Dynamic Conditional Correlation Models with Asymmetric Multivariate Laplace Innovations*

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Abstract

In this paper we propose a multivariate (GARCH) asymmetric generalised dynamic conditional correlation (AGDCC) model where the vector of standardised residuals is assumed to follow an asymmetric multivariate Laplace (AML) distribution. This multivariate distribution is able to capture leptokurtosis and asymmetry which characterise returns from financial assets. It preserves, under general conditions, desirable properties such as finiteness of moments and stability under geometric summation. The empirical validity of this form is tested in the context of a Value-at-Risk (VaR) model. We illustrate the methodology by fitting a sample of 21 FTSE All-World stock indices and 12 bond return indices. We provide clear evidence that in our data set this distribution overwhelmingly outperforms the case in which we assume normality of innovations.

PRELIMINARY, PLEASE DO NOT QUOTE. COMMENTS WELCOME.

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1 Introduction

A good understanding of the dynamic properties of cross-market correlation (or dependence across markets) is vital for assessing the level of integration between international markets both for investment purposes and for increasing the capacity to produce reliable forecasts. Modelling the dynamics of volatilities of returns from financial assets has been one of the working horses in the development of financial econometrics over the last years (Bollerslev, 2001; Engle, 2001). Nonetheless, most of the advances, especially if we consider the use of the proposed framework for practical purposes, have been seen almost exclusively in univariate cases. The growth in techniques modelling the dynamics of covariances and correlations has lagged considerably behind the growth in modelling time-varying volatility, as evidenced by the shortage of the literature on time-varying correlations compared to that of modelling time-varying volatility. One of the main reasons for this uneven expansion is the "curse of dimensionality", due to the extremely cumbersome problems faced in estimating unrestricted multivariate GARCH (MGARCH) models in highly dimensioned settings.

Among the various MGARCH specifications recently proposed in the literature[1], one in particular has proved to be particular suitable to provide a parsimonious, flexibility, and feasible model that significantly reduces the "curse of dimensionality". This is the Dynamic Conditional Correlation (DCC) model proposed in Engle (2002) and Engle and Sheppard (2001). In this model, the dynamic variance-covariance matrix of conditional returns is specified as a function of univariate variances and linear correlations. When the model is estimated by maximum likelihood this framework allows to "break" the log-likelihood function in two parts, one for the parameters determining univariate volatilities and another for the parameters determining the correlations (the so-called DCC two-step estimation technique). By using this technique large systems can be consistently estimated with limited computational costs without imposing too many restrictions like in the case of factor models.

A vital assumption of the DCC model is that standardized residuals are normally distributed. Nevertheless, financial time series do not favour this assumption. Where time-varying volatilities are estimated by assuming a normal-GARCH process for the innovations, it is easy to show that even for correctly specified models, statistically significant levels of leptokurtosis and excess kurtosis can still be found. This has important repercussions for risk management activities if the estimated variance is employed in a Value-at-Risk (VaR) analysis. If excess kurtosis is ignored then the probability of extreme events will almost surely be underestimated.

As already pointed out, returns from financial assets show well defined patterns of leptokurtosis and skewness which cannot be captured by the normality assumption. There are several multivariate distributions in the literature that present high levels of kurtosis as well as asymmetries and that could be used in a

[1] For a survey on MGARCH models see Bauwens et al (2003). Kroner and Ng (1998) also analyses several models.

MGARCH framework[2]. However, the majority of these distributions are either too complicated to be estimated for GARCH purposes or present undesirable properties (like an infinite variance) that impede their use for financial applications. One multivariate distribution that parsimoniously captures the main features of financial returns and keeps flexibility is the Asymmetric Multivariate Laplace (AML) distribution, as recently proposed by Kotz, Kozubowski, and Podgorski (2003). In the univariate context, the Laplace or double-exponential distribution has been widely used in financial modelling. Some applications include Madan and Seneta (1990), Madan et al. (1988), Linden (2001), Kou and Wang (2001), Hanson and Zhu (2004), Sepp (2004), Heyde and Kou (2004), Komunjer (2005) among many others. The asymmetric multivariate version used in this paper is defined as a subclass of geometric stable distributions (stable under geometric summation), a characteristic that in the case of the AML distribution can be used to model linear combinations of random variables with univariate symmetric Laplace distributions. This feature is extremely important as it permits the use of this distribution in the computation of the parametric-VaR of *portfolios* of financial assets, characteristic that was thought exclusive of the Pareto-stable distribution and of its most widely used limiting case such as the normal distribution.

Our work is in the spirit of Mencia and Seneta (2004) who use a generalised hyperbolic distribution in a model where the variance matrix dynamics follow a conditionally heteroskedastic single factor model and the conditional variance of the factor obeys a univariate GQARCH (1,1) process; and Bawens and Laurent (2004) who uses a type of multivariate skewed Student-t distribution to fit a DCC (1,1) model to two sets of three assets data. As far as we are concerned this is the first work where the AML distribution is used to model the returns of financial assets in a MGARCH setting.

The main aim of this paper is to develop a multivariate time-varying framework for modelling and forecasting cross-market correlations where innovations are assumed to follow an AML distribution. The outline of this paper is as follows. In Section 2, we review the dynamic conditional correlation (DCC) model of Engle (2002) and Engle and Sheppard (2001) and the extensions that allow for asymmetries in the dynamics and asset-specific correlations, as proposed by Cappiello, Engle and Sheppard (2004). In Section 3, we present a framework where the DCC is enriched by the asymmetric multivariate Laplace (AML) distribution. In Section 4, we discuss the implications of the estimation of the DCC model by maximum likelihood under the AML assumption. In Section 5 we report the results from an empirical application using a sample of 21 FTSE All-World stock indices and 12 bond return indices. In Section 6 we present the conclusions.

[2] Bauwens and Laurent (2004) briefly reviews the literature of multivariate asymmetric densities.

2 Dynamic Conditional Correlation (DCC) Models

Consider the $n-$dimensional returns process $\mathbf{X}_t \in \Re^{Txn}$, $t = 1, .., T$ generated as,

$$\mathbf{X}_t = \boldsymbol{\mu}_t(\boldsymbol{\theta}) + \mathbf{H}_t^{1/2}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \tag{1}$$

$$\boldsymbol{\mu}_t(\boldsymbol{\theta}) = E\left(\mathbf{X}_t \mid \Omega_{t-1}\right) \tag{2}$$

$$\mathbf{H}_t = Var\left(\mathbf{X}_t \mid \Omega_{t-1}\right) \tag{3}$$

where Ω_τ is the information set at time τ, and $\boldsymbol{\varepsilon}_t$ is an i.i.d. process. In the DCC setting $\mathbf{H}_t$ is modelled directly as a function of dynamic univariate variances and dynamic linear correlations,

$$\mathbf{H}_t \equiv \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t \tag{4}$$

$$\mathbf{R}_t = \left(\mathbf{Q}_t^*\right)^{-1}\mathbf{Q}_t\left(\mathbf{Q}_t^*\right)^{-1} \tag{5}$$

$$\mathbf{Q}_t = \left(1 - \sum_{l=1}^{L}\alpha_l - \sum_{s=1}^{S}\beta_s\right)\overline{\mathbf{Q}} + \sum_{l=1}^{L}\alpha_l\boldsymbol{\varepsilon}_{t-l}\boldsymbol{\varepsilon}_{t-l}^{'} + \sum_{s=1}^{S}\beta_s\mathbf{Q}_{t-s} \tag{6}$$

where

$$\mathbf{Q}_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & ... & 0 \\ 0 & \sqrt{q_{22}} & 0 & ... & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & ... & \sqrt{q_{nn}} \end{bmatrix}, \tag{7}$$

$\mathbf{D}_t \in \Re^{Txnxn}$ is a diagonal matrix with elements $\sqrt{h_{it}}, i = 1, ..., n,\ t = 1, .., T$, $\overline{\mathbf{Q}} \in \Re^{nxn}$ is the unconditional variance-covariance matrix of $\boldsymbol{\varepsilon}_t$, i.e. $\overline{\mathbf{Q}} = E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')$, and α_l and β_s are scalar parameters satisfying $\sum_{l=1}^{L}\alpha_l + \sum_{s=1}^{S}\beta_s < 1$. The specification in (5) secures that $\mathbf{R}_t$ will be a valid correlation matrix while (4) and (6), in addition to the condition of stationarity, secure $\mathbf{H}_t$ to be a positive definite matrix.

The dynamics in (4) are specially appealing because allow for a two step estimation that makes feasible the estimation of highly dimensioned $\mathbf{X}_t$ processes, estimation that for many non-factor models is usually not possible because of the "curse of dimensionality".

To illustrate the two-step estimation technique let us assume first normality for the vector of standardise residual, i.e. $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{R}_t)$. Denoting $\boldsymbol{\theta}$ as the vector of parameters in the conditional variance-covariance matrix $\mathbf{H}_t$, the log-likelihood $L_T(\boldsymbol{\theta})$ for the T observations of this estimator,

$$L_T(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log f(\mathbf{X}_t \mid \boldsymbol{\theta},\Omega_{t-1}) \tag{8}$$

is given by,

$$L_T(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T} \left\{ n\log(2\pi) + \log|\mathbf{H}_t| + \mathbf{r}_t^{'}\mathbf{H}_t^{-1}\mathbf{r}_t \right\} \tag{9}$$

Following (4) we have,

$$L_T(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T} \left\{ n\log(2\pi) + \log|\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t| + \mathbf{r}_t^{'}(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{r}_t \right\} \tag{10}$$

$$L_T(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T} \left\{ n\log(2\pi) + \log\left|\mathbf{D}_t^2\right| + \log|\mathbf{R}_t| + \boldsymbol{\varepsilon}_t^{\prime}\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t \right\} \tag{11}$$

where in (11) we replace by the standardise residual $\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$.

Engle (2002) proposes to estimate the first stage by assuming $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0},\mathbf{I})$ where $\mathbf{I} \in \Re^{nxn}$ is an identity matrix. By partitioning the vector of parameters in two subsets $\boldsymbol{\theta} = (\boldsymbol{\zeta},\boldsymbol{\varphi})$, where $\boldsymbol{\zeta}$ contains the parameters of the n univariate volatilities and $\boldsymbol{\varphi}$ contains the parameters of the correlations, the log-likelihood function can be expressed as,

$$L_T(\boldsymbol{\theta}) = L_T(\boldsymbol{\zeta}) + L_T(\boldsymbol{\varphi} \mid \boldsymbol{\zeta}) \tag{12}$$

The estimation of the first stage consists in the maximization of the function,

$$L_T(\boldsymbol{\zeta}) = -\frac{1}{2}\sum_{t=1}^{T} n\log(2\pi) + \log\left|\mathbf{D}_t^2\right| + \mathbf{r}_t^{'}\mathbf{D}_t^{-2}\mathbf{r}_t \tag{13}$$

$$L_T(\boldsymbol{\zeta}) = -\frac{1}{2}\sum_{t=1}^{T} \left[n\log(2\pi) + \sum_{i=1}^{n}\left(\log h_{it} + \frac{r_{it}^2}{h_{it}}\right)\right] \tag{14}$$

$$L_T(\boldsymbol{\zeta}) = -\frac{1}{2}\sum_{t=1}^{T} \left[\sum_{i=1}^{n}\left(\log(2\pi) + \log h_{it} + \frac{r_{it}^2}{h_{it}}\right)\right] \tag{15}$$

Once the vector $\boldsymbol{\zeta}$ is estimated, the vector of standardise residuals $\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$ is employed in the second stage, which corresponds to the maximization of the function,

$$L(\boldsymbol{\varphi} \mid \boldsymbol{\zeta}) = -\frac{1}{2}\sum_{t=1}^{T} \log|\mathbf{R}_t| + \boldsymbol{\varepsilon}_t^{\prime}\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t, \tag{16}$$

under the assumption $\varepsilon_t \sim N(\mathbf{0}, \mathbf{R}_t)$.

The novelty of this technique is that estimation is speeded up by employing $n+1$ log-likelihood functions[3] instead of one single (but nonetheless extremely flat) log-likelihood function.

To show consistency of the two-step DCC estimator Engle (2002) employs the results in Newey and McFadden (1994) for the two-step Generalised Method of Moments (GMM). The result follows from the fact that Maximum Likelihood estimation can be considered a special case of the GMM when the moment conditions are set equal to the scores of the log-likelihoods,

$$\nabla_{\boldsymbol{\zeta}} L_T(\boldsymbol{\zeta}) = \mathbf{0} \tag{17}$$

$$\nabla_{\boldsymbol{\varphi}|\boldsymbol{\zeta}} L_T(\boldsymbol{\varphi} \mid \boldsymbol{\zeta}) = \mathbf{0} \tag{18}$$

2.1 Extensions of the DCC Model: Asymmetries and Asset Specific Correlations

The specification in (6) can be enriched by allowing for asymmetries in conditional correlations and covariances as well as for asset-specific correlations. We will refer to this general model as the Asymmetric Generalised Dynamic Conditional Correlation (AGDCC) (L,S,U) model, where L corresponds to the number of autoregressive lags, S corresponds to the number of persistence lags, and U corresponds to the number of asymmetric shock lags[4]. The specification of the matrix $\mathbf{Q}_t$ for the AGDCC (1,1,1) case is given by,

$$\begin{aligned}\mathbf{Q}_t \;=\; & \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + \mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A} + \mathbf{B}'\mathbf{Q}_{t-1}\mathbf{B} + \\ & \mathbf{G}'\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}'\mathbf{G}\end{aligned} \tag{19}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{G}$ are diagonal parameter matrices ($\mathbf{A}, \mathbf{B}, \mathbf{G} \in \Re^{nxn}$) with elements a_{ii}, b_{ii} and g_{ii} respectively, $\boldsymbol{\eta}_\tau = I\left[\varepsilon_\tau < 0\right] \circ \varepsilon_\tau$, " $\circ$ " denotes the Hadamard product and $\overline{\mathbf{N}} = E(\boldsymbol{\eta}_t \boldsymbol{\eta}_t')$. $\mathbf{Q}_t$ will be positive-definite if

$$\left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right)$$

is positive definite. Because this cannot always be guaranteed, we will follow the approach in Hafner and Franses (2003) and replace this expression by $\left(1 - \overline{a}^2 - \overline{b}^2 - \overline{g}^2\right)\overline{\mathbf{Q}}$. The correlation-targeting approach implicit in (19) is sacrificed with this substitution but $\mathbf{Q}_t$ will be positive-definite.

[3] The first stage implies the estimation of n independent univariate volatility processes and the second stage the estimation of one single correlation process.

[4] The extension to an Asymmetric Dynamic Conditional Correlation (ADCC) (L,S,U) model is presented in Cappiello et al (2004). Alternative generalisations that allow for asset-specific correlations and group-specific correlations are presented in Billio et al (2004, "Flexible DCC" model), Hafner and Franses (2003, "Generalised DCC" model), and also in Cappiello et al (2004, "Asymmetric Generalised DCC" model).

The AGDCC (L,S, U) model nests several specifications:

- DCC (L,S) model : $\mathbf{G} = [0], \mathbf{A} = \sqrt{a}, \mathbf{B} = \sqrt{b}$
- ADCC (L,S, U) model: $\mathbf{G} = \sqrt{g}, \mathbf{A} = \sqrt{a}, \mathbf{B} = \sqrt{b}$
- GDCC (L,S) model: $\mathbf{G} = [0]$.

3 The Asymmetric Multivariate Laplace Distribution

Asymmetric Laplace laws were introduced in Kozubowski and Podgorski (2001) as a subclass of geometric stable distributions. Kotz et al (2003) generalised these laws to the multivariate case.

In the geometric stable model, the return $r_{f(p)}$ is consider to be the sum of smaller returns $r^{(i)}$ over the period of time $f(p)$ which is a stopping time random variable with geometric probability function $P(f(p) = j) = p(1-p)^{j-1}, \quad k = 1, 2, ...$ The geometric stable distribution can be approximated to a normalised geometric stable model sum when the p parameter of the stopping time function $f(p)$ approaches zero. More formally, the random array $\mathbf{X}$ has a geometric stable distribution in $\Re^n$ if and only if,

$$a(p) \sum_{i=1}^{f(p)} \left(\boldsymbol{\kappa}(p) + \mathbf{r}^{(i)} \right) \xrightarrow{D} \mathbf{X}, \qquad \text{as } p \longrightarrow 0 \tag{20}$$

where $\left\{ \{\mathbf{r}^{(d)} = \left(r_1^{(d)}, ..., r_n^{(d)} \right), d \geq 1 \right\}$ is a sequence of i.i.d. random vectors in $\Re^n$ independent of $f(p)$, $a(p) > 0$, $\boldsymbol{\kappa}(p) \in \Re^n$, and $\xrightarrow{D}$ denotes convergence in distribution. The AML distribution appears in this context when the distributional limit (20) is restricted to have a finite second moment. More precisely, Kozubowski and Podgorski (2001) shows that when each vector in $\mathbf{r}$ has a mean $m_i, i = 1, ...n$, a variance $\sigma_{ij}, i = 1, ...n, j = 1, ...n$, and when we let $a(p) = \sqrt{p}$ and $\boldsymbol{\kappa}(p) = \mathbf{m} \left(\sqrt{p} - 1 \right)$, the random variable $\mathbf{X}$ defined by the convergence in distribution in (20) will have an AML distribution with the characteristic function,

$$\Psi(\mathbf{t}) = \frac{1}{1 + \frac{1}{2}\mathbf{t}'\mathbf{H}\mathbf{t} - i\mathbf{t}'\mathbf{m}} \tag{21}$$

where $\mathbf{t} \in \Re^n$, and $\mathbf{H} \in \Re^{nxn}$ is a positive-definite matrix with elements $\sigma_{ij}, i = j$ in the diagonal and $\sigma_{ij}, i \neq j$ in the off-diagonal.

The density function of the AML distribution allowing for time dependency in $\mathbf{H}$ and $\mathbf{r}$ is given by,

$$f(\mathbf{r}) = \frac{2\exp\left(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{m}\right)}{(2\pi)^{n/2}\left|\mathbf{H}_t\right|^{1/2}}\left(\frac{\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t}{2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m}}\right)^{v/2} K_v\left(\sqrt{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t)}\right) \tag{22}$$

where $v = (2-n)/2$ and $K_v(u)$ is the modified Bessel function of the third kind defined by $K_v(u) = \frac{(u/2)^v\Gamma(1/2)}{\Gamma(v+1/2)}\int_1^\infty e^{-ut}(t^2-1)^{v-1/2}dt$, $u > 0, v \geq -1/2$. The vector $\mathbf{m}$ is the location parameter and the matrix $\mathbf{H}$ is the scale parameter of this distribution.

A very important characteristic of the AML distribution is that it is unimodal with the mode equal to zero. Because of this the $\mathbf{m}$ parameter does not only determines the mean of the distribution, but also its level of asymmetry. When $\mathbf{m} = \mathbf{0}$ the distribution is symmetric collapsing as it can clearly be seen in equation (21) to the elliptical case (discussed in Johnson and Kotz (1972)).

Figures 1-6 present alternative bivariate AML densities with various $\mathbf{m}$ vectors and correlation levels.

[INSERT FIGURE 1 TO 6 HERE]

3.1 Generalised Hyperbolic Distributions and Asymmetric Multivariate Laplace distributions

As shown in Kotz et al (2003), AML distributions can also be obtained as a limiting case of the Generalised Hyperbolic (GH) distribution[5]. These are location-scale mixtures of normal distributions, i.e. if $\mathbf{X}$ has a GH distribution in $\Re^n$ then,

$$\mathbf{X} \overset{D}{=} \boldsymbol{\mu} + \mathbf{m}\boldsymbol{\xi} + \boldsymbol{\xi}^{1/2}\mathbf{Z} \tag{23}$$

where $\mathbf{Z} \sim N_n(\mathbf{0}, \mathbf{H})$, $\boldsymbol{\mu} \in \Re^n$, and $\boldsymbol{\xi}$ is a generalised inverse Gaussian variable with parameters ν, γ, and δ, i.e. $\boldsymbol{\xi} \sim GIG(\nu, \gamma, \delta)$. AML distributions appear when $\boldsymbol{\mu} = \mathbf{0}$ and when $\boldsymbol{\xi}$ is not $GIG(\nu, \gamma, \delta)$ but standard exponential, i.e. $\boldsymbol{\xi} \sim EXP(1)$[6].

Mencia and Sentana (2004) analyse the GH distribution in multivariate conditionally heteroskedastic dynamic regression models. The dynamics of the conditional covariance matrix $\mathbf{H}_t$ are given by a single factor model with a GQARCH(1,1) specification for the common factor, and a time-invariant diagonal matrix for the idiosyncratic terms. Given that $\boldsymbol{\mu} = \mathbf{0}$ because the mean of the returns has been removed prior to estimation, the only difference with the AML distribution resides in the employed mixing distribution. The generalised

[5] GH distributions were introduced in Barndorff-Nielsen (1977)

[6] The limiting case $GIG(1, 0, 2)$ is equivalent to $EXP(1)$

inverse Gaussian distribution allows for flexible tail modelling but at the cost of limiting the inclusion of rich dynamics for the conditional variance matrix because of the "course of dimensionality". For the case of a highly parameterised specification like the AGDCC model the estimation using the GH distribution is extremely difficult.

The representation of the AML distribution as a location-scale mixture of normal distributions is given by,

$$\mathbf{X} \stackrel{D}{=} \mathbf{m}\boldsymbol{\xi} + \boldsymbol{\xi}^{1/2}\mathbf{Z} \tag{24}$$

where in this case $\boldsymbol{\xi} \sim EXP(1)$. From this it can easily be seen that $E(\mathbf{X}) = \mathbf{m}$ and $Var(\mathbf{X}) = \mathbf{H} + \mathbf{mm}'$. This is of particular importance for the estimation of the MGARCH model. Contrary to the Gaussian case, the variance of a random variable with AML distribution does not coincide with the scale parameter of the distribution[7].

In contrast with the majority of GH distributions, the AML distribution in the special case $\mathbf{m} = \mathbf{0}$ is stable[8]. This condition implies an important property necessary for the modelling of financial portfolios known as the *additivity property*, which is basically the concept that a linear combination of independent random variables with stability index α is also stable with the same parameter α[9].

Pareto stable distributions are stable under random summation. Formally, the random variable $\mathbf{X}$ is said to be Pareto stable if for any $a_i > 0$, $i = 1, ..., d$, there exist a constant $c = d^{1/\alpha}$ and $\mathbf{u}_d \in \Re^n$ for any $d \geq 2$ such that,

$$a_1\mathbf{X}^{(1)} + ... + a_d\mathbf{X}^{(d)} \stackrel{D}{=} c\mathbf{X} + \mathbf{u}_d \tag{25}$$

where $\mathbf{X}^{(1)}, ..., \mathbf{X}^{(d)}$ are independent copies of $\mathbf{X}$. In an alike way Laplace laws are stable, but not under random summation but under geometric summation. To be able to preserve stability we have to constraint the normalising constants $a(p)$ and $\boldsymbol{\kappa}(p)$ in (20) to,

$$a(p) = \sqrt[\alpha]{p}, \quad \boldsymbol{\kappa}(p) = \mathbf{0} \tag{26}$$

The first condition implies that for the case of the AML distribution $\alpha = 2$. This is the same alpha value of the normal distribution which is the only Pareto-stable distribution with a finite second moment. The second condition $\boldsymbol{\kappa}(p) = \mathbf{0}$ implies $\mathbf{m} = \mathbf{0}$, restricting the use of the distribution for portfolio-VaR applications to the symmetric case.

[7] $Var(\mathbf{X}) = \mathbf{H}$ only when the distribution is elliptical, i.e. when $\mathbf{m} = \mathbf{0}$.

[8] The only GH distribution besides the AML distribution that possesses this property is the normal distribution.

[9] See Khindanova et al (2001)

4 New likelihood function and estimation

We now turn to the details of the estimation of the DCC models which employ the AML distribution described in the previous section. The log-likelihood function $L_T^{AML}(\boldsymbol{\theta})$ assuming a AML distribution for the conditional returns is proportional to,

$$
\begin{aligned}
L_T^{AML}(\boldsymbol{\theta}) &= \sum_{t=1}^{T} \Big\{ \mathbf{r}_t' \mathbf{H}_t^{-1} \mathbf{m} - \frac{1}{2} \ln |\mathbf{H}_t| + \\
& \frac{v}{2} \left(\ln(\mathbf{r}_t' \mathbf{H}_t^{-1} \mathbf{r}_t) - \ln(2 + \mathbf{m}' \mathbf{H}_t^{-1} \mathbf{m}) \right) \\
& + \ln \left[K_v \left(\sqrt{(2 + \mathbf{m}' \mathbf{H}_t^{-1} \mathbf{m})(\mathbf{r}_t' \mathbf{H}_t \mathbf{r}_t)} \right) \right] \Big\}
\end{aligned} \tag{27}
$$

Following the transformations explained in Section 2 equation (27) is converted to,

$$
\begin{aligned}
L_T^{AML}(\boldsymbol{\theta}) &= \sum_{t=1}^{T} \Big\{ \mathbf{r}_t' (\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t)^{-1} \mathbf{m} - \frac{1}{2} \ln |(\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t)| \\
& + \frac{v}{2} \left(\ln(\mathbf{r}_t' \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \mathbf{r}_t) - \ln(2 + \mathbf{m}' (\mathbf{D_t R_t D_t})^{-1} \mathbf{m}) \right) \\
& + \ln \left[K_v \left(\sqrt{(2 + \mathbf{m}' (\mathbf{D_t R_t D_t})^{-\mathbf{1}} \mathbf{m})(\mathbf{r}_t' \mathbf{D_t R_t D_t} \mathbf{r}_t)} \right) \right] \Big\}
\end{aligned} \tag{28}
$$

Denoting again $\boldsymbol{\zeta}$ as the set of parameters in the matrix $\mathbf{D}_t$ we have,

$$
\begin{aligned}
L_T^{AML}(\boldsymbol{\zeta}) &= \sum_{t=1}^{T} \big\{ \mathbf{r}_t' \mathbf{D}_t^{-2} \mathbf{m} - \ln |\mathbf{D}_t| + \\
& \frac{v}{2} \left(\ln(\mathbf{r}_t' \mathbf{D}_t^2 \mathbf{r}_t) - \ln(2 + \mathbf{m}' \mathbf{D_t^{-2}} \mathbf{m}) \right) \\
& + \ln K_v \left(\sqrt{(2 + \mathbf{m}' \mathbf{D_t^{-2}} \mathbf{m})(\mathbf{r}_t' \mathbf{D_t^2} \mathbf{r}_t)} \right) \Big\}
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
L_T^{AML}(\boldsymbol{\zeta}) &= \sum_{t=1}^{T} \sum_{i=1}^{n} \left\{ \left(\frac{r_{it} m_i}{h_{it}} - \ln \sqrt{h_{it}} \right) + \right. \\
& \frac{v}{2} \left(\ln \sum_{i=1}^{n} \frac{r_{it}^2}{h_{it}} - \ln \left(2 + \sum_{i=1}^{n} \frac{m_i^2}{h_{it}} \right) \right) \\
& \left. + \ln K_v \left(\sqrt{\left(2 + \sum_{i=1}^{n} \frac{m_i^2}{h_{it}} \right) \sum_{i=1}^{n} \frac{r_{it}^2}{h_{it}}} \right) \right\}
\end{aligned} \tag{30}
$$

Contrary to (15), $L_T^{AML}(\boldsymbol{\zeta})$ cannot be expressed as the sum of n-log-likelihood functions, i.e. the parameters in $\boldsymbol{\zeta}$ have to be estimated maximizing *one single* log-likelihood function. This does not impede the use of the two-step estimation technique[10] although it does extend the time for estimation.

Defining $\boldsymbol{\varepsilon}_t = \mathbf{r}_t'\mathbf{D}_t^{-1}$ and $\boldsymbol{\varepsilon}_t^* = \mathbf{m}'\mathbf{D}_t^{-1}$ the second-stage log-likelihood is given by,

$$\begin{aligned} L_T^{AML}(\boldsymbol{\varphi}|\,\boldsymbol{\zeta}) &= \sum_{t=1}^{T}\Bigg\{\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\left(\boldsymbol{\varepsilon}_t^*\right)' - \frac{1}{2}\ln|\mathbf{R}_t| + \\ &\quad \frac{v}{2}\left(\ln(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t') - \ln(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}\left(\boldsymbol{\varepsilon}_t^*\right)')\right) \\ &\quad + \ln K_v\left(\sqrt{(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}\left(\boldsymbol{\varepsilon}_t^*\right)')(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t')}\right)\Bigg\} \end{aligned} \tag{31}$$

In the case $n = 2s+3$, where $s = 0, 1, 2, ...,$ we have $v = -(s+1)/2$ and $K_v = K_{-v}$, in which case the Bessel function has a close form that simplifies expression (22) to,

$$\begin{aligned} f(\mathbf{r}) &= \frac{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})^{s/2}\exp\left(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{m}-\sqrt{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t)}\right)}{\left(2\pi\sqrt{\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t}\right)^{s+1}|\mathbf{H}_t|^{1/2}} \\ &\bullet\sum_{k=0}^{s}\frac{(s+k)!}{(s-k)!k!}\left(2\sqrt{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t)}\right)^{-k} \end{aligned} \tag{32}$$

This form is very convenient as it avoids the computation of the Bessel function. The benefits are evident in the routines that maximise both log-likelihood functions. Of course it can only be applied in circumstances where the number of series in the matrix of returns is odd, but nonetheless its use in these particular situations yields enormous computational benefits.

Following a similar process as in equations (27) to (31) we have,

$$\begin{aligned} L_T^{AML^*}(\boldsymbol{\theta}) &= \sum_{t=1}^{T}\Bigg\{\frac{s}{2}\ln(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})-\frac{(s+1)}{2}\ln(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t) - \frac{1}{2}\ln|\mathbf{H}_t| \\ &\quad +\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{m}-\sqrt{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t)} \\ &\quad +\ln\left(\sum_{k=0}^{s}\frac{(s+k)!}{(s-k)!k!}\left(2\sqrt{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t)}\right)^{-k}\right)\Bigg\} \end{aligned} \tag{33}$$

[10] The parameter estimates in the vector $\widehat{\boldsymbol{\zeta}}$ will still be consistent if the specification for the distribution of the returns is correct

$$
\begin{aligned}
L_T^{AML^*}(\boldsymbol{\theta}) \;=\; & \sum_{t=1}^{T}\Bigg\{\frac{s}{2}\ln(2+\mathbf{m}'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{m}) - \frac{(s+1)}{2}\ln(\mathbf{r}_t'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{r}_t) \\
& -\frac{1}{2}\ln|(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)| + \mathbf{r}_t'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{m} - \\
& \sqrt{(2+\mathbf{m}'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{m})(\mathbf{r}_t'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{r}_t)} \\
& + \ln\left(\begin{array}{c}\sum_{k=0}^{s}\frac{(s+k)!}{(s-k)!k!}\cdot \\ \left(2\sqrt{(2+\mathbf{m}'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{m})(\mathbf{r}_t'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{r}_t)}\right)^{-k}\end{array}\right)\Bigg\}
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
L_T^{AML^*}(\boldsymbol{\zeta}) \;=\; & \sum_{t=1}^{T}\Bigg\{\frac{s}{2}\ln(2+\mathbf{m}'\mathbf{D}_t^{-2}\mathbf{m}) - \frac{(s+1)}{2}\ln(\mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t) \\
& -\ln|\mathbf{D}_t| + \mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{m} - \sqrt{(2+\mathbf{m}'\mathbf{D}_t^{-2}\mathbf{m})(\mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t)} \\
& + \ln\left(\sum_{k=0}^{s}\frac{(s+k)!}{(s-k)!k!}\left(2\sqrt{(2+\mathbf{m}'\mathbf{D}_t^{-2}\mathbf{m})(\mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t)}\right)^{-k}\right)\Bigg\}
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
L_T^{AML^*}(\boldsymbol{\zeta}) \;=\; & \sum_{t=1}^{T}\Bigg\{\frac{s}{2}\ln\left(2+\sum_{i=1}^{n}\frac{m_i^2}{h_{it}}\right) - \frac{(s+1)}{2}\ln\sum_{i=1}^{n}\frac{r_{it}^2}{h_{it}} \\
& + \sum_{i=1}^{n}\left(\frac{r_{it}m_i}{h_{it}} - \ln\sqrt{h_{it}}\right) - \sqrt{\left(2+\sum_{i=1}^{n}\frac{m_i^2}{h_{it}}\right)\left(\sum_{i=1}^{n}\frac{r_{it}^2}{h_{it}}\right)} \\
& + \ln\left(\sum_{k=0}^{s}\frac{(s+k)!}{(s-k)!k!}\left(2\sqrt{\left(2+\sum_{i=1}^{n}\frac{m_i^2}{h_{it}}\right)\left(\sum_{i=1}^{n}\frac{r_{it}^2}{h_{it}}\right)}\right)^{-k}\right)\Bigg\}
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
L_T^{AML^*}(\boldsymbol{\varphi}|\,\boldsymbol{\zeta}) \;=\; & \sum_{t=1}^{T}\Bigg\{\frac{s}{2}\ln(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)') - \frac{(s+1)}{2}\ln(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t') \\
& -\frac{1}{2}\ln|\mathbf{R}_t| + \boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)' - \sqrt{(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)')(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t')} \\
& + \ln\left(\sum_{k=0}^{s}\frac{(s+k)!}{(s-k)!k!}\left(2\sqrt{(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)')(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t')}\right)^{-k}\right)\Bigg\}
\end{aligned} \tag{37}
$$

Finally, the functions (36) and (37) were estimated via maximum likelihood estimation method.

5 Empirical Application

This section is divided in two parts. First we estimate the AGDCC (1,1,1) model and its nested models (GDCC (1,1), ADCC (1,1), and DCC (1,1)) assuming that the conditional innovations are distributed as an AML. We employed the same data set of Cappiello et al (2004)[11] and thus it will be interesting to compare our results with those where normality is assumed. We consider shares indices of 21 countries listed in the FTSE All-World Indices and bond indices of 12 countries constructed by datastream. We refer the interested reader to Cappiello et al (2004) for a detailed description of the data. The frequency is weekly and spans over the period 08/01/1987-07/02/2002 (785 observations). The 21 countries of the share indices are: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, United Kingdom, and the United States. The 12 countries of the bond indices are Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Japan, Netherlands, Sweden, Switzerland, and the United Kingdom.

In the second part of this section we make use of the conditional variances estimated in section 5.1 to perform a conditional Value-at-Risk (VaR) exercise. We use the backtesting duration-based approach of Christoffersen and Pelletier (2003) and the Monte Carlo tests of Dufour (2004) to evaluate the validity of the different estimated MGARCH models and the assumption of a AML distribution for conditional returns.

5.1 Estimation of Models

Weekly returns were calculated through log differences using Friday to Friday closing prices and filtered by removing the mean,

$$r_{jt} = \log\left(\frac{P_{jt}}{P_{jt-1}}\right) - \frac{1}{T}\sum_{i=1}^{T}\log\left(\frac{P_{ji}}{P_{ji-1}}\right), \quad j = 1, ...n \tag{38}$$

where P_{jt} is the price of assets j at time t.

We estimate the four models described in Section 2.1: AGDDC (1,1,1), GDCC(1,1), ADCC (1,1,1), and the DCC (1,1)[12]. Table 1 presents the parameter estimates of the GARCH (1,1) processes for the univariate volatilities, the parameter estimates of the asymmetry vector $\mathbf{m}$, and the skewness and kurtosis of the returns standardised by their estimated standard deviation.

[INSERT TABLE 1 HERE]

To evaluate the assumption of normality for the studied data we first compare the Kolmogorov-Smirnov distances between the data and the two implicit univariate normal and asymmetric Laplace distributions. Table 2 shows how the

[11] We wish to thank Kevin Sheppard for providing us with the dataset.

[12] For simplicity from this point we drop the number of lags when we refer to the MGARCH models estimated in the exercise.

normal distribution performs distinctively worse than the asymmetric Laplace distribution for each one of the indices. Overall, the Kolmogorov-Smirnov distances are 2.85% smaller for the group of share indices and 1.58% smaller for the group of bond indices when the AML distribution is used instead of the normal distribution.

[INSERT TABLE 2 HERE]

To evaluate normality in a multivariate framework we employed the omnibus test of Doornik and Hansen (1994). Multivariate normality was overwhelmingly rejected for the raw and standardised data after fitting the normal DCC, ADCC, GDCC, and AGDCC models (all p-values ≈ 0).

Before estimating the models for the conditional correlation we evaluated the constancy of correlation performing the LM test of Tse (2000). We overwhelmingly reject the null of constant correlation with a p-value = 0.000. This result justifies further analysis where a time-dependency in the correlation is included.

Table 3 reports the correlation parameter estimates for the DCC and ADCC models.

[INSERT TABLE 3 HERE]

We found that the parameter estimates of the DCC (1,1) model are very similar to those reported in Cappielo et al (2004). This is not the case of the ADCC model. For this model Cappiello et al (2004) report a much higher level for the persistence parameter (0.94816 for the normal case against 0.5913 for the AML case).

The correlation parameter estimates for the GDCC and AGDCC specifications are reported in Table 4. Overall, we found high levels of persistence but not as pronounced as in Capiello et al (2004). For the case of normal innovations the range of the beta parameter in the GDCC model goes from 0.9186 (Canada shares) to 0.9759 (Austria bonds), while for the case of AML innovations is much more open; it goes from 0.1764 (Germany shares) to 0.9748 (New Zealand shares). The parameter estimates of the AGDCC model also show a higher degree of heterogeneity across indices when the AML distribution is assumed.

[INSERT TABLE 4 HERE]

All asymmetric parameters in the AGDCC model were highly significant. Table 5 reports the Log-likelihood values for each one of the four models.

[INSERT TABLE 5 HERE]

In contrast to the case described in Cappiello et al where the innovations are assumed normal, the inclusion of asymmetric terms does not increase the log-likelihood. In addition, the diagonal models are not always superior to the scalar versions; the augment in the log-likelihood is only seen in the DCC-GDCC pair.

We follow Hafner and Franses (2003) and employ the minimum variance portfolio criterion as a specification test of the models. We compare the variance of the portfolios formed by all the securities in the array $\mathbf{X}_t$ estimated with the eight models (four models assuming normality and four models assuming the AML distribution). The weight vector at time t for each one of the portfolios is given by,

$$^{m_i}\mathbf{w}_t = \frac{^{m_i}\mathbf{H}_t^{-1}\boldsymbol{\iota}}{\boldsymbol{\iota}'^{m_i}\mathbf{H}_t^{-1}\boldsymbol{\iota}} \tag{39}$$

where $i = 1, ..., 8$ and $\boldsymbol{\iota}$ is an (n x 1) vector of ones. The variance of each portfolio will be given by $\mathbf{V}_t = \mathbf{w}_t'\mathbf{H}_t\mathbf{w}_t$. If $^{m_i}\mathbf{H}_t$ is accurately specified, then model m_i should give the minimum variance portfolio. Figures 7a and 7b shows the eight series of $^{m_i}\mathbf{V}_t$ and Table 6 presents the average portfolio volatilities.

[INSERT TABLE 6 HERE]
[INSERT FIGURES 7a AND 7b HERE]

According to this criterion the AML-AGDCC model is the best specified followed by the normal-AGDCC. Interestingly, the remaining three AML models under-performed in relation to their normal counterparts.

As explained earlier in the paper, one of the main motivations for the use of the AML distribution is its applicability in the computation of the parametric VaR of a portfolio of financial assets. In our setting the calculation of the VaR is not only improved by assuming a more realistic distribution for the return of financial assets, but also by the inclusion of a time-dependent specification for the variance of the portfolio. This time-varying variance will also be a function of the time-varying correlations between the assets composing the portfolio (besides the weights and variances of the individual assets). It is clear then that the distributional features of these correlations will have a great impact in the computed VaR, and in particular, the level of kurtosis in the correlation time series will in some degree indicate the capability of the model to capture the so called "correlation breakdown" effect[13].

Plots with the distribution of conditional correlations for four pairs of correlations as well as descriptive statistics for the estimated series are presented in Figure 8 to Figure 11.

[INSERT FIGURES 8 TO 11 HERE]

First, we observe that the distribution of the correlation across models changes significantly. Across the four pairs the ADCC model with AML innovations highlights for its extreme level of kurtosis (225.11 for the UK shares-US

[13] Correlation breakdow is what in the literature is described as the fenomenon where after controlling for the change in volatility of the individual assets composing the portfolio, the empirical regularity of the measure of co-movement between series changes drastically over periods of time. See Kaplanis (1988), Ratner (1992), Bertero and Mayer (1990), Longin and Solnik (1995), Karolyi and Stulz (1995), Longin and Solnik (1998), Goetzman et al (2001), and Forbes and Rigobon (2002).

shares pair, 24.79 for the Japan bonds-UK bonds pair, 30.49 for the UK shares-Mexico shares pair, and 20.92 for the UK bonds-Switzerland bonds pair). This leptokurtosis is a result of very small volatilities (0.84%, 0.72%, 0.82%, and 0.52% respectively) and one single positive jump registered on Black Monday in 1987.

Plots with the conditional correlation series for four pairs of correlations are presented in Figure 12 to Figure 15.

[INSERT FIGURES 12 TO 15 HERE]

In general, the kurtosis registered for the dynamic correlation estimated assuming an AML distribution is higher than that estimated assuming normality. In Table 7 we present a comparison of the levels of kurtosis between correlations assuming the two types of distributions for the asset pairs considered in Figures 8 to 15.

[INSERT TABLE 7 HERE]

5.2 Value-at-Risk (VaR) application

A frequently used methodology for estimation of market risks that appear from variations in prices of shares, exchange rates, bonds, and interest rates is the Value-at-Risk (VaR), which is defined as the maximum possible loss in a portfolio over a particular period of time at a specified confidence level. There are several techniques to approximate the distribution of returns in the VaR framework: Parametric methods, historical simulation, Monte Carlo simulation, and stress-testing. For the application of the AGDCC models estimated in section 5.1 we will focus on the model-based parametric method were variations in the portfolio are characterized by a parametric distribution. Formally, consider the portfolio return,

$$\mathbf{r}_p = \sum_{i=1}^{n} w_i r_i = \mathbf{w}'\mathbf{r} \tag{40}$$

where $w_1 + ... + w_n = 1$. The VaR at the α level is the solution to,

$$\alpha = \int_{-\infty}^{VaR} f(\mathbf{r}_p) d\mathbf{r}_p \tag{41}$$

where $f(\mathbf{r}_p)$ is the density function of $\mathbf{r}_p$. As explained in Section 3, stable and geometric stable distributions have the additivity property that allow us to use them in the modelling of portfolio returns. In the special case where $f(\mathbf{r}_p)$ is the density of the AML distribution and assuming that the magnitude of the mean vector $\boldsymbol{\mu}_t$ is considerably less significant than the size of $\mathbf{H}_t^{1/2}$ and, therefore, can be ignored, the conditional VaR implicit in (41) reduces to,

$$VaR_t = \left(\mathbf{w}_t'\mathbf{H}_t\mathbf{w}_t\right)^{1/2} L_\alpha \tag{42}$$

where L_α is the α-th quantile of the univariate standard Laplace distribution. We consider three invariant vectors of weights $\mathbf{w}$:

$$\mathbf{w^1} = \begin{bmatrix} w_1^1 = 0.035714286 \\ . \\ . \\ . \\ w_{21}^1 = 0.035714286 \\ w_{22}^1 = 0.020833333 \\ . \\ . \\ . \\ w_{33}^1 = 0.020833333 \end{bmatrix} \tag{43}$$

$$\mathbf{w^2} = \begin{bmatrix} w_1^2 = 0.03030303 \\ . \\ . \\ . \\ w_{33}^2 = 0.03030303 \end{bmatrix} \tag{44}$$

$$\mathbf{w^3} = \begin{bmatrix} w_1^3 = 0.011904762 \\ . \\ . \\ . \\ w_{21}^3 = 0.011904762 \\ w_{22}^3 = 0.0625 \\ . \\ . \\ . \\ w_{33}^3 = 0.0.0625 \end{bmatrix} \tag{45}$$

$\mathbf{w}^1$ corresponds to the case where the 21 FTSE All-World indices constitute the 75% of the portfolio and the 12 Bond indices constitute the remaining 25%. $\mathbf{w}^2$ corresponds to the case where the 33 indices have the same weight in the portfolio, and $\mathbf{w}^3$ corresponds to the case where the 21 FTSE All-World indices constitute the 25% of the portfolio and the 12 Bond indices constitute the remaining 75%. Given that we are considering four MGARCH models for the dynamics of $\mathbf{H}_t$ and two different innovation densities (the normal and the AML), we have in total 24 different variance portfolios to analyse.

We computed the conditional VaR for the 24 cases at the 1% and 5% levels using the entire sample of 785 observations.

5.2.1 Backtesting Analysis

To evaluate the goodness of the conditional VaR under the dynamics of the different MGARCH models and alternative distributions we used the duration-based approach proposed by Christoffersen and Pelletier (2003). The Markov tests proposed in this work are designed to detect clustering in the violations of the VaR measures, where a violation is defined as the event where the ex-post portfolio loss exceeds the ex-ante VaR. Clearly, given the parametric model-based nature of the VaR methodology employed in this exercise, a correct dynamic specification of the portfolio volatility and a correct distribution for conditional returns are necessary to secure a right specification of the VaR technique.

We consider the unconditional coverage (*uc*), independence (*ind*), and conditional coverage (*cc*) test of Christoffersen and Pelletier (2003). Consider the hit sequence of VaR violations defined as,

$$I_t = \begin{cases} 1, \text{if } r_t < -VaR(\alpha) \\ 0, \text{ else} \end{cases} \tag{46}$$

In the *uc* test we test the null hypothesis that I_t is iid Bernoulli with parameter α, against the alternative that the sequence is iid Bernoulli with parameter π, where π is the ratio of the number of violations over the number of observations. If the VaR method is correct the empirical failure rate π must be equal to α.

The *ind* test tests explicitly the assumption of independence of the hit sequence,

$$H_{0,ind:}\pi_{01} = \pi_{11} \tag{47}$$

where π_{ij} is the probability of an i on day $t-1$ being followed by a j on day t. Neither the *uc* test nor the *ind* test are complete by their own, the first one test that on a average the coverage implicit by the VaR model is correct, while the second tests the clustering effect on the failures without testing the correct number of failures. The *cc* test combines both tests:

$$H_{0,cc:}\pi_{01} = \pi_{11} = \alpha \tag{48}$$

Under the null the likelihood ratio test of unconditional coverage (LR_{uc}) and the likelihood ratio test of independence (LR_{ind}) are χ^2 with one degree of freedom. Under the null the likelihood ratio test of conditional coverage (LR_{cc}) is χ^2 with two degrees of freedom.

While the large-sample distribution of the LR tests described above is theoretically correct, the dearth of violations of 1% VaR or even 5% VaR make the effective sample size rather small, even when the nominal size is large. To overcome this problem and obtain p-values robust to finite sample scenarios we employed as in Christoffersen and Pelletier (2003) the Monte Carlo tests of Dufour (2004). Tables 8, 9, and 10 present the failure rates, p-values, and Monte Carlo p-values of the *uc*, *ind*, and *cc* tests for the four MGARCH models and for the three portfolios $\mathbf{w}^1, \mathbf{w}^2$, and $\mathbf{w}^3$.

[INSERT TABLES 8 TO 10 HERE]

The main findings from the VaR analysis can be summarised as follows:

- The Laplace model usually overestimates the 99% VaR, case in general favoured by financial regulators, while the Gaussian model consistently underestimates the 99% VaR.
- The performance of the scalar models (DCC and ADCC) across portfolios is very similar.
- The estimated VaR models in general capture quite well the clustering of violations. The models with AML innovations are superior to the models with normal innovations for the cases of the $\mathbf{w}^2$ and $\mathbf{w}^3$ portfolios. For the $\mathbf{w}^1$ portfolio the results for the independence test are quite mixed.
- For the $\mathbf{w}^1$ portfolio we found a very poor performance of the models regarding the unconditional and conditional coverage.

6 Conclusions

In this paper we propose a multivariate (GARCH) asymmetric generalised dynamic conditional correlation model where the vector of standardised residuals is assumed to follow an asymmetric multivariate Laplace distribution. This multivariate distribution is able to capture leptokurtosis and asymmetry which characterise returns from financial assets; it is the only distribution (besides the normal) with desirable properties such as additivity and finiteness of moments. In addition, contrary to the majority of (geometric) stable distributions, it has a density function with a closed-form that makes the maximum likelihood estimation method easy to implement. Very importantly, we show that the two-step approach of the DCC model is preserved when innovations are modelled via non-normal multivariate distributions.

We empirical validity of the model we propose is tested by fitting the sample of 21 FTSE All-World stock indices and 12 bond return indices of Cappiello, Engle and Sheppard (2004). We provide clear evidence that this distribution overwhelmingly outperforms the case in which we assume normality of innovations. The empirical validity of this form is also tested in the context of a Value-at-Risk (VaR) model. By performing a conditional-VaR analysis, we obtained mixed results. Though all models capture quite well the clustering of violations of the VaR levels, they performed quite poorly when they were tested for the level of failure rates. But when we evaluate the independence of hit sequences, once again the models with asymmetric multivariate Laplace innovations outperforme models where normality of the innovations is assumed.

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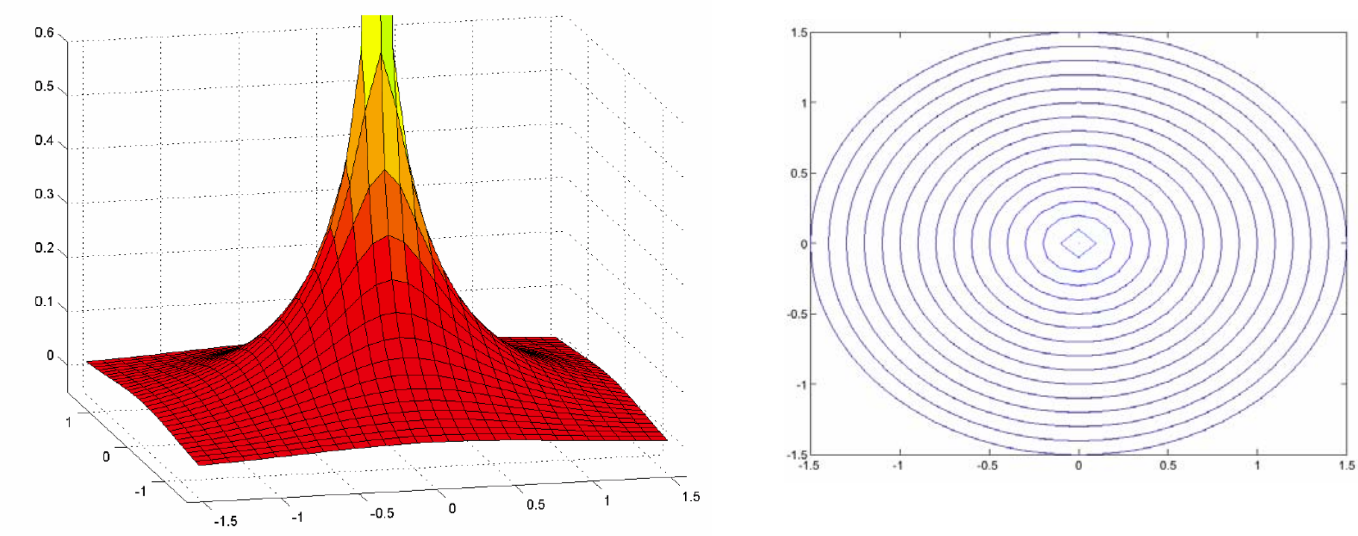


Figure 1. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 0$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0$

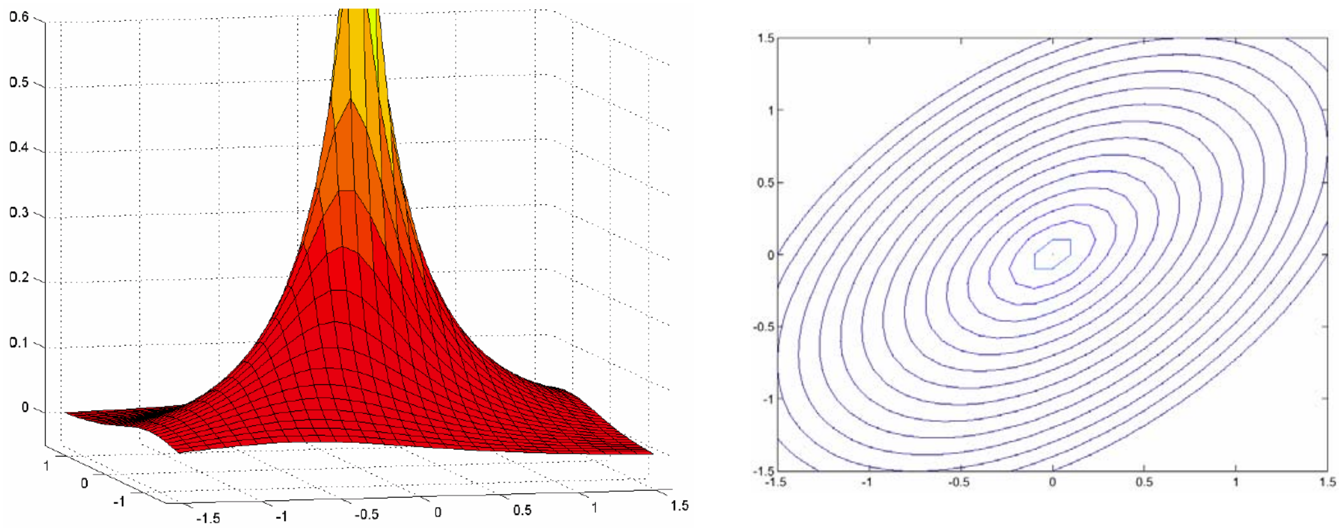


Figure 2. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 0$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0.5$

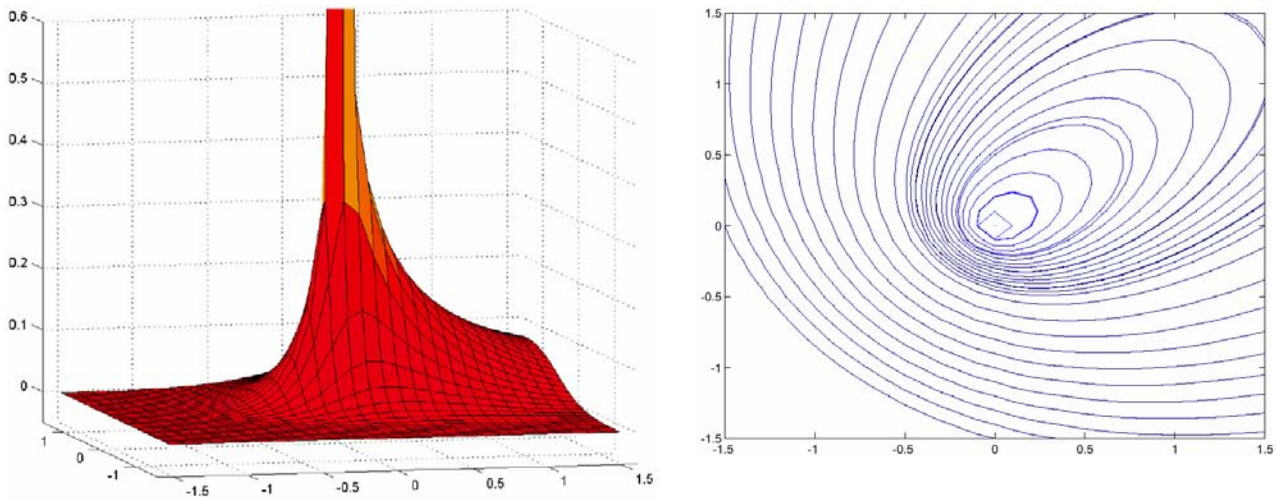


Figure 3. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 2$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0$

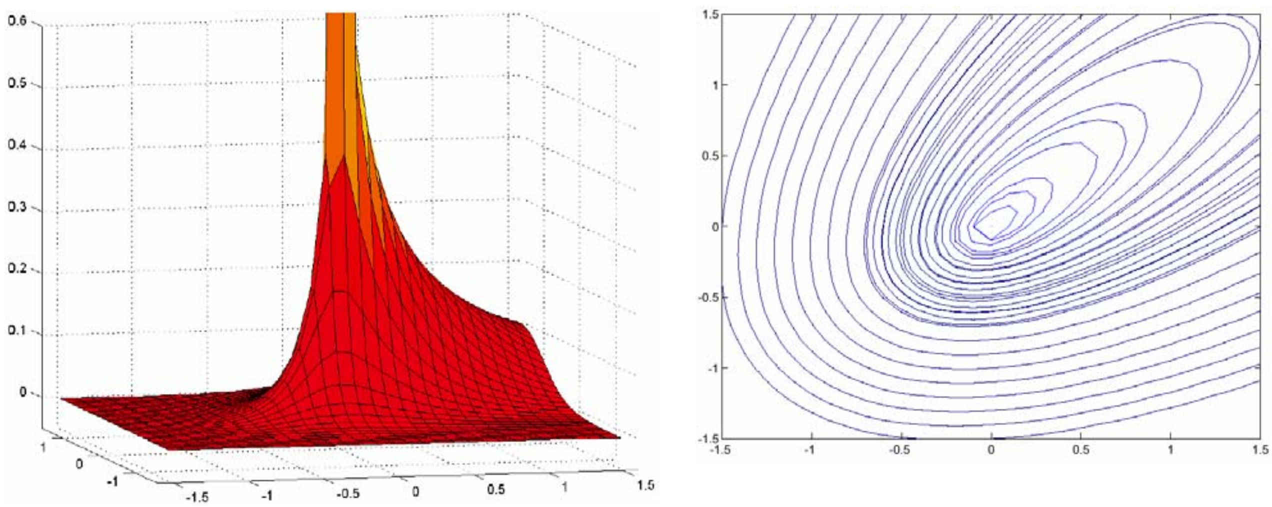


Figure 4. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 2$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0.5$

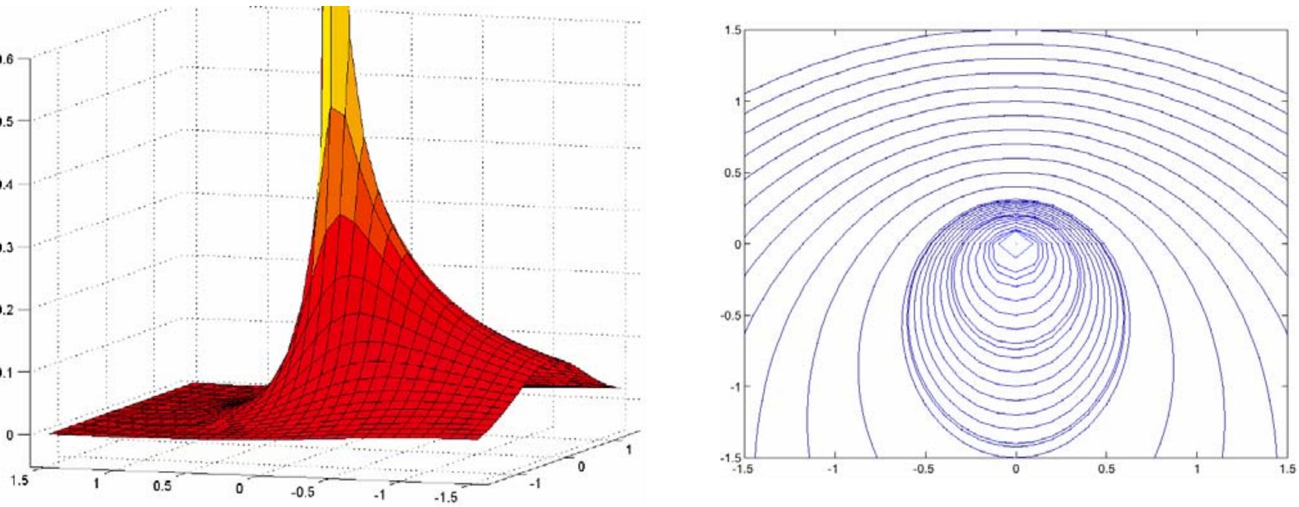


Figure 5. Bivariate Asymmetric Laplace density and contours with m_1 = -2, m_2 = 0, σ_1 = σ_2 = 1, and ρ = 0

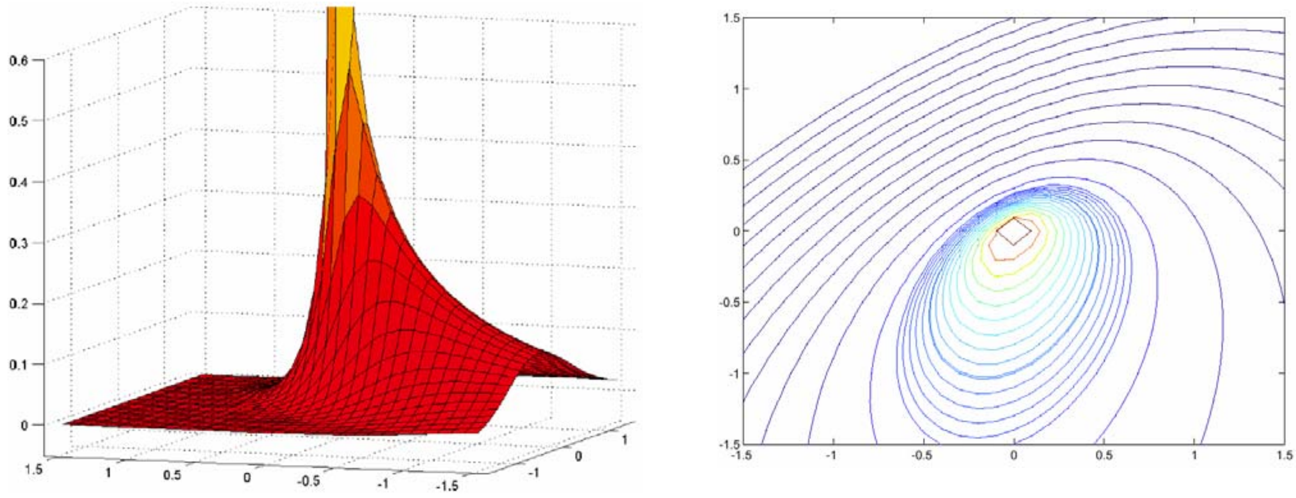


Figure 6. Bivariate Asymmetric Laplace density and contours with m_1 = -2, m_2 = 0, σ_1 = σ_2 = 1, and ρ = 0.5

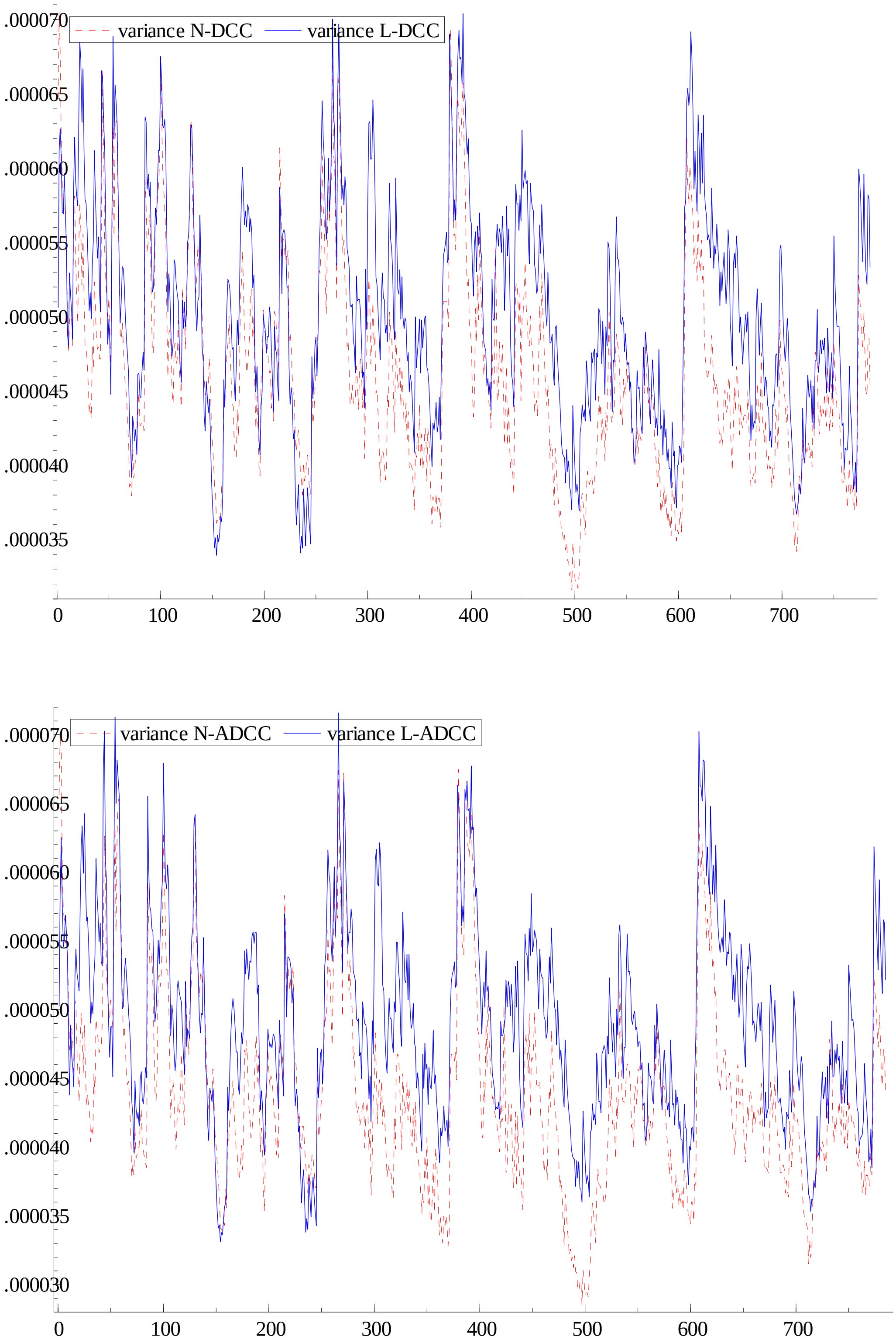
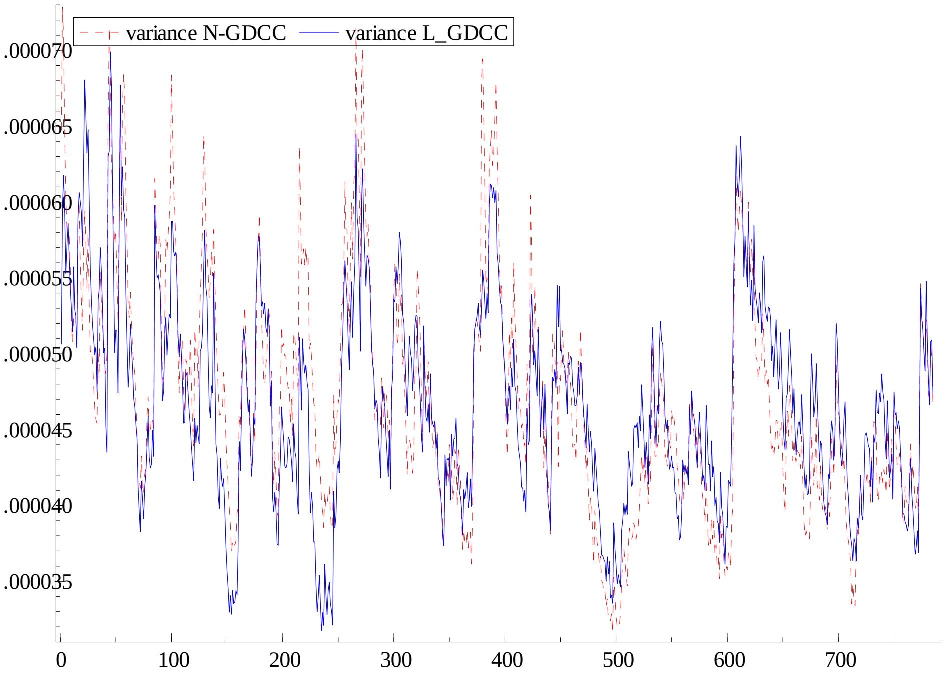


Figure 7A. Series of the variances of the portfolios composed of all the assets in the sample data for the Normal-DCC, AML-DCC, Normal-ADCC and AML-ADCC models.



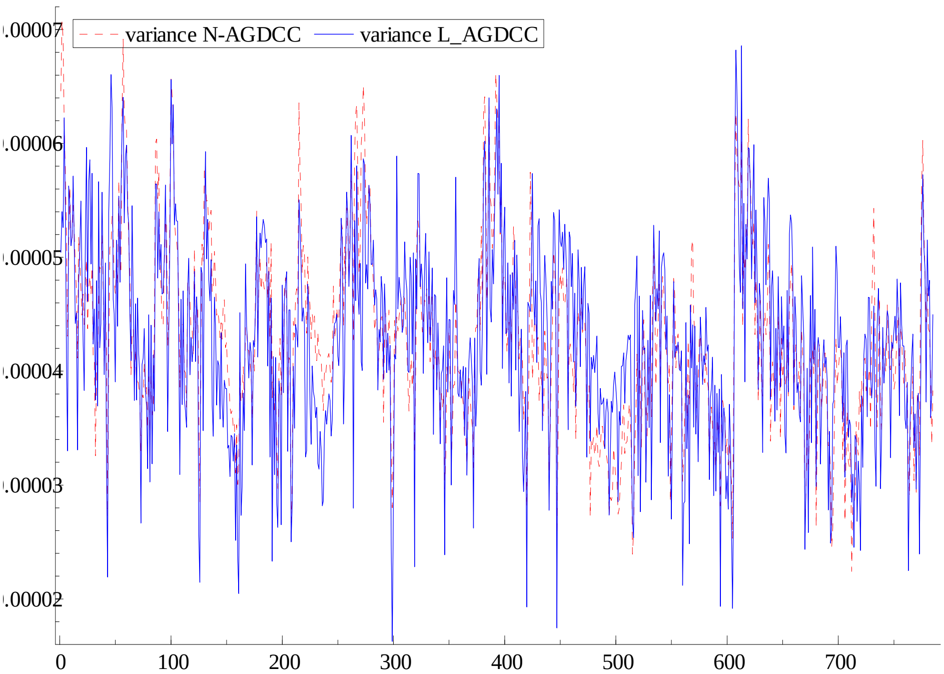
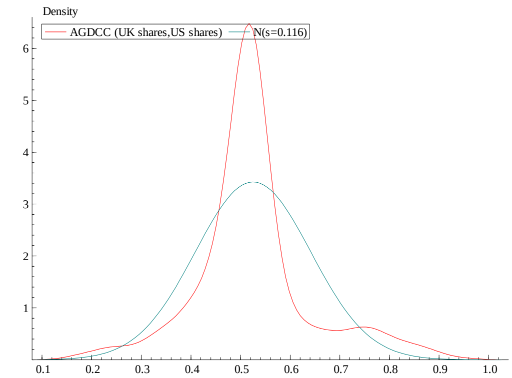
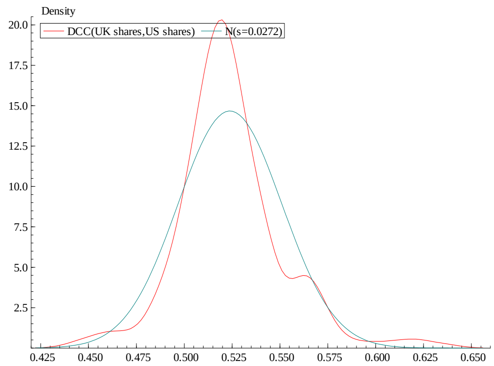


Figure 7B. Series of the variances of the portfolios composed of all the assets in the sample data for the Normal-GDCC, AML-GDCC, Normal-AGDCC and AML-AGDCC models.

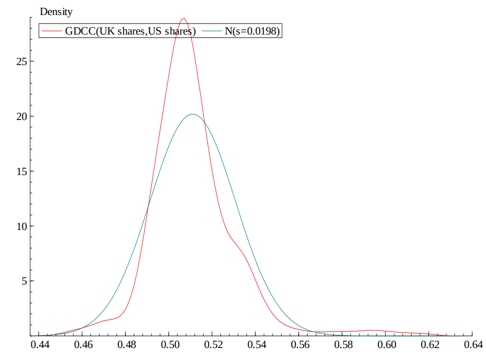
Mean 0.52519
Std.Devn. 0.11643
Skewness 0.59744
Excess Kurtosis 1.9738
Minimum 0.16910
Maximum 0.95840
Asymptotic test: Chi^2(2) = 174.12 [0.0000]**
Normality test: Chi^2(2) = 60.841 [0.0000]**



Mean 0.52390
Std.Devn. 0.027194
Skewness 0.72778
Excess Kurtosis 2.5516
Minimum 0.43860
Maximum 0.64040
Asymptotic test: Chi^2(2) = 282.24 [0.0000]**
Normality test: Chi^2(2) = 78.158 [0.0000]**



Mean 0.51106
Std.Devn. 0.019759
Skewness 1.5059
Excess Kurtosis 5.7961
Minimum 0.45240
Maximum 0.61900
Asymptotic test: Chi^2(2) = 1395.5 [0.0000]**
Normality test: Chi^2(2) = 175.42 [0.0000]**



Mean 0.52361
Std.Devn. 0.0084942
Skewness 10.454
Excess Kurtosis 222.11
Minimum 0.47520
Maximum 0.69710
Asymptotic test: Chi^2(2) =1.6279e+006 [0.0000]**
Normality test: Chi^2(2) = 5200.4 [0.0000]**

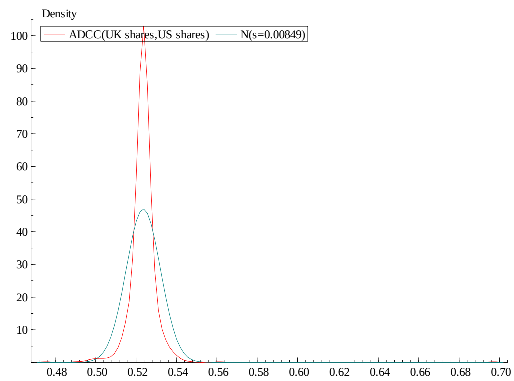
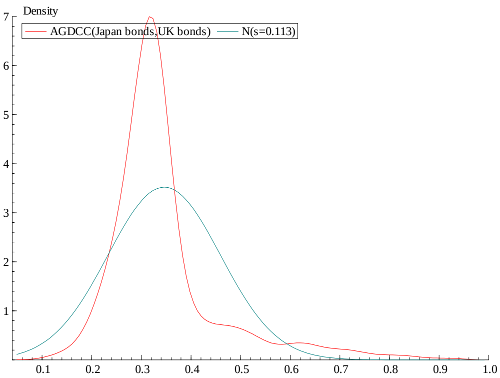
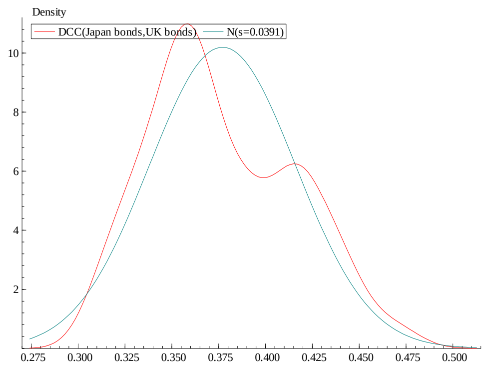


Figure 8. Distribution of the dynamic correlation between the return of UK shares and US shares employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

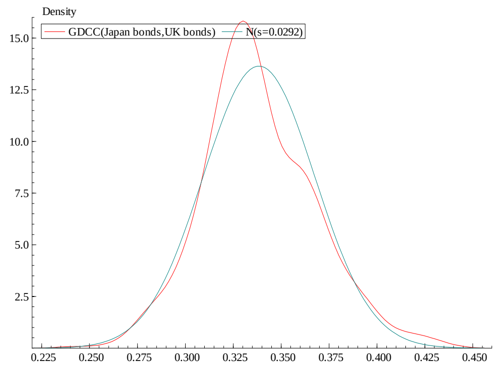
Mean 0.34610
Std.Devn. 0.11333
Skewness 2.0983
Excess Kurtosis 5.3426
Minimum 0.12380
Maximum 0.92800
Asymptotic test: Chi^2(2) = 1509.7 [0.0000]**
Normality test: Chi^2(2) = 995.61 [0.0000]**



Mean 0.37703
Std.Devn. 0.039126
Skewness 0.37672
Excess Kurtosis -0.65933
Minimum 0.30290
Maximum 0.48150
Asymptotic test: Chi^2(2) = 32.786 [0.0000]**
Normality test: Chi^2(2) = 66.152 [0.0000]**



Mean 0.33844
Std.Devn. 0.029234
Skewness 0.35843
Excess Kurtosis 0.44919
Minimum 0.23960
Maximum 0.43860
Asymptotic test: Chi^2(2) = 23.408 [0.0000]**
Normality test: Chi^2(2) = 16.522 [0.0003]**



Mean 0.38789
Std.Devn. 0.0072434
Skewness 1.7742
Excess Kurtosis 21.791
Minimum 0.34220
Maximum 0.45930
Asymptotic test: Chi^2(2) = 15944. [0.0000]**
Normality test: Chi^2(2) = 895.29 [0.0000]**

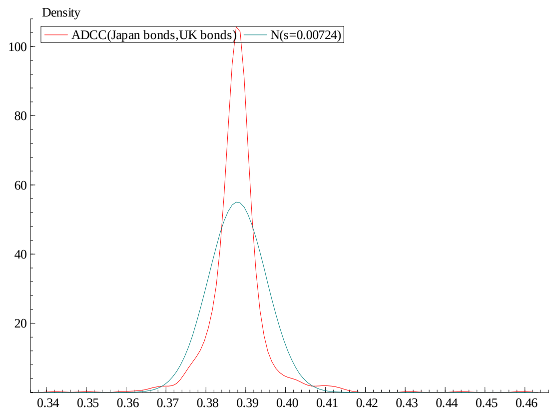
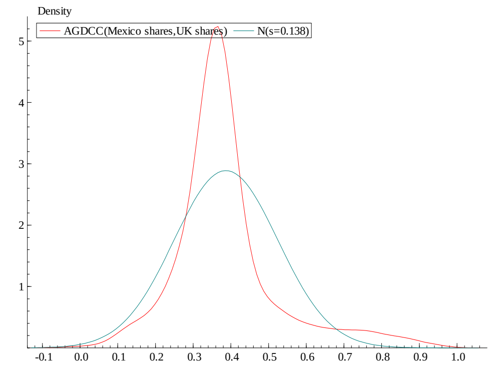
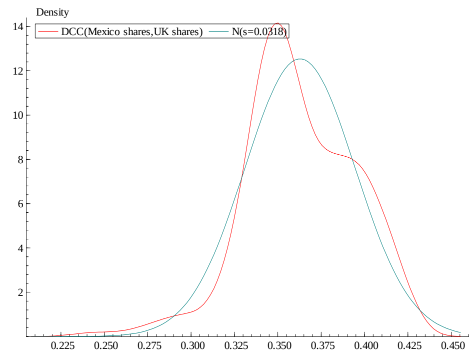


Figure 9. Distribution of the dynamic correlation between the return of Japan bonds and UK bonds employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

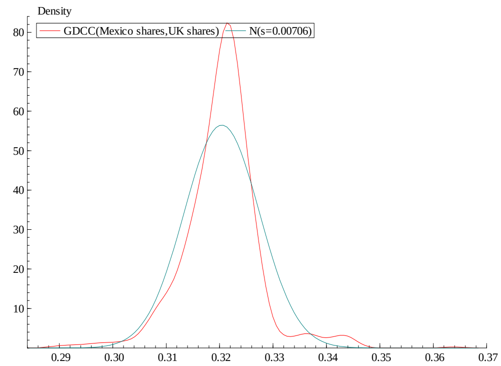
Mean 0.38693
Std.Devn. 0.13806
Skewness 1.3358
Excess Kurtosis 3.0517
Minimum -0.029900
Maximum 0.94950
Asymptotic test: Chi^2(2) = 538.06 [0.0000]**
Normality test: Chi^2(2) = 220.01 [0.0000]**



Mean 0.36282
Std.Devn. 0.031816
Skewness -0.32559
Excess Kurtosis 0.58357
Minimum 0.23340
Maximum 0.43110
Asymptotic test: Chi^2(2) = 24.786 [0.0000]**
Normality test: Chi^2(2) = 16.112 [0.0003]**



Mean 0.32040
Std.Devn. 0.0070628
Skewness 0.23460
Excess Kurtosis 4.5504
Minimum 0.28930
Maximum 0.36390
Asymptotic test: Chi^2(2) = 684.45 [0.0000]**
Normality test: Chi^2(2) = 281.82 [0.0000]**



Mean 0.36575
Std.Devn. 0.0082164
Skewness -0.16322
Excess Kurtosis 27.493
Minimum 0.29340
Maximum 0.44070
Asymptotic test: Chi^2(2) = 24726. [0.0000]**
Normality test: Chi^2(2) = 2597.8 [0.0000]**

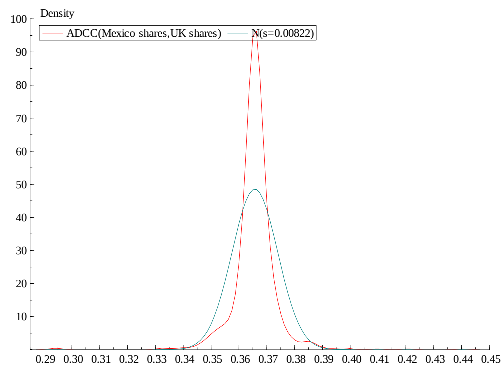
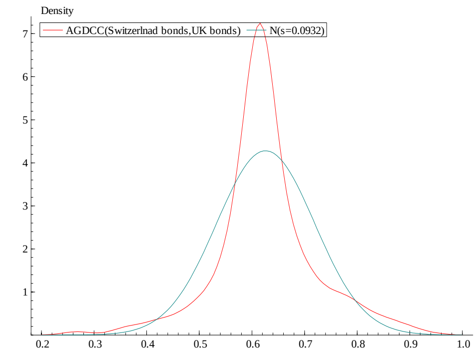
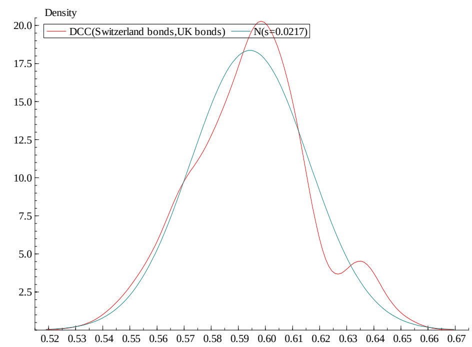


Figure 10. Distribution of the dynamic correlation between the return of UK shares and Mexico shares employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

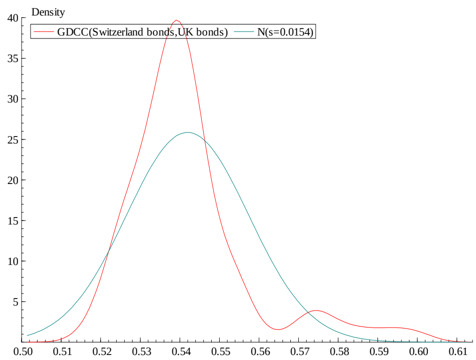
Mean 0.62607
Std.Devn. 0.093218
Skewness 0.050799
Excess Kurtosis 2.1194
Minimum 0.25990
Maximum 0.94750
Asymptotic test: Chi^2(2) = 147.26 [0.0000]**
Normality test: Chi^2(2) = 94.468 [0.0000]**



Mean 0.59432
Std.Devn. 0.021719
Skewness 0.058313
Excess Kurtosis 0.0034808
Minimum 0.53060
Maximum 0.65380
Asymptotic test: Chi^2(2) = 0.44528 [0.8004]
Normality test: Chi^2(2) = 0.47967 [0.7868]



Mean 0.54196
Std.Devn. 0.015424
Skewness 1.6521
Excess Kurtosis 3.2051
Minimum 0.51350
Maximum 0.60200
Asymptotic test: Chi^2(2) = 693.11 [0.0000]**
Normality test: Chi^2(2) = 579.90 [0.0000]**



Mean 0.62043
Std.Devn. 0.0052324
Skewness 0.30366
Excess Kurtosis 17.926
Minimum 0.58740
Maximum 0.67080
Asymptotic test: Chi^2(2) = 10523. [0.0000]**
Normality test: Chi^2(2) = 1609.0 [0.0000]**

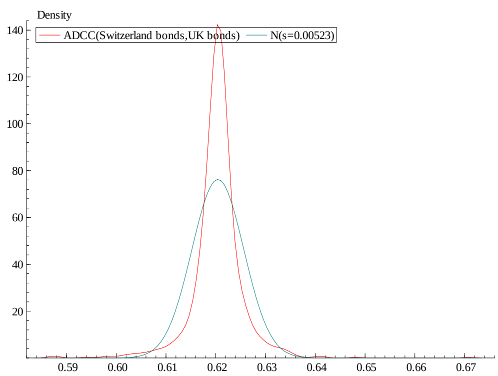


Figure 11. Distribution of the dynamic correlation between the return of UK bonds and Switzerland shares employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

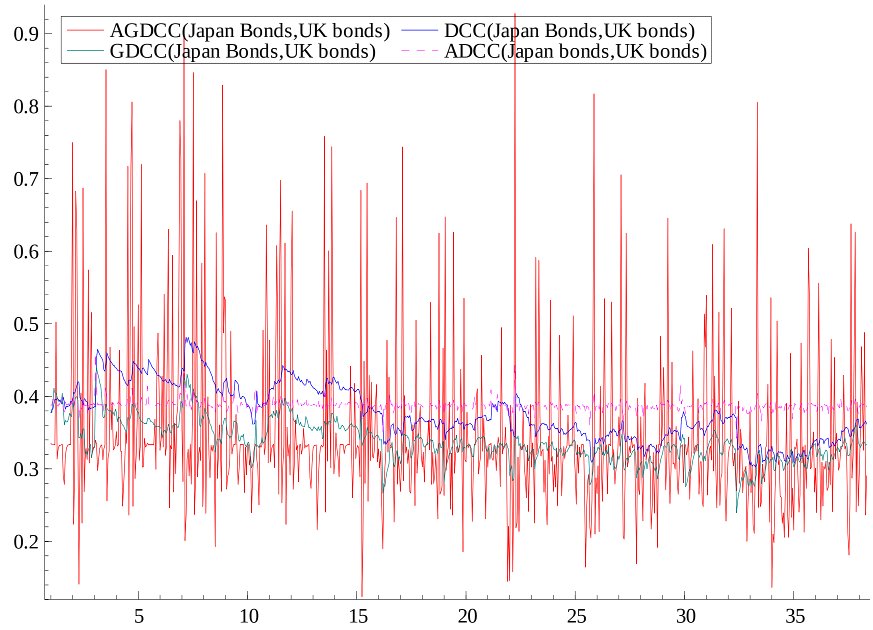


Figure 12. Plot of the correlation series between the returns of "Japan shares" and "UK shares" estimated with the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

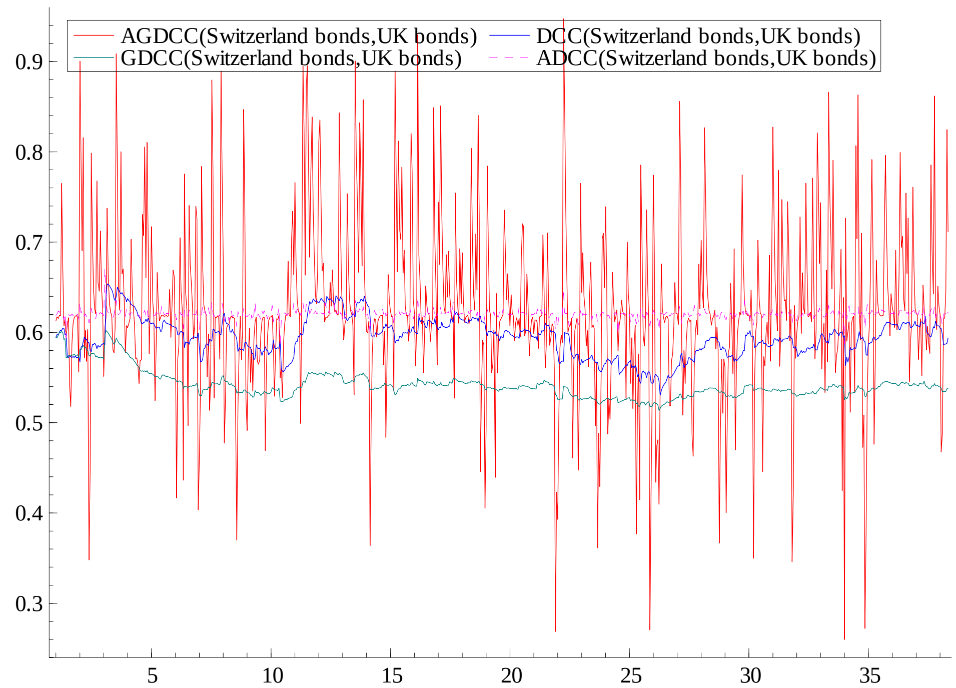


Figure 13. Plot of the correlation series between the returns of "Switzerland bonds" and "UK bonds" estimated with the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

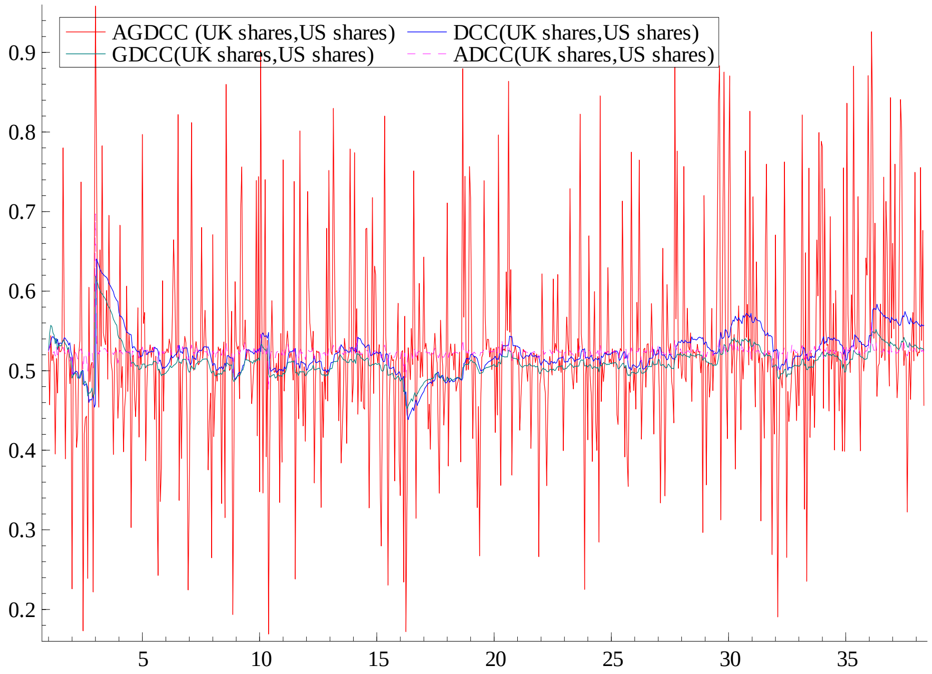


Figure 14. Plot of the correlation series between the returns of "US shares" and "UK shares" estimated with the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

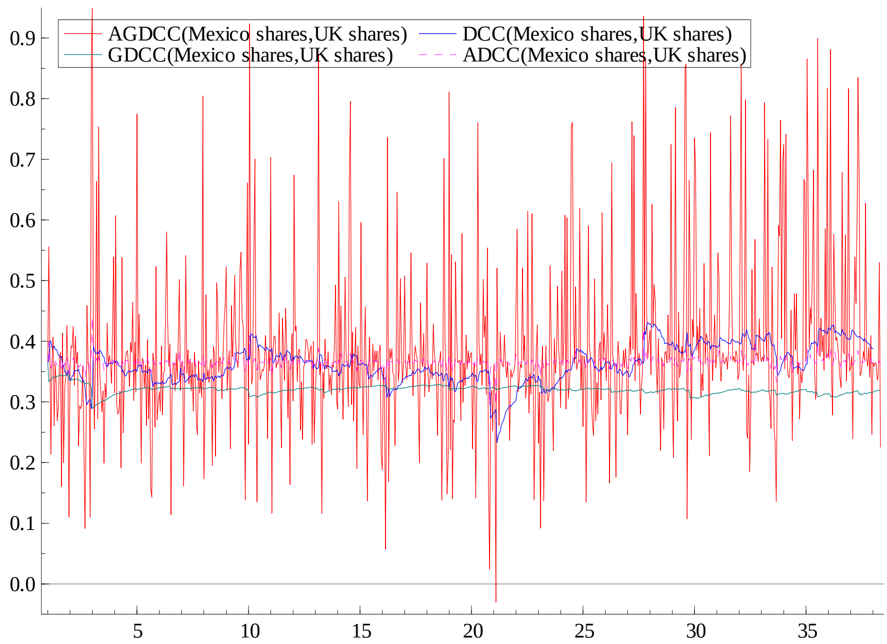


Figure 15. Plot of the correlation series between the returns of "Mexico shares" and "UK shares" estimated with the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

Panel A (Shares)

	ω	α	β	*m*	Standardised Skewness	Standardised Kurtosis
Australia	0.000019	0.0256	0.9507	0.006616	-1.27	11.23
Austria	0.000045	0.1010	0.8556	0.009245	-0.34	4.18
Belgium	0.000013	0.0490	0.9287	0.001413	-0.48	4.52
Canada	0.000036	0.0860	0.8490	0.00343	-1.08	10.65
Denmark	0.000040	0.0801	0.8682	0.007016	0.07	4.02
France	0.000031	0.0492	0.9080	0.006149	-0.27	3.62
Germany	0.000019	0.0527	0.9204	0.004176	-0.57	4.90
H.K.	0.000227	0.1319	0.7401	0.009199	-1.18	9.05
Ireland	0.000026	0.0395	0.9349	0.00284	-1.05	10.74
Italy	0.000010	0.0311	0.9651	0.011139	-0.23	4.47
Japan	0.000169	0.1555	0.7435	0.014386	0.05	3.91
Mexico	0.000172	0.1011	0.8527	0.016857	-0.49	6.61
Netherlands	0.000024	0.0502	0.8980	0.004782	-1.10	9.65
New Zealand	0.000407	0.0625	0.5798	0.011886	-0.53	5.68
Norway	0.000056	0.0656	0.8859	0.008844	-1.03	10.95
Singapore	0.000145	0.1198	0.7973	0.010878	-1.26	11.60
Spain	0.000222	0.1111	0.6570	0.002206	-0.47	5.61
Sweden	0.000048	0.0727	0.8928	0.007722	-0.56	5.53
Switzerland	0.000095	0.0370	0.8077	0.004581	-1.10	12.62
UK	0.000016	0.0526	0.9191	0.005786	-0.99	7.60
USA	0.000024	0.0209	0.9287	0.006477	-1.31	14.14

Panel B (Bonds)

	ω	α	β	*m*	Standardised Skewness	Standardised Kurtosis
Belgium	0.000017	0.0617	0.8421	0.005741	0.14	3.69
Canada	0.000011	0.1076	0.8468	0.006068	0.00	3.65
Denmark	0.000008	0.0377	0.9158	0.005637	0.13	3.68
France	0.000034	0.0746	0.7290	0.005702	0.10	3.27
Germany	0.000055	0.0785	0.6346	0.005905	0.22	3.83
Ireland	0.000065	0.0897	0.6039	0.005164	-0.33	4.57
Japan	0.000102	0.1027	0.5528	0.003929	0.62	6.02
Netherlands	0.000026	0.0566	0.8007	0.00622	0.27	4.05
Sweden	0.000001	0.0286	0.9700	0.002111	0.02	3.43
Switzerland	0.000119	0.0435	0.4896	0.004316	0.26	3.92
UK	0.000004	0.0274	0.9570	0.003306	-0.24	5.03

Table 1. Parameter estimates for the univariate GARCH models, asymmetric vector ***m***, and skewness and kurtosis of the returns standardised by their estimated standard deviation using the AML distribution. Estimates for shares are presented in PanelA and estimates for bonds in Panel B

Panel A (Shares)

	Normal	Asymmetric Laplace
Australia	0.4714	0.4561
Austria	0.4652	0.4485
Belgium	0.4722	0.4550
Canada	0.4718	0.4579
Denmark	0.4681	0.4545
France	0.4693	0.4529
Germany	0.4678	0.4516
H.K.	0.4571	0.4375
Ireland	0.4632	0.4541
Italy	0.4592	0.4396
Japan	0.4580	0.4383
Mexico	0.4359	0.4227
Netherlands	0.4728	0.4619
New Zealand	0.4623	0.4617
Norway	0.4669	0.4571
Singapore	0.4523	0.4378
Spain	0.4604	0.4384
Sweden	0.4610	0.4613
Switzerland	0.4711	0.4606
UK	0.4740	0.4600
USA	0.4756	0.4779

Panel B (Bonds)

	Normal	Asymmetric Laplace
Austria	0.4824	0.4710
Belgium	0.4795	0.4735
Canada	0.4872	0.4799
Denmark	0.4813	0.4798
France	0.4824	0.4763
Germany	0.4817	0.4804
Ireland	0.4809	0.4781
Japan	0.4782	0.4634
Netherlands	0.4807	0.4674
Sweden	0.4795	0.4763
Switzerland	0.4784	0.4672
UK	0.4792	0.4679

Table 2. Kolmogorov-Smirnov distances between the data and the normal and asymmetric Laplace fits. Statistics for shares are presented in Panel A and statistics for bonds in Panel B

Table 3

Model	α	β	γ
DCC (1,1)	0.0075	0.9484	
	[0.0000]	[0.0000]	
ADCC (1,1)	0.0060	0.5913	0.0529
	[0.0018]	[0.0000]	[0.0000]

Table 3. Parameter estimates for the DCC (1,1) and ADCC (1,1,1) models using the AML distribution. P-values are reported in brackets.

Panel A (Shares)

	GDCC model		AGDCC model		
	a_i	b_i	a_i	g_i	b_i
Australia	0.0130	0.9106	0.0006*	0.0568	0.8440
Austria	0.0205	0.9793	0.0027*	0.0742	0.9231
Belgium	0.0002*	0.5437	0.0002*	0.1990	0.6655
Canada	0.3563	0.6414	0.0015*	0.2283	0.7701
Denmark	0.0766	0.7250	0.0050*	0.0946	0.9003
France	0.0760	0.8274	0.0050*	0.0864	0.8346
Germany	0.0147*	0.1764	0.0004*	0.1369	0.6508
H.K.	0.0004*	0.4351	0.1014	0.0053*	0.6299
Ireland	0.0090	0.7565	0.0002*	0.1135	0.6294
Italy	0.3110	0.5661	0.0025*	0.0309	0.9666
Japan	0.1250	0.3615	0.0260	0.0000*	0.9740
Mexico	0.0000*	0.8883	0.0376*	0.0080	0.7502
Netherlands	0.0000*	0.7923	0.0145	0.0524	0.7463
New Zealand	0.0252	0.9748	0.0002*	0.0093	0.9905
Norway	0.0002*	0.6731	0.0002*	0.0073*	0.7881
Singapore	0.0007*	0.8896	0.0518	0.0201	0.9280
Spain	0.0150*	0.5150	0.0237	0.0105*	0.5845
Sweden	0.0222	0.7109	0.0386*	0.0540	0.8854
Switzerland	0.0595	0.2155	0.0171	0.1301	0.8528
UK	0.0456	0.9498	0.0822	0.0228	0.6062
USA	0.0399	0.9202	0.0070*	0.0379	0.8427

Panel B (Bonds)

	GDCC model		AGDCC model		
	a_i	b_i	a_i	g_i	b_i
Austria	0.0735	0.7585	0.1238	0.1482	0.6814
Belgium	0.0318	0.8927	0.0854	0.1572	0.7059
Canada	0.0208	0.9786	0.0238	0.0000*	0.9762
Denmark	0.0550	0.8268	0.0641	0.1767	0.7592
France	0.0627	0.8444	0.2588	0.0000*	0.6494
Germany	0.1022	0.7773	0.1305	0.1449	0.6906
Ireland	0.1536	0.8069	0.0796	0.1684	0.7518
Japan	0.0888	0.8189	0.0004*	0.0000*	0.5485
Netherlands	0.0950	0.7977	0.1268	0.1473	0.7247
Sweden	0.0003*	0.8813	0.0003*	0.1929	0.6866
Switzerland	0.0318	0.9670	0.0036*	0.2820	0.7142
UK	0.0228	0.9203	0.0007*	0.2625	0.7002

Table 4. Parameter estimates for the GDCC and AGDCC models using the AML distribution. * indicates insignificant at the 5% level. Estimates for shares are presented in Panel A and estimates for bonds in Panel B

Table 5

Model	Log_likelihood
DCC (1,1)	95995
GDCC	97312
ADCC (1,1,1)	93556
AGDCC	92049

Table 5. Log-likelihood values for the four estimated models.

Table 6

Model	Average Portfolio variance
Normal-DCC	4.61E-05
Normal-ADCC	4.39E-05
Normal-GDCC	4.71E-05
Normal-AGDCC	4.28E-05
AML-DCC	5.03E-05
AML-ADCC	4.90E-05
AML-GDCC	4.68E-05
AML-AGDCC	4.27E-05

Table 6. Average variance of the portfolios formed by all the securities in the sample data estimated with the four models assuming normality and the four models assuming the AML distribution.

Table 7

Model	UK shares-US shares	UK bonds-Japan bonds	UK shares-Mexico shares	UK bonds-Switzerland bonds
Normal-DCC	3.93	2.15	3.14	2.97
AML-DCC	5.55	2.34	3.58	3.00
Normal-ADCC	11.69	5.66	8.50	6.04
AML-ADCC	225.11	24.79	30.49	20.92
Normal-GDCC	3.42	2.06	2.82	2.93
AML-GDCC	8.79	3.45	7.55	6.20
Normal-AGDCC	5.03	5.29	7.09	4.60
AML_AGDCC	4.97	8.34	6.05	5.12

Table 7. Kurtosis of the conditional correlation series created with the eight models for four pairs of assets.

Portfolio W1

		Failure rate	*uc* p-value	*uc* MC p-value	*ind* p-value	*ind* MC p-value	*cc* p-value	*cc* MC p-value
DCC	N-VaR 5%	4.459%	0.0000	0.0000	0.0188	0.0255	0.0000	0.0000
	L-VaR 5%	4.586%	0.0000	0.0000	0.0047	0.0045	0.0000	0.0000
	N-VaR 1%	1.529%	0.0000	0.0000	0.0106	0.0033	0.0000	0.0000
	L-VaR 1%	0.764%	0.0000	0.0000	0.0323	0.0180	0.0000	0.0000
ADCC	N-VaR 5%	4.459%	0.0000	0.0000	0.0188	0.0255	0.0000	0.0000
	L-VaR 5%	4.713%	0.0002	0.0003	0.0011	0.0010	0.0000	0.0000
	N-VaR 1%	1.401%	0.0000	0.0000	0.0071	0.0031	0.0000	0.0000
	L-VaR 1%	0.764%	0.0000	0.0000	0.0323	0.0118	0.0000	0.0000
GDCC	N-VaR 5%	4.586%	0.0000	0.0000	0.0047	0.0045	0.0000	0.0000
	L-VaR 5%	6.369%	0.0000	0.0000	0.0130	0.0154	0.0000	0.0000
	N-VaR 1%	1.783%	0.0000	0.0000	0.0214	0.0101	0.0000	0.0000
	L-VaR 1%	1.274%	0.0000	0.0000	0.0045	0.0023	0.0037	0.0024
AGDCC	N-VaR 5%	4.076%	0.0000	0.0000	0.1796	0.2417	0.0000	0.0000
	L-VaR 5%	4.204%	0.0000	0.0000	0.2097	0.2680	0.0000	0.0000
	N-VaR 1%	1.019%	0.2408	0.2067	0.0652	0.0270	0.3635	0.2625
	L-VaR 1%	0.637%	0.0000	0.0000	0.8001	0.8962	0.0000	0.0000

Table 8. Portfolio w^1: 75% formed by the 21 FTSE All-World indices and 25% by the 12 Bond indices. Failure rates and p-values for the hit sequence Markov tests: unconditional coverage (*uc*) test, independence (*ind*) test, and conditional coverage (*cc*) tests of Christoffersen and Pelletier (2003). The MC p-values were obtained with the Monte Carlo test methodology of Dufour (2004). Number of replications: 20,000.

Portfolio W2

		Failure rate	*uc* p-value	*uc* MC p-value	*ind* p-value	*ind* MC p-value	*cc* p-value	*cc* MC p-value
DCC	N-VaR 5%	4.968%	0.2247	0.2554	0.1665	0.2322	0.1838	0.2672
	L-VaR 5%	4.968%	0.2247	0.2554	0.0486	0.0863	0.0684	0.0807
	N-VaR 1%	1.274%	0.0000	0.0000	0.0046	0.0022	0.0037	0.0020
	L-VaR 1%	0.764%	0.0000	0.0000	0.0323	0.0180	0.0000	0.0000
ADCC	N-VaR 5%	4.968%	0.2247	0.2554	0.1665	0.2322	0.1838	0.2672
	L-VaR 5%	4.968%	0.2247	0.2554	0.0486	0.0863	0.0685	0.0808
	N-VaR 1%	1.274%	0.0000	0.0000	0.0046	0.0022	0.0037	0.0020
	L-VaR 1%	0.892%	0.0049	0.0062	0.0008	0.0007	0.0000	0.0000
GDCC	N-VaR 5%	5.096%	0.0359	0.0415	0.0600	0.1122	0.6490	0.6518
	L-VaR 5%	6.624%	0.0000	0.0000	0.0664	0.1258	0.0000	0.0000
	N-VaR 1%	1.401%	0.0000	0.0000	0.0071	0.0032	0.0000	0.0000
	L-VaR 1%	1.274%	0.0000	0.0000	0.0045	0.0023	0.0037	0.0024
AGDCC	N-VaR 5%	4.076%	0.0000	0.0000	0.5536	0.5858	0.0000	0.0000
	L-VaR 5%	4.204%	0.0000	0.0000	0.6086	0.6383	0.0000	0.0000
	N-VaR 1%	0.892%	0.0049	0.0064	0.7226	0.6858	0.0180	0.0133
	L-VaR 1%	0.510%	0.0000	0.0000	0.8395	0.9552	0.0000	0.0000

Table 9. Portfolio w^2: The 33 indices have the same weight. Failure rates and p-values for the hit sequence Markov tests: unconditional coverage (*uc*) test, independence (*ind*) test, and conditional coverage (*cc*) tests of Christoffersen and Pelletier (2003). The MC p-values were obtained with the Monte Carlo test methodology of Dufour (2004). Number of replications: 20,000.

Portfolio W3

		Failure rate	*uc* p-value	*uc* MC p-value	*ind* p-value	*ind* MC p-value	*cc* p-value	*cc* MC p-value
DCC	**N-VaR 5%**	5.096%	0.0359	0.0404	0.3994	0.4373	0.1579	0.2472
	L-VaR 5%	4.968%	0.2247	0.2554	0.4383	0.4774	0.3544	0.3877
	N-VaR 1%	1.274%	0.0000	0.0000	0.6114	0.3107	0.0000	0.0000
	L-VaR 1%	0.382%	0.0000	0.0000	0.8794	0.9861	0.0000	0.0000
ADCC	**N-VaR 5%**	4.713%	0.0002	0.0003	0.5232	0.5676	0.0010	0.0020
	L-VaR 5%	4.713%	0.0002	0.0003	0.5232	0.5676	0.0010	0.0020
	N-VaR 1%	1.274%	0.0000	0.0000	0.6114	0.3086	0.0000	0.0000
	L-VaR 1%	0.382%	0.0000	0.0000	0.8794	0.9856	0.0000	0.0000
GDCC	**N-VaR 5%**	5.223%	0.0013	0.0013	0.3628	0.4143	0.0091	0.0149
	L-VaR 5%	6.242%	0.0000	0.0000	0.4939	0.5287	0.0000	0.0000
	N-VaR 1%	1.401%	0.0000	0.0000	0.5760	0.2220	0.0000	0.0000
	L-VaR 1%	0.637%	0.0000	0.0000	0.8001	0.8966	0.0000	0.0000
AGDCC	**N-VaR 5%**	4.459%	0.0000	0.0000	0.6170	0.6626	0.0000	0.0000
	L-VaR 5%	4.968%	0.2247	0.2554	0.4383	0.4779	0.3544	0.3899
	N-VaR 1%	1.146%	0.0012	0.0007	0.6477	0.4234	0.0061	0.0031
	L-VaR 1%	0.382%	0.0000	0.0000	0.8794	0.9857	0.0000	0.0000

Table 10. Portfolio w^3: 25% formed by the 21 FTSE All-World indices and 75% by the 12 Bond indices. Failure rates and p-values for the hit sequence Markov tests: unconditional coverage (*uc*) test, independence (*ind*) test, and conditional coverage (*cc*) tests of Christoffersen and Pelletier (2003). The MC p-values were obtained with the Monte Carlo test methodology of Dufour (2004). Number of replications: 20,000.