_{01} + \mu_j - z_{02}, \tag{23}
\]

which means that the net cost of woman \(j\) getting educated exceeds the benefit to the couple. Thus, there is no joint net gain from such a rearrangement of investment choices.

Next, consider a strictly assortative equilibrium and a married couple \((i, j)\) such that neither spouse is educated. Could this couple have been better off had the partners coordinated their educational investments so that they both had acquired education? If they had done so, the partners together would have gained \(z_{22} - z_{11}\) in terms of marital output but their total cost of schooling would have been the sum of \(2z_{01} - z_{02} + \mu_j\) and \(2z_{10} - z_{20} + \mu_i\). Then, by (14) and (15), we would have had

\[
z_{02} - \mu_j + U_2 < 2z_{01} + U_1, \tag{24}
\]

\[
z_{20} - \mu_i + V_2 < 2z_{10} + V_1,
\]

and by (12)

\[
U_2 - U_1 = V_2 - V_1 = \frac{z_{22} - z_{11}}{2}. \tag{25}
\]

Therefore,

\[
z_{22} - z_{11} < 2z_{01} + \mu_j - z_{02} + 2z_{10} - z_{20} + \mu_i, \tag{26}
\]

16
which again means that there is no joint gain from such a rearrangement of investment choices.

Such calculations can be carried out for all types of matches and they yield the same result; the equilibrium shares that individuals expect to receive within marriage induce them to fully internalize the social gains from their premarital investments. An important piece of this argument is that the marriage market is large in the sense that individual perturbations in investment do not affect the equilibrium shares. In particular, a single agent cannot tip the market from excess supply to excess demand of educated men or women. This efficiency property of large and frictionless marriage markets has been noted by Cole et al. (2001), Felli and Roberts (2002) Peters and Siow (2002) and Iyigun and Walsh (forthcoming). In contrast, markets with frictions or small number of traders are usually characterized by inefficient premarital investments (Lommerud and Vagstad, 2000, Baker and Jacobsen, 2005).13

4 Gender Differences in the Incentive to Invest

In this section, we discuss differences between women and men that can cause them to invest at different levels. We discuss two possible sources of asymmetry:

- In the labor market, women may receive lower wages than men; this would lower the schooling return for single women.

- In marriage, women may be required to take care of the children; this would lower the schooling return for married women.

Either of the above causes can induce women to invest less in schooling. Therefore, the lower incentives of women to invest can create equilibria with mixing, where educated men are in excess supply and some of them marry less-educated women.

To illustrate these effects we shall perform several comparative statics results, starting from a benchmark equilibrium with strictly positive assortative matching, resulting from a complete equality between the sexes in wages and household roles such that $w_{1m} = w_{1w} = w_1, w_{2m} = w_{2w} = w_2$ and $\tau = 0$.

13Peters (2005) formulates premarital investments as a Nash game in which agents take as given the actions of others rather than the expected shares (as in a market game). In this case, inefficiency can persist even as the number of agents approaches infinity. The reason is that agents play mixed strategies that impose on other agents the risk of being matched with an uneducated spouse, leading to under-investment in schooling.
4.1 The Household

We use a rudimentary structural model to trace the impact of different wages and household roles of men and women on the marital output and surplus. We assume that, irrespective of the differences in wages or household roles, men and women have the same preferences given by

\[ u = cq + \theta, \tag{27} \]

where \( c \) is a private good, \( q \) is a public good that can be shared if two people marry but is private if they remain single, and \( \theta \) is the emotional gain from being married (relative to remaining single). The household public good is produced according to a household production function

\[ q = e + \gamma t, \tag{28} \]

where \( e \) denotes purchased market goods, \( t \) is time spent working at home and \( \gamma \) is an efficiency parameter that is assumed to be independent of schooling.\(^{14}\)

This specification implies transferable utility between spouses and allows us to trace the impact of different market wages or household roles on the decisions to invest and marry. Time worked at home is particularly important for parents with children. To simplify, we assume that all married couples have one child and that rearing it requires a specified amount of time \( t = \tau \), where \( \tau \) is a constant such that \( 0 \leq \tau < 1 \). By assumption, individuals who never marry have no children and for them we set \( \tau = 0 \).\(^{15}\)

Initially, we shall assume that, due to social norms, all the time provided at home is supplied by the mother. Hence, women spend a fraction \( \tau \) of their time on child rearing and \( 1 - \tau \) of it on market work. In Section 4.4 below, we shall discuss how endogenous specialization in home work by the husband or the wife could change our analysis.

If man \( i \) of class \( I \) with wage \( w^m_{I(i)} \) marries woman \( j \) of class \( J \) with wage \( w^w_{J(j)} \), their joint income is \( w^m_{I(i)} + (1 - \tau)w^w_{J(j)} \). Any efficient allocation of the family resources maximizes the partners’ sum of utilities given by \( [w^m_{I(i)} + (1 - \tau)w^w_{J(j)} - e](e + \tau \gamma) + \theta_i + \theta_j \). In an interior solution with a positive money expenditure on the public good, the

\(^{14}\)A plausible generalization is to allow the mother’s schooling level to affect positively child quality. This would be consistent with the findings of Behrman (1997) and Glewwe (1999), for example. However, the qualitative results will be unaffected as long as schooling has a larger effect on market wages than on productivity at home. The fact that educated women participate more in the labor market than uneducated women supports such an assumption.

\(^{15}\)We make no distinction here between cohabitation and marriage. So either no one cohabitates, or, if two individuals cohabitate, they behave as a married couple.
maximized marital output is

\[ o_{ij} = \frac{[w_i^m + \tau \gamma + (1 - \tau)w_j^w]^2}{4} + \theta_i + \theta_j. \]  \hspace{1cm} (29)

Note that the wages of the husband and wife complement each other in generating marital output, which is a consequence of sharing the public good.\(^{16}\)

An unmarried man \(i\) solves

\[ \max_{c_i,e_i} c_i e_i \]  \hspace{1cm} (30)

subject to

\[ c_i + e_i = w_i^m, \]  \hspace{1cm} (31)

and his optimal behavior generates a utility level of \((w_i^m/2)^2\). A single woman \(j\) solves an analogous problem and obtains \((w_j^w/2)^2\). Therefore, the total material surplus generated by the marriage in the second period is

\[ s_{ij} = \frac{[w_i^m + \tau \gamma + (1 - \tau)w_j^w]^2 - (w_i^m)^2 - (w_j^w)^2}{4} + \theta_i + \theta_j \equiv z_{I(i)J(j)} + \theta_i + \theta_j. \]  \hspace{1cm} (32)

The surplus of a married couple arises from the fact that married partners jointly consume the public good. If the partners have no children and \(\tau = 0\), the gains arise solely from the pecuniary expenditures on the public good. In this case, the surplus function is symmetric in the wages of the two spouses. If the couple has a child, however, and the mother takes care of it then the mother’s contribution to the household is a weighted average of her market wage and productivity at home. We assume that \(w_2^w > \gamma > w_1^w\) so that having children is costly for educated women but not for uneducated women. The surplus function in (32) maintains complementarity between the wages of the husband and wife, which is a consequence of sharing the public good. However, the assumed asymmetry in household roles between men and women implies that a higher husband’s wage always raises the surplus but a higher

\(^{16}\)The first order condition for \(e\) is

\[ [w_i^m + (1 - \tau)w_j^w - e] - (e + \tau \gamma) \leq 0. \]

Hence, \(e = [w_i^m + (1 - \tau)w_j^w - \gamma] / 2\) in an interior solution. The maximized material output in this case is \([w_i^m + \gamma + (1 - \tau)w_j^w]^2 / 4\). If \(e = 0\), the maximal material output is \([w_i^m + (1 - \tau)w_j^w] \tau \gamma\), which would imply an additive surplus function, contradicting our assumption of complementarity. A sufficient condition for a positive \(e\) is \(w_i^m + (1 - \tau)w_j^w > \tau \gamma\) and we assume hereafter that this condition holds.
mother’s wage can reduce the surplus. In other words, it may be costly for a high-wage woman to marry and have a child because she must spend time on child care, while if the mother does not marry, her utility as a single remains \( w_{J(j)}^2 / 4 \). In addition, it is no longer true that \( z_{21} = z_{12} \) as in the case with wage equality and \( \tau = 0 \). In particular, when the wages of men and women are equal but \( \tau > 0 \), we have

\[
z_{21} - z_{12} = \frac{\tau(w_2 - w_1)}{2} [(1 - \tau) \frac{w_2 + w_1}{2} + \tau \gamma] > 0,
\]

which means that the marriage of an educated man with an uneducated woman creates a larger marital surplus than the marriage of an educated woman with an uneducated man.

### 4.2 The Impact of the Wage Gap

We are now ready to examine the implications of gender wage differences. The gender difference in wages can be an outcome of discrimination associated, for instance, with fewer opportunities for investment on the job. Such discrimination can reduce or increase the incentives of women to invest, depending on whether discrimination is stronger at the low or high levels of schooling.

Define the (relative) wage gap among educated individuals as \( d_2 = \frac{w_2^m}{w_2^m} \) and let the gender wage gap between uneducated individuals be \( d_1 = \frac{w_1^w}{w_1^w} \). Starting from the benchmark equilibrium with strictly assortative mating and equal shares (point e in Figure 8), we examine the impact of a difference in the market returns from schooling of women and men. Specifically, we consider an increase in the wage of educated men, \( w_2^m \), combined with a reduction in the wage of educated women, \( w_2^w \), holding the wage of uneducated men at the benchmark value, \( w_1 \). To isolate the role of market returns, we assume that the increase in the wage of educated men exactly compensates the reduction in the wage of educated women so that marital output is unaffected and symmetry is maintained.\(^{17}\) In other words, the change in wages affect directly only the returns as singles, \( R^m \) and \( R^w \). For now, we assume that discrimination is uniform across schooling levels so that \( d_1 = d_2 \equiv d < 1 \) and women have a lower market return from schooling investment than men.\(^{18}\)

---

\(^{17}\) When wages change \( z_{(i)J(j)} \) usually changes. Also, when wages differ by gender, we generally do not maintain symmetry in the contribution of men and women to marriage so that \( z_{12} \neq z_{21} \). It is only in the special case in which the product \( w_{J(j)}^m w_{J(j)}^w \) remains invariant under discrimination that the marital surplus generated by all marriages is intact. The qualitative results for shares are not affected by this simplification.

\(^{18}\) In standard human capital models where the only cost of investment is forgone earnings and the only return is higher future earnings, uniform discrimination has no impact on investment. In
shall discuss a case in which discrimination against educated women is weaker so that \( d_1 < d_2 < 1 \).

With uniform discrimination, the returns to investment in schooling for never married men and women, respectively, are

\[
R^m = z_{20}^m - 2z_{10}^m = \left( \frac{w_2^m}{2} \right)^2 - 2 \left( \frac{w_1^m}{2} \right)^2, \tag{34}
\]

and

\[
R^w = z_{02}^w - 2z_{01}^w = \left( \frac{w_2^w}{2} \right)^2 - 2 \left( \frac{w_1^w}{2} \right)^2 = d^2 R^m < R^m. \tag{35}
\]

The higher market return from schooling of men encourages their investment in schooling and also strengthens their incentives to marry, because schooling obtains an additional return within marriage. In contrast, the lower return to schooling for women reduces their incentives to invest and marry. These changes create excess supply of men who wish to invest and marry. Consequently, to restore equilibrium, the rates of returns that men receive within marriage must decline implying that, for any \( V_1 \), the value of \( V_2 \) that satisfies conditions (19) and (20) must decline. These shifts in the equilibrium lines are represented by the broken blue and red lines in Figure 8.

For moderate changes in wages, strictly positive assortative mating continues to hold. However, the equilibrium value of \( V_2 \) declines and educated men receive a lower share of the surplus than they do with equal wages in any marriage. That is, as market returns of men rise and more men wish to acquire education the marriage market response is to reduce the share of educated men in all marriages. When the gap between \( R^m \) and \( R^w \) becomes large, the equilibrium shifts to a mixed equilibrium, where some educated men marry uneducated women. That is, because of their higher tendency to invest, some educated men must “marry down.” This equilibrium is represented by the point \( e^f \) in Figure 8, where the broken red line representing equality in the numbers of men and women that wish to marry (condition (19)) intersects the green line representing the lower bound on the share that educated men obtain in the marital surplus, \( z_{21} - z_{11} \). As seen, both \( V_1 \) and \( V_2 \) are lower in the new equilibrium so that all men (women), educated and uneducated, receive lower (higher) shares of the material surplus when men have stronger market incentives to invest in schooling than women.

[Figure 8 about here.]
These results regarding the shares of married men and women in the *material surplus* must be distinguished from the impact of the shares in the *material output*. If men get a higher return from schooling as singles (due to the fact that their labor-market return from schooling is higher than that of women), then their share of the material output can be higher even though they receive a lower share of the surplus. The same remark applies to our subsequent analysis as well; one can obtain sharper comparatives statics results on shares of the surplus than on shares of the material output.

### 4.3 The Impact of Household Roles

Recall that we assume that the wife alone spends time on child care. To investigate the impact of this constraint, we start again at the benchmark equilibrium and examine the impact of an increase in $\tau$, holding the wages of men and women at their benchmark values, that is $w^m_1 = w^w_1 = w_1$ and $w^m_2 = w^w_2 = w_2$. Such an increase reduces the contribution that educated women make to marital output and raises the contribution of uneducated women. That is, $z_{11}$ and $z_{21}$ rise because uneducated women are more productive at home, $\gamma > w_1$, while $z_{12}$ and $z_{22}$ decline because educated women are less productive at home, $\gamma < w_2$. Consequently, both equilibrium lines corresponding to conditions (19) and (20) shift down so the $V_2$ is lower for any $V_1$. At the same time, the boundaries on the rate of returns from schooling that men can obtain within marriage shift as $z_{21} - z_{11}$ rises and $z_{22} - z_{12}$ declines. These changes are depicted in Figure 9.

For moderate changes in $\tau$, strictly positive assortative mating with equal sharing continues to hold. As long as a symmetric equilibrium is maintained, the returns to schooling that men and women receive within marriage, $V_2 - V_1$ and $U_2 - U_1$, are equal. Hence, men and women have the same incentives to invest. But because the material surplus (and consequently utilities within marriage) of educated men and women, $z_{22}/2$, declines with $\tau$, while the material surplus of uneducated men and women, $z_{11}/2$, rises, both men and women will reduce their investments in schooling by the same degree.

As $\tau$ rises further, the difference in the contributions of men and women to marriage can rise to the extent that an educated man contributes to a marriage with uneducated woman more than an educated woman contributes to a marriage with an
educated man.\textsuperscript{19} That is, 
\[ z_{21} - z_{11} > z_{22} - z_{21}. \] (36)

Condition (36) implies that the lower bound on the return to schooling that men receive within marriage exceeds the upper bound on the return to schooling that women receive within marriage. In this event, the symmetric equilibrium in Figure 9 is eliminated and instead there is a mixed equilibrium with some educated men marrying uneducated women (point \( e \) in Figure 9). This outcome reflects the lower incentive of educated women to enter marriage and the stronger incentive of men to invest because their return from schooling within marriage, \( V_2 - V_1 = z_{21} - z_{11} \), exceeds the return to schooling that women can obtain within marriage. Consequently, some educated men must “marry down” and match with uneducated women.

[Figure 9 about here.]

### 4.4 Division of Labor and Career Choice

We can further refine the family decision problem by letting the partners decide who shall take care of the children. Reinterpreting \( \tau \) as a temporal choice, imagine that one of the partners must first spend \( \tau \) units of time during marriage on the child and later enter the labor market and work for the remainder of the period (length \( 1 - \tau \)).

An important idea of Becker (1991, ch. 2) is that wage differences among identical spouses can be created endogenously and voluntarily because of learning by doing and increasing returns. Thus, it may be optimal for the household for one of the spouses to take care of the child and for the other to enter the labor market immediately, thereby generating a higher wage in the remainder of the period. Thus, by choosing schooling ahead of marriage one can influence his/her household role within marriage.

\textsuperscript{19} Consider the expression 
\[ h(w_1, w_2, \tau) \equiv 2z_{21} - z_{11} - z_{22} = 2[w_2 + \tau \gamma + (1 - \tau)w_1]^2 - [w_1 + \tau \gamma + (1 - \tau)w_1]^2 - [w_2 + \tau \gamma + (1 - \tau)w_2]^2 \]

as a function of \( w_1 \) and \( w_2 \) and \( \tau \). For \( w_1 = w_2 = \gamma \), \( h(\gamma, \gamma, \tau) = 0 \) and 
\[ h_1(\gamma, \gamma, \tau) = -4\gamma \tau, \]
\[ h_2(\gamma, \gamma, \tau) = 4\gamma \tau. \]

Therefore, for a positive \( \tau \), \( w_1 \) slightly below \( \gamma \) and \( w_2 \) slightly above \( \gamma \), \( h(w_1, w_2, \tau) > 0 \). Also 
\[ h_3(w_1, w_2, \tau) = (w_2 - w_1)[w_2(4 - 2\tau) + 2\tau(2\gamma - w_1)] > 0 \]

and for all \( w_2 > \gamma > w_1 \), \( h(w_1, w_2, 0) < 0 \) and \( h(w_1, w_2, 1) > 0 \). Hence, the larger is \( \tau \) the broader will be the range in which \( h(w_1, w_2, 0) > 0 \).
Because we assume transferable utility between spouses, household roles will be determined efficiently by each married couple, as long as there is ability to commit on the transfers to the party that sacrifices outside options when he/she acts in a manner that raises the total surplus. In particular, the partners will assign the spouse with the lower wage to take care of the child. In the previous analysis, there was no need for such a commitment because the division of the surplus was fully determined by attributes that are determined prior to marriage via competition over mates who can freely replace partners. However, the costs of caring for the child can differ between the two spouses, if time spent on child care affects one’s labor market wages subsequently. Thus, implementing the efficient outcome might require some form of commitment even if (re)matching is frictionless. A simple, enforceable, prenuptial contract is one in which both partners agree to pay the equilibrium shares $V_I$ to the husband and $U_J$ to the wife in case of divorce. By making those shares the relevant threat points of each spouse, this contract sustains the equilibrium values $V_I$ and $U_J$ in marriage, which is sufficient to attain the efficient household division of labor.

If there is discrimination against women and women receive lower market wages than men, then it is the wife that will be typically assigned to stay at home, which will erode her future market wage and reinforce the unequal division of labor. Similarly, if there are predetermined household roles such that women must take care of the child, then women will end up with lower market wages. Thus, inequality at home and the market are interrelated. Models of statistical discrimination tie household roles and market wages through employers’ beliefs about female participation. Typically, such models generate multiple equilibria and inefficiency (Hadfield, 1999, Lommerud and Vagstad, 2002). Here, we do not require employers’ beliefs to be correct. Instead, we think of household roles and discrimination as processes that evolve slowly and can be taken as exogenous in the medium run.

### 4.5 Why Women May Acquire More Schooling than Men

We have examined two possible reasons why women may invest less than men in schooling. The first is that women may receive lower return from schooling investment in the market because of discrimination. The second reason is that women may receive a lower return to schooling in marriage because of the need to care of children (due to social and cultural norms or the biological time requirements of child care), in which case the contribution of schooling to marital output is lower.

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1 For related papers that emphasize the same dual-feedback mechanism between the intensity of homework and labor market wages we discuss here, see Albanesi and Olivetti (2005, 2006) and Chichilnisky (2005).
Over time, fertility has declined and women’s wages have risen in industrialized countries, a pattern being replicated in many developing countries too. This is consistent with increased investment in education by women. The fact that women are now slightly more educated than men, on average, appears surprising given the fact that women still earn substantially less than men. However, in dealing with investments in education, the crucial issue is whether the gender wage gap rises or declines with schooling, or equivalently, whether women obtain a higher rate of return from schooling. There is some evidence that this is indeed the case and that the gender wage gap declines with schooling (Dougherty, 2005).

Now consider a comparison of the following two situations. An “old” regime in which married women must spend a relatively large fraction of their time at home and a “new” regime in which, because of reductions in fertility and improved technology in home production, married women spend less time at home and work more in the market (Greenwood, Seshadri and Yorukoglu, 2005). Assume further that women suffer from statistical discrimination because employers still expect them to invest less on the job. However, this discrimination is weaker against educated women because they are expected to stay longer in the labor market than uneducated women. It is then possible that in the new regime women will invest in schooling more than men. The presence of discrimination raises the return of women relative to men because schooling serves as an instrument for women to escape discrimination. The fact that women are still tied up in homework lowers their return from schooling relative to men because women obtain lower returns from schooling within marriage. However, as women raise their labor force participation, this second effect weakens and the impact of discrimination can dominate.

One issue that we shall need to address is whether partners act efficiently or whether household roles are determined by norms. We shall provisionally assume that in the old regime household roles obeyed the rule that the mother always works at home but in the new regime the roles are determined efficiently. We shall later discuss some alternative assumptions.

In Figure 10, we display the transition between the two regimes. We assume that \( d_2 > d_1 \) so that discrimination against women is lower at the higher level of schooling. This feature generates stronger incentives for women than men to invest in schooling. However, the fact that women must spend time working at home has the opposite effect. We then reduce the amount of time that the mother has to spend at home, \( \tau \), and raise the wage that educated women receive (so that \( d_2 \) rises), which strengthens the incentives of women to invest in schooling and to marry. Therefore, holding the marriage surplus \( z_{1,1} \) constant, an increase in \( V_2 \) relative to \( V_1 \) is required to maintain equality between the number of men who wish to invest and marry and the number
of women who wish to invest and marry. This effect is represented by the upwards shifts in the broken red and blue lines in Figure 10.\textsuperscript{21} The impact is assumed to be large enough to generate an equilibrium in which the two equilibrium requirements—equality of the numbers of men and women who acquire no schooling and marry (the broken blue line) and equality of the total numbers of men and women who wish to marry (the broken red line)—yield an intersection above the upper bound on the returns from schooling that men can receive within marriage. Therefore, strictly positive assortative mating cannot be sustained as an equilibrium and the outcome is a mixed equilibrium in which there are more educated women than men among the married and some educated women marry uneducated men. This new mixed equilibrium is indicated by the point $e''$ in Figure 10.

[Figure 10 about here.]

The result that there are more educated women among the married does not, by itself, imply that women invest in schooling more than men. But, in this example, we assume that women have a higher return from schooling as singles so that there are also more educated women among the singles. The basic logic is that the gender with higher labor market returns invests more in schooling and, as a consequence, has higher incentives to marry, which can drive the returns to schooling within marriage of that gender to the lower bound, where an excess supply of the educated members of that gender among the married as well as among the singles is created.

### 4.5.1 A Numerical Example

Suppose that $\mu$ and $\theta$ are uniformly distributed on the interval $[-4, 4]$ and $[-6, 6]$ respectively and that wages of men and women are given by

<table>
<thead>
<tr>
<th></th>
<th>Uneducated</th>
<th>Educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>men</td>
<td>$w_1^m = 2$</td>
<td>$w_2^m = 3$</td>
</tr>
<tr>
<td>women</td>
<td>$w_1^w = 1$</td>
<td>$w_2^w = 2.1$</td>
</tr>
</tbody>
</table>

Thus women earn less than men, $d_1 = .5$ and $d_2 = .7$. However, the absolute and relative wage gains from schooling are larger for women. We set $\gamma = 2$ so that

\textsuperscript{21}Because the marital surplus matrix, $z_{IJ}$, also changes, it is not always the case that the equilibrium curves shift up. In fact, for the parameters of Figure 10, there is a range over which the equilibrium line representing market-clearing in the marriage market shifts down. This, however, has no bearing on the equilibrium outcome.
educated women are more productive in the market and uneducated women are more productive at home. Note that educated women can earn more than uneducated men.

The assumed wages imply the utility levels of single men and women given below:

<table>
<thead>
<tr>
<th></th>
<th>Uneducated</th>
<th>Educated</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>$z_{10} = 1.00$</td>
<td>$z_{20} = 2.25$</td>
<td>$R^{m} = z_{20} - 2z_{10} = .25$</td>
</tr>
<tr>
<td>Women</td>
<td>$z_{01} = .25$</td>
<td>$z_{02} = 1.10$</td>
<td>$R^{w} = z_{02} - 2z_{01} = .60$</td>
</tr>
</tbody>
</table>

Thus, women enjoy higher returns from schooling as singles (in utility terms), reflecting the reduction in the gender wage gap as schooling rises.

Now consider a change in regimes such that the education premium rises for both sexes, the gender wage gap narrows, and the time requirement for child care falls. In particular, while the wage of uneducated men remains constant at $w_{1}^{m} = 2$, the wage of educated men rises to $w_{2}^{m} = 3.2$ and the wages of uneducated and educated women respectively become $w_{1}^{w} = 1.3$ and $w_{2}^{w} = 2.7$. These relative changes in men’s and women’s wages reflect a narrower gender wage gap (i.e., now we have $d_{1} = 0.65$ and $d_{2} = 0.85$). In addition, $\tau$ declines from 0.8 to 0.6. A direct effect of these changes is an increase in the returns to schooling of single women from $R^{w} = .60$ to $R^{w} = 1.00$ and an increase in that of men from $R^{m} = .25$ to $R^{w} = .56$. The marriage market implications of these changes are summarized in Tables 1-3 below.

### Table 1: Impact of parameter changes on marital surplus

**Old regime: $\tau = .8$, $\gamma = 2$, $d_{1} = .5$, $d_{2} = .7$**

<table>
<thead>
<tr>
<th></th>
<th>Uned. wife</th>
<th>Educ. wife</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uned. husband</td>
<td>$z_{11} = 2.36$</td>
<td>$z_{12} = 1.98$</td>
</tr>
<tr>
<td>Educ. husband</td>
<td>$z_{21} = 3.26$</td>
<td>$z_{22} = 2.95$</td>
</tr>
</tbody>
</table>

**New Regime: $\tau = .6$, $\gamma = 2$, $d_{1} = .65$, $d_{2} = .85$**

<table>
<thead>
<tr>
<th></th>
<th>Uned. wife</th>
<th>Educ. wife</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uned. husband</td>
<td>$z_{11} = 2.04$</td>
<td>$z_{12} = 2.72$</td>
</tr>
<tr>
<td>Educ. husband</td>
<td>$z_{21} = 3.07$</td>
<td>$z_{22} = 3.12$</td>
</tr>
</tbody>
</table>

A decrease in the amount of time worked at home, raises the contribution of an educated woman to the material surplus and lowers the contribution of an uneducated woman. In the old regime with $\tau = .8$, the material surplus declines with the education of the wife when the husband is uneducated, while in the new regime with $\tau = .6$, it rises. This happens because educated women are more productive in the
market than uneducated women but, by assumption, equally productive at home.\textsuperscript{22} Even if the wife has the higher wage, which is the case when the husband is uneducated, it is still costly to have an educated wife, because the opportunity cost is the wage of an uneducated woman that could replace the husband in home work if he would marry her.

\textbf{Table 2: Impact of parameter changes on the equilibrium shares}

<table>
<thead>
<tr>
<th>Parameter Changes</th>
<th>Uneducated</th>
<th>Educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old regime: $\tau = .8$, $\gamma = 2$, $d_1 = .5$, $d_2 = .7$</td>
<td>$V_1 = .84$</td>
<td>$V_2 = 1.74$</td>
</tr>
<tr>
<td></td>
<td>$U_1 = 1.51$</td>
<td>$U_2 = 1.20$</td>
</tr>
<tr>
<td>New regime: $\tau = .6$, $\gamma = 2$, $d_1 = .65$, $d_2 = .85$</td>
<td>$V_1 = 1.12$</td>
<td>$V_2 = 1.51$</td>
</tr>
<tr>
<td></td>
<td>$U_1 = 0.92$</td>
<td>$U_2 = 1.61$</td>
</tr>
</tbody>
</table>

The implied returns from schooling within marriage in the old regime are

\[ U_2 - U_1 = 1.20 - 1.51 = z_{22} - z_{21} = 2.95 - 3.26 = .31, \]

\[ V_2 - V_1 = 1.74 - .84 = z_{21} - z_{11} = 3.26 - 2.36 = .90. \]

That is, men receive the lower bound on their return from schooling within marriage while women receive the upper bound on their return from schooling. This pattern is reversed in the new regime:

\[ U_2 - U_1 = 1.61 - .92 = z_{12} - z_{11} = 2.72 - 2.03 = .69, \]

\[ V_2 - V_1 = 1.51 - 1.12 = z_{22} - z_{12} = 3.11 - 2.72 = .39, \]

where women receive their lower bound and men receive their upper bound. In the new regime, women receive higher returns from schooling and men receive lower returns within marriage, because the effective wage of educated women, $\tau \gamma + (1-\tau)w^e_2$, has risen. As a result, the highest gains accrue to the marriages between educated

\textsuperscript{22}For an important extension where the education level of women influences their productivity at home, see footnote 14.
women and uneducated men. This, in turn, makes educated men less valuable and uneducated men relatively more valuable in the marriage market.

**Table 3**: Impact of parameter changes on the investment and marriage rates*

<table>
<thead>
<tr>
<th></th>
<th>Old regime: ( \tau = .8, \gamma = 2, d_1 = .5, d_2 = .7 )</th>
<th>New regime: ( \tau = .6, \gamma = 2, d_1 = .65, d_2 = .85 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Married</td>
<td>Unmarried</td>
</tr>
<tr>
<td>Educ.</td>
<td>.423, .340</td>
<td>.147, .193</td>
</tr>
<tr>
<td>Uned.</td>
<td>.245, .328</td>
<td>.185, .139</td>
</tr>
<tr>
<td>All</td>
<td>.668, .668</td>
<td>.332, .332</td>
</tr>
</tbody>
</table>

* First and second entries in each cell refer to men and women resp.

In the old regime, more men invest in schooling than women and some educated men marry down to match with uneducated women. This pattern is reversed in the new regime and women invest in schooling more than men and some educated women marry down to join uneducated men. Moreover, fewer men choose to get educated in the new regime than in the old one, despite the fact that the education premium has risen for men too.\(^{23}\) Educated women are less likely to marry than educated men, because their return for schooling in marriage is lower. In anticipation of that, they will engage in home work if they marry educated men. This effect is stronger in the old regime in which the time requirements at home are relatively larger. In the new regime, the marriage market returns for educated men is lower and that for uneducated men is higher. Hence, in the new regime, fewer educated men choose to marry and more educated men remain single, while more uneducated men marry and fewer uneducated men remain single.

\(^{23}\)In fact, the education premium has risen more for men than it has for women: in the old regime, women’s education premium, \( w_2^w / w_1^w \), equaled 2/1 = 2 and men’s education premium, \( w_2^m / w_1^m \), equaled 3 / 2 = 1.5. In the new regime, they have respectively become 3.2 / 2 = 1.6 and 2.72 / 1.3 = 2.09. Hence, men’s education premium has risen by more than six percent, while women’s education premium has increased by less than five percent.
We can use these examples to discuss the impact of norms. To begin with suppose that in the old regime couples acted efficiently and if the wife was more educated than her husband, she went to work full time and the husband took care of the child. In that case the matrix of the marital surplus would be

Table 4: Impact of norms on material surplus

<table>
<thead>
<tr>
<th>Regime</th>
<th>Husband</th>
<th>Wife</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old regime: $\tau = .8, \gamma = 2, d_1 = .5, d_2 = .7$, without norms</td>
<td>Uned.</td>
<td>Edu.</td>
</tr>
<tr>
<td></td>
<td>z_{11} = 2.36</td>
<td>z_{12} = 2.10</td>
</tr>
<tr>
<td>New Regime: $\tau = .6, \gamma = 2, d_1 = .65, d_2 = .85$, with norms</td>
<td>Uned.</td>
<td>Edu.</td>
</tr>
<tr>
<td></td>
<td>z_{11} = 2.03</td>
<td>z_{12} = 1.83</td>
</tr>
<tr>
<td></td>
<td>Edu.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>z_{21} = 2.89</td>
<td>z_{22} = 2.93</td>
</tr>
</tbody>
</table>

In contrast to the case in which the mother always works in home, we see in Table 4 that the education levels now become substitutes, namely $z_{11} + z_{22} < z_{12} + z_{21}$, implying that we can no longer assume that there will be always some educated men married to some educated women and some educated women married to educated men. More specifically, an educated woman contributes more to an uneducated man than she does to an educated man (i.e. $z_{12} - z_{11} > z_{22} - z_{21}$) so that uneducated men can bid away the educated women from educated men. Thus acting efficiently can have a marked difference on the patterns of assortative mating.\(^{24}\)

Consider now the possibility that the norms persist also in the new regime and the mother must work at home even if she is more educated than her husband. However, the mixing is reversed from one in which women invest more than men in the efficient case (when norms are removed) to the case in which men invest more than women in the inefficient case (when norms remain in effect). This is reflective of the fact that, when norms remain in effect and there exist married couples for whom the wife is more educated, the presence of norms would significantly reduce the gains from mixed marriages (as $z_{12}$ drops from 2.72 to 1.83 and as displayed in Tables 4 and 5).

Hence, fewer women and more men choose to get educated when norms remain in effect and educated women go from being in surplus to being in shortage.

\(^{24}\)We note in passing, that, if we hold household roles fixed, the second cross derivative of the surplus function with respect to wages is still positive. But the relevant measure of complementarity is embedded in the maximized marital gains that can change discontinuously as household roles change.
The marriage and investment rates as well as the proportion of educated women who wish to marry decline when educated women are forced to stay at home. Men, however, increase their investment in schooling and uneducated men are less inclined to marry, due to the loss of efficiency in mixed marriages in which the wife who is more educated than her husband.

**Table 5: Impact of norms on investment and marriage rates (new regime)**

<table>
<thead>
<tr>
<th>Efficient Allocations</th>
<th>Married</th>
<th>Unmarried</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.399, .447</td>
<td>.170, .175</td>
<td>.569, .622</td>
</tr>
<tr>
<td>Educ.</td>
<td>.269, .221</td>
<td>.163, .158</td>
<td>.432, .379</td>
</tr>
<tr>
<td>All</td>
<td>.668, .668</td>
<td>.332, .332</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wife always works at home</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Educ.</td>
</tr>
<tr>
<td>All</td>
</tr>
</tbody>
</table>

* The first and second entry in each cell refer to men and women resp.

Changes in the equilibrium shares are associated with the changes in marriage and investment patterns described above. Because educated women and uneducated men together generate a lower surplus, they both lose surplus shares in all marriages (including the marriages of educated women with educated men). Educated men gain from the reduced share of their wives, while uneducated women who are married to uneducated men also obtain a larger surplus share. All of these changes reflect the weakened competitiveness of educated women. Thus, the return to schooling that men receive within marriage rises while the return to schooling that women receive declines, supporting the increase in the proportion of men who acquire schooling and the reduction in the proportion of women who do the same.
Table 6: Impact of norms on the equilibrium shares in the new regime

<table>
<thead>
<tr>
<th>Efficient allocation</th>
<th>Uneducated</th>
<th>Educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>$V_1 = 1.12$</td>
<td>$V_2 = 1.51$</td>
</tr>
<tr>
<td>Women</td>
<td>$U_1 = 0.92$</td>
<td>$U_2 = 1.61$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wife always works at home</th>
<th>Uneducated</th>
<th>Educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>$V_1 = 0.52$</td>
<td>$V_2 = 1.89$</td>
</tr>
<tr>
<td>Women</td>
<td>$U_1 = 1.52$</td>
<td>$U_2 = 1.23$</td>
</tr>
</tbody>
</table>

As Table 6 shows, the removal of social norms do not benefit all women (or harm all men). In particular, while educated women and uneducated men gain, uneducated women and educated men lose. In this exercise, women still earn less than men in both scenarios and for all education classes. Thus, for couples with identically educated spouses, the household division of labor is not affected by the norms; for all such couples, the husband works in the market and the wife takes care of the child. However, the norm does affect the division of labor for couples where the wife has a higher education level than her husband. This is due to the fact that educated women have a higher wage than uneducated men in the labor market and their market wage exceeds their productivity at home. For all such couples the marital surplus rises and both spouses obtain a larger (absolute) share of the marital surplus when the norm is removed and couples act efficiently. Because of competition, all uneducated men and all educated women receive higher shares, implying that all educated men get a lower share and all uneducated women get a higher share.

This example illustrates the differences between the predictions of general equilibrium models with frictionless matching, like the one we present here, and partial equilibrium models that rely on bargaining. The latter would predict that no woman would lose from the removal of norms that forces women in general to stay at home and take care of the child, but as this example demonstrates, market competition could benefit some women and hurt others depending on their level of education.

5 Conclusions

In standard models of human capital, individuals invest in schooling with the anticipation of being employed at a higher future wage that would compensate them for the current foregone earnings. In this paper, we add another consideration: the
anticipation of being married to a spouse with whom one can share consumption and coordinate work activities. Schooling has an added value in this context because of complementarity between agents, whereby the contribution of the agents' schooling to marital output rises with the schooling of his/her spouse. In the frictionless marriage market considered here, the matching pattern is fully predictable and supported by a unique distribution of marital gains between partners. Distribution is governed by competition, because for each agent there exists a perfect substitute that can replace him/her in marriage. There is thus no scope for bargaining and, therefore, premarital investments are efficient. This simple framework allows us to jointly determine investment and marriage patterns as well as the welfare of men and women under a variety of circumstances.

Using our framework, we find that when the market return to education and household roles are gender neutral, men and women acquire education in equal proportions and, under the assumption that the schooling of the two spouses complement each other, a strictly positive assortative matching arises in the marriage markets. That is, educated men marry only educated women and uneducated men marry only uneducated women. But if the returns are not gender neutral, then there is mixing in equilibrium where some educated individuals marry uneducated spouses and individuals of the gender that marries down obtains a lower return from schooling within marriage. In that case, we show that a transition from an old regime in which women are required to work at home and expect lower market wages to a new regime where less work is required at home and the wages of educated women become more in line with that of educated men, women may overtake men in terms of schooling, despite their lower market wage rate and higher amount of housework compared with men. Moreover, we find that, while women can raise their levels of educational attainment in response to these changes, men can lower theirs even if their education premium rises.

From the perspective of family economics, gender differences in investment in schooling are of particular interest because assortative mating based on schooling is a common feature of marriage patterns in modern societies. However, schooling is an acquired trait that responds to economic incentives. We mentioned two interrelated causes that may diminish the incentives of women to invest in schooling: lower market wages and larger amount of household work. Although we did not fully specify the sources of discrimination against women in the market, we noted that such discrimination tends to decline with schooling and therefore increases the incentive to invest. This is a possible explanation for the slightly higher investment in schooling by women that we observe today. We do not view this outcome as a permanent phenomenon but rather as a part of an adjustment process, whereby women who now
enter the labor market in increasing numbers, following technological changes at home and in the market that favor women, must be “armed” with additional schooling to overcome norms and beliefs that originate in the past.

We should add that there are other possible reasons for why women may invest in schooling more than men. One reason is that there are more women than men in the marriage market at the relatively young ages at which schooling is chosen, because women marry younger. Iyigun and Walsh (forthcoming) have shown, using a similar model to the one discussed here, that in such a case women will be induced to invest more than men in competition for the scarce males. Another reason is that divorce is more harmful to women, because men are more likely to initiate divorce when the quality of match is revealed to be low. This asymmetry is due to the higher income of men and the usual custody arrangements (see Chiappori and Weiss, 2005). In such a case, women may use schooling as an insurance device that mitigates their costs from unwanted divorce.
References


6 Appendix

We provide here **sufficient** conditions that ensure that if any of the 3 equilibrium types (strict or mixed) exists then the equilibrium shares are all strictly positive. The conditions are based on the monotonicity of the line that represents the equilibrium condition (19) that ensures the clearing of the marriage market. We have already shown that this condition defines a line with a negative slope in the $V_1$, $V_2$ space. We now want to state conditions that ensure that, for any $V_1$ satisfying $0 \leq V_1 \leq z_{11}$, equation (19) has a solution that satisfies $0 < V_2 < z_{22}$.

Consider, first, the solution for $V_2$ in equation (19) when we set $V_1 = z_{11}$ (and $U_1 = 0$) and $0 < V_2 < z_{22}$.

$$F(z_{11}) + \int_{z_{11}}^{V_2} G(R^m + V_2 - \theta) f(\theta) d\theta = F(0) + \int_{0}^{z_{22} - V_2} G(R^m + z_{22} - V_2 - \theta) f(\theta) d\theta. \quad (A1)$$

We want to show that this solution which we denote by $V_2^u$ is positive. Now because the LHS of (A1) rises with $V_2$ and the RHS of (A1) declines with $V_2$, it is sufficient that the RHS exceeds the LHS when we evaluate them at $V_2 = 0$. That is,

$$F(z_{11}) - \int_{0}^{z_{11}} G(R^m - \theta) f(\theta) d\theta < F(0) + \int_{0}^{z_{22}} G(R^m + z_{22} - \theta) f(\theta) d\theta. \quad (A2)$$

The economic meaning of condition (A2) is that if under the situation in which uneducated men get all the surplus when married to uneducated women but educated men get none of the surplus when married to educated women, there is excess supply of women who wish to marry and the market adjusts by a rise in $V_2$, because $V_1$ is already at its maximal value, $z_{11}$.

Consider, next, the solution for $V_2$ in equation (19) when we set $V_1 = 0$ (and $U_1 = z_{11}$); and $z_{22} - V_2 > 0$.

$$F(0) + \int_{0}^{V_2} G(R^m + V_2 - \theta) f(\theta) d\theta = F(z_{11}) + \int_{z_{11}}^{z_{22} - V_2} G(R^m + z_{22} - V_2 - \theta) f(\theta) d\theta. \quad (A3)$$

We want to show that this solution which we denote by $V_2^l$ is below $z_{22}$. Now, because the LHS of (A3) rises in $V_2$ and the RHS of (A3) declines with $V_2$, it is sufficient that the LHS exceeds the RHS when we try the value, $V_2 = z_{22}$. That is,

$$F(0) + \int_{0}^{z_{22}} G(R^m + z_{22} - \theta) f(\theta) d\theta > F(z_{11}) - \int_{0}^{z_{11}} G(R^m - \theta) f(\theta) d\theta. \quad (A4)$$
The economic meaning of condition (A4) is that if under the situation in which uneducated men get none of the surplus when married to uneducated women but educated men get all of the surplus when married to educated women, there is excess supply of men who wish to marry and the market adjusts by a decrease in $V_2$, because $V_1$ is already at its minimal value, 0.

Because the equilibrium line (19) has a negative slope, conditions (A2) and (A4) together ensure that, for any $V_1$ satisfying $0 \leq V_1 \leq z_{11}$, we have $0 \leq V_2 \leq z_{22}$. Using symmetry, conditions A2 and A4 can be rewritten more compactly as

$$\int_0^{z_{11}} G(\theta - R_m^m) f(\theta) d\theta < \int_0^{z_{22}} G(R_m^w + z_{22} - \theta) f(\theta) d\theta; \quad (A2')$$

$$\int_0^{z_{22}} G(R_m^m + z_{22} - \theta) f(\theta) d\theta > \int_0^{z_{11}} G(\theta - R_m^w) f(\theta) d\theta. \quad (A4')$$

We can make two observations about these conditions. For any distributions $F(.)$ and $G(.)$, conditions (A2) and (A4) hold if $z_{22} + R_m^m + R_m^w > 2z_{11}$, because then $G(R_m^w + z_{22} - \theta) > G(\theta - R_m^m)$ and $G(R_m^m + z_{22} - \theta) > G(\theta - R_m^w)$ for all $\theta$ such that $\theta \leq z_{11}$. When $2z_{11} > z_{22} > z_{11}$, conditions (A2) and (A4) hold if the distributions $F(.)$ and $G(.)$ are sufficiently widely spread, because then there will be a relatively large mass in the range $z_{22} > \theta > z_{11}$.

To ensure that at least one equilibrium exist in which $0 < V_1 < z_{11}$, we need to impose one of the the following:

In a mixed equilibrium at the lower bound, the equilibrium shares are all strictly positive if $V_2^l < z_{21}, V_2^h > z_{21} - z_{11}$.

In a mixed equilibrium at the upper bound, the equilibrium shares are all strictly positive if $V_2^h > z_{22} - z_{12}, V_2^l < z_{11} + z_{22} - z_{12}$.

In a strict assortative equilibrium, we shall have $0 < V_1 < z_{11}$ if $\bar{V}_2^h > V_2^l$ and $\bar{V}_2^l < V_2^h$, where $\bar{V}_2^h$ and $\bar{V}_2^l$ are the solutions for $V_2$ of the equilibrium condition (20)

$$F(V_1)G(-R_m^m + V_1 - V_2) = F(U_1)G(-R_m^w + U_1 - U_2). \quad (20)$$

$V_1$ is set at $z_{11}$ and 0, respectively.
Figure 1: Completed Education by Sex, 30-40 years old, US 1968-2005 (CPS)

Figure 2: Impacts of higher degrees (relative to high school) on log-wages, adjusted for (potential) experience by sex, US 1968-2003 (CPS)

Figure 3: Educational Attainment of Spouses by Husbands’ Year of Birth (United States)

Figure 4.a: Spousal Education by Own Education, Ages 30-40, United States, 1970-1979


Figure 4.b: Spousal Education by Own Education, Ages 30-40, United States, 1996-2005

Figure 5: Regions for Marriage and Investment
Figure 6: Equilibrium with Strictly Positive Assortative Matching
Figure 7: Mixed Equilibrium with More Educated Men than Educated Women
Figure 8: The Impact of an Increase in the Wage of Educated Men Combined with a Reduction in the Wage of Educated Women
Figure 9: The Impact of an Increase in the Wife’s Work at Home
Figure 10: The Impact of a Decrease in the Wife’s Work at Home Combined with an Increase in the Wage of Educated Women