

COMPARATIVE ECONOMIC CYCLES

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Abstract

The income cycles that have been experienced by six OECD countries over the past 24 years are analysed. The amplitude of the cycles relative to the level of aggregate income varies amongst the countries, as does the degree of the damping that affects the cycles. The study aims to reveal both of these characteristics. It also seeks to determine whether there exists a clear relationship between the degree of damping and the length of the cycles. In order to estimate the parameters of the cycles, the data have been subjected to the processes of detrending, anti-alias filtering and subsampling.

1 Introduction

This paper compares the economic cycles that have beset a group of six OECD countries, which have experienced similar socioeconomic conditions over the past quarter century. The shocks and the disturbances that have impinged on their economies are bound to have had differing intensities; and the effects have been mitigated by a wide variety of fiscal and monetary policies. The aim is to determine the extent to which the consequences of such differences are perceptible in the aggregate indices of economic activity.

For the purposes of this assessment, we shall employ a stylised model of the economic cycle, for which we shall attempt to estimate the parameters from the data of each country. These parameters will provide the basis for our comparisons. We shall employ some simple methods of estimation, but we

shall discover a need to modify them in novel ways in order to accommodate some peculiar characteristics of the data.

Within the framework of our analysis, there can be two explanations for the strength of an economic cycle. The first explanation lies in the power of the disturbances that are the driving force of the cycle. The second explanation lies in the frictional forces of the economy which include the forces of economic regulation. If these are weak, then the economy is liable to fluctuate widely under the impact of the disturbances. We shall attempt to discriminate between these two explanations.

The countries that we shall examine have experienced high levels of economic growth throughout the period in question. The economic recessions that have occurred have been characterised more often by diminutions in the rates of growth than by absolute reductions in the levels of output. In other words, the economic cycles have been carried on the backs of rising trends. The problem of separating the trends from the cycles is a difficult one that has generated much debate, and we feel bound to offer our own opinions.

Some of the distinctions that have arisen in the course of this debate, such as the distinction between deterministic and stochastic trends, may have been drawn too firmly. In practice, the data cannot be relied upon to distinguish unequivocally between such stylised models as the polynomial trend and the unit-root stochastic trend, when both are buried in noise.

It may also be true that our own epigram concerning the causes of the economic cycle, which is based on a mechanical analogy, is drawn too simply and that the underlying realities are far more complex than we shall be proposing for the sake of argument. Nevertheless, our model does take us into realms that have not been fully explored hitherto by economists.

2 A Schematic Model of the Economic Cycle

In proposing simple macroeconomic models, we are liable to assume that the relative proportions of the economic aggregates are maintained, approximately, despite variations in the levels. Examples are provided by the ratios of consumption and investment to gross national product (GNP), of which the underlying constancy is frequently postulated. It is reasonable to assume that the relative amplitude of the economic fluctuations is also maintained in the long run.

A schematic model of a macroeconomic index might, therefore, set

$$Y(t) = \Xi(t)B(t) \quad \text{with} \quad B(t) = 1 + \sum_j \sigma_j \cos(\omega_j t + \theta_j), \quad (1)$$

where $Y(t)$ stands for an aggregate economic index, such as GNP, and where $\Xi(t)$ is its underlying trend. Modulating this trend is the factor $B(t)$, which comprises a sum of sinusoids. The j th sinusoid has an amplitude of σ_j , a phase angle of θ_j radians and a period of $\tau_j = 2\pi/\omega_j$, where ω_j is an angular velocity, or frequency, measured in radians per period.

For statistical purposes, we might amend this model by replacing $B(t)$ by the factor $1 + \beta(t)$, where $\beta(t)$ is generated by a linear stochastic process of an autoregressive (AR) or autoregressive moving-average (ARMA) variety. Over a finite period, the output of such a process can also be expressed as a sum of sinusoids, the parameters of which are assumed to have been drawn from statistical distributions.

However, there are some problems with this formulation. First, as we have defined it, there is no limit on the range of the stochastic process $\beta(t)$, whereas it is necessary, at least, that its negative deviations should be limited in order to prevent $Y(t)$ from becoming negative. Secondly, it is likely that we should wish to include an additional stochastic factor that is unrelated to the cycles.

The first difficulty is answered by setting

$$B(t) = e^{\beta(t)} = \left(1 + \beta(t) + \frac{\{\beta(t)\}^2}{2!} + \dots \right), \quad (2)$$

where $\beta(t)$ follows a linear stochastic process of variance σ^2 . Then $H(t)B(t)$ can be multiplied by the additional stochastic factor $\mathcal{E}(t)$, bounded in the same manner as $B(t)$, and the amended model becomes

$$Y(t) = \Xi(t)B(t)\mathcal{E}(t) \quad \text{or} \quad y(t) = \xi(t) + \beta(t) + \varepsilon(t), \quad (3)$$

where $y(t) = \ln Y(t)$, $\xi(t) = \ln \Xi(t)$, $\beta(t) = \ln B(t)$ and $\varepsilon(t) = \ln \mathcal{E}(t)$ are the logarithms of the factors.

It transpires that, in practice, the economic cycle can be represented by an ordinary second-order autoregressive AR(2) process. The equation of an AR(2) process can be compared with a second-order differential equation, which is associated with numerous physical processes that provide good analogies for the cycle.

Consider a damped harmonic oscillator driven by a sinusoidal forcing function with a frequency of ω_f . A spring-mass system with viscous damping is an example. The equation of motion is

$$m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + hy(t) = \phi \cos(\omega_f t), \quad (4)$$

where m is the oscillating mass, c is the coefficient of viscous damping, and h is the return force per unit of displacement, which can be described as the stiffness of the spring. The variable $y(t)$ is the displacement of the system about its stationary point. The solution of this differential equation is

$$y(t) = A \cos(\omega_f t - \theta). \quad (5)$$

The system dissipates energy at an average rate that is equal to the input of power from the driving force. It also stores a quantity of energy. At the maximum amplitude, this consists of potential energy, stored in the form of the strain energy of the spring, whereas, at the point of maximum velocity, where the spring is unstretched, it consists of kinetic energy, stored in the mass by virtue of its velocity.

A measure of the damping properties of the system is provided by its loss ratio, which is defined as the energy W dissipated per radian, averaged over the cycle, divided by the peak potential energy U of the system. For a second-order system, there is

$$W = \frac{c\omega_f A^2}{2} \quad \text{and} \quad U = \frac{hA^2}{2}. \quad (6)$$

Therefore, the loss ratio is

$$\frac{W}{U} = \frac{c}{h} \omega_f = \Lambda \omega_f. \quad (7)$$

The loss coefficient $\Lambda = c/h$ is a characteristic of the system that is independent of the frequency ω_f at which it is driven. It can be expressed in terms of the solution of the following auxiliary equation:

$$\begin{aligned} 0 &= ms^2 + cs + h = m(s - \kappa)(s - \kappa^*) \\ &= m\{s^2 - (\kappa + \kappa^*)s + \kappa\kappa^*\}, \end{aligned} \quad (8)$$

wherein $\kappa = \gamma + i\omega$ and $\kappa^* = \gamma - i\omega$ are conjugate complex roots. We observe that $c = -2\gamma m$ and that $h = m(\gamma^2 + \omega^2)$. Therefore,

$$\Lambda = \frac{-2\gamma}{\gamma^2 + \omega^2}. \quad (9)$$

It is necessary to express this coefficient in terms of the parameters of the second-order difference equation that will be fitted to the data. This may be achieved by comparing the solution of the homogeneous differential equation, derived from (4) by setting the driving force to zero, via $\phi = 0$, with the solution of the corresponding difference equation. The solutions must agree at the points in discrete time over which the difference equation is defined.

The solution of this differential equation is

$$y(t) = \sigma e^{\gamma t} \cos(\omega t - \theta), \quad (10)$$

where σ relates to the initial amplitude at time $t = 0$, ω is the resonant frequency of the system and $\gamma = -c/2m$ is the exponential rate at which the system converges to its point of rest.

The homogeneous second-order difference equation takes the form of

$$(\alpha_0 + \alpha_1 L + \alpha_2 L^2)y(t) = 0, \quad (11)$$

where L denotes the lag operator such that $Ly(t) = y(t - 1)$. The auxiliary equation is

$$\begin{aligned} 0 &= \alpha_0 z^2 + \alpha_1 z + \alpha_2 = \alpha_0(z - \mu)(z - \mu^*) \\ &= \alpha_0\{z^2 - (\mu + \mu^*)z + \mu\mu^*\}, \end{aligned} \quad (12)$$

wherein $\mu = \beta + i\delta$ and $\mu^* = \beta - i\delta$ are conjugate complex roots. We define $\rho^2 = \beta^2 + \delta^2 = \alpha_2$ and $\omega = \tan^{-1}(\delta/\beta)$. Then the solution of the homogeneous equation can be denoted

$$y(t) = \sigma \rho^t \cos(\omega t - \theta). \quad (13)$$

The comparison of (10) and (13) indicates that they agree when

$$e^{\gamma t} = \rho^t, \quad \text{which is when } \gamma = \ln \rho. \quad (14)$$

Therefore, if we know ρ from the difference equation, we know $c/m = -2\gamma$. Also the parameter ω is common to both equations so, knowing this, we can

proceed to find $h/m = \gamma^2 + \omega^2$. Then the loss parameter $\Lambda = c/h$ can be calculated.

It is worth noting the manner in which the resonant frequency of an oscillating system, free from a driving force, declines as the degree of damping, represented by c , increases. Thus, in terms of the differential equation, there is

$$\omega^2 = \frac{4mh - c^2}{4m^2} = \omega_n^2 - \gamma^2, \quad (15)$$

where $\omega_n = \sqrt{h/m}$ is the natural frequency of the undamped system and $\gamma = -c/2m$. It will be interesting to discover whether a similar feature emerges from the comparison of the national economies.

In comparing the economies, we are dealing with the relative amplitude of the cycles. Therefore, the absolute size of the economy is not a factor in the analysis. For the differential equation this is tantamount to setting $m = 1$, whereas, for the difference equation, the corresponding normalisation is to set $\alpha_0 = 1$. As regards the process that drives the cycles, we can only make the stylised assumption that it is a white-noise process. The strength of the process will be reflected in an estimate of its variance.

3 The Trend Component

We shall now explore two of the alternative methods that are available for extracting the trend from the data. What remains will contain the economic cycle as well some other motions that will need to be removed and discarded.

The first and the simplest method of extracting the trend is to fit a polynomial function of time. A linear trend fitted to the logarithms of the data corresponds to a constant exponential growth path, which some authors would describe as a deterministic trend. Although we do not wish to invoke the concept of a true underlying growth path, such a trend does provide a firm benchmark against which the cyclical activities of the economy can be measured.

We begin by considering the matrix version of the difference operator and its inverse, which is the cumulation operator. These will be useful in portraying both the method of polynomial regression and the Hodrick–Prescott filter. Indeed, our purpose is to depict the linear trend as a limiting case of the Hodrick–Prescott trend.

Consider, therefore, the identity matrix of order T defined by

$$I_T = [e_0, e_1, \dots, e_{T-1}], \quad (16)$$

where e_j represents a column vector that contains a single unit preceded by j zeros and followed by $T - j - 1$ zeros. Then, the finite-sample lag operator is the matrix

$$L_T = [e_1, \dots, e_{T-1}, 0], \quad (17)$$

which has units on the first subdiagonal and zeros elsewhere. This is obtained from the identity matrix by deleting the leading column and by appending a column of zeros to the end of the array.

The matrix that takes the d -th difference of a vector of order T is given by

$$\nabla_T^d = (I - L_T)^d. \quad (18)$$

Using a notation that we shall apply to other matrix operators, we may partition the matrix so that $\nabla_T^d = [Q_*, Q]'$, where Q_* has d rows. The inverse matrix is partitioned conformably to give $\nabla_T^{-d} = [S_*, S]$. We may observe that

$$\begin{bmatrix} S_* & S \end{bmatrix} \begin{bmatrix} Q_*' \\ Q' \end{bmatrix} = S_* Q_*' + S Q' = I_T, \quad (19)$$

and that

$$\begin{bmatrix} Q_*' \\ Q' \end{bmatrix} \begin{bmatrix} S_* & S \end{bmatrix} = \begin{bmatrix} Q_*' S_* & Q_*' S \\ Q' S_* & Q' S \end{bmatrix} = \begin{bmatrix} I_d & 0 \\ 0 & I_{T-d} \end{bmatrix}. \quad (20)$$

The matrix $\nabla_T^{-d} = [S_*, S]$ is a lower-triangular Toeplitz matrix, which is completely characterised by its leading column. The elements of that column are the ordinates of a polynomial of degree $d - 1$ of which the argument is the row index $t = 0, 1, \dots, T - 1$. Moreover, the leading d columns of the matrix ∇_T^{-d} , which constitute the submatrix S_* , provide a basis for all polynomials of degree $d - 1$ that are defined on the integer points $t = 0, 1, \dots, T - 1$.

The ordinates of a polynomial of degree $d - 1$ defined over the integers $t = 0, 1, \dots, T - 1$ are given by

$$p = S_* r_* \quad \text{where} \quad r_* = Q_*' p. \quad (21)$$

Since the polynomial is fully determined by the elements of the starting-value vector r_* , fitting it to the data in the vector $y = [y_0, \dots, y_{T-1}]'$ according to the least-squares criterion is a matter of minimising

$$(y - p)'(y - p) = (y - S_* r_*)'(y - S_* r_*) \quad (22)$$

with respect to r_* . The resulting values are

$$r_* = (S'_*S_*)^{-1}S'_*y \quad \text{and} \quad p = S_*(S'_*S_*)^{-1}S'_*y. \quad (23)$$

For an alternative expression, we may use the identity

$$S_*(S'_*S_*)^{-1}S'_* = I - Q(Q'Q)^{-1}Q', \quad (24)$$

which follows from the fact that Q and S_* are complementary matrices such that $Q'S_* = 0$ and $\text{Rank}[Q, S_*] = T$.

Using (24) in (23) gives the following expression for the vector of polynomial ordinates:

$$p = y - Q(Q'Q)^{-1}Q'y. \quad (25)$$

Figure 1 shows the quarterly sequence of the logarithms of aggregate income in the U.K. for the period 1955 to 1994, through which a linear trend has been interpolated. This is an instance of the simplest form of polynomial regression. Figure 2 shows the periodogram of the residuals, which are the deviations of the sequence from this trend.

The spectral signature of the low-frequency cycles that surround the trend is clearly represented in the periodogram. It occupies a range of frequencies extending from zero to $\pi/8$ radians at least. In the vicinities of $\pi/2$ and π , there appears the somewhat incoherent signature of the rough seasonal variations that affect the income sequence. The remainder of the periodogram may be described as dead space punctuated by small elements of noise.

The periodogram of the original trended data is of little use in discerning the low-frequency structure. The latter is concealed within the slew of spectral power that is attributable to the disjunctions which occur in the periodic extension of the data where the end of one replication of the sample is joined to the start of another.

It is also the case that none of the low-frequency spectral structure will be evident in the periodograms of either the first or the second differences of the data. The gain factor of the second-differencing operator at $\pi/10$, for example, is 0.00958, which means that the ordinates of the periodogram are so severely attenuated by the differencing operation as to become invisible.

It is evident from equation (25) that the residuals from fitting a polynomial of degree $d - 1$, which are found in the vector $y - p$, contain exactly the same information as the differences of order d within the vector $Q'y$. Nevertheless, they serve to reveal the spectral structure over the entire range of

frequencies. It should be emphasised that the use of polynomial residuals as a means of revealing the spectral structure does not imply any decision to model the trend via a polynomial function.

The second method of trend extraction entails the notion of a stochastic trend. This represents the cumulative effects of stochastic elements that impart an upward drift to the economy. The usual statistical model of such a trend is a first-order random walk with drift or a second-order, or integrated, random walk that happens to be generating a rising path. A common device for extracting such trends is the Hodrick–Prescott (1980, 1997) filter.

The Hodrick–Prescott (H–P) filter is derived in reference to an equation

$$y(t) = \xi(t) + \eta(t), \quad (26)$$

in which the data sequence $y(t)$ is expressed as the sum of a trend component $\xi(t)$, which follows a second-order random walk, and a residual component $\eta(t)$, which is white noise. (We shall use the corresponding roman letters $x(t)$ and $h(t)$ to denote the estimates of $\xi(t)$ and $\eta(t)$, respectively.) The random walk is described by the equation $(1 - L)^2\xi(t) = \zeta(t)$, where $\zeta(t)$ is a white-noise process that is independent of $\eta(t)$. Therefore, the differenced data sequence

$$\begin{aligned} (1 - L)^2y(t) &= (1 - L)^2\xi(t) + (1 - L)^2\eta(t) \\ &= \zeta(t) + \kappa(t) \end{aligned} \quad (27)$$

constitutes a stationary process; and the autocovariance generating functions of the differenced components are

$$\gamma_\zeta(z) = \sigma_\zeta^2 \quad \text{and} \quad \gamma_\kappa(z) = \sigma_\eta^2(1 - z)^2(1 - z^{-1})^2. \quad (28)$$

According to the Wiener–Kolmogorov principle, the detrending highpass filter is derived by setting $z = L$ in the following ratio of autocovariance generating functions:

$$\psi(z) = \frac{\gamma_\kappa(z)}{\gamma_\kappa(z) + \gamma_\zeta(z)} = \frac{\sigma_\eta^2(1 - z)^2(1 - z^{-1})^2}{\sigma_\eta^2(1 - z)^2(1 - z^{-1})^2 + \sigma_\zeta^2}. \quad (29)$$

The residual component is estimated by $h(t) = \psi(L)y(t)$. The complementary lowpass filter, which estimates the trend, is derived by setting $z = L$ within the function $1 - \psi(z)$.

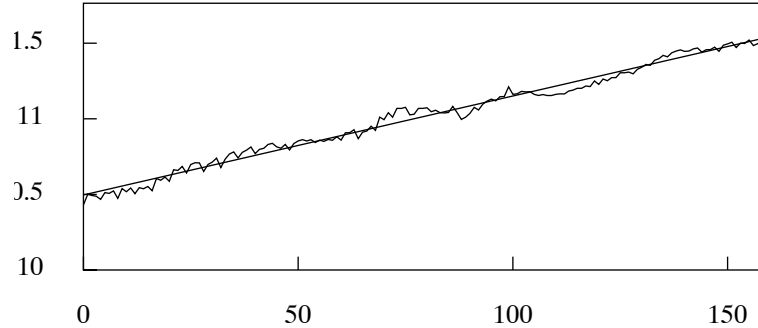


Figure 1: The quarterly sequence of the logarithms of income in the U.K. for the years 1955 to 1994, together with a linear trend interpolated by least-squares regression.

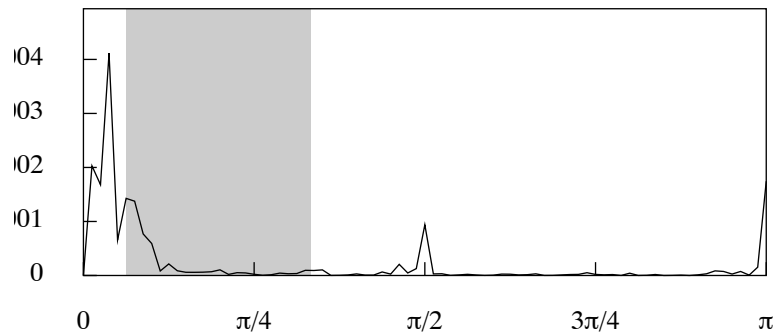


Figure 2: The periodogram of the residuals obtained by fitting a linear trend through the logarithmic sequence of U.K. income. A band, with a lower bound of $\pi/16$ radians and an upper bound of $\pi/3$ radians, is masking the periodogram.

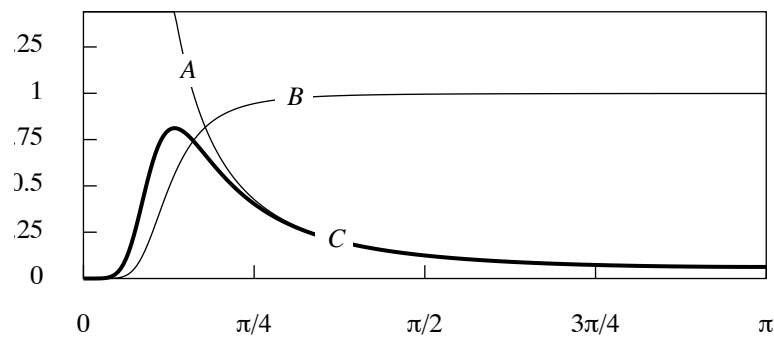


Figure 3: The effect of applying the Hodrick–Prescott filter to a random walk. The smoothing parameter is $\lambda = 100$ and the variance of the white-noise process driving the random walk is $\sigma_\varepsilon^2 = 0.25$

Setting $z = e^{i\omega}$ in $\psi(z)$ gives the frequency response of the filter which, in this instance, is a real-valued function on account of the symmetry of the filter in respect of z and z^{-1} . The squared modulus of the frequency response function, which, in this case, is just the square, constitutes its squared gain. This is plotted in Figure 3, for a particular value of $\lambda = \sigma_\eta^2/\sigma_\zeta^2$, as the curve that is labelled *B*. Also plotted on the diagram is the pseudo spectrum of a first-order random walk labelled *A*.

The curve labelled *C* in the diagram, is the spectral density function of a detrended series derived from the random walk by applying a filter with a smoothing parameter of $\lambda = 100$. In place of this single curve, one can imagine a family of curves generated by varying the value of λ . In that case, one would discern that the functions associated with lower values of λ have peaks of lesser height located at higher frequency values. The inference is that the lower the value of the smoothing parameter λ the shorter are the durations of the cycles in the detrended series and the less is their amplitude.

In practice, the data are available as a finite sequence, which constitutes a vector $y = \xi + \eta$, where ξ is the trend and η is the noise. Therefore, filters must be derived that operate on finite sequences. Recall that Q' , defined by (19), denotes the matrix version of the second-difference operator. Then

$$\begin{aligned} Q'y &= Q'\xi + Q'\eta \\ &= \zeta + Q'\eta, \end{aligned} \quad (30)$$

where

$$\begin{aligned} E(\zeta) &= 0, & D(\zeta) &= \sigma_\zeta^2 I_{T-2}, \\ E(\eta) &= 0, & D(\eta) &= \sigma_\eta^2 I_T, \\ & & \text{and } C(\zeta, Q'\eta) &= 0. \end{aligned} \quad (31)$$

The independence of ξ and η implies that $D(Q'y) = \sigma_\eta^2 Q'Q + \sigma_\zeta^2 I$.

On the assumption that the components have a normal distribution, there is the following joint density function:

$$N(\zeta, \eta) = (2\pi)^{1-T} \sigma_\zeta^{2-T} \sigma_\eta^{-T} \exp\left\{-\frac{1}{2}(\sigma_\zeta^{-2} \xi' Q Q' \xi + \sigma_\eta^{-2} \eta' \eta)\right\}. \quad (32)$$

The maximum-likelihood estimate x of the trend component ξ is found by minimising the following criterion function, which is derived from the quadratic exponent of the density function by setting $\eta = y - \xi$:

$$S(\xi) = \sigma_\zeta^{-2} \xi' Q Q' \xi + \sigma_\eta^{-2} (y - \xi)' (y - \xi). \quad (33)$$

The minimising value of ξ is

$$x = \sigma_\eta^{-2}(\sigma_\zeta^{-2}QQ' + \sigma_\eta^{-2}I)^{-1}y. \quad (34)$$

According to the matrix inversion lemma, there is

$$(\sigma_\zeta^{-2}QQ' + \sigma_\eta^{-2}I)^{-1} = \sigma_\eta^2\{I - Q(Q'Q + (\sigma_\zeta^2/\sigma_\eta^2)I)\}^{-1}Q'. \quad (35)$$

Using this in (34) and writing $\sigma_\zeta^2/\sigma_\eta^2 = \lambda^{-1}$, we get

$$x = y - Q(Q'Q + \lambda^{-1}I)^{-1}Q'y. \quad (36)$$

This is the appropriate finite-sample version of the H–P trend estimation filter.

We should make two observations in respect of this equation. First, if y were to follow a linear trend, then the equation would deliver $x = y$ since, in that case, $Q'y = 0$. Secondly, as $\lambda \rightarrow \infty$, the equation will tend to that of the least-squares estimator of a linear trend, which is represented by (25). We note that λ is conventionally described as the smoothing parameter. It is also a noise/signal variance ratio. When the noise is strong relative to the signal, the Hodrick–Prescott filter is also liable to deliver a trend that is approximately linear within wide neighbourhoods.

The H–P filter has been used as a lowpass smoothing filter in numerous macroeconomic investigations (see, for example, Hartley et. al. 1998) where it has been customary to set the smoothing parameter to certain conventional values. Thus, for example, the econometric computer package *Eviews 4.0* (2000) imposes the following default values:

$$\lambda = \begin{cases} 100 & \text{for annual data,} \\ 1,600 & \text{for quarterly data,} \\ 14,400 & \text{for monthly data.} \end{cases} \quad (37)$$

These are high values, and their effect is to ensure that, in many practical cases, the trend that is extracted by the Hodrick–Prescott filter is virtually linear. It is not surprising, therefore, that there should be difficulty in distinguishing between a linear deterministic trend, that would be fitted via a polynomial regression, and a stochastic trend, that might be extracted by the H–P filter.

An alternative to specifying the smoothing parameter λ is to specify a frequency value ω_c such that $\psi(\omega_c) = 0.5$. This frequency corresponds to the

midpoint in the transition of the gain of the lowpass H–P filter from the value unity, which is attained when $\omega = 0$ to the value of zero, which is attained when $\omega = \pi$. The closer is ω_c to 0, the higher is the value of λ . One might be tempted to describe ω_c as the nominal cut-off point of the filter, but, in view of the gradual transition of the gain of the H–P filter from unity to zero, this could be regarded as a misnomer.

The correspondence between ω_c and λ is as follows:

$$\lambda = 1/4\{1 - \cos(\omega_c)\}^2 \quad \text{and} \quad \omega_c = \cos^{-1}(1 - 1/\sqrt{4\lambda}). \quad (38)$$

Instead of specifying ω_c directly, it may be easier to specify the duration of the cycles of this frequency. For a duration of τ years, the frequency is $\omega_c = 2\pi/(\tau s)$, where s is the number of observations per year.

It has become customary to define the business cycle as a composite of sinusoidal motions of durations not exceeding 8 years and not less than one-and-a-half years. When the nominal value of the limiting duration is set at 8 years, this frequency response of the lowpass H–P filter has considerable leakage across the boundary, with the effect that large proportions of some of the business-cycle components are removed from the residue. Therefore, in order to achieve a good representation of the business cycle, the smoothing parameter of the H–P filter should greatly exceed the value that corresponds to a duration of 8 years.

4 The Issue of Spurious Cycles

The H–P filter has been subject to an oft-repeated aspersion that it is liable to induce spurious cycles in the detrended data. The argument has been made, for example, by Cogley and Nason (1995) and it has been supported by Harvey and Jaeger (1993), amongst others. Contrary opinions have been offered by Pollock (1997, 2000) by Pedersen (2001) and by Valle e Azevedo, (2002). There is a semantic issue at the root of these differences of opinion, but there is also evidence of a widespread misunderstanding.

The highpass H–P filter $\psi(L)$ has a frequency response function for which the gain never exceeds unity. This means that its effect is either to preserve or to attenuate the sinusoidal elements of which a data sequence is composed. It never amplifies any sinusoidal elements and it never introduces any. If a sinusoid is present in the processed data, then it must also be present to no

lesser extent in the original data. Therefore, it seems wrong to describe such cyclical components as spurious.

The proposal of Cogley and Nason that the H–P filter induces cycles is based upon an analysis of the frequency response of a filter that results from conflating the H–P filter with the unit-root summation operator belonging to the model of a random walk. The random walk $y(t)$ is modelled by the equation $(1 - L)y(t) = \varepsilon(t)$, where $\varepsilon(t)$ stands for a white-noise process. The filtered sequence is

$$h(t) = y(t) - x(t) = \{\psi(L)/(1 - L)\}\varepsilon(t). \quad (39)$$

Cogley and Nason attribute to the H–P filter the gain of the filter $\psi(L)/(1 - L)$, instead of the gain of $\psi(L)$.

An example of the squared gain of the filter $\psi(L)/(1 - L)$ is given by the curve labelled C in Figure 3. This curve also represents the spectrum of the filtered sequence $h(t)$. It certainly manifests a strong spectral peak which indicates the presence of cyclical motions within $h(t)$. Such motions are attenuated versions of those that are present in the data process $y(t)$; and they are not induced by the filter.

Notwithstanding the nature of its Fourier decomposition, which comprises sinusoidal motions at all frequencies, there is some justification for the assertion that a true random walk contains no genuine cycles. The process is the product of an accumulation of statistically independent increments; and it has no inherent central tendency. On the other hand, genuine cyclical motions are commonly regarded as the products of centralising forces that increase in proportion to the distance of an object from the point to which it is tethered. It is on this basis that the cycles that are generated by filtering a random walk might be regarded as illusory artefacts.

Economic trends are often modelled as random-walk processes. Nevertheless, such models need not be interpreted in a literal manner. Whereas a random walk evolves in an unbridled manner, economic trends are subject to evident constraints. They are driven by the buoyant forces of entrepreneurial endeavour and consumer aspirations, and they are constrained by the more-or-less pliable limits of productive capacity and resource availability. In a thriving economy, they alternately press against the constraints and rebound from them in a manner that is undeniably cyclical.

There may be doubts about the applicability to economic circumstances of an analysis, such as the one that underlies Figure 3, that postulates a

random walk defined over a doubly infinite sequence of integers. Such a process is unbounded in mean, and it does not have a finite variance. It is expected, at any point in time, to be infinitely remote from the origin. By contrast, the random walks that are postulated in applied econometrics are defined over a finite interval, and they have starting values at a finite distance from the origin.

A measure of the difficulties in interpreting an infinite random walk is provided by the limiting case of the infinite-sample Hodrick–Prescott filter, where $\lambda \rightarrow \infty$. This is when the finite-sample filter delivers a linear trend. In the limit, the corresponding infinite-sample filter has a frequency response function with a unit gain everywhere except at zero frequency. Therefore, it is virtually an allpass filter; which suggests that the infinite random walk should be unaffected by a process of linear detrending. This is contrary to what we expect from the linear detrending of a finite random-walk sequence.

To resolve these issues, we may observe that fitting a straight line by least-squares regression to a finite segment of a random walk will result in a residual sequence of mean zero that will inevitably show a reversion to the mean. From a global perspective, which views the sample as whole, this central tendency will not be affected by a growing sample duration. There will be the same number of crossings of the trend line on average, regardless of the length of the sample; and the number of crossings will be few.

On the other hand, if we look myopically at the sampled sequence, then the effect of increasing the sample size will be to reduce the rate of mean reversion, as measured from one point to the next. Therefore, eventually, the linearly detrended random walk will become a random walk itself. Thus, we are able to reconcile the behaviour of the limiting case of the infinite-sample H–P filter with the behaviour of the finite-sample filter.

We may consider the random walk to be the product of a regular process of sampling applied to a continuous Wiener process. The Wiener process is self-similar in the sense that short segments, viewed in detail, have the same appearance as a longer segments, viewed more distantly. Therefore, allowing for a change of scale, the effect of increasing the size of the sample by allowing a growing number of points to accumulate with the passage of time is no different from the effect of increasing their number by increasing the rate at which a finite segment of the process is sampled. Thus, it may be easier to see these things in the small than in the large.

The effect of fitting a linear trend by least-squares to a finite segment of a random walk has been analysed by Chan, Hayya and Ord (1971). They

have found a formula for the autocovariances of the residuals. They have also provided a formula for the expected value of the estimated autocovariances of the residuals. This has been corrected by Nelson and Kang (1981), who have extended the analysis. The latter have revealed that the spectral density function, derived from the expected values of the sample autocovariances, has a peak at a frequency that corresponds to a cycle of a duration that is 0.83 times the number of sample periods.

Nelson and Kang have emphasised the risk of finding spurious dynamics in the residuals from the inappropriate detrending of a random walk. A myopic analysis that looks only at the values of the parameters of an estimated autoregressive model is clearly at risk of drawing false conclusions. However, an analysis that measures the rate of mean reversion relative to the length of the sample can avoid the risk.

The frequencies of the fluctuations around their interpolated linear trends of the sequences that we shall analyse are far greater than the frequency that is characteristic of a random walk. Therefore, the sequences appear to manifest a genuine cyclicity. Figure 4 provides such evidence.

5 Alternative Methods of Business-Cycle Estimation

We shall now consider briefly some of the other methods that have been used by economists to extract the business cycle from the data. The business cycle has been defined as the component of the data that is composed of cyclical elements of durations no less of one-and-a-half years and no greater than eight years.

This definition, which has been adopted by Baxter and King (1999), for example, is derived from a definition given by Burns and Mitchell (1946) in their early and influential study of the business cycle in the U.S. In fact, the duration of eight years is that of the longest cycle that they observed; and it does not seem to have been an immutable part of their definition.

The modern definition of Baxter and King is reflected in the band that has been imposed on the periodogram of Figure 2 which runs from the frequency of $\pi/16$ radians per period up to $\pi/3$ radians per period. There is nothing in the spectral structure of macroeconomic cycles to suggest that this is anything but an arbitrary definition. Indeed, Figure 2 and others like it,

indicate that much of the power of the low-frequency economic cycles lies outside the band.

Problems arise in the attempt to isolate the components that correspond to the definition. Baxter and King have proposed to use a moving-average filter that is obtained from the Fourier transform of the frequency response of an ideal bandpass filter. This response is represented by a rectangle of unit height based on the frequency interval $[\pi/16, \pi/3]$, which constitutes its pass band. The complementary stop band, where the gain of the filter is zero, comprises the remainder of the frequency range.

The Fourier transform of the frequency response of the ideal bandpass filter generates a doubly-infinite sequence of coefficients that needs to be truncated to produce a practical moving-average filter. However, the truncated filter is affected by a substantial spectral leakage, whereby a filtered sequence is liable to contain elements whose frequencies lie well outside the designated pass band.

The leakage can be mitigated, to some extent, by applying a cosine window to the filter coefficients, such as the Blackman window or the Hamming window. (See, for example, Pollock 1999.) However, whereas one effect of windowing the coefficients is to limit the range of the frequencies of the elements that enter the filtered sequence, another effect is to replace the sharp cut-offs at the limits of the pass band by gentle transitions that extend well into the stop band.

A gradual transition at the lower end of the designated pass band is bound to admit to the filtered sequence some powerful low-frequency elements. Therefore, the implementation of the band pass filter via a short-span moving average will lead to a doubtful realisation of the definition.

A more precise realisation of the bandpass definition of the business cycle can be obtained via a Fourier-based method. This entails extracting the Fourier ordinates that lie within the designated frequency band from a data sequence that has been reduced to stationarity by first or second differencing. From these ordinates, a sequence can be synthesised which can be reinflated by a process of summation that reverses the differencing.

The summation process requires some initial conditions; and it is reasonable to select the values that minimise the sum of squares of the deviations from zero of elements of the reinflated sequence. The method had been pursued by Pollock (1997, 2000, 2001a, 2001b) in a series of papers devoted to the filtering of nonstationary sequences.

The problems affecting the bandpass definition of the business cycle, as

well as the difficulties of implementing it, discourage us from adopting the definition. Instead, we are inclined to adopt a broader definition of the economic cycle that includes both the business cycle, however it is defined, and those cycles of longer duration that are commonly described as growth cycles.

In the empirical analysis of the following section, we shall detrend the logarithmic data by interpolating a quadratic function, and we shall also use the H–P filter. A quadratic function is used instead of a linear function in order to accommodate cases where there is some indication of a gradual increase or decrease, over the sample period, in the rate of the underlying growth of GDP; but we shall find that, in most cases, the quadratic function is virtually a linear function. We shall pursue the method of estimating the trend via the H–P filter, mainly for comparative purposes; and we shall be interested to see the effect of varying the smoothing parameter.

It is an unusual circumstance when the data are amenable to a linear or a quadratic detrending, as is the case for each of the six OECD countries throughout our sample period. In different eras and over longer periods, we would expect to resort, instead, to a flexible method of trend estimation that employs the H–P filter and that accommodates structural breaks via local variations in the smoothing parameter.

6 The Empirical Results

The results from fitting an AR(2) to the data of each of the six OECD countries, for the period 1980Q1–2003Q4, are displayed in Table 1. These estimates are the outcome of a multistep procedure.

For a start, a quadratic trend—which serves as a benchmark against which the cyclical activity of the national economies can be measured—has been fitted to the logarithms of the data. The residuals from the extracted trend, which are liable to be examined for their low-frequency content, contain seasonal and irregular components at higher frequencies, which are not directly attributable to the business cycle.

To remove these components, the data from each country is subjected to a process of lowpass filtering and subsampling, which serves to discard all except the information that lies in the frequency interval $[0, \pi/4]$ in the spectrum of the original data. In effect, we apply to the data a filter that has a pass band over the interval $[0, \pi/8]$, a transition band over the interval

$[\pi/8, 3\pi/16]$ and a stop band over the remainder of the frequency range, which is limited by the maximum Nyquist frequency of π .

The filter is implemented by weighting the Fourier coefficients of the data. The weights of the passband are unity. The weights in the transition band, which have a cosine decrement, are governed by the function $[1 + \cos(x)]/2$, where $x = 0$ at the end of the passband and $x = \pi$ at the beginning of the stop band. The weights of the stop band are all zeros.

The filter is also effective in overcoming any aliasing that could arise from the process of subsampling that selects every fourth data point. In the absence of an anti-aliasing filter, subsampling by a factor of 4 would serve to map the information content in the upper three quarters of the frequency range into its lowest quarter, whereafter the latter would be expanded by a factor of 4 to occupy the interval $[0, \pi]$. With proper anti aliasing, the effect of the subsampling is to map the unimpaired contents of the original data in the frequency interval $[0, \pi/4]$ onto the wider interval $[0, \pi]$, which corresponds to the full range of frequencies within any sampled data sequence.

The processes of filtering and subsampling the data are illustrated in Figures 5 and 6. Figure 5 displays the periodogram of the detrended data over the interval $[0, \pi/4]$, and it corresponds to a segment of Figure 2. The figure also superimposes the frequency response of the lowpass anti-aliasing filter with a pass band over the range $[0, \pi/8]$ and a transition band over $[\pi/8, 3\pi/16]$. (The transition band is the highlighted strip in the diagram.) Figure 6 is the periodogram of the data that has been processed by the lowpass filter and thereafter subsampled by a factor 4.

Figure 6 also displays the parametric spectrum of an AR(2) model that has been fitted to the processed data. The estimates have been derived by maximising the Whittle likelihood function, which entails the assumption that the data are the product of a circular process. (It is equivalent to propose that the sample represents one cycle of a periodic function.) To sustain this assumption, some attention has to be paid to the problem of the disjunction that can occur at the point where the end of the sample is joined to its beginning.

We believe that an appropriate way of dealing with this matter is to extend the sample at both the ends by forecasting and backcasting the data. In particular, our procedure uses the preliminary estimates of an AR(2) model to lengthen the sample by 25 per cent. Then, the extrapolated sample is subjected to a tapering operation based on a split cosine bell. The latter is

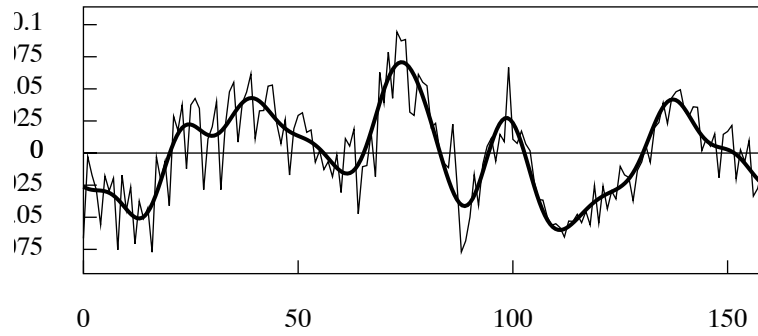


Figure 4: The residual sequence from fitting a linear trend to the income data of Figure 1. The interpolated line, which represents the business cycle, has been obtained from the Fourier ordinates that generate the periodogram of Figure 6.

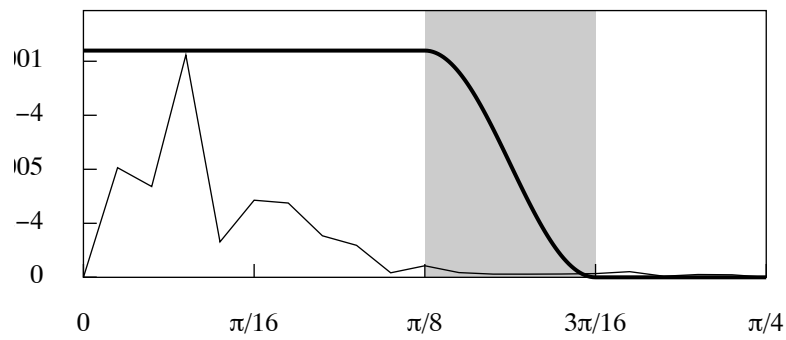


Figure 5: A segment of Figure 2, which is the periodogram of the residuals from fitting a linear trend to the logarithmic data. The gain of the lowpass filter with a transition over the interval $[\pi/8, 3\pi/16]$ is superimposed on the diagram.

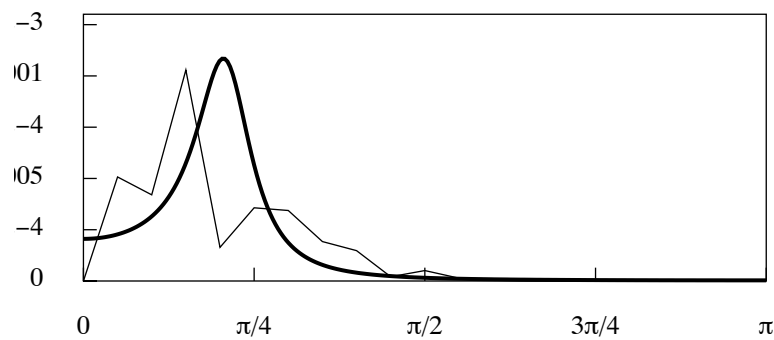


Figure 6: The periodogram of the sub sampled anti aliased data with the parametric spectrum of an estimated AR(2) model superimposed.

Table 1: The business-cycle parameters obtained via quadratic detrending

	α_1	α_2	ρ	ω	Λ	σ_x	b
FRANCE	-1.51735	0.89095	0.94390	36.51	0.28	0.01760	0.02040
ITALY	-1.26764	0.73203	0.85558	42.20	0.55	0.01414	0.02520
NORWAY	-1.36220	0.75611	0.86955	38.44	0.60	0.02302	0.02720
SPAIN	-1.50901	0.86237	0.92864	35.66	0.38	0.02345	0.02720
U.K.	-1.45279	0.84039	0.91673	37.59	0.40	0.02214	0.02520
U.S.A.	-1.09085	0.66619	0.81621	48.07	0.55	0.01924	0.03280

nothing but a cosine bell with an inserted stretch of units¹. The sample elements are given unit weights and the extrapolated elements are tapered. This procedure gives rise to the estimates in Table 1. Table 2 gives the ranking of the estimated values, which should assist in making the comparisons.

Our first concern is to assess the degree of damping to which each economy has been subject over the sample period. This is revealed by the estimated value of the damping factor $\rho = e^\gamma$, which is entailed in the analytic solution of a second-order difference equation. The inherent damping properties of the economy are revealed by the estimated loss parameter Λ , which, according to our analysis of section 2, is independent of the driving forces of the system. The rankings of the six economies according to ρ are not perfectly correlated with those according to Λ . This is a consequence of the fact that the estimated value of ω , which characterises the length of the economic cycles, differs amongst the countries.

A question which was posed at the end of section 2 was whether the frequency value ω , or equivalently the length of the cycle, that is implied by the estimated AR(2) equation is related to the degree of damping. Table 2 shows a high degree of correlation between the rankings of the values of ρ and ω . An active policy of counter-cyclical macro economic regulation might

¹The *Split Cosine Bell* function is given by:

$$w_t = \begin{cases} 0.5\{1 - \cos \pi(t - 0.5)/m\} & \text{for } t = 0, 1, \dots, m - 1, \\ 1 & \text{for } t = m, \dots, N - m - 1, \\ 0.5\{1 - \cos \pi(N - t + 0.5)/m\} & \text{for } t = N - m, \dots, N - 1. \end{cases}$$

where m is chosen so that $2m/n$ is equal to the proportion p of the data to be tapered.

Table 2: The rankings of the parameters of Table 1 in descending order of magnitude and in ascending order—via the numbers in parenthesis

	ρ	ω	Λ	σ_x	b
FRANCE	1(6)	5(2)	6(1)	5(2)	6(1)
ITALY	5(2)	2(5)	2(4)	6(1)	4(2)
NORWAY	4(3)	3(4)	1(6)	2(5)	2(4)
SPAIN	2(5)	6(1)	5(2)	1(6)	2(4)
U.K.	3(4)	4(3)	4(3)	3(4)	4(2)
U.S.A.	6(1)	1(6)	2(4)	4(3)	1(6)

be expected to manifest itself both in a high degree of damping and in a shortening of the cycles; and this may well explain some of this correlation.

Table 1 also indicates, via the values of σ_x , the relative amplitude of the economic cycles for each of the six countries. This is measured as the standard deviation of the logarithmic income series from the interpolated trend line. This set of figures constitutes a factual reportage, which may be at variance with our presuppositions. The final column of the Table 1 gives the values of b which is the slope parameter of a linear trend interpolated through the logarithmic series by least-squares regression. The values represent the average rates of growth of the countries for the period in question.

There is a supposition that the lower the relative amplitude of the business cycle the better are the prospects for the growth of the economy. This idea was famously propounded in a pamphlet published in the U.K. by the National Economic Development Council (1963) under the title of *Conditions Favourable to Faster Growth*. It was proposed that the economic stop-go policies of the United Kingdom in the preceding decade had created a series of booms and slumps that had inhibited the growth of the economy. Table 2 gives no indication of a negative relationship between the rates of growth of the countries and the relative amplitude of their economic cycles. However, the question remains of whether such a relationship could be found by observing the same economy in different epochs.

We now proceed to examine the effect of using the lowpass Hodrick–Prescott filter, instead of a quadratic function, to remove the trend from the data, in the attempt to reveal the business cycle. We shall attribute a range of alternative values to the smoothing parameter λ . The higher the

Table 3: Parameters for smoothing parameter equal to 8000

	α_1	α_2	ρ	ω	Λ	σ_x
FRANCE	-1.38422	0.86706	0.93116	41.99	0.26	0.01304
ITALY	-1.09024	0.69650	0.83456	49.22	0.47	0.01049
NORWAY	-1.25688	0.76454	0.87438	44.05	0.44	0.01612
SPAIN	-1.40937	0.82478	0.90817	39.11	0.41	0.01703
U.K.	-1.30833	0.81649	0.90360	43.62	0.34	0.01612
U.S.A.	-0.90703	0.68439	0.82728	56.76	0.37	0.01581

value of the smoothing parameter, the more rigid is the estimated trend. The analysis of section 3 has shown that, as λ increases, the resulting trend tends to a linear function that is interpolated through the data by a least-squares regression

Intuition suggests that the more flexible is the interpolated trend the lower will be the amplitudes of the fluctuations in the residual sequence and the shorter will be the duration of its cycles. This intuition has been supported by an analysis in the frequency domain of the effect of applying the H-P filter to a random walk. Figure 3 is relevant to that analysis.

We begin by setting $\lambda = 8000$. This value corresponds nominally to the frequency value of $\omega = \pi/30$ radians per quarter and to a duration of 15 years. In fact, the resulting trends that are interpolated through the data are virtually linear. When the AR(2) model is fitted to the detrended data, we obtain the parameter values that are recorded in Table 3.

The next value to be assigned to the smoothing parameter is $\lambda = 1600$, which is the value that is commonly used in extracting macroeconomic trends from quarterly data, and it has been proposed by Hodrick and Prescott (1980, 1997). This value produces a trend that strongly reflects the cyclical pattern of the original time series. For this reason, it provides an attenuated version of the business cycle. It will be observed from the comparison of Tables 3 and 4 that the relative amplitudes of the business cycles measured by σ_x are considerably reduced and the duration of the cycles, as reflected in the angular velocity ω , is systematically reduced. This is in accordance with our presuppositions.

The final value to be investigated is $\lambda = 677.13$, which corresponds nominally to an angular velocity of $\pi/16$ radians per quarter and to a duration

Table 4: Parameters for smoothing parameter equal to 1600

	α_1	α_2	ρ	ω	Λ	σ_x
FRANCE	-1.22796	0.85235	0.92323	48.31	0.22	0.008367
ITALY	-0.84551	0.71086	0.84312	59.91	0.30	0.007071
NORWAY	-1.07407	0.79689	0.89269	53.02	0.26	0.010488
SPAIN	-1.22190	0.74364	0.86235	44.89	0.47	0.009487
U.K.	-1.13421	0.80470	0.89705	50.79	0.27	0.010488
U.S.A.	-0.67159	0.76647	0.87548	67.45	0.19	0.011402

Table 5: Parameters for smoothing parameter equal to 677.13

	α_1	α_2	ρ	ω	Λ	σ_x
FRANCE	-1.10779	0.85086	0.92242	53.10	0.19	0.005477
ITALY	-0.65877	0.76005	0.87181	67.80	0.19	0.004472
NORWAY	-0.95635	0.81477	0.90265	58.01	0.20	0.007071
SPAIN	-1.00325	0.66526	0.81564	52.05	0.47	0.005477
U.K.	-0.99001	0.82177	0.90651	56.90	0.20	0.007071
U.S.A.	-0.52275	0.83719	0.91498	73.40	0.11	0.008367

of 8 years. These values correspond to the upper limit of the business cycles duration according to the definition of Baxter and King (1999). These authors have also attributed a minimum duration of 1.5 years to the business cycles. Artis, Marcellino and Proietti (2004), have implemented the definition of Baxter and King via a bandpass filter that comprises a lowpass H-P filter with $\lambda = 0.52$ followed by a highpass filter with $\lambda = 677.13$. The initial lowpass filter may have little effect upon deseasonalised data for the reasons that there is liable to be very little spectral power within the corresponding stop band.

The parameters derived by fitting an AR(2) to these data are given in table 5. We are disinclined to give much credence to these results for the reason that the 8-year limit on the duration of the cycles is an artificial one that does not appear to correspond to any evident feature in the spectral structure of the data.

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